

# Statistical performance assessment of a bistable vibration energy harvester

*How to evaluate and compare non-linear generators?*

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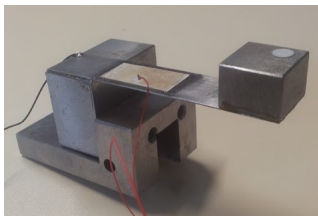


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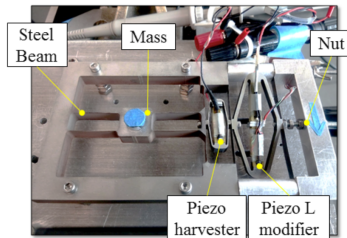


# Vibration energy harvesting

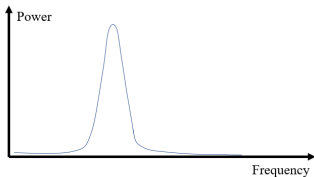
*How can the bandwidth of vibration energy harvesters be increased?*



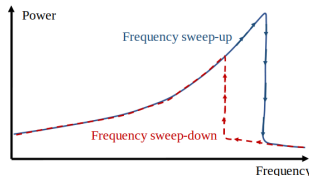
Simple linear generator: a beam



Bistable generator



Narrow bandwidth



Large bandwidth

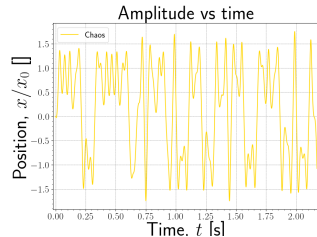
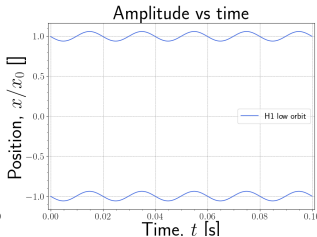
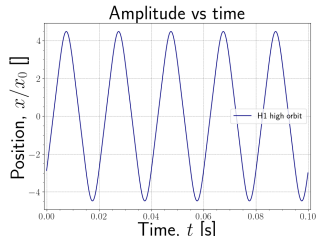
→  
adding non linearities



# Piezoelectric bistable generators

$$\begin{cases} \ddot{x} = -\frac{\omega_0^2}{2} \left( \frac{x^2}{x_0^2} - 1 \right) x - \frac{\omega_0}{Q} \dot{x} - 2 \frac{\alpha}{ML} xv + A \sin(2\pi f_d t) \\ \dot{v} = 2 \frac{\alpha}{LC_p} x \dot{x} - \frac{1}{RC_p} v \end{cases}$$

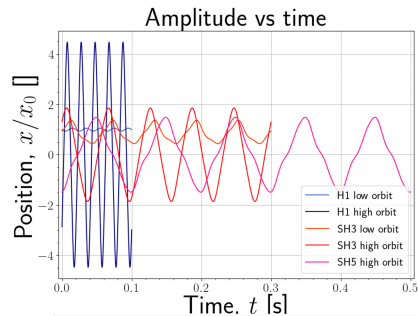
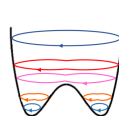
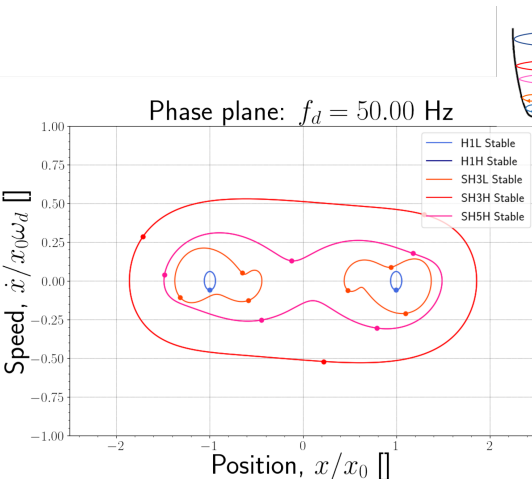
Complex behaviors: multiple orbits and chaos



*How to evaluate and compare non-linear piezoelectric generators?*



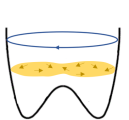
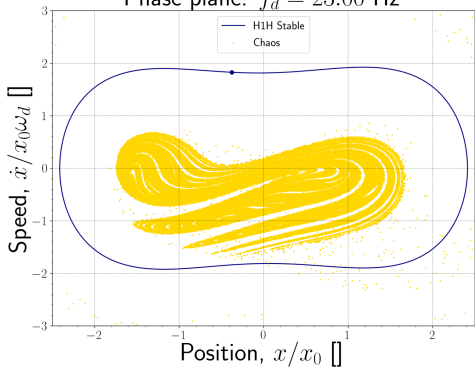
## Behaviors at 50 Hz



