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Carnap and Gödel, again

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Abstract

Difficulties and ambiguities of Carnap's conception of logic and mathematics are the main target of Gödel's analysis in his famous drafts of 'Is mathematics Syntax of Language?'. In a recent article Gregory Lavers discusses two main recent dismissive analyses of Gödel's drafts, concluding at a defense of Gödel's arguments against Carnap's position in the 1930. Lavers partially integrates in his examination an argument that I presented in a paper published in 2003. Yet, there are three important differences between my understanding of Gödel's argument and Lavers's interpretation of it. These differences concern the appreciation of a) Gödel's strategy of using, in any critical examination of his opponents, only arguments that can be accepted by them; b) Gödel's analysis of Carnap's position in the 1950s; c) Gödel's understanding of Carnap's philosophical project. The paper aims to argue that, contrary to Lavers's opinion, Gödel takes seriously the details of Carnap's original conception and does not overlook the novelty of its solutions in the 1930s and 1950s. In particular, the comparison between Gödel's arguments in the Gibbs lecture of 1951 and in version III of his draft of 'Is mathematics Syntax of Language?' shows how Gödel had in 1953-55 a careful concern with Carnap's philosophical project of LSL.

Introduction: where the controversy lies

The development of Carnapian studies over the last years has highlighted the wide range of reasons for analyzing the intellectual and theoretical relations between Carnap and Gödel.¹ It appears clearly from these studies how the wrong historical perspective on Carnap's alleged "dogmatic empiricism", was actually motivated by the naturalistic rejection of two key elements in the European philosophical debate at the beginning of the 20th century. These key elements, that Carnap shared with Gödel, are the importance of clarification of meaning in the analysis of concepts of science and the correlated distinction between *a priori* and *posteriori* knowledge. These two ideas were at the heart of the search for a new scientific philosophy, a project that was forged in the crucible of neo-Leibnizian and neo-Kantian studies of the beginning of the XX century. On the one hand, the idea that one of the main tasks of philosophy should be the clarification of concepts of science, for constructing an Encyclopedia of the Unified Science with the help of the new logic, belongs certainly to Leibniz's legacy. On the other hand, the distinction between *a priori* and *posteriori* is not only a necessary component for a theory of knowledge aimed at establishing what of knowledge comes from us and what comes from experience. It is also a task that philosophy can assume, redefining its critical role with respect to science, in conformity to the Kantian tradition.

Gödel and Carnap certainly shared the desire to realize such a scientific philosophy, that through the critical analysis of science would allow them to contribute to the progress, the unity, and the understanding of our knowledge. They both believed that in order to realize such a program the analysis of meaning was an essential element. Nevertheless, they conceive it in a very different way.

¹ See [Goldfarb 2005], [Awodey & Carus 2004] et [Awodey & Carus 2010], but also [Heinzmann & Proust 1988] and [Crocco 2002].

For Carnap, the analysis of meaning must lead to the neutralization of philosophical disputes by isolating, through conceptual clarification, a metaphysically neutral core of scientific theories on which universal agreement can be reached. I will call in the sequel this program the program of conciliation through neutralization, because neutralization would lead to reconciliation between competing philosophical trends.²

For Gödel, the analysis of meaning is a tool for exploring the plurality of metaphysics (of the worldviews as he calls them in his *Vortrag Konzept* [Gödel *1961]). Such a plurality is a fact on which he never ceases to reflect.³ But instead of neutralizing this plurality, Gödel's aim is to organize metaphysics in a hierarchically manner, in terms of their principles and of their values giving justice to the right part of each of them. Gödel was convinced that metaphysics is important not only for creating science and proposing a rational foundation of it, but also for contributing to the understanding of its role for human kind. Although he certainly elaborated his own metaphysics, the Leibnizian idea of a possible harmony (*unitas in varietatis*) of their plurality is not absent from his perspective.⁴

At the heart of Carnap's and Gödel's opposite way of understanding the task of clarification of meaning we find their different interpretation of the nature and the role of logic (and of mathematics), the appropriate tool for the analysis of meaning. Carnap wants to use logic in order to isolate the metaphysically neutral core of science via the analysis of meaning of its propositions; but in order to accomplish that, he must make sure that logic is an appropriate means of analysis for such a task. He must make sure that the metaphysical disputes that plague physics, cosmology, biology, and the other natural sciences as well as the social sciences and psychology,⁵ once eliminated through translation into the formal mode of speech, do not come back through logic and mathematics. He must make sure that no problem of foundation of mathematics and in particular no dispute between formalism and logicism can undermine the project from the very beginning.⁶ Syntax, with its conventional axioms and its contentless propositions, is his own way out from the philosophical plagues that afflict logic and mathematics. Such a strategy cannot but discomfort Gödel, whose way of conceiving the analysis of meaning implies to recognize content to logical concepts. Nevertheless both Carnap and Gödel agree on the fact that a satisfying analysis of logic and of mathematics should take into account their application and therefore their contribution to the whole of science. For Gödel applicability to whatever (objects or concepts of any domain) is the central feature of logical concepts, but also in mathematics the constraint of applicability plays a key role in the

² For a very clear presentation of Carnap's fundamental interest in neutralizing and reconciling traditional philosophical ways of thinking, see [Carnap 1963, p. 17-18]. See also [Carnap 1928, I, A, 5] and [Carnap 1928, 175-178], where, respectively, Carnap expresses the idea that the doctrine of constitution offers a neutral language vis-à-vis idealism and realism and formulates the distinction between metaphysical and constitutional problems. Finally see [Carnap 1937, §78, 301] where the controversy between positivism and realism is interpreted as an idle dispute about pseudo-theses expressed in the material mode of speech and [Carnap 1937, §80 ff] where the disappearance of the pseudo-questions of ontology in the "so-called foundations of sciences" is accomplished thanks to the formal mode of speech transforming contradictory ontological assertions in linguistic suggestions. Concerning Carnap's attitude against metaphysics, see Gottfried Gabriel's analysis of Nohl, Dilthey and Lange's influence on the young Carnap [Gabriel 2004, 10-17].

³ See, for example, [Gödel *2019, p. 2-3], [Gödel *2020, 78].

⁴ The possible influence of Heinrich Gomperz's *Weltanschauungslehre* on his student Gödel is still largely to explore.

⁵ See [Carnap 1928, §20-2, §24], where, at the end of §22 and of §24, Carnap discusses the desolating situation of conflict about the problem of the relation between psychological and physical phenomena on one side, and psychological and cultural phenomena on the other.

⁶ The discussion of the problem is explicit in [Carnap 1937, §84]

foundational task.⁷ Concerning Carnap, the most explicit assertion about the importance of applicability can be found in his intellectual autobiography:

“[...] it is the task of logic and of mathematics within the total system of knowledge to supply the forms of concepts, statements, and inferences, forms which are then applicable everywhere, hence also to non-logical knowledge. It follows from these considerations that the nature of logic and mathematics can be clearly understood only if close attention is given to their application in non-logical fields, especially in empirical sciences. Although the greater part of my work belongs to the field of pure logic and the logical foundations of mathematics, nevertheless great weight is given in my thinking to the application of logic to non-logical knowledge. This point of view is an important factor in the motivation for some of my philosophical positions, for example for the choice of forms of languages, for my emphasis on the fundamental distinction between logical and non-logical knowledge.” [Carnap 1963, p. 12-3] (emphasis mine).

The difficulties and ambiguities in Carnap’s conception of logic, related to the notions of “syntax” and of “convention”, are one of the main points of Gödel’s criticism on Carnap in his 1953-1959 drafts of *Is mathematics Syntax of Language?* (*ISML?*). One of the main theses of the present work is that the neglecting of the importance for both Carnap and Gödel of the applicability of mathematics is one of the reasons for the numerous negative assessment of Gödel’s argument that we find in the literature.

The philosophical analyses of the two main editors of Gödel 1953-59 (W. Goldfarb for versions III and V [Gödel *1953-59] and R. Consuegra, for versions II and VI [Gödel *1953-59a]) stress the importance of the second incompleteness theorem on Gödel’s argument and criticize the “old fashioned” foundationalist approach of Gödel to Carnap’s work: what about asking for demonstrable coherence of our mathematical knowledge? This is a foundationalist pretention. If we abandon reductionism and foundationalism as tasks for epistemology, it is said, we should accept the intrinsic uncertainty of our knowledge, we should accept, with Neurath and Quine, to be like a sailor forced to rebuild his boat while he is inside it. Gödel’s argument is therefore considered as faulty, because based on the misinterpretation of Carnap’s aims. In a paper published in 2003 I tried to answer to such criticisms, stressing that Gödel’s argument is not foundationalist: Gödel wants to show that, in the liberalized frame of Carnap’s *Logical syntax of language* (LSL) [Carnap 1934/1937], any *explication* of **our trusting the applications** of logic and mathematics is incompatible with the thesis of their syntactical nature.⁸

Recently, Gregory Lavers integrates in some sense this conclusion⁹ in a general argument that deals also with two recent dismissive analyses of Gödel’s *ISML?* formulated by Awodey & Carus (2004, 2010). Nevertheless, there are some important differences between Lavers understanding of *ISML?* and the one I proposed in 2003.

Firstly, Lavers suggests that, because Gödel had no intention to enter in the technical details of Carnap’s analysis, it is incorrect to charge him for not taking Carnap’s argument on its own

⁷ See for Gödel the very beginning of [Gödel 1933o]: “The problem of giving a foundation for mathematics (and by mathematics I mean here the totality of the methods of proofs actually used by mathematicians) can be considered as falling into two different parts. At first these methods of proofs have to be reduced to a minimum number of axioms and primitive rules of inference, which have to be stated as precisely as possible, and then secondly a justification in some sense or other has to be sought for these axioms, i.e., a theoretical foundation of the fact that they lead to results agreeing with each other *and with empirical facts*.” K. Gödel The present situation in the foundations of mathematics (*1933o) CWIII, p. 45., emphasis mine.

