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# Self-scheduled $H_\infty$ control of autonomous vehicle in collision avoidance maneuvers

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**Abstract:** This article presents the development of a control law for vehicle steering, acceleration and braking in the autonomous driving context. A self-scheduled robust feedback control design strategy is proposed for the stabilization of the vehicle along a predefined reference collision avoidance maneuver. A non-linear vehicle model with tire forces saturation is developed. Its linearization along the reference trajectory leads to a Linear Parameter-Varying (LPV) model. An LPV  $H_\infty$  control law with pole placement is computed for the LPV model. Simulation results illustrate the stabilization of the vehicle along the reference trajectory.

*Keywords:* Autonomous Vehicles, Linear parametrically varying (LPV) methodologies, Robust control, Pole placement.

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## 1. INTRODUCTION

Autonomous driving is widely considered a possible solution to several private transportation challenges, as for example efficiency, energy consumption and safety, as described in Litman (2020). Several *Advanced Driver-Assistance Systems* (ADAS) have been developed in the recent years, see Bengler et al. (2014). Some of these systems are able to control the vehicle under the driver supervision in specific environmental conditions. They constitute the first basic components of autonomous driving. To move towards increased levels of autonomous driving, defined in SAE (2018), less driver supervision is required. Hence safety functions have to be developed, to control the vehicle in emergency conditions. The development of methods for driving automation in collision avoidance maneuvers are of great interest for both academical and industrial communities.

The ADAS systems dealing with driving automation that are currently available on commercial vehicles mainly handle low dynamics maneuvers, i.e. maneuvers with a maximum vehicle acceleration of 0.3g. In these conditions, longitudinal and lateral vehicle dynamics can be considered uncoupled, and the tire forces behave linearly. The control of the longitudinal and lateral dynamics can hence be treated separately.

The longitudinal control architecture is usually designed hierarchically, with an higher level controller, generating the reference value for the acceleration, and a lower level controller that tracks the desired acceleration through throttle and brakes. A common solution for the higher level controller is the *Linear Proportional Integral Derivative* (PID) control law, as proposed in Persson et al. (1999), Naus et al. (2010) and Hima et al. (2011). For the lower level controller the sliding mode method can be used for

engine control to deliver the required torque, as proposed in Hedrick et al. (1991) and Sei-Bum Choi and Hedrick (1995).

Lateral control is often based on the *linear bicycle model*, Rajamani (2012). There exist several solutions that propose  $H_2$  and  $H_\infty$  control laws, as in Shimakage (2002) and Hima et al. (2011), to track lateral and yaw errors with respect to a reference trajectory.

In high dynamics maneuvers, i.e. with accelerations greater than 0.3g, other phenomena occur. The longitudinal and lateral dynamics cannot be considered decoupled. Moreover the tire forces are subject to saturation and combined longitudinal and lateral slip. As a consequence in literature a large majority of the solutions developed to control the vehicle in high dynamics conditions consider vehicle models with coupled longitudinal and lateral dynamics.

There exist several solutions based on *Model Predictive Control* (MPC) for collision avoidance maneuvers. In Falcone et al. (2007) an MPC controller is developed based on a non-linear bicycle model with both longitudinal and lateral dynamics. An alternative controller based on local linearizations of the non-linear model is also proposed to reduce the computational load. Falcone et al. (2008) and Yin et al. (2015) propose a similar controller based on a non-linear four-wheel vehicle model. In Gao et al. (2014) *tube-based* MPC is used to address also robustness criteria. The main advantage of MPC is the possibility to take into consideration the inputs and states constraints in the underlying optimization problem. It is then easy to represent actuators limits. However the framework requires a large computational load. This is a crucial point for the integration of the controller on a vehicle *Electronic Control Unit* (ECU), which for economic reasons has a limited computational capacity.

On the contrary, the *Linear Parameter-Varying* (LPV)

framework offers methods for the synthesis of controllers that do not require a large computation load; see Wu (1995), Bianni (1996), Blanchini and Miani (2015). Moreover it has already been proven effective in the vehicle dynamics control domain. In literature there exist several solutions that employ LPV controllers for *Global Chassis Control* (GCC) systems, which aim is either assist the driver by increasing vehicle stability in high dynamics conditions or improve comfort through several chassis actuators. In Fergani (2006), Gáspár et al. (2007) and Poussot-Vassal et al. (2011) different solutions for vehicle stabilisation through combinations of active steering, braking and active suspensions are proposed. They are all based on robust LPV  $H_\infty$  control law.