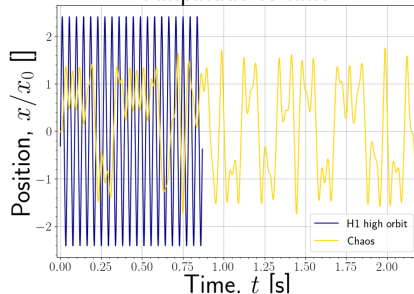
$$P_h^{H1H} \approx 5.3 \text{ mW}, P_h^{H1L} \approx 6.8 \cdot 10^{-4} \text{ mW}, P_h^{SH3H} \approx 1.2 \cdot 10^{-1} \text{ mW} \dots$$



## Behaviors at 23 Hz

Phase plane:  $f_d = 23.00$  Hz

Amplitude vs time

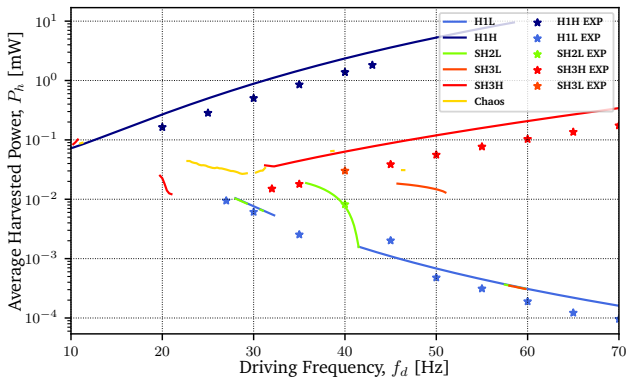


$$P_h^{Chaos} \approx 0.044 \text{ mW} \text{ and } P_h^{H1H} \approx 0.39 \text{ mW}$$



# Random initial grid: orbits detection

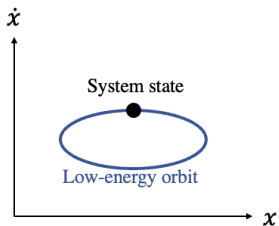
$$G = \{(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{v}) \mid (x_i, \dot{x}_i, v_i) \hookrightarrow \mathcal{U}(l_x \times l_{\dot{x}} \times l_v), i \in \llbracket 0, N_o - 1 \rrbracket\} \subset \mathbb{R}^3$$



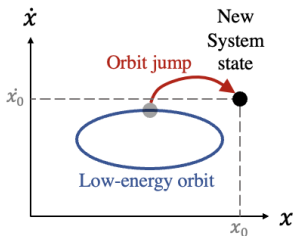
- ▶ Definition of a random initial grid to detect all possible orbits
- ▶ Each orbit is associated to an average harvested power
- ▶ Necessity to converge to the higher orbit



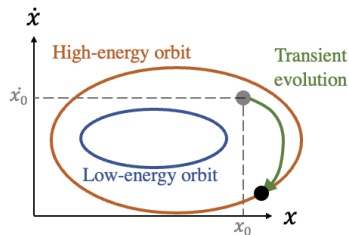
# Links between initial conditions and orbit jump



Initial state  
Initial orbit



Orbit jump:  
Force initial condition  $(x_0, \dot{x}_0, v_0)$



Convergence toward  
a new orbit

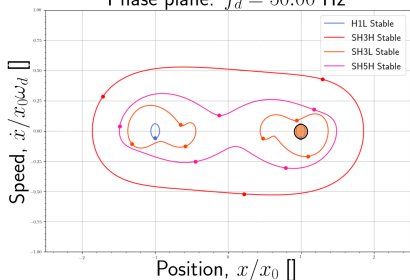
# Probability of occurrence of each orbit

Lowest orbit (I.C. 1)

- ▶ probability of occurrence depends on the distribution of the initial condition (I.C.)





- ▶ initial conditions depend on the orbit jump strategy

Phase plane:  $f_d = 50.00$  Hz



Orbit probability depending on the I.C.

Initial condition:  $(x, \dot{x}, v)^T = (x_0, 0, 0)^T$

- ▶ H1L:  $r = 100\%$ , 
- ▶ H1H:  $r = 0\%$ , 
- ▶ SH3H:  $r = 0\%$ , 
- ▶ SH5H:  $r = 0\%$ , 





# Probability of occurrence of each orbit

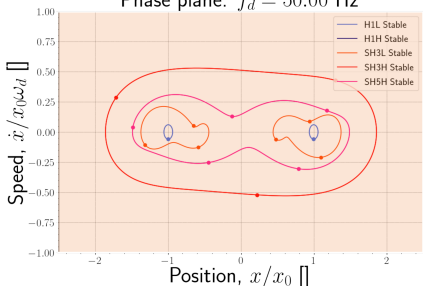
Random initial grid at 50 Hz (I.C. 2)

- ▶ probability of occurrence depends on the distribution of the initial condition (I.C.)
- ▶ initial conditions depend on the orbit jump strategy

Orbit probability depending on the I.C.