⁸ [Crocco 2003], section 2.4, pp.35-37

⁹ See for example the assertion that according to Gödel, without intuition « we would have no reason to believe that a valid reasoning appealing to both mathematics and true empirical claims will lead to something we ought expect to be true. » [Lavers 2019, 222] or the argument on [Lavers 2019, 229-230].

term.¹⁰ Lavers stresses essentially the begging-question definition of “syntax” and “content” used by Carnap in *LSL*, placing no emphasis on the use of the second incompleteness theorem, which is important in Goldfarb’s and Goldfarb and Ricketts’s interpretation of Gödel’s argument [Goldfarb 1995], [Goldfarb & Ricketts 1992]. In this paper I would like to show that an attentive reading of the differences between Gödel’s argument in his 1951 conference and version III of *ISML?* shows the contrary of what Lavers affirms. This is a crucial point because Gödel’s conciliatory attitude toward the plurality of possible “worldviews”, requires him to use, in any critical examination of his opponents, only arguments that can be accepted by these same opponents. This is an internal strategy that Gödel applies for example in the discussion with Hilbert, Einstein and Bernays.¹¹ Therefore, Gödel’s analysis of Carnap should not be an exception.

Secondly, Lavers affirms that Carnap’s early philosophy is nominalist and conventionalist, but Carnap, by the fifties, abandons nominalism and Gödel overlooks this change. On the contrary, I would like to stress the strong continuity between Carnap’s position in 1935 and in 1950, as Gödel presents it. According to Gödel ontological theses can be corroborated or refuted on the bases of our scientific knowledge. On the contrary Carnap’s early and last philosophy is completely opposed to such an idea.

Thirdly, Lavers affirms that Gödel’s point is that Carnap’s attenuated sense of *syntax* induces a conception of mathematics “which is of no philosophical consequences” [Lavers 2019 section 3, p. 229]. I will argue that Gödel’s point is that Carnap’s own philosophical project, which implies to explain applicability,¹² of mathematics cannot be carried out on the bases of his own liberalized syntactical program.

This paper will be also the occasion to give a revised version of my argument of [Crocco 2003], correcting some parts of it,¹³ and offering it in a different perspective, more attentive to Carnap’s and Gödel’s own philosophical projects.

I will proceed in three steps.

First, after recalling how Gödel conceived at the beginning *ISML?* as a simple rewriting of his 1951 lecture (sec. 1), I carefully examine the content of the latter (sec. 2) and show how the contrast between Gödel’s foundationalism and Carnap’s anti-foundationalist and conciliatory attitude becomes clear when considering Carnap’s concerns between 1928 and 1934 (sec. 3).

Second, I analyze the differences between the 1951 conference and the third version of the 1953-59 draft. I argue that Gödel shows here a finer understanding of Carnap’s point of view of *LSL* and justifies plainly his conclusion: any *explication* of our trusting the **applications** of logic and mathematics is incompatible with the thesis that they are syntactical conventions. Without such an explication Carnap’s project of conciliation between formalism and logicism, as explained in Chapter V of *LSL*, becomes impossible (sec 4).

¹⁰ See footnote 4 of [Lavers 2019] “Crocco (2003) argues that Goldfarb demonstrates a ‘deep misunderstanding’ of Gödel’s argument. Crocco argues that Goldfarb is mistaken to see Gödel’s position as a foundationalist one (taking there to be an absolute distinction between empirical and logico-mathematical sentences) and Carnap as non-foundational. I will argue that the fault with Goldfarb and Ricketts is in not taking on Gödel’s argument on its own term”.

¹¹ See on this point Audureau (2015), p. 58-59.

¹² It implies in turn to explain “what does ‘five’ mean in contexts like ‘I have five fingers in my right hand’” [Carnap 1963, 48]

¹³ In section 2.3 [Crocco 2003], I wrongly assumed that §25 of Gödel’s version III of *ISML?* aimed to Carnap’s explicationist program. I think now that, in this section, Gödel is concerned by Kalmar’s work as it seems proved by the content of Gödel’s footnote 42 (Gödel 1995, p 352). More generally the whole argument of my section 2.3 has now a different content, explicitly related to Carnap’s notion of explication. I have deliberately left section 2.4 almost unchanged, because I still think that it gives the right interpretation of Gödel’s argument.

1. Some facts and a question

When answering on July 2, 1953 to Schilpp's proposition¹⁴ to contribute the planned volume on Carnap, with an article on "Carnap and the ontology of mathematics", Gödel declines the invitation and writes:

I regret that I shall not be able to contribute a long article to the Carnap-Volume of the Library of Living Philosophers. However, if that serves your purpose, I shall be glad to ~~send you~~ write a few pages under the title

Some observations on the nominalistic view of the nature of mathematics.

I am purposely avoiding to mention Carnap's name in the title, because I don't know to which extent the view I am going to criticize is (or still is) his. I want to leave that for him to decide in his reply.

I don't know if you are aware that I have discussed exactly the same question in my Gibbs-lecture of 1951, which will be published soon. My paper may therefore in part, literally agree with what I said there. [Gödel 2003b, 239].

The end of the story, with the six versions of Gödel's essay never sent to Schilpp, and Gödel's letter on February 3, 1959 announcing his withdrawal from the Carnap-volume, is widely known.¹⁵

I would just observe and recall some facts that will help us to set up our analysis.

None of the six versions of the 1953/9 paper conserved in Gödel's *Nachlass* bears the title announced by Gödel in his letter and the title "Is mathematics Syntax of Language?" appears only from the third version on. Nevertheless, Gödel's cautious attitude concerning Carnap's position expressed in the letter to Schilpp are echoed by note 9, in version 3. After having quoted a passage from *Formalwissenschaft und Realwissenschaft* [Carnap 1935], Gödel adds:

I would like to say right here that Carnap today would hardly uphold the formulations I have quoted (cf. §45). Moreover, some of them were given only by Hahn or Schlick and probably would never have been subscribed to by Carnap. However, I am not concerned in this paper with a detailed evaluation of what Carnap has said about the subject, but rather my purpose is to discuss the relationship between syntax and mathematics from an angle which, I believe, has been neglected in the publications about the subject. For, while the syntactical program itself and its elaboration, as far as it is possible, have been presented in detail the negative result as to its feasibility in its straightforward and philosophically most interesting sense have never been discussed sufficiently. [Gödel 1953-59, 335-6]¹⁶.

Moreover, the first three versions conserved by Gödel contain extended notes of the passage which, in the posthumous published version III, appears as §45 and concerns Carnap's position around 1950. Actually, the first three versions of the draft discuss the three periods of Carnap reflection on the nature of mathematics, before, during and after the formulation of the program of *LSL*. Versions V and VI of *ISML?* are certainly the 1/3 and 1/4 reduction of the third version,

¹⁴ [Gödel 2003b, p. 238].

¹⁵ See Goldfarb introduction to 1953-59 [Goldfarb 1995, 324-34], the correspondence with Schilpp in volume V of the Collected Works and Goldfarb introduction to it [Gödel 2003b, 214-216].

¹⁶ Several times in his paper Lavers quotes the last part of this sentence ("in its straightforward and philosophically most interesting sense") as expressing Gödel's aim to discuss the syntactical program as a whole and not in its technical details. It seems clear from the context that Gödel is talking about the consequences of "the negative result as to its feasibility". Discussing the non-feasibility of the syntactical program in its philosophically most interesting sense does not imply the intention to disregard Carnap's own way to cast the program.

announced by Gödel in his letter of September 9, 1957 [Gödel 2003b, 243-4].¹⁷ They contain no precise references to Carnap's works, who, except from the opening sentence of the essay, is mentioned explicitly and briefly only about his notion of "factual content" [Gödel *1953-59, 360]. These facts allow us a conjecture: in the first three versions, despite his explicit intentions not to go into the details of Carnap's position, Gödel finds himself considering the complex aspects of Carnap's conception of logic and mathematics. As we know from the Gödel-Carnap correspondence [Gödel 2003a, 335-59], Gödel mastered the mathematical details of LSL, which he discussed in detail with Carnap, but his perception of Carnap project (see [Wang 1987, 49-52]) can have been filtered by its knowledge of the Vienna Circle's theses. Facing the philosophical part of the project Gödel was pushed more and more towards a reconsideration of Carnap's writings. This slide from a theoretical critique of a conception of logic and mathematics widely admitted by the Vienna Circle (but above all advocated by Hahn, Gödel's *Doctorvater*, and Schlick) to a comprehensive and overall critique of the different forms in which the linguistic program has been developed, including Carnap's more sophisticated and complex philosophical project, was probably one of the reasons for the intense and meticulous work of the first two years (from July 1953 to end of 1955). After that, Gödel's health problems and the preparation of the *Dialectica* paper published in 1958 (although the mathematical ideas of (Gödel 1958) were written in 1941) could have been caused the interruption of the drafting of the essay. Versions IV-VI are attempts to come back to the initial intention, readjusting the argument, which remains substantially the same but avoids precise reference to Carnap.

Now our first question concerns the relations between Gödel's argument in 1951 and in 1953-55. More precisely is there any difference between the version of the argument given in the 1951 paper and the long third version of *ISML*?

Let's try to analyze the problem starting with the Gibbs lecture.