This paper proposes a self-scheduled LPV  $H_\infty$  controller for vehicle stabilization along a reference collision avoidance maneuver. The controller is based on a LPV model for the vehicle dynamics along the maneuver, presented by the authors in Penco et al. (2021). This LPV model is obtained from the linearization along the reference maneuver of a nonlinear model that considers the coupling between longitudinal and lateral dynamics and the tire nonlinear behavior. In particular, due to an algebraic loop introduced by the saturation function of the tires, we propose a strategy to obtain an uncertain parameter related to the slope of sector non-linearities.

The paper is organized as follows. In Section 2 the non-linear vehicle model and the procedure to obtain an LPV model are discussed. The method used for the synthesis of the controller is illustrated in Section 3. In Section 4 the results of the application of the controller in a simulation environment are shown. The conclusions and future work are discussed in Section 5.

## 2. PRELIMINARY RESULTS

In this section we discuss the development of a non-linear vehicle model and the procedure to obtain an LPV model. A more detailed description of the non-linear model is available in Penco et al. (2021). The difference between the model presented in Penco et al. (2021) and the model studied in this paper is the influence of the road slope and camber in the dynamical model.

### 2.1 Non-linear vehicle model

To obtain a model of the vehicle we consider that the two wheels of each axle have been regrouped at the center of the axle track. The vehicle degrees of freedom allow it to move on the  $xy$  plane, so it can translate along the  $x$  and  $y$  axis and rotate around the  $z$  axis. The vehicle axis are shown in Figure 1.

In Table 1 there are the model state variables and inputs. The road slope and camber angles are shown in Figure 2. Table 2 contains the forces shown in Figures 1 and 3. The forces applied on the front wheel have each a component on the  $x$  and  $y$  axis of the vehicle, that depends on  $\delta_f$ . We denote the components of  $F_{xf}$  on the  $xy$  plane:

$$\begin{aligned} F_{xxf} &= F_{xf} \cos \delta_f \\ F_{xyf} &= F_{xf} \sin \delta_f. \end{aligned}$$

The model equations of motion in the vehicle frame are:

$$\begin{aligned} m(\dot{v} - r u) &= 2(F_{xxf} + F_{xr} - R_{xxf} - R_{xr} \\ &\quad - F_{xyf}) - F_{aero} - P_x \end{aligned} \quad (1a)$$

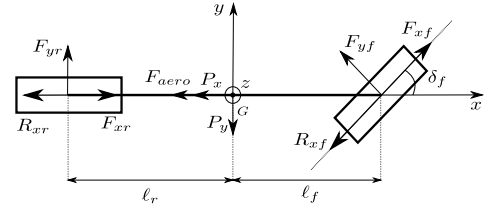


Fig. 1. Vehicle's  $xy$  plane, with the forces listed in Table 2.

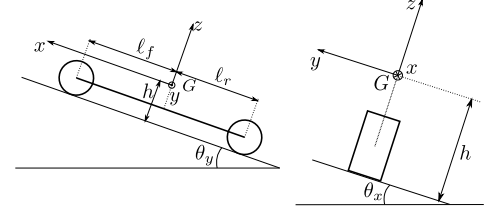


Fig. 2. Road inclination.

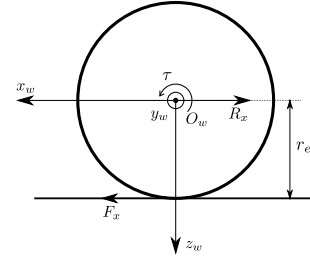


Fig. 3. Wheel's  $x_w z_w$  plane.

Table 1. Model state variables and inputs

State variables	
$v$	[m/s] longitudinal speed
$u$	[m/s] lateral speed
$r$	[rad/s] yaw rate
$\omega_{wf}$	[rad/s] front wheel rotational speed
$\omega_{wr}$	[rad/s] rear wheel rotational speed
$x$	[m] absolute longitudinal position
$y$	[m] absolute lateral position
$\psi$	[rad] yaw angle
System's inputs	
$\delta_f$	[deg] front wheel steering angle
$\tau_{wf}$	[Nm] front wheel torque
$\tau_{wr}$	[Nm] rear wheel torque
$\theta_x$	[rad] road camber angle
$\theta_y$	[rad] road slope angle