Initial condition:  $(x, \dot{x}, v)^T \in G$

Phase plane:  $f_d = 50.00$  Hz



▶ H1L:  $r = 64.24\%$ ,



▶ H1H:  $r = 10.25\%$ ,



▶ SH3H:  $r = 16.22\%$ ,



▶ SH5H:  $r = 4.60\%$ ,



# Probability of occurrence of each orbit

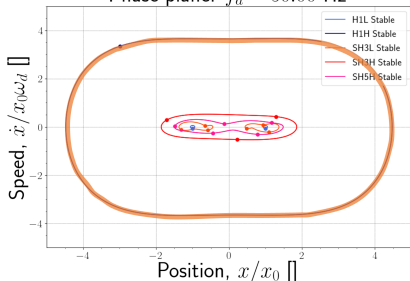
Higher orbit (I.C. 3)

- ▶ probability of occurrence depends on the distribution of the initial condition (I.C.)
- ▶ initial conditions depend on the orbit jump strategy

Orbit probability depending on the I.C.

Initial condition:  $(x, \dot{x}, v)^T \in \max_{n \in \mathbb{N}} \text{SH}n$

Phase plane:  $f_d = 50.00 \text{ Hz}$



▶ H1L:  $r = 0\%$ ,



▶ H1H:  $r = 100\%$ ,



▶ SH3H:  $r = 0\%$ ,



▶ SH5H:  $r = 0\%$ ,



# Power expectation

	H1L	H1H	SH3H	SH3L	SH5H	Chaos	$\mathbb{E}$ [mW]
I.C. 1	1	0	0	0	0	0	$6.8 \cdot 10^{-4}$
I.C. 2	0.64	0.10	0.16	0.05	0.05	0	0.55
I.C. 3	0	1	0	0	0	0	5.3
$P_h$ [mW]	$6.8 \cdot 10^{-4}$	5.3	$1.2 \cdot 10^{-1}$	$1.5 \cdot 10^{-2}$	$3.6 \cdot 10^{-2}$	—	—

Table: Probabilities for different initial conditions

## Power expectation

$$\mathbb{E}[P_h] = \sum_k r_k P_{h,k}$$

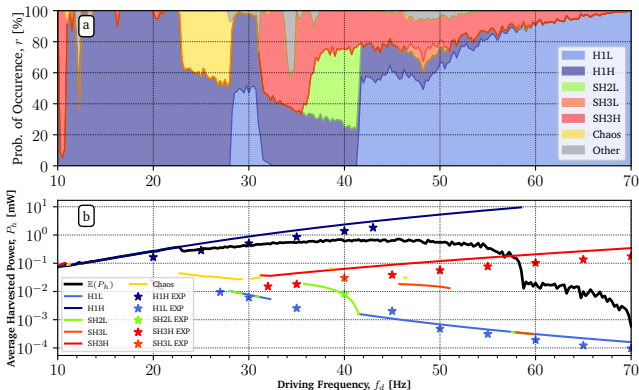


# Power expectation for all driving frequencies

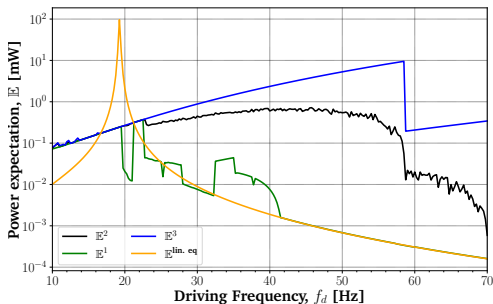
## Several jumping orbits strategies

- ① always on the lowest orbit
- ② random jump
- ③ always on the higher orbit

Probabilities in case ②:



# Comparison of orbit jump strategies



## Figure of Merit (FoM)

$$FoM = \int_{f_d} \mathbb{E} df / \int_{f_d} \mathbb{E}^{\text{lin. eq}} df$$

	Case 1	Case 2	Case 3	Linear eq.
$FoM$	0.06	0.5	3.3	1



# Conclusion and perspectives

## Conclusion

- 1 Comparison of orbit jump strategies for a given nonlinear generator:
  - 1 The worst case is 17 times lower than the linear equivalent
  - 2 is 2 times lower than the linear equivalent
  - 3 is 3 times larger than the linear equivalent
- 2 Evaluation of the performance of nonlinear generators by taking into account subharmonic regimes

## Perspectives

- 1 Test more orbit jump strategies
- 2 Comparison of nonlinear generators



# Thank you !



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