2. Gödel's foundationalist argument in the Gibbs lecture

As announced in Gödel's first letter to Schilpp, his arguments in *ISML*? rest on the theoretical consequences of the second incompleteness theorem and develop the thesis discussed in the Gibbs-Lecture of 1951 [Gödel 1951, 304-323] against the idea that mathematics could be our "free creation" or "free invention". In this later, after having explained that the second incompleteness theorem reveals what he calls *the inexhaustibility of mathematics*,¹⁸ Gödel develops his argument by showing that the inexhaustible character of mathematics is related to the non-eliminability of its content, since this content consists of abstract concepts such as the concepts of set, function of integers, demonstrability, derivability, and existence (footnote 27, [Gödel 1951, 318]). More precisely Gödel shows [Gödel 1951, 315-321] how his own metamathematical results disprove what he qualifies the most radical formulation that has been given of the "free invention" thesis i.e., the one "which interpret mathematical propositions as expressing solely certain aspects of syntactical (or linguistic) conventions that is, they simply repeat parts of these conventions". Gödel explicitly mentions the works of Hahn [Hahn 1935] and Carnap [Carnap 1935] as "very lucid expositions of the philosophical aspects of this nominalistic view" of mathematics, and says that such a conception includes, as a special elaboration, the formalistic foundation of mathematics [Gödel *1951, 315, footnote 23]. According to Gödel, the simplest version of the view in question consists in the assertion that

¹⁷ Because of the content of this letter there is no reason to acknowledge Lavers's idea [Lavers 2019 section 3, 229] that versions V and VI were incorrectly labeled by Gödel.

¹⁸ It can be described as follows: as soon as we try to makes precise the analysis of our mathematical practice and try to comprise it in a formal system of rules, this very fact induces us to recognize new evident propositions which cannot be derived by the chosen rules, as proved by the second incompleteness theorem. See [Gödel *1951, 309].

mathematical propositions are true owing solely to the definitions¹⁹ of the terms occurring in them, in such a way that they can be reduced to the explicit tautology $a=a$, by successively replacing their terms by their *definitia*. This simple version is disproved by the fact that it implies the existence of a mechanical procedure for deciding every mathematical proposition. The more refined version (for which Carnap's result in *LSL* is quoted ([Gödel *1951, 317 footnote 25] together with Ramsey work [Ramsey 1926])), consists in affirming the "tautological"²⁰ nature of mathematical propositions on the basis of their derivability from rules stating the meaning of the sentences of the chosen language i.e., stipulating in the precisely formulated chosen language the conditions under which they are true or false. Mathematical propositions are therefore tautological in the sense that they are true owing to the definition of the meaning given by these rules and that they can be asserted on the bases of nothing else except these rules. Now, Gödel says, it is true that, with suitably chosen rules, the tautological character of mathematical axioms can be shown. It is therefore possible to derive the axioms of mathematics from these rules:

however, (and *this is the great stumbling block*), in this derivation the mathematical and logical concepts and axioms themselves must be used in a special application, namely, as referring to symbols, combinations of symbols, sets of such combinations, etc. Hence, this theory, if it wants to prove the tautological character of the mathematical axioms, must first assume theses axioms to be true [Gödel *1951, 317] (emphasis mine).

In other words, a proof for the tautological character of mathematical axioms (i.e. for their being true in virtue of the definitions provided by the rules of the well specified language to which they belong) requires the same resources than the proof of their consistency. From the second incompleteness theorem, we know that such a proof cannot be achieved with any weaker means of proof than are contained in the axioms themselves. We need *abstract concepts* for accomplishing the proof, i.e., not concepts which refer to concrete objects (such as symbols) but abstract ones referring to *all possible* combinations of symbols. In footnote 25 Gödel makes clear that in *LSL* Carnap has to consider infinite sets, sets of sets etc., of the finite propositions in which his system consists of.

The argument is exactly the one discussed in the Gödel-Carnap correspondence, concerning Carnap's notion of valuation [Gödel 2003a, 354]. It is also the one which opens the *Dialectica* paper, dedicated to Bernays, where the works of Bernays and Gentzen, considered amongst the "the leading formalists" [Gödel 1958, 318, footnote 27] of the Hilbert school, are quoted and the philosophical impact of Gentzen's coherence proof of arithmetic are discussed. The way in which Ramsey, Carnap and Gentzen obtain proofs of consistency using respectively infinite propositions, valuations through infinite sets of formulas and induction up to ε_0 show that the content of mathematics, what it has to be presupposed to master it, is something abstract, ineliminable and impossible to be captured by the rules of a formal system.

In the 1951 paper Gödel draws some consequences from this fact and one of them is important for our purpose. It states that no "rational justification"²¹ of our practice can be given through the syntactical interpretation of mathematics. The fact that we apply mathematics to the empirical words, and the very fact that we perform our calculations and believe that these applications and calculations can be trusted (if performed with enough accuracy) can be justified only by our intimate conviction of the coherence of mathematics. Critical analysis of

¹⁹ Explicit or contextual but eliminable definitions.

²⁰ Gödel opposes here the term "tautological" to the term "analytical" to characterize propositions which are true "owing to the property occurring in them". See also footnote 18 below.

²¹ Gödel affirms: "*there exists no rational justification of our precritical beliefs concerning the applicability and consistency of classical mathematics (nor even its undermost level, number theory) on the basis of the syntactical interpretation*" [Gödel *1951, 318].

the nature of the latter should give a foundation of these pre-critical beliefs unless one of the most important aspect of mathematics (that is its applicability) remains a mystery. Any interpretation of mathematics incapable to explain our mathematical practice, which includes its applicability, is from Gödel's point of view philosophically irrelevant.

3. The different tasks of the logical analysis of meaning: foundation *versus* neutralization

One aspect²² of Gödel's analysis of 1951 appears inappropriate to fit with Carnap position in *LSL*. It concerns the foundationalist²³ character of the analysis of 1951 and its foundationalist interpretation of the syntactic program both in terms of its simple version and in terms of its refined version.

Firstly, "the great stumbling block" stressed by Gödel hints at the circularity in foundations of mathematics attempted by the reduction of mathematical propositions to "tautologies".

Secondly, Gödel openly speaks about the necessity of a *rational justification* of the beliefs concerning *the applicability and consistency of classical mathematics*. We can understand the term "rational justification" of our beliefs as consisting of a critical foundation of what justify these beliefs. We need to know if and how we are justified to have them and to use them for guiding the activities implying their application. The task of a rational justification is to help us to answer yes or no questions arising inside the different domain of science, to develop a coherent and general analysis about the ontological foundations of our theories and to unify and organize our knowledge in developing an overall and coherent worldview. Worldviews can be plural, but Gödel is convinced that it is exactly from the success, expediency, and fruitfulness of our scientific theories (logical, mathematical or of any other domain) that we can draw ontological and metaphysical conclusions corroborating a worldview. From the analysis of the foundations of our theories we can bring out arguments in favor of a worldview, although such arguments should be corroborated by the examination and possible disproof of the concurrent philosophical worldviews.²⁴ This is essentially Gödel's ideal of scientific philosophy.

On the contrary, Carnap's project can be interpreted in a very different, not foundationalist perspective: that is conciliation through neutralization.²⁵ Carnap is convinced that the

²² There are certainly other aspects of Gödel's argument, which show how it is essentially Hahn's position, Gödel's *Doctorvater*, with its peculiar mix of conventionalism and Wittgensteinian absolutism that is targeted here. One of the clues for such an assertion is Gödel's use of the term "tautological". Carnap, as many member of the Vienna Circle before Gödel's results of 1931, was convinced that the nature of mathematical truths implies their decidability, i.e. the possibility of knowing them through finitary linguistic procedures. The Wittgensteinian term "tautological" was generally used to characterize this aspect of mathematical and logical propositions. In Carnap's *LSL*, the term "tautology" is opposed to the term analytic, defined in terms of the consequence-relation where consequence is the result of an eventually infinite series of infinite classes of formulae. Carnap here openly criticizes Wittgenstein's conception of logic and of mathematics: "When Wittgenstein says (Tractatus p. 164): 'It is possible...to give at the outset a description of all 'true' logical propositions. Hence then can never be surprises in logic. Whether a proposition belongs to logic can be determined' he seems to overlook the *indefinite* character of the term 'analytic' — apparently because he has defined 'analytic' ('tautology') only for the elementary domain of sentential calculus, where this term is actually a definite term". [Carnap 1937, 101]

²³ Gödel's foundationalism has nothing to do with reductionism in so far it is not at all motivated by the necessity to ensure our practice, or to eliminate the risk of contradiction because anyway: "[w]e have no absolute knowledge of anything" [Wang 1995, 9.2.35, p 302]. Nevertheless, foundationalism has essentially the philosophical and epistemological task to give the instruments for understanding, unifying, and critically evaluating this practice from the different philosophical options at our disposal.

²⁴ After "having disproved the nominalistic view" in mathematics, Gödel says: "There are, however, other alternative to Platonism, psychologism and Aristotelian realism. In order to establish Platonistic realism, these theories have to be disproved one after the other" [Gödel *1951, 322]. The examination and "disproof" of concurrent philosophical systems is not at all incompatible with the acknowledgement of the part of truth which each of them highlights, and therefore with the possibility of their "hierarchical organization" of the systems we mentioned in our introduction.

²⁵ See Carnap's intellectual autobiography [Carnap 1963, 17-18] and footnote 2 above.

philosophical controversies that run through the science of his time (in the fields of physics, mathematics but also biology, psychology, and sociology) are obstacles to the development of knowledge. Therefore, his philosophical aim is to find a ground of conciliation, freed from all disputes, a neutral core in every domain of science from which universal agreement can be established and a complete and coherent *scientific* philosophy can be developed.