Table 2. Model's forces

$F_{xf}$	front tire longitudinal force	$F_{xr}$	rear tire longitudinal force
$F_{yf}$	front tire lateral force	$F_{yr}$	rear tire lateral force
$R_{xf}$	front wheel rolling friction	$R_{xr}$	rear wheel rolling friction
$F_{aero}$	aerodynamic drag	$P_z$	weight component in z axis
$P_x$	weight component in x axis	$P_y$	weight component in y axis
$N_f$	front wheel normal force	$N_r$	rear wheel normal force

$$m(\dot{u} + r v) = 2(F_{yxf} - R_{yxf} + F_{yyf} + F_{yr}) - P_y \quad (1b)$$

$$I_{zz}\dot{r} = 2(F_{yxf} + F_{yyf} - R_{yxf})l_f - 2F_{yr}l_r \quad (1c)$$

$$I_{wy}\dot{\omega}_{wf} = \tau_{wf} - 2F_{xf}r_e \quad (1d)$$

$$I_{wy}\dot{\omega}_{wr} = \tau_{wr} - 2F_{xr}r_e. \quad (1e)$$

In high dynamics maneuvers, the tires are subject to saturation and combined slip (Pacejka (2012)). It is important then to take into consideration these phenomena by using a non-linear model for the tire forces. The tire

model in Kissai et al. (2017), developed for control purposes, is a variation of the *Dugoff's model*, proposed in Dugoff et al. (1969). It is simpler and requires less parameters than other commonly used models, as the *Pacejka's magic formula*, in Bakker et al. (1987), and the *brush model*, in Svendenius (2003). We rely on this model since it allows to represent tire forces saturation and combined slip.

The longitudinal and lateral forces are linear respectively on the longitudinal and lateral slip,  $\kappa$  and  $\alpha$  as

$$\hat{F}_x = c_\kappa^* (\mu, N, \alpha) \kappa \quad (2a)$$

$$\hat{F}_y = c_\alpha^* (\mu, N, \kappa) \alpha. \quad (2b)$$

Longitudinal and lateral slip  $\kappa$  and  $\alpha$  are defined in Pacejka (2012). The coefficients  $c_\kappa^*$  and  $c_\alpha^*$  are non-linear with respect to the  $\alpha$  and  $\kappa$  respectively, the wheel normal force  $N$  and the tire-ground friction coefficient  $\mu$ .

For the sake of brevity we will not discuss the tire model in details. It is however important to discuss of the tire force saturation. Here we use a circle to approximate the saturation of the tire forces as in Schuring et al. (1996), thus simplifying the more general ellipse model, discussed in Svendenius (2003). The radius of the saturation circle is given by  $F_{max} = \mu N$ . The logistic function, shown in Figure 4, is used to express the saturation

$$\sigma(\hat{F}, u, \ell) = \frac{u - \ell}{1 + e^{-\frac{4}{u-\ell}(\hat{F} - \frac{u+\ell}{2})}} + \ell \quad (3)$$

where  $u$  and  $\ell$  are the upper and lower saturation bounds. The unsaturated forces  $\hat{F}_x$  and  $\hat{F}_y$  of (2) are the inputs of (3), the outputs are the saturated tire forces  $F_x$  and  $F_y$ . The normal force  $N$  on the tire is present both in the coefficients  $c_\kappa^*$  and  $c_\alpha^*$  and the saturation bound.

Considering the importance of  $N$  in the computation of the tire forces, a variable normal force model has then been developed. The procedure to obtain it has been discussed in Penco et al. (2021) and for brevity is not shown in this paper. The expressions of  $N_f$  and  $N_r$  depend on the longitudinal acceleration  $\dot{v}$ .  $N_f$  and  $N_r$  are saturated between 0 and  $P_z$  using the logistic function.

To be brief the other forces are not discussed in this paper, if needed the reader can consult Rajamani (2012).

Finally we complete the vehicle model by adding the vehicle position and orientation. Their dynamics on the inertial frame are governed by:

$$\dot{x} = v \cos \psi - u \sin \psi \quad (4a)$$

$$\dot{y} = u \cos \psi + v \sin \psi \quad (4b)$$

$$\dot{\psi} = r. \quad (4c)$$

## 2.2 Model linearization

The state and input vectors of the model in (1) are:

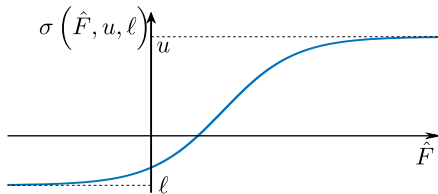


Fig. 4. Logistic function.

$$\underline{\mathbf{x}} = \begin{bmatrix} v \\ u \\ r \\ \omega_{wf} \\ \omega_{wr} \end{bmatrix} \quad \underline{\mathbf{u}} = \begin{bmatrix} \delta_f \\ \tau_{wf} \\ \tau_{wr} \\ \theta_x \\ \theta_y \end{bmatrix}.$$

The model expressed by (1) is in implicit form  $f(\dot{\underline{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}) = 0$ . Due to the saturation functions used for the tire forces, it is not possible to express the model in explicit form.