3.1 Carnap's neutralization by the means of *Logic* and the problem of logical pluralism

In the *Aufbau* [Carnap 1928], the search for conciliation implies the elimination of ontological and metaphysical assertions from science, and hence, the clarification of the meaning of the terms of sciences, through the strict and absolute distinction between analytic and synthetic propositions. Logical and mathematical truths are considered tautologies in a sense which is very close to the Wittgensteinian conception, implying the idea of Language, as *the* absolute and unique frame which offer us the condition of possibility of the determination of the content of our scientific assertions. But this analysis of logic is very instable at the time of the *Aufbau*. To the Wittgensteinian idea that the laws of logic and mathematics reveal the inner structure of *the* language, which mirrors the structure of the world, Carnap prefers the idea that they are “conventional stipulations concerning the use of symbols and tautologies based on these conventions” [Carnap 1928, §107]. Carnap uses at the same time the Fregean notion of *Festsetzung* (employed to qualify definitions) and Hahn's idea that tautologies are derivable by these conventions. The instability comes from the fact that conventions are here at the same time the result of our free choices for abbreviating language in the most efficient way (Hahn's position) and the unique invariant inner structure of the language, which logical analysis reveals (Wittgenstein). As Carnap's writings on the foundations on logic and mathematics around 1930 show, this ambiguity troubles him, exactly because logical analysis is the crucial tool for conciliation. The importance of the method of logical analysis for a scientific philosophy which aims the elimination of metaphysical disputes is constantly reaffirmed in Carnap's works at the end of 1930.²⁶ The very problem is that disputes arise inside logic and exactly between the two tendencies which Carnap tries to compose in a unique frame: that is logicism and formalism. As Carnap claims in the Königsberg Conference of September 1930:

Logicism has a methodological affinity with formalism. Logicism proposes to construct the logical-mathematical system in such a way that although **the axioms and rules of inference are chosen with an interpretation of the primitive symbols in mind**, nevertheless inside the system the chains of deductions and of definitions are carried though formally as in a pure calculus, i.e., without reference to the meaning of the primitive. [Carnap 1931/1983, 52] (emphasis mine).

But the question of pluralism, which appears with Carnap's following discussion of Ramsey's interpretation of impredicative definitions, cannot be settled down without conflicts: What should be the correct interpretation of generality? How can a general statement be verified? Should generality be defined as running through all possible values or in terms of general derivability, or in terms of substitution? More generally how a philosophical method based on logic, which should be neutral and above conflicts, could be constructed from a given interpretation of logical symbols? If the tool of neutralisation is not in turn neutral, how can we hope to found the agreement on the neutral core of science?

After the discussions with Gödel in August 1930, the idea for the solution was finally found, at the beginning of 1931, as Carnap retrospectively recounts in his autobiography [Carnap 1963, 53]. *LSL* is a systematization of this idea.

²⁶ See for example [Carnap 1932/ 1959, 77].

3.2 Setting up the new pluralistic and anti-foundationalist frame of LSL

In *LSL*, Carnap's project of conciliation invests also the domain of logic and mathematics, with the effect that there is no absolute correct language, from which we cannot get out and about which we cannot speak. The result is a pluralistic conception of logic and mathematics, with no room for indeterminacy of meaning because of the possibility of representing the syntactical aspects of a language in the language itself through Gödel's technic of arithmetization. Carnap says:

[...] let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols. By this method, also, the conflict between divergent points of view on the problem of the foundation of mathematics disappears. For language in its mathematical form can be constructed according to the preferences of any one of the points of view represented; so that *no question of justification arises at all*, but only the question of the syntactical consequences to which one or the other of the choices leads, including the question of the non-contradiction" [Carnap 1937, XV] (emphasis mine).

This sailing in the "boundless ocean of unlimited possibilities"²⁷ of the logical forms of language, has two important consequences that Carnap states in a very holistic Duhemian mood:

- a) There is no more possibility to justify the necessity of mathematical truths. Every mathematical proposition is in principle revisable when we consider the total language of mathematics and physics, because we can decide to maintain our Physical language to the detriment of the Logical language in front of an empirical disproof [Carnap 1937 §82, p 318, and §78, p. 299].
- b) There is no more an absolute difference between analytical and empirical propositions. The opposition is only relative to the choice of language which has been made [Carnap 1937, §82, end of p. 318].

Notwithstanding, Carnap is still convinced that there is "the possibility of differentiating pseudo-concepts and pseudo-sentences from real scientific concepts and sentences and thus of eliminating the former" [Carnap 1937, 322] using the notion of testability. Therefore, the project of replacing philosophical and epistemological inquiries by syntax of language still aims to establish the possibility of the distinction amongst empirical concepts and sentences and non-empirical but logical ones, in order to offer a clear criterion of demarcation between the latter two and propositions of metaphysics. It is certainly true that the opposition between empirical and non-empirical is at the beginning internal and relative to a language. Nevertheless, it is also true that language forms are defined in order to be compared and eventually discharged, if ineffective. The overall aim of this comparison is the conciliation in a "total language"²⁸ containing the proposition of all sciences and the rules for their applications. As it has been noted²⁹, the nature of the project if not absolutist remains universalist, i.e. it tries to establish a universal frame for discussing and deciding genuine philosophical problems with the help of linguistic (syntactical) analysis. To make this universal frame possible the internalization of syntax, that is the possibility of judging language forms inside syntax, plays an essential role. Metaphysical quarrels can be solved in terms of syntactical relations between languages only when a formal mode of speech is adopted, one which disregards completely the content (the

²⁷ [Carnap 1937, XV].

²⁸ That such a total language must be in some extent *indefinite* is what Carnap considers the great consequence of Gödel theorems; but indefinite, for him does not mean non-syntactical. The expression "total language" occurs in several places of *LSL* for example: §78, p. 299; §84, p. 327. See also [Carnap 1963, 12-13] quoted at the end of section 4.3 below.

²⁹ See [Rivenc 1993, p. 150].

Sinn) and the reference (the *Bedeutung*) of the linguistic expression of our science [Carnap 1937, 1). More precisely, the part of the *Sinn* which is independent from psychological or historical considerations, and that is necessary to compare sentences and theories can be defined solely in terms of deductive consequence. What a sentence means, its content, depends solely on what sentences can be derived from it. In this way we can establish if two sentences are equipollent in a language, that is, if they have the same sense, being synonymous. All these questions, including the construction of a “total language”, are explicitly sketched by Carnap in part V of the *LSL*. There Carnap explains that the overall aim of the project is to transform the “so called” problems of philosophical or logical foundations of mathematics, physics, biology, psychology, sociology and whatsoever, in questions of Syntax of language, i.e., questions that a formal syntactical analysis can decide.³⁰

In this sense, the task of logical syntax in respect to biology is to verify if, yes or no, it is possible to define a “total language” S3, [Carnap 1937, 323) which contains both the language of biology S1 (concerning organic process) and the language of physics S2 (concerning inorganic process):

In syntactical form: is every descriptive primitive symbol of S1, synonymous in S3 with a symbol which is definable in S2? [...] Is every primitive law of S1 equipollent in S3 to a law which is valid in S2 [...]. [Carnap 1937, §83, 323-324]

Concerning the foundation of mathematic, the conciliation between formalism and logicism is stated as before (see section 3.1 above) but is now solved in terms of syntax. Carnap considers that formalism and logicism are opposed mainly relative to the question of what a foundation of mathematics should achieve. Formalism, considers that it is achieved by a construction of a formal system, a calculus, which makes possible the proof of the formulae of classical mathematics and where the meaning of the symbols is not to be taken into considerations [Carnap 1937, §84, 325]. Moreover, Hilbert considered that Mathematics and Logic have to be constructed together in a common calculus for which the question of freedom from contradiction is made the center of investigation. Frege maintained that the logical foundation of mathematics has the task not only of setting up a calculus but also of giving an account of the meaning of mathematical symbols and sentences. Such an analysis *should therefore give rules for the application of mathematics, that is for the application of mathematics in extra-mathematical context*.

According to Carnap the apparent conflict disappears when the formal mode of speech is used and meaning (content) is expressed in terms of the consequence relation. When translated in syntactical terms there is no more antithesis between the two approach when the requirement of logicism is so interpreted:

the task of logical foundation of mathematics is not fulfilled by a metamathematics (that is, by a syntax of mathematics) alone, but only by a syntax of *the total language*, which contains both logico-mathematical and synthetic sentences [Carnap 1937, §84, 327] (emphasis mine).

The translation from the material mode of speech concerning reference and content to the formal mode of speech allows us to distinguish between legitimate answerable questions, internal to a specific language or to the total language of knowledge (questions concerning symbols the way they can be arranged, used, derived and replaced) and unanswerable questions concerning ontology, metaphysics and values. These questions are dangerous because they divide science and represent an obstacle to his progress.

In the pluralistic and ecumenical solution of Carnap’s project, one condition is again essential to escape to relativism. If logic (in the sense of logic plus mathematics) can be the frame of conciliation for all scientific disputes, in all domain, it must be formulated as syntax, that is the

³⁰ See in particular §82-84 of *LSL*.

neutral and unified hearth of our language. Logical sentences must have no content as they are consequences of the null set of sentences and therefore of every sentence. That means that they are what follows solely from the rules governing the use of our symbols. Allowing us to compare languages inside languages, they are the most efficient, fruitful and expedient tools enabling us to construct the total language of our knowledge, freed from every foundational quarrel.

It is important to stress that this freedom from quarrel, this neutrality, is not obtained through a nominalistic *foundation* of mathematics or a *reduction* of mathematical concepts to linguistic symbols but thanks to their *interpretation*³¹ in syntactical terms.

It has been argued³² that Carnap's technical solution in *LSL* relies on four methodological principles, made explicit in the discussion of the notion of **explication** contained in the first chapter of *Logical Foundation of Probability* (1950) and mentioned explicitly by Carnap for the first time in [Carnap 1945]. In this perspective, the logical syntax offers not a foundation or a justification but an explication of the nature of mathematics, substituting vague concepts (as “*logisch-gültig*”, “*aus logischen grunden wahr*”, “*Folge*”) with syntactical ones. Logical syntax allows us **to replace** vague notions (*explicanda*) with new technical ones (*explicata*) made precise by rules of use.

Carnap [Carnap 1950b] stresses that:

- 1) The *explicatum* is to be *similar* to the *explicandum* in such a way that, in most cases in which the *explicandum* has so far been used, the *explicatum* can be used; however, this similarity does not imply the possibility of substitution in every context
- 2) The characterization of the *explicatum* by the rules of its use, is to be given in an *exact* form, in such a way that all we need to understand the *explicatum* is in the rules defining it.
- 3) the *explicatum* should be a *fruitful* concept, that is useful for the formulation of empirical laws or logical theorems.
- 4) The *explicatum* should be as *simple* as possible

These four requirements are essential for the understanding of Gödel's arguments on the unfeasibility of Carnap's project.

4. Gödel's 1953-1955 argument

Gödel disagrees with many aspects of Carnap's analysis. He thinks that the axioms and rules of inference cannot be arbitrarily chosen. Mathematical axioms and rules describe the property of the concepts that we use to describe the structures of reality. Because these concepts “are composed of primitive ones, which as well as their properties, we can create as little as the primitive constituents of matter” [Gödel *193-59, p. 360], mathematical axioms and rules cannot be arbitrary. He disagrees also with Carnap abdication to the ideal of mathematical necessity and to his relativization of the distinction between analytic and empirical proposition. Gödel is also constant in his foundationalism. Logical and philosophical analyses must arrive at “fundamental conceptual advances” which should strive us to a rational justification of knowledge and to see more sharply to the fundamental primitive concepts by which all other concepts are conceived. Considering these fundamental divergences, the question is the following: does Gödel criticize Carnap's thesis from his own foundationalist point of view or does Gödel accept or assume in his argument Carnap's anti-foundationalist thesis in order to show that starting from Carnap's own frame of analysis his own aims cannot be achieved?

³¹ Interpretation is a key term in section IV, of *LSL* §62 [Carnap 1937, 228]

³² F. Rivenc (1993, p. 157) stresses the relationship between Carnap's notion of explication and Husserl's « synthesis of identification ». Rivenc's interpretation is confirmed by the facts reported in (Atten & Kennedy 2003, p 426-7): Carnap studied with Husserl in 1925-6, (as reported by K. Schumann (1977), p.82) and he probably attended Husserl's conference in Vienna. See, also [Beaney 2004] and [Lavers 2013] for further analysis on the subject.

4.1 To reduce/ To replace

As in the 1951 conference, in version III of *ISML*? Gödel distinguishes two versions of what he calls now the syntactical program, but he is more cautious in the way he describes each of them.

The first version appears around 1930. Largely under the influence of Wittgenstein, he says, Carnap, Hahn and Schlick developed a conception of the nature of logic and mathematics which is a “combination of nominalism and conventionalism”. He stresses that it was one of the main objectives of Hahn and Schlick to conciliate strict empiricism with the *a priori* certainty of mathematics. Strict empiricism is therefore correctly not mentioned as one of Carnap aims. This first version affirms that mathematics can be **reduced** to syntax of language [Gödel *1953-59 note 6, 335] and Gödel stresses that only if such a reduction was possible the syntactical program could be used in support of nominalism and conventionalism [Gödel *1953-59, 343]. As a matter of fact, this reduction is **disprovable** [Gödel *1953-59, 337], when we take the words “Mathematics”, “syntax” and “language” in the usual way, i.e., when mathematics includes what is necessary for the application to empirical sciences and language and syntax are intended in the sense of finitary combinatorics.

The second version arises from the different methods deployed for carrying out in detail the syntactical program. Gödel mentions *LSL*, Ramsey’s paper of 1926 and the works of the Hilbert school (Gentzen and Bernays) “about the formalization and the consistency of mathematics that can be interpreted as partial elaboration of this view”. [Gödel *1953-59 p. 336] In the sequel Gödel makes also reference to Karl Menger’s implicationism and to Laszlo Kalmar’s finitism. The second version affirms that mathematics can be **replaced** by Syntax of language, “i.e., what is asserted in mathematics can be *interpreted* to be syntactical conventions and their consequences and [...], on the basis of this interpretation, *the same conclusions as to ascertainable facts* can be drawn, if mathematical theorems are applied” [Gödel *1953-59 footnote 6, 335]. By ‘ascertainable facts’ Gödel means mathematical concrete calculus and results of the applications of mathematics to the physical world.

In *IMSL*?, the distinction between the two versions is crucial for Gödel and he chooses very carefully his words in talking about them. When he addresses himself to the Vienna Circle version of 1930, he says that according to it mathematics can be **reduced** to language. When he talks about Carnap’s *LSL* Gödel says that according to it mathematical propositions can be interpreted or **replaced** by syntactical sentences. So, the term “to replace” is strictly reserved for the description of the second conception and for Carnap’s *LSL*. This is very important if we consider that “to replace”, “to take the place”, “to substitute” are the very terms used by Carnap in *Logical Foundation of Probability* where the notion of explication is discussed [Carnap 1950, 3]. We will call, in the sequel, the first version of the syntactical program “reductionist program” and the second version “explicationist program”.

The explicationist program, according to Gödel, has developed interesting technical results which “have contributed much to the clarification of some fundamental concepts” (p 336). Nevertheless, all these developments:

[...] prove that: (1) *Mathematics can be interpreted to be syntax of language* only if the terms “language” or “syntax” or “interpreting” are taken in a very generalized or attenuated sense, or only if a small part of what is commonly regarded as “mathematics” is acknowledge as such. And they prove that (2) *Mathematical sentences have no content*, only if the term content is taken from the beginning in a sense acceptable only to empiricists and not well founded even from the empirical standpoint”. [Gödel *1953-59, 337]

In other words, in this second version the syntactical program **is not disprovable**, but with an attentive analysis we can prove that it becomes unfit to sustain the original objective of the program. What should consist this analysis in? Firstly, it should consist in pointing out the conceptual shifts from the ordinary usual sense of the terms involved in the definition of the reductionist program to the unusual technical sense, necessary to the explicationist program in order to avoid the disproof. Secondly, it should consist in a critical evaluation of the consequences of these conceptual shifts for the whole Carnap's syntactical program.

In each of the six items, clarifying the usual meaning of the terms occurring in the assertion (1) above, Gödel points out a conceptual shift and gives a critical evaluation of the consequences of such a shift. In each item, when he stresses an aspect of the usual meaning of these terms, he points to the fact that some developments of the syntactical program have given a new **explication** of these terms, i.e. have replaced the old usual sense with a new technical one. But, according to Gödel, precisely these shifts taken together show, indirectly, how the program is unfeasible. It is exactly from the analysis of these conceptual shifts, when correctly interpreted, that we can evaluate the attention paid by Gödel to the explicationist, non-foundationalist and conciliatory attitude of Carnap in *LSL*. Many of the alleged Gödel's misinterpretations of Carnap come from a misleading interpretation of these items. Stressing the 6 items, Gödel is not asking Carnap to conform to the standard use of the terms implied in assertion (1) above,³³ but he wants to evaluate the consequences of these non-standard use on Carnap's project of *LSL*. Gödel is setting up the steps for an unambiguous understanding of his final critical argument on the assertion that *Mathematics can be interpreted to be syntax of language*. This final critical argument is resumed in §29 of the third version:

On the grounds of these results it can be said that *the scheme of the syntactical program to replace mathematical intuition by rules for the use of symbols fails because this replacing destroys any reason for expecting consistency which is vital for both pure and applied mathematics, and because for the consistency proof one needs a mathematical intuition of same power as for the discerning the truth of the mathematical axioms or a knowledge of empirical facts involving an equivalent mathematical content*". [Gödel *1953-59, 346]

Such argument occurs before the elucidation of the question as to whether mathematics is void of content (assertion 2). It is therefore independent from it, contrary to what is affirmed in Lavers's reconstruction of Gödel's argument [Lavers 2019, section 4].

4.2 Conceptual shifts

Gödel is very clear on the fact that Carnap's solution "by no means" satisfies the 6 requirements expressed by the items, and this is evident because "Carnap uses non-finitary syntactical rules and arguments" [Gödel *1953-59, 343]. But what are the implications of this assertion concerning the way Gödel judges the unfeasibility of Carnap's own program? No correct answer can be given to this question without entering in the analysis of the items listed by Gödel. This analysis will show that the shift on the notions of "syntax", in the liberalized frame of *LSL*, implies important displacements in the meaning of the notions of "existence", "factual" sentence and "proof". These displacements make *LSL* unfit to sustain the original combination of nominalism and conventionalism of the syntactical program. They also show that Carnap's own *LSL* program of a total language of knowledge become unfeasible, because, on the bases of such displacements of meaning we cannot explain our trust in the application of logic and mathematics, which is an essential element of Carnap's strategy of conciliation.

³³ I stressed in [Crocco 2003], that Goldfarb's criticism of Gödel's argument relies on such a misunderstanding.

The aim of the list of items appears to be the evaluation of *the* “philosophical questions” [Gödel *1953-59, 337] involved in the syntactical program and of the way it was carried out in its explicationist version. Of the six items, the first two aim to evaluate Menger’s and Ramsey position. The four subsequent items directly involve Carnap’s way out from the reductionist program. They are analyzed in the sequel.

Items 3 says that, since the syntactical program aims at dispensing with mathematical intuition, in favor of syntactical rules:

[...] it will have to require of the *rules of syntax* that they be “finitary”, i.e., that they must not contain, they must not contain phrases such as ‘If there exists an infinite set with a certain property’, nor even: ‘If all expressions of a certain infinite set have a certain property’ [Gödel *1953-59, 338]

Carnap’s conceptual shift discussed in item 3 concerns the notion of “rules of syntax”. Gödel says that, as the usual notion of syntax coincide with that of finitary combinatorics, also the rules of syntax should be finitary, i.e., they must not contain phrases such as ‘If there exists an infinite set of expression with a certain property’, or: ‘If all expressions of a certain infinite set have a certain property. But this is exactly what Carnap’s definitions of “analytic” and “contradictory” for Language II, in section 34d of *LSL*, presuppose. Gödel mentions (footnote 14) Carnap’s defense of the syntactical nature of this infinitary rules as presented at page 114 of *LSL*. “Syntactical” means (as we know) independent from content (*Sinn*) and from reference or denotation (*Bedeutung*). Carnap considers that such rules are syntactical because one may know how to handle transfinite concepts without making any metaphysical assumption about the objective existence (as reference) of the abstract entities concerned.