We can write the model in a feedback form as in Figure 5:

$$\dot{\underline{\mathbf{x}}} = g(\underline{\mathbf{x}}, \underline{\mathbf{u}}) + B_\sigma(\underline{\mathbf{u}}) \underline{\sigma}(\underline{\mathbf{h}}) \quad (5)$$

where:

$$\underline{\sigma}(\underline{\mathbf{h}}) = \begin{bmatrix} \sigma(h_1(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}})) \\ \sigma(h_2(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}})) \\ \sigma(h_3(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma})) \\ \sigma(h_4(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma})) \\ \sigma(h_5(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma})) \\ \sigma(h_6(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma})) \end{bmatrix}$$

$$\underline{\mathbf{h}}(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma}) = \begin{bmatrix} h_1(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}) \\ h_2(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}) \\ h_3(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma}) \\ h_4(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma}) \\ h_5(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma}) \\ h_6(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma}) \end{bmatrix} = \begin{bmatrix} \hat{N}_f \\ \hat{N}_r \\ \hat{F}_{xf}(\sigma(\hat{N}_f)) \\ \hat{F}_{xr}(\sigma(\hat{N}_r)) \\ \hat{F}_{yf}(\sigma(\hat{N}_f)) \\ \hat{F}_{yr}(\sigma(\hat{N}_r)) \end{bmatrix}.$$

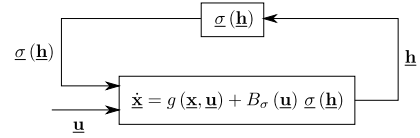


Fig. 5. Isolation of the implicit part of (1).

The vectors  $\underline{\mathbf{h}}(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma})$  and  $\underline{\sigma}(\underline{\mathbf{h}})$  contain respectively the inputs and the outputs of the saturation functions. The model non-linearities due to the saturation functions have then been isolated and they result as an input of the model. The expressions of  $g(\underline{\mathbf{x}}, \underline{\mathbf{u}})$  and  $B_\sigma(\underline{\mathbf{u}})$  for brevity are not detailed here.

The state dynamics expressed as in (5) is then linearized along a reference trajectory  $(\underline{\dot{\mathbf{x}}}_0(t), \underline{\mathbf{x}}_0(t), \underline{\mathbf{u}}_0(t), \underline{\sigma}_0(t))$ , with  $\underline{\sigma}_0(t) = \underline{\sigma}(\underline{\mathbf{h}}(\underline{\dot{\mathbf{x}}}_0(t), \underline{\mathbf{x}}_0(t), \underline{\mathbf{u}}_0(t)))$ . For simplicity, in the following, the reference trajectory, containing both the states and the inputs, will be denoted by  $\mathcal{T}(t)$ .

$$\Delta \dot{\underline{\mathbf{x}}} = A(t) \Delta \underline{\mathbf{x}} + B(t) \Delta \underline{\mathbf{u}} + B_\sigma(t) \Delta \underline{\sigma}$$

$$\Delta \underline{\mathbf{h}} = C(t) \Delta \underline{\mathbf{x}} + D(t) \Delta \underline{\mathbf{u}} + D_\sigma(t) \Delta \underline{\sigma}$$

$$A(t) = \left. \frac{\partial g(\underline{\mathbf{x}}, \underline{\mathbf{u}})}{\partial \underline{\mathbf{x}}} \right|_{\mathcal{T}(t)}$$

$$B(t) = \left. \frac{\partial g(\underline{\mathbf{x}}, \underline{\mathbf{u}})}{\partial \underline{\mathbf{u}}} \right|_{\mathcal{T}(t)} + \left. \frac{\partial (B_\sigma(\underline{\mathbf{u}}) \underline{\sigma}(\underline{\mathbf{h}}))}{\partial \underline{\mathbf{u}}} \right|_{\mathcal{T}(t)}$$

$$B_\sigma(t) = B_\sigma(\underline{\mathbf{u}})|_{\mathcal{T}(t)}$$

$$C(t) = \left. \frac{\partial \underline{\mathbf{h}}(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma})}{\partial \underline{\mathbf{x}}} \right|_{\mathcal{T}(t)} A(t) + \left. \frac{\partial \underline{\mathbf{h}}(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma})}{\partial \underline{\mathbf{x}}} \right|_{\mathcal{T}(t)}$$

$$D(t) = \left. \frac{\partial \underline{\mathbf{h}}(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma})}{\partial \underline{\mathbf{x}}} \right|_{\mathcal{T}(t)} B(t) + \left. \frac{\partial \underline{\mathbf{h}}(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma})}{\partial \underline{\mathbf{u}}} \right|_{\mathcal{T}(t)}$$

$$D_\sigma(t) = \left. \frac{\partial \underline{\mathbf{h}}(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma})}{\partial \underline{\mathbf{x}}} \right|_{\mathcal{T}(t)} B_\sigma(t) + \left. \frac{\partial \underline{\mathbf{h}}(\underline{\dot{\mathbf{x}}}, \underline{\mathbf{x}}, \underline{\mathbf{u}}, \underline{\sigma})}{\partial \underline{\sigma}} \right|_{\mathcal{T}(t)}$$

In this paper we consider the reference trajectory of Figure 6. It corresponds to a two lanes change maneuver without braking, starting at the speed of 70km/h.