Gödel’s reply is also confined in note 14, but his argument is crucial as it is repeated in the criticism of Carnap’s position of the fifties [Gödel 1953-50, 335]. If Carnap wants to ensure ontological neutrality to syntax, shifting from the ordinary sense of the expression “syntactic rule” to the new sense, which includes infinite rules, he must either give up a notion crucial to the application of logic and mathematics to the analysis of science, or introduce an ad hoc dissymmetry in the treatment of physical and mathematical entities.

In applying philosophy (or epistemology) to the analysis e.g. of physical theories, we have to distinguish a metaphysical absolute notion of existence, from an immanent (“kantian” Gödel says) provisional one. The latter is necessary in order to differentiate existential assumptions of objects belonging to a corroborated theory, from existential assumptions of objects belonging to a wrong, i.e. disprovable, theory. Such a notion of existence is essential to explain why modern chemistry is to be preferred to phlogistic theories and in general how experimentation can dismiss physical theories, denying existence to the entities they presuppose. If philosophy, in all its proper epistemological tasks,³⁴ can be replaced by the logic of science, i.e. by syntax, it is impossible to renounce to such a notion of provisional existence.

But then, mathematical entities should be treated as physical entities. Refutations of physical theories come from experimentation; refutations of mathematical ones come from the emergence of contradictions in concrete calculations. There is no reason to consider the sentences of the formers as containing terms having reference whereas the latter are said to have terms without any object-designation. Therefore, either we renounce to the notion of provisional existence, but then “philosophy” is not replaceable in a very crucial context of application by “syntax”, and therefore “syntax” is not a good *explicatum* of philosophy

³⁴ When we have removed from epistemology all the tasks that belong to special sciences such as history and psychology, we are left with the analysis of science, and thus of the rational decisions that lead us to abandon one theory in favor of another. See [Carnap 1937, §72,]

(condition 1, section 3.2 above) or we introduce an *ad hoc* distinction in order to force ontological neutrality.

This argument is quite similar to the one addressed to the later semantical publications of Carnap. In [Carnap 1950a], Carnap associates mathematical objects to formulae as denotations but he continues to consider that the question of their “external existence” is devoid of cognitive content. In *LSL Bedeutungen* were denied to mathematical terms occurring in mathematical sentences; in 1950 *Bedeutungen* are admitted but, still, Carnap thinks that the very (external) question of their existence has no cognitive meaning.³⁵ Against such an assertion Gödel argues on §45 as follows:

In general, if by using concepts and their relations among each other and with the sensations, one arrives at verifiable consequences, it is exactly from the *existence of objects* having these relations that the verifiable consequences follow. [...] That the existential assertions, also in mathematics are not mere “façon de parler” follows from the fact that they can be disproved (by inconsistencies derived from them) and that they have consequences as to ascertainable facts. [Gödel *1953-59, 355].

Lavers argues that Carnap’s later philosophy was misunderstood by Gödel. He is convinced that the position Gödel argues against is nominalism. According to him Carnap’s position in *LSL* is nominalist but in 1950 Carnap clearly wants to distance himself from nominalism and conventionalism. If my interpretation of Carnap’s project as aiming to neutralize metaphysical positions is correct, Carnap cannot be charged in *LSL* as endorsing nominalism. Carnap tries to clearly explain the different role played in knowledge by logical mathematical concepts and empirical ones. Nevertheless, in order to sustain at the same time an ambiguous mixt of a transcendentalist and conventionalist conception of logic, he denies existence to mathematical and logical objects. According to Gödel, Carnap’s semantical analysis in the fifties suffers of a similar prejudice, because questions of “external” existences are considered by him as devoid of cognitive content. This is not to mean, for Gödel, that, in a certain sense, Carnap is not right in pointing out that there is a difference between mathematical and space-time objects. Carnap is right in stressing out that this difference appears, from the syntactical or the semantical point of view, in the different role which the terms denoting them play in the formalism of science. But the true difference, according to Gödel, becomes fully visible only if the *meaning* of both mathematical and empirical terms is considered, and the difference in the concepts they denote is correctly analyzed. Here we see very clearly the kind of Gödel’s conciliatory strategy, we mentioned in our introduction: taking care as far as possible of the arguments of his opponents, in its own terms.

Items four insists on what Gödel considers the wrong Carnapian distinction between “syntactical” and “non-syntactical” rules and which leads to a conceptual shift about the usual meaning of the term “factual”.

³⁵ Carnap in the fifties restores the parallelism between mathematical and physical objects, but he denies for all the questions relative to existence of the latter, the status of cognitive questions: “[...] the external questions of the reality of physical space and physical time are pseudo-questions. A question like ‘are there (really) space-time points?’ is ambiguous. It may be meant in the external sense: ‘Shall we introduce such and such forms into our language?’; in this case is not a theoretical question but a practical question, a matter of decision rather than assertion, and here the proposed formulation would be misleading. Or, finally, it may be meant in the following sense: ‘are our experiences such and such that the use of the linguistic forms in question will be expedient and fruitful?’. This is a theoretical question of a factual empirical nature. But it concerns matter of degree; therefore, a formulation in the form ‘real or not?’ would be inadequate” [Carnap 1950a, 248-9]. According to Gödel all theoretical questions are in general matter of degree. See [Wang 1996, 8.1]

Gödel says:

4. Moreover a rule about the truth of sentences can be called *syntactical* only if it is clear from its formulation, or if it somehow can be known beforehand, that it does not imply the truth or falsehood of any "factual" sentence (i.e., one whose truth, owing to the semantical rules of the language, depends on extralinguistic facts). This requirement not only follows from the concept of a convention about the use of symbols, but also from the fact that it is the lack of content of mathematics upon which its a priori admissibility in spite of strict empiricism is to be based. The requirement under discussion implies that the rules of syntax must be demonstrably consistent, since from an inconsistency *every* proposition follows, all factual propositions included. [Gödel *1953-59, 339].

It seems to us that is impossible to ignore the specific way in which Gödel expresses himself. The first occurrence of the term "factual" is in quotation marks (line three) and the following parenthesis explains the meaning of the quotation marks: *LSL* prescribes that the difference between "factual" and "not factual" sentences depends on the structure of the language. On the contrary, in the last line, the term occurs without quotation-marks.

Let's start from the end of this passage to explain this puzzle. The end of item 4 states a prerequisite that the systems of *LSL* do not satisfy. Language I and Language II of *LSL* are not demonstrably coherent in the usual sense of the terms. Carnap explicitly affirms that we can employ a system of syntactic rules even if we cannot prove their consistency within the system. Concerning his theorem 34.i24, which states that Language II is not contradictory, Carnap remarks that we should not overestimate its importance. Theorem 34.i24 does not give us any assurance that Language II is free from contradictions; actually, to prove this theorem we need a language richer than language II, and the consistency proof of this metalanguage requires in turn an even richer language. The consistency of this latter is at least as doubtful as that of language II itself. But there are philosophical consequences of such a strategy that Carnap does not mention. As the syntactical rules are not demonstrably consistent (in the usual sense of the term, where a proof is something which gives such an assurance of what is proved), the appearance of a contradiction reveals each statement derivable from the empty set and therefore analytic. Carnap would say that such a specific situation is a reason to consider the chosen syntactical rules as inexpedient or ineffective. Nevertheless, such a strategy cannot be considered giving an "explication" of the notion of "factual" sentences. In the original Carnap's project, the partition between formal and factual served to eliminate metaphysical propositions from the language of science, by a norm based on the analysis of meaning. This objective is still that of *LSL*, as we saw. However, in this new theoretical framework the term "factual" designates something that depends on the syntactical rules of the system. Gödel is saying that, in a liberalized syntactical interpretation of mathematics, admitting rules of syntax whose "proof of coherence" gives no assurance, implies a shift from the usual concept of factuality to a technical term "factual" which has nothing to do with it. With this shift, the whole partition between logical and non-logical sentences of science, which was one of the aims of the project, becomes ineffective. The usual notion of factual sentence as Gödel says, means : "one whose truth, owing to the semantical rules of the language, depends on extralinguistic facts". In a contradictory syntactical frame, or in a syntactical frame just containing a contradictory portion of mathematics, every sentence is analytic, including those which usually we consider factual. Therefore, we have to conclude that the factuality of a given sentence depends in a certain sense from the coherence of mathematics, and this new technical notion of factuality does not seem to be of much help in deciding between competing theoretical frameworks. Suppose we have two different theories that attempt to represent the same physical phenomena and both turn out to be contradictory. Syntactic analysis could give us no help in assessing, on the basis of factual

truths and analysis of their meaning, how to make the right conceptual changes to formulate a new coherent theory. Every sentence in both of these contradictory theories becomes analytic, and the extreme holism implicit in Carnap's strategy destroys the usefulness of this new technical notion of factuality, which, on the contrary, constitutes the main interest of the usual notion. In Carnap's terms [Carnap 1950], the explication of the notion of "factual" in the liberalized syntactical version destroys any similarity with the usual notion (see condition 1 characterizing Carnap's notion of explication). For this reason it is unacceptable.

Items 5 and 6 both insist on the shifts on the notion of "proof" that the syntactical interpretation of mathematics implies. In footnote 20 Gödel explains what is, according to him, the original epistemic concept of proof from which the strict syntactical one has been formed: a proof should be a sequence of thoughts convincing a sound mind. With the reductionist version of the syntactical program a proof becomes a sequence of formulae respecting some formal conditions, which can be recognized mechanically. The explicationist version induces a new shift in the notion of proof. Items 5 and 6 stress that if a mathematical "proof" is such as the liberalized syntactical view of mathematics says it is, then it cannot support our trust on its conclusions.

Item 5 hints at the consequences of the shift on the derivation of the principles (axioms and procedure of proofs) from the "syntactical rules" and stresses that the conclusions of a "proof", in a system where the axioms and the procedure of proof are so conceived, cannot support any theorems which could be applied to ascertainable facts.

5. [...] the phrase "Mathematics can be *interpreted* to be the syntax of language" will have to mean (1) that the formal axioms and the procedures of proof of mathematics can be deduced from suitably chosen rules of syntax, and (2) that the conclusion as to ascertainable facts which are obtained on applying mathematical theorems and which formerly were based on the intuitive truth of mathematical axioms can be justified by syntactical considerations. [Gödel *1953-59, 339].

Requisite 5 part 1 states a condition that LSL does not satisfy. The proofs of the analyticity for the Axiom of choice (or of selection as Carnap calls it) and the Axiom of induction, belonging to language II, need the use of these same axioms in the metalanguage. Carnap presents the axioms at section 34h and discusses the problem at page 123-4 of *LSL*. In proving that every instance of, respectively, the principles of Selection and the principle of Induction are analytics, he has to use these very principles in the "syntax-language, which we use in our syntactical investigations" [Carnap 1937, 123]. He claims that no circularity is implied by such a "proof" because of the distinction between object-language and metalanguage (the syntax-language). Then Carnap adds:

The proofs of theorems 1 and 2 must not be interpreted as though by means of them we have proved that the principle of induction and the principle of selection were materially true. They only show that our definition of 'analytic' effects on this point what it is intended to effect, namely, the characterization of a sentence as analytic, if in material interpretation, it is regarded as logically valid". [Carnap 1937, 124]

The problem is that such a proof cannot effect what it is also intended to effect, i.e., to produce an explication of our belief that the axioms and the procedure of proof we are concerned with, are logically valid. There is nothing in such a "proof" which can epistemically constrain us to trust in the conclusion that we could draw from such axioms and rules of proof. How can we use our theorems in mathematical calculations, or in the conception of experimental protocols,

if such theorems come from proofs and procedures of proofs, whose only justification is that they are the object language correlates of a metalinguistic infinitary principle? In other words, a liberalized syntactical interpretation of mathematics should not destroy the possibility to explain the importance and the usefulness of mathematical concepts in science. Carnap's *explication* of the axioms and procedures of proofs of mathematics cannot afford its own requirement of fruitfulness (requirement 3 on the notion of explication). If we really consider that the syntactical program should propose an explication of the use of mathematical concepts for the derivations of empirical laws and mathematical theorems, Carnap notion of "proof" cannot afford it.

Items 6 hints at a second shift on the notion of "proof" which concerns the nature of the steps in the chain of the proof.

6.[...] it will have to be required that (1) not only in the rules of syntax, but also in the derivation of the mathematical axioms from them and in the proof of their consistency, only syntactical concepts be used [...] (i.e., only finitary concepts referring to finitary combinations of symbols) and (2) only procedures of proof which cannot be rejected by anyone who knows how to apply these concepts at all. [Gödel *1953-59, 341]

Here, as in item 5, Gödel points out a double shift operated by the explicationist program. Traditionally, it is part of the usual notion of proof the fact that starting with evident axioms and evident procedures of proofs, we construct the sequence of thoughts of the proof in such a way that each step is simpler and more evident (epistemically convincing) than that we want to prove. The abstract evidence of the axioms, through the evidence of the steps, justify the evidence of the conclusion.

When the reductionist program transforms the proof in a syntactical object of finitary combinatorics the simplicity and evidence of the steps rely on the formal inspection of the sentences, based on a mechanical recognition. Hilbert's concrete intuition is the only thing required by the formal notion of proof. But this is not the case for the explicationist program, where through the notion of valuation, as we saw, infinitary concepts applied to infinitary sets of symbols are used in the steps of the proof. No evidence of knowledge is conveyed anymore by such proofs.

Gödel repeats here the argument of circularity presented in the 1951 paper as the "stumbling block", but here it seems not any more casted in foundationalist terms : when we try to replace mathematical intuition which support evidence, by a liberalized "syntax" the whole original syntactical program "completely changes in meaning and is turned into its downright opposite: instead of *clarifying the meaning of the non-finitary mathematical terms by explaining* them in syntactical rules, non-finitary terms are used in order to formulate the syntactical rules". (p 341-2) (emphasis is mine). In Carnap's terms [Carnap 1950], we could say that the *explicatum* (the notion of "proof" containing infinitary concepts and rules in its steps, is not *simpler* than the *explicandum*, the epistemic notion of proof (requirement 4 of the notion of explication). So, the explication loses its interest.

The second part of item 6 is important for Gödel final argument in §29, against the thesis that *Mathematics is syntax of language*. Discussing item 6.2, Gödel takes into consideration the possibility that, for mathematics, a kind of indirect inductive proof of consistency could be used. Considering that no contradiction has arisen so far, we could explain our trust in the

application of mathematical concepts and proofs. But, Gödel concludes, in this case, mathematical propositions cannot be said to be syntactical conventions, because they acquire indeed, through their application so far considered, an empirical meaning. So, such inductive considerations concerning the notion of proof should be rejected by anyone who considers that the only thing to know about a rule of syntax is how to apply it. In other words, if for applying mathematics we need to use some inductive argument to explain our trust in the application, then the *explicandum* (the consistency of a system and the implied applicability of mathematics) is not explained by its *explicatum* (the syntactical consistency proof of the liberalized syntax). Something more is required than the syntactical consistency proof in order to understand and therefore use it, i.e., inductive arguments. In Carnap's terms [Carnap 1950], the *explicatum* is not an *exact* explanation of the *explicandum* (the epistemic notion of proof), (see Carnap's requirement 2 in the characterization of the notion of explication).

4.3 Summing up Gödel's argument

We can now correctly interpret Gödel's argument as presented at §29. Let's suppose that, for replacing the vague notions of truth, analyticity, and consequence in mathematics, we choose a system of rules not syntactical in the strictest sense of this word; i.e., we choose a system of rules which is not demonstrably consistent in the usual sense of the term and uses infinitary rules and axioms which are not strictly syntactical or cannot be deduced by strictly syntactical means. Could we then affirm that the syntactical program in the explicationist version has succeeded? Could we affirm that we have given a syntactical explication of mathematics?

Without any doubt we have obtained by this replacement explicit rules of use, such that, even if they cannot give a foundation of mathematics, they allow us to explain the linguistic nature of mathematics in an appropriate and clear language.

Nevertheless, if mathematical activity is really to be explained by the notion of convention and if mathematical intuition is really to be replaced by the simple "inferential" activity from conventional sets of rules, this explanation and replacement should lead to the same conclusions about the "ascertainable facts" that can be arrived at using mathematics.

In other words, the syntactical explication of mathematics should not destroy our trust in the predictive power of mathematics, predictive power vital both for pure mathematics and for applied mathematics. Using mathematical proofs of theorems, we can normally predict certain properties for certain natural numbers. A proof of Goldbach's conjecture, for example would be sufficient to affirm that every mechanical calculation by reliable means (a well-functioning computer, for example) will give the attended result. On the basis of a proof of Goldbach's conjecture, we can predict with a plain conviction that, given as input an even number to such a computer with an appropriate calculating program, after a certain amount of time the computer will stop and give us two prime numbers whose sum will be equal to the input. Using the physical theory of elastic body, which can be formulated only using a certain portion of mathematics, we can predict whether a certain bridge, constructed according to these laws, falls or not. Any trust in these predictions would be unjustified if the rules which allow us to formulate these predictions were simple syntactical conventions. Actually:

the scheme of the syntactical program to replace mathematical intuition by rules for the use of the symbols fails because this *replacing destroys any reason for expecting consistency, which is vital for both pure and applied mathematics*. [Gödel *1953-59, 346). Emphasis mine

The argument seems conclusive and not foundationalist. We need the consistency of our syntactical rules just because we use them to make predictions and we need to trust such predictions when applying mathematics. An explication which will not explain this trust will just be unacceptable because it would not explain the fruitfulness of our mathematics and its usefulness for applications.

Why is this trust destroyed by the syntactical program?

What can explain our trust in the predictive power of mathematics?

i) Empirical induction, based on the fact that no contradiction appeared until now. But then:

[...] it is true that, if consistency is interpreted to refer to the handling of physical symbols, it is empirically verifiable like a law of nature. However, if this empirical consistency is used, mathematical axioms and sentences completely lose their "conventional" character, their "voidness of content" and their "apriority" [...]. [Gödel *1953-59, 342)

ii) Mathematical intuition. This leads us to trust in the existence of mathematical concepts and objects. The trust in the existence of those entities, produces the trust in the properties of those entities (as no object could satisfy inconsistent properties). It is evident that this trust could be invalidated by the emergence of contradictions, because our intuition and our perception of mathematical objects and concepts are fallible. Notwithstanding, this trust in the existence of mathematical objects is a sufficient condition to explain our trust in the predictions which can be made using mathematics;

iii) A mathematical proof of consistency. But every mathematical proof of consistency will need transfinite or abstract concepts; if it is possible to prove the consistency of a system of rules in such a syntax, this proof, as Carnap admits, cannot explain our trust in the consistency unless we admit the mathematical intuition of the abstract concepts and of the infinitary rules needed to produce the proof.

Therefore, the syntactical program makes impossible to explain our trusting in mathematical applicability.

We can certainly wonder whether an explanation or description of the nature of mathematics should really explain or describe its applicability as well. The answer seems to be generally negative, especially considering the formalist view of mathematics, but Gödel seems to be aware that applicability is a central feature of Carnap's philosophical project from the very beginning, as Carnap explicitly states later in his autobiography.

More specifically, we can wonder whether an explanation of the nature of mathematics should really account for our trusting in mathematical applicability. It can be argued that "trusting" is a psychological attitude that Carnap, at least after his turn from *Erkenntnistheorie* to the Logic of science, should have considered as belonging to the psychological part of epistemology and thus not to the project of *Logical Syntax*. Nevertheless, the analysis of the conditions of possibility of mathematical practice can hardly be excluded from the scope of Logic of Science, if, in agreement with Carnap, we consider that : "the nature of logic and mathematics can be clearly understood only if close attention is given to their application in non-logical fields, especially in empirical sciences" [Carnap 1963, p. 13]. As "Logical questions may be concerned [...] with the meaning and content of the sentences, terms etc," [Carnap 1937, §72, 277] it is legitimate to expect, considering the example discussed in section 3.2 above, to be able to decide whether or not every descriptive primitive symbol of a language S1 for biology, is synonymous in a new language S3 with a symbol which is definable in a language S2 for physics or to decide whether or not a primitive law of S1 is equipollent in S3 to a law which is

valid in S2. But if logic (including mathematics) is plural, according to the principle of Tolerance, and if any question of correctness is eliminated, how a rational choice can be settled out where the notions of existence, of factuality and of proof loose their usual sense?