The saturation functions can be approximated as sector-

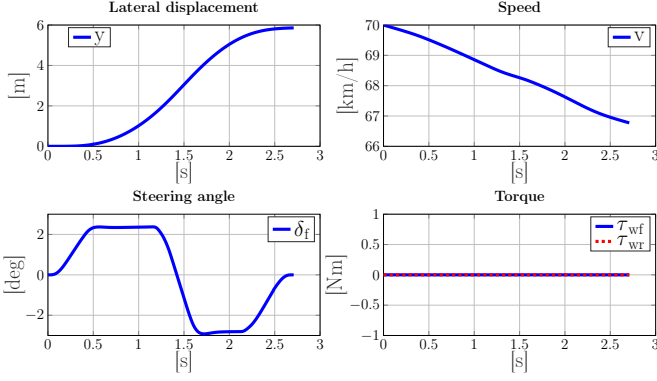


Fig. 6. Reference trajectory.

bounded non-linearities, whose slope varies according to the system working point. Figure 7 shows the slope sector for each element of  $\sigma(\mathbf{h})$  along the reference trajectory. The wheels torques are equal to zero, hence the longitudinal tire forces and the load transfer are small and their sectors are then narrow. On the contrary the lateral tire forces magnitude in the reference trajectory includes a wider sector.

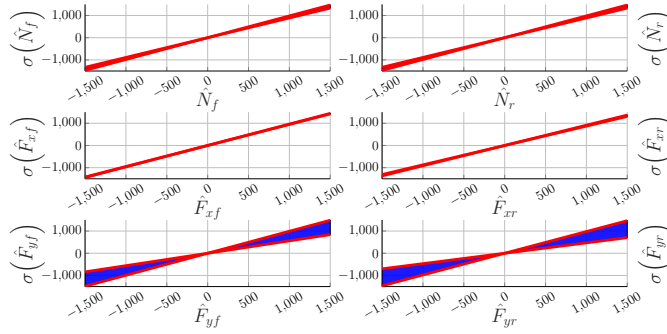


Fig. 7. Slope sector for reference trajectory of Figure 6.

We then approximate the non-linear function by a time-varying affine function as  $\sigma(\mathbf{h}) \approx K_\sigma \mathbf{h}$ .  $K_\sigma$  is a time-varying diagonal matrix, with the slope of the saturation functions along the diagonal. Applying this approximation in the expression of  $\Delta \mathbf{h}$  in (6) we obtain:

$$\Delta \mathbf{h} = (I - D_\sigma(t) K_\sigma)^{-1} C(t) \Delta \mathbf{x} + (I - D_\sigma(t) K_\sigma)^{-1} D(t) \Delta \mathbf{u}.$$

Replacing this expression of  $\Delta \mathbf{h}$  in (6), the linearized state dynamics with the sector-bounded approximation of the saturation functions is:

$$\Delta \dot{\mathbf{x}} = \tilde{A}(t) \Delta \mathbf{x} + \tilde{B}(t) \Delta \mathbf{u} \quad (7a)$$

$$\tilde{A}(t) = A(t) + B_\sigma(t) K_\sigma (I - D_\sigma(t) K_\sigma)^{-1} C(t) \quad (7b)$$

$$\tilde{B}(t) = B(t) + B_\sigma(t) K_\sigma (I - D_\sigma(t) K_\sigma)^{-1} D(t). \quad (7c)$$

### 2.3 LPV polytopic model for control synthesis

Given that the linearized model is obtained along a reference trajectory and the saturation functions are approximated by time-varying gains, the matrices  $\tilde{A}(t)$  and  $\tilde{B}(t)$  in (7b) and (7c) are time-varying.

In the following, the elements of the matrix  $K_\sigma$  will be considered as the average slope coefficients of the slope

sector shown in Figure 7.

It is possible to consider the elements of  $\tilde{A}(t)$  and  $\tilde{B}(t)$  that vary with time as varying parameters. We call  $\theta_i$ ,  $i = 1, \dots, q$  these parameters.  $\mathbf{p}$  is the vector containing all the parameters. We then express the linearized model in (7) as an LPV model, with the state matrix  $A$  and the input matrix  $B$  affinely dependent on the varying parameters.

Along the reference trajectory shown in Figure 6, a total of 23 elements of  $\tilde{A}(t)$  and  $\tilde{B}(t)$  vary with time. However several of these elements have a small variation. It is possible to consider just the 6 elements of  $\tilde{A}(t)$  and  $\tilde{B}(t)$  that vary the most to constitute the vector  $\mathbf{p}$  of parameters for synthesis purposes. The range of each parameter  $\theta_i$ ,  $i = 1, \dots, 6$  is between a minimum  $\underline{p}_i$  and a maximum  $\bar{p}_i$ .  $\underline{p}_i$  and  $\bar{p}_i$  are the minimum and maximum values of  $p_i$  along the reference trajectory  $\mathcal{T}(t)$ .

For the control synthesis we add the longitudinal, lateral and yaw angle errors with respect to  $\mathcal{T}(t)$  to the state of the model. They are defined as:

$$\begin{aligned} x_L &= x - x_0 \\ y_L &= y - y_0 \\ \Delta \psi &= \psi - \psi_0. \end{aligned}$$

$x_0$ ,  $y_0$  and  $\psi_0$  are the vehicle's absolute position and attitude along the reference trajectory  $\mathcal{T}(t)$ .

The vehicle's position dynamics in (4a) and (4b) can be simplified neglecting the vehicle's lateral speed  $u$ , which is considerably smaller than the longitudinal speed  $v$

$$\dot{x} = v \cos \psi \quad (9a)$$

$$\dot{y} = v \sin \psi. \quad (9b)$$

Considering that  $v = v_0 + \Delta v$ ,  $u = u_0 + \Delta u$  and  $\psi = \psi_0 + \Delta \psi$  and that the  $\Delta \psi$  is small along the reference trajectory, the dynamics of the error variables are:

$$\begin{aligned} \dot{x}_L &= \Delta v \\ \dot{y}_L &= \Delta v \psi_0 + \Delta \psi v_0 \cos \psi_0 \\ \Delta \dot{\psi} &= \Delta r. \end{aligned}$$

The expressions  $\psi_0$  and  $v_0 \cos \psi_0$  are added to  $\mathbf{p}$ .

It is finally possible to define the polytopic domain  $\mathcal{P}$  as the hypercube of dimension 8, each dimension being the parameter  $p_i$ ,  $i = 1, \dots, 8$ , between  $\underline{p}_i$  and  $\bar{p}_i$ . The polytopic model is defined at its  $N = 2^8$  vertices:

$$S_i = (A_i \ B_i) \quad i = 1, \dots, N. \quad (11)$$

The model inside  $\mathcal{P}$  is defined as:

$$S(\mathbf{p}) = \sum_{i=1}^N \alpha_i (A_i \ B_i)$$

where  $\alpha_i$  for  $i = 1, \dots, N$  are the interpolation parameters and  $\sum_{i=1}^N \alpha_i = 1$ .

## 3. MAIN RESULTS

A dynamic LPV  $H_\infty$  state feedback controller has been designed starting from the polytopic LPV model described in Section 2.3.

We consider the system of order  $n$  in the form:

$$\dot{\mathbf{x}} = A_i \mathbf{x} + B_{1i} \mathbf{w} + B_2 \mathbf{u} \quad (12a)$$

$$\mathbf{z} = C_{1i} \mathbf{x} + D_{11i} \mathbf{w} + D_{12} \mathbf{u} \quad (12b)$$

$$\mathbf{y} = C_2 \mathbf{x} + D_{21} \mathbf{w} \quad (12c)$$

for  $i = 1, \dots, N$ , where  $\underline{\mathbf{y}}$  is the controller input,  $\underline{\mathbf{z}}$  contains the controlled variables,  $\underline{\mathbf{w}}$  the exogenous inputs and  $\underline{\mathbf{u}}$  the control inputs. The system in the polytopic domain  $\mathcal{P}$  is then expressed with respect to the  $N$  vertices:

$$S(\underline{\mathbf{p}}) = \sum_{i=1}^N \alpha_i \begin{pmatrix} A_i & B_{1i} & B_2 \\ C_{1i} & D_{11i} & D_{12} \\ C_2 & D_{21} & 0 \end{pmatrix} \quad (13)$$

where  $\alpha_i$  for  $i = 1, \dots, N$  are the interpolation coefficients. In this article, the solution proposed in Chilali and Gahinet (1996) is used for the synthesis of the LPV  $H_\infty$  controller. This method allows to specify limits to the dynamics of the closed loop. This is obtained by computing a full-order LPV controller  $K(\underline{\mathbf{p}})$  that forces the closed loop eigenvalues into some LMI region for all admissible values of (13) in  $\mathcal{P}$ .