Stated in non-psychological words, Carnap succeeds on giving a linguistic account of the nature of mathematics but not in explaining how this syntactic formulation leads to the same predictions to which usual mathematics leads. If we want to give rules *for the application of mathematics in extra-mathematical context*, we need to be sure that these rules are applicable with the same rational consequences, unless we only trivially satisfy the central request of logicism. But, Carnap's rejection of abstract intuition, and the impossibility to grant the application of the infinitary syntactical rules on a formal consistency proof prevent the adoption of solutions (ii) and (iii), stated above in this section, both based on the use of abstract intuition or on provisional ontological commitments.

Condition i, based on empirical induction, is still available, but it goes opposite to Carnap's idea of the conventional and "void of content" nature of the logical mathematical frame of our knowledge and it leads dangerously, toward Quine's naturalism.

Bibliography

[Atten & Kennedy 2003] M. Van Atten & J. Kennedy, On the philosophical development of Kurt Gödel, *Bulletin of Symbolic Logic* 9 (4):425-476 (2003)

[Audureau 2016] E. Audureau. Gödel: From the Pure Theory of Gravitation to Newton's Absolute. In Crocco & Engelen 2016, 57-79

[Awodey & Carus 2004] S. Awodey and A. Carus. How Carnap could have replied to Gödel. In [Awodey & Klein 2004], 203-223.

[Awodey & Carus 2010] S. Awodey and A. Carus. Gödel and Carnap. In *Kurt Gödel: Essays for His Centennial*, S. Feferman, Ch. Parsons & S. G. Simpson (eds.). Association for Symbolic Logic (2010), 252-254.

[Awodey & Klein 2004] S. Awodey and C. Klein. *Carnap Brought Home. The View from Jena*. Chicago and La Salle Illinois, Open Court (2004).

[Beaney 2004] M. Beaney. Carnap's conception of Explication: From Frege to Husserl? In [Awodey & Klein 2004] 117-151.

[Crocco 2002] G. Crocco. Gödel, Quine et la conception des mathématiques chez Carnap. In *Carnap Aujourd'hui*, F. Lepage and F. Rivenc (eds.), Paris (Bellarmin/Vrin), (2002), 215-242.

[Crocco 2003] G. Crocco. Gödel, Carnap and the Fregean Heritage, *Synthese*, **137**, n°1-2, (2003), 21-41.

[Crocco & Engelen 2016] G. Crocco and E.M. Engelen. *Kurt Gödel Philosopher scientist*, Presses Universitaires de Provence, Aix-en-Provence (2016).

[Crocco& Engelen 20016] G. Crocco and Eva-Maria Engelen, Kurt Gödel's Philosophical remarks (Max Phil). In [Crocco & Engelen 2016], 33-54.

[Carnap 1928] R. Carnap. *Der Logische Aufbau der Welt*, Berlin Weltkreis (1928)

[Carnap 1931] R. Carnap. Die logizistische Grundlegung der Mathematik, *Erkenntnis* Bd 2, vol 2/3, 91-105 (1931). Presented at the Königsberg Symposium on Foundation of Mathematics, 1930. Translated as The logicist foundation of mathematics in *Philosophy of mathematics. Selected readings*, P. Benacerraf & H. Putnam (eds), Prentice Hall, Inc, (1964) first edition ; Cambridge University Press second edition (1983).

[Carnap 1932] R. Carnap. Überwindung der Metaphysik durch logische Analyse der Sprache, *Erkenntnis* (Leipzig) Band 2, vol 4, 219-241, (revised version of a lecture delivered in Warsaw in Nov 1930). Translated as: The elimination of metaphysics through logical analysis, in *Logical Positivism*; A.J. Ayer (ed), Free Press, (1959), 60-81.

[Carnap 1934a] R. Carnap. *Logische Syntax der Sprache*, Wien, Julius Springer, (1934). Translation by A. Smeaton London, *The logical Syntax of Language*, Kegan Paul (1937).

[Carnap 1934b] R. Carnap. Die Aufgabe der Wissenschaftlogie, *Einheitswissenschaft* 3, Vienna Gerold

[Carnap 1935] R. Carnap. Formalwissenschaft un Realwissenschaft, *Erkenntnis* 5, 30-37 (1935)

[Carnap 1945] R. Carnap. The two concepts of probability, *Philosophy and Phenomenological Reasearch* (Buffalo) vol 5, n4, 513-532.

[Carnap 1950a] R. Carnap. Empiricism Semantic and Ontology, *Revue Internationale de Philosophie*, (Bruxelles)4, n.11, 20-40

[Carnap 1950b] R. Carnap. *Logical Foundation of Probability*. Chicago) Univ. of Chicago Press. (1950)

[Carnap 1963] R. Carnap. Intellectual Autobiography in *The Philosophy of Rudolf Carnap*, (P .A. Schilpp (ed) La Salle, Illinos, Cambridge University Press, 3- 86.

[Gabriel 2004] G. Gabriel Introduction : Carnap Brought Home, in [Awodey & Klein 2004], 3-23.

[Gödel 1958] K. Gödel. Über eine bisher noch nicht benützte Erwaiterung des finiten Standpunktes, *Dialectic* 12, (1958) 280-87. Reprinted with English translation in *Kurt Gödel's Collected Works vol II*, S. Feferman *et alii*, New York Oxford University Press, (1990), 240-251.

[Gödel *1951] K. Gödel. Some basics theorems on the foundations of mathematics and their philosophical implications. In *Kurt Gödel's Collected Works vol III, Unpublished Essays and Lectures*. S. Feferman *et alii*, New York Oxford University Press. 304-323.

- [Gödel *1953-59] K. Gödel. Is Mathematics Syntax of Language ? versions III and V. In *Kurt Gödel's Collected Works vol III, Unpublished Essays and Lectures*. S. Feferman et alii, New York Oxford University Press.(1995), 334-362.
- [Gödel *1953-59a] K. Gödel. Is Mathematics Syntax of Language ? versions II and VI. In *Kurt Gödel's Unpublished Philosophical Essays*. F.R. Consuegra, Birkhäuser Verlag. (1995),
- [Gödel 1961] K. Gödel. The modern development of the foundations of mathematics in the light of philosophy. In *Kurt Gödel's Collected Works vol III, Unpublished Essays and Lectures*. S. Feferman et alii, New York Oxford University Press, (1995), 374-388.
- [Gödel *2003a] *Kurt Gödel's Collected Works vol IV, Correspondence A-G*. S. Feferman et alii, New York Oxford University Press, (2003).
- [Gödel *2003b] *Kurt Gödel's Collected Works vol V, Correspondence H-Z*. S. Feferman et alii, New York Oxford University Press, (2003).
- [Gödel *2019] Kurt Gödel. *Philosophische Notizbücher. Philosophie I, Maximen 0*. E.M. Engelen (ed). De Gruyter 2020.
- [Gödel *2020] *Kurt Gödel Maxims and Philosophical Remarks, vol. IX*. G. Crocco, M.van Atten, P.Cantù, R.Rollinger (eds) Hal Archives Ouverts (hal-02892852)
- [Goldfarb 1995] W. Goldfarb. Introductory note to *1953-59. In *Kurt Gödel's Collected Works vol III, Unpublished Essays and Lectures*. S. Feferman et alii, New York Oxford University Press. (1995), 324-334.
- [Goldfarb 2005] W. Goldfarb. On Gödel's way in: The influence of Rudolf Carnap, *Bulletin of Symbolic Logic* 11 (2):185-193 (2005)
- [Goldfarb & Ricketts 1992] W. Goldfarb and T. Ricketts. Carnap and the philosophy of mathematics. In D. Bell&W Vossenkuhl (eds) *Science and Subjectivity: The Vienna Circle and Twentieth Century Philosophy*. Berlin AkademieVerlag. (1992), 61-78.
- [Hahn 1935] H. Hahn. Logique, Mathématique et connaissance de la réalité. *Actualités Scientifiques et industrielles* 226. Paris Hermann (1935). French translation of Logik, Mathematik und Naturerkenntnis. *Einheitswissenschaft* 2. Vienna Gerold (1935).
- [Heinzmann & Proust 1988] G. Heinzmann and J. Proust, Carnap et Gödel: échange de lettres autour de la définition d'analyticité. Introduction, Traduction et Notes. *Logique et Analyse*, (1988), vol 31, n. 123/124, 257-291.
- [Lavers 2013] G. Lavers. Frege, Carnap and Explication: 'Our Concern Here is to arrive at a Concept of Number Usable for the Purpose of Science.' *History and Philosophy of Logic*, vol. 34, N. 3, (2013), 225-241.
- [Lavers 2019] G. Lavers. Hitting a moving target: Gödel, Carnap, and Mathematics as Logical Syntax. *Philosophia Mathematica*, Volume 27, Issue 2, June 2019, Pages 219–243.

[Ramsey 1926] F. P. Ramsey. The Foundations of Mathematics. Proceedings of the London Mathematical Society (2) 25, 338-384.

[Rivenc 1993] F. Rivenc *Recherche sur l'Universalisme logique*, Pari, Payot (1993).

[K. Schumann 1977], K. Schumann. *Husserl-Chronik. Denk- und Lebensweg Edmund Husserls*, Martinus Nijhoff, Den Haag, (1977).

[Wang 1987] H. Wang. *Reflections on Kurt Gödel*. MIT Press Cambridge Massachusetts (1987).

[Wang 1995] H. Wang. *A Logical Journey. From Gödel to Philosophy*. MIT Press Cambridge Massachusetts (1995).