An LMI region  $\mathcal{D} = \{z \in \mathbb{C} : L + zM + \bar{z}M^T < 0\}$  of order  $s$  is a subset of the complex plane, with  $L = L^T \in \mathbb{R}^{s \times s}$  and  $M \in \mathbb{R}^{s \times s}$ .

Given a polytopic plant (12), an LMI region  $\mathcal{D}$  and the  $H_\infty$  performance  $\gamma > 0$ , we search an LPV controller such that: (A) the closed loop poles for all admissible values of (13) in  $\mathcal{P}$  are in  $\mathcal{D}$ ; (B) the  $\mathcal{L}_2$ -norm of the input/output mapping from  $\underline{\mathbf{w}}$  to  $\underline{\mathbf{z}}$  is bounded by  $\gamma$ . It exists if there exist two symmetric matrices  $R, S \in \mathbb{R}^{n \times n}$  and matrices  $A_K, B_K, C_K$  and  $D_K$  such that:

$$\begin{pmatrix} R & I \\ I & S \end{pmatrix} > 0$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{21}^T \\ \Psi_{21} & \Psi_{22} \end{bmatrix} < 0$$

$$L \otimes \begin{pmatrix} R & I \\ I & S \end{pmatrix} + M \otimes \Phi + M^T \otimes \Phi^T < 0$$

where, for  $i = 1, \dots, N$

$$\Phi = \begin{bmatrix} A_i R + B_2 C_K & A_i + B_2 D_K C_2 \\ A_K & S A_i + B_K C_2 \end{bmatrix}$$

$$\Psi_{11} = \begin{bmatrix} A_i R + R A_i^T + B_2 C_K + C_K^T B_2^T & B_{1i} + B_2 D_K D_{21} \\ (B_{1i} + B_2 D_K D_{21})^T & -\gamma I \end{bmatrix}$$

$$\Psi_{21} = \begin{bmatrix} A_K + (A_i + B_2 D_K C_2)^T & S B_{1i} + B_K D_{21} \\ C_{1i} R + D_{12} C_K & D_{11i} + D_{12} D_K D_{21} \end{bmatrix}$$

$$\Psi_{22} = \begin{bmatrix} A_i^T S + S A_i + B_K C_2 + C_2^T B_K^T & (C_{1i} + D_{12} D_K C_2)^T \\ C_{1i} + D_{12} D_K C_2 & -\gamma I \end{bmatrix}$$

We compute any square matrices  $M$  and  $N$  such that  $MN^T = I - RS$ . Solving the system of equation:

$$B_K = N B_{K_i} + S B_2 D_K$$

$$C_K = C_{K_i} M^T + D_K C_2 R$$

$$A_K = N A_{K_i} M^T + N B_{K_i} C_2 R + S B_2 C_{K_i} M^T + S (A_i + B_2 D_K C_2) R$$

for  $i = 1, \dots, N$ , for  $B_{K_i}$ ,  $C_{K_i}$  and  $A_{K_i}$  we find the controller matrices at the vertices of the polytopic domain  $\mathcal{P}$ . The full-order LPV  $H_\infty$  controller is:

$$K(\underline{\mathbf{p}}) = \sum_{i=1}^N \alpha_i \begin{pmatrix} A_{K_i} & B_{K_i} \\ C_{K_i} & D_K \end{pmatrix}.$$

For the control synthesis an LMI region has been used. It guarantees that the closed loop eigenvalues for all admissible values in  $\mathcal{P}$  are between  $\underline{\lambda} = -8500$  and  $\bar{\lambda} = -1.8$ , to bound slow and fast dynamics, and  $\theta = 1.12\text{rad}$ , to have a minimum damping ratio  $\zeta > 0.9$ .

To bound the bandwidth of the control signals a diagonal weighting function  $W_u$  as been used, where the

elements on the diagonal are  $W_{u1}(s) = \frac{100s+31.57}{s+315.7}$  and  $W_{u2}(s) = W_{u3}(s) = \frac{100s+315.7}{s+3157}$ . The control architecture is represented in Figure 8.

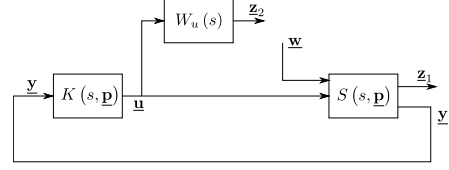


Fig. 8. Control architecture.

We define  $\underline{\mathbf{z}} = [\underline{\mathbf{z}}_1, \underline{\mathbf{z}}_2]^T$ ,  $\underline{\mathbf{y}} = \underline{\mathbf{x}}$  and:

$$\underline{\mathbf{x}} = \begin{bmatrix} \Delta v \\ \Delta u \\ \Delta r \\ \Delta \omega_{wf} \\ \Delta \omega_{wr} \\ x_L \\ y_L \\ \Delta \psi \end{bmatrix} \quad \underline{\mathbf{w}} = \begin{bmatrix} \Delta \theta_x \\ \Delta \theta_x \end{bmatrix} \quad \underline{\mathbf{u}} = \begin{bmatrix} \Delta \delta_f \\ \Delta \tau_{wf} \\ \Delta \tau_{wr} \end{bmatrix} \quad \underline{\mathbf{z}}_1 = \begin{bmatrix} x_L \\ y_L \\ \Delta \psi \end{bmatrix}. \quad (17)$$

The matrix  $B_{1i}$  and  $B_2$  are obtained from  $B_i$  in (11), given the definitions of  $\underline{\mathbf{w}}$  and  $\underline{\mathbf{u}}$  in (17).

#### 4. NUMERICAL RESULTS

In this Section some numerical results of the controller are shown. The controller has been tested on MATLAB/Simulink on the non-linear bicycle model described in Section 2.1. The trajectory considered is the reference trajectory shown in Section 2.2, Figure 6.

Several simulations have been carried out, with a set of different initial conditions for  $\Delta v$  and  $\Delta u$ , to establish empirically the region of initial conditions for which the controller is able to stabilize the system and converge to the reference trajectory. Figure 9 shows the empirical region of attraction of the controller for non-zero initial conditions for the variables  $\Delta v$  and  $\Delta u$ . The capacity of the controller to stabilize the system and bring back the vehicle along the reference trajectory is limited for  $-5.2\text{m/s} \lesssim \Delta v(0) \lesssim 4.3\text{m/s}$  and  $-5\text{m/s} \lesssim \Delta u(0) \lesssim 8.4\text{m/s}$ .

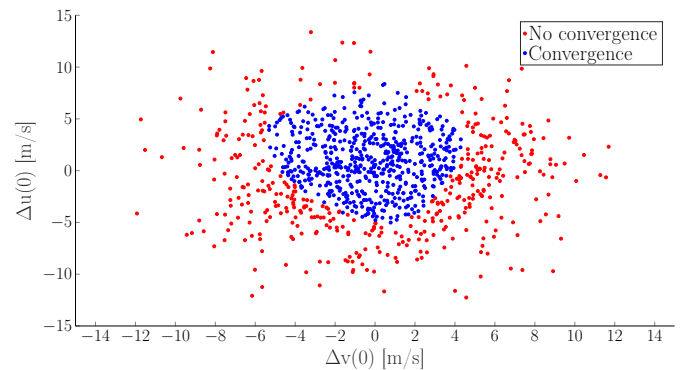


Fig. 9. Empirical region of attraction for  $\Delta v$  and  $\Delta u$ .

A case with initial conditions of  $\Delta v(0) = 3\text{m/s}$ ,  $\Delta u(0) = 2\text{m/s}$  and  $\Delta r(0) = 15\text{deg/s}$  is simulated and shown in detail. Figure 10 shows the commands computed by the controller, the vehicle's position and attitude error and the vehicle overall longitudinal and lateral accelerations. Due to the large initial conditions, the position errors are initially large, but they eventually converge to zero. Indeed the vehicle acceleration belong to the high dynamics category.

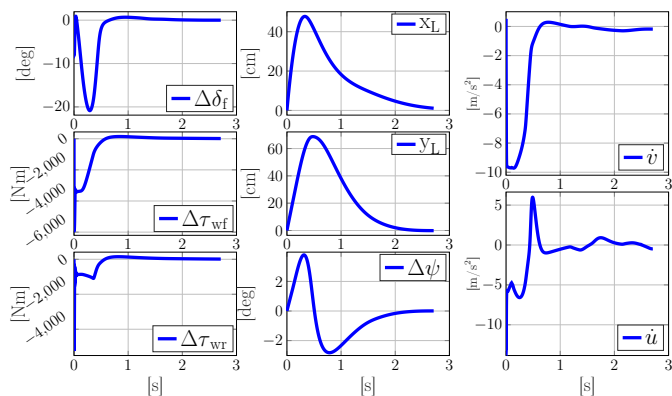


Fig. 10. Simulation results.

## 5. CONCLUSION AND PERSPECTIVES

The LPV model discussed in Section 2 had already been used by the authors for the synthesis of a static state feedback controller in Penco et al. (2021). The results of Section 4 show that the LPV  $H_\infty$  dynamic controller developed in Section 3, compared to the static controller developed in Penco et al. (2021), considerably enlarges the range of initial conditions for which the controller is able to stabilize the system along the reference trajectory. Moreover this has been obtained considering also the road inclination as system perturbation. A more thorough methodology to estimate the region of attraction of the closed-loop systems, without simulations, will be proposed by considering sector inequalities associated to the saturation function.

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