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# Investment games in Crowdfunding platforms - Extended version

V. S. Varma, L. Salahaldin and S.E Elayoubi

**Abstract**—We analyze the behavior of investors who have to decide on which projects to support in a crowdfunding platform. We model the behaviour of different sets of funders, as reported by crowdfunding literature, and also the behavior of utility-maximizing funders as a stochastic game. The founders' behaviour is also modeled as a Stackelberg game. Our results are illustrated by a numerical performance analysis which provides several insights into the choice of investment strategies.

**Keywords:** *Game theory, Crowdfunding.*

## I. INTRODUCTION

Crowdfunding allows entrepreneurs who are willing to overcome classical funding channels such as venture capital and credit loans to address directly the crowd via Internet-based Crowdfunding platforms (CFP) for collecting financial resources either in the form of donation or in exchange for the future product or some form of reward [1].

The literature on crowdfunding is already rich and covers wide aspects, from the profile of funders and their behavior to the characteristics of projects that make them more successful. [1] compares two forms of CF, reward-based and equity-based and shows that the choice made by the entrepreneur to select between these two forms depends essentially on the amount of required capital and that equity-based CF is more suitable for large projects. [2] models investors choices when facing asymmetric information about the projects quality (good or bad) and showed that higher proportions of informed investors do not always lead to more good projects being funded, while higher proportion of bad projects can paradoxically increase the number of good projects that end up funded. Other literature works give an overview on the key factors that contribute to the success of the project. For instance, [3] shows that the size of the network of the project holder is positively correlated with the success and that the quality signals are also success factors. These signals have been identified as the advertisement videos, the update frequency and even the absence of typos, for reward-based crowdfunding [3], [4], and the clarity of the financial roadmap, the transparency with respect to the risks, the professional experience and the education level of the funders for equity-based crowdfunding [5].

Most of the crowdfunding literature is based on empirical observation from data collected on different crowdfunding

platform, and these observations lead sometimes to contradictory conclusions. For example, concerning the impact of the duration of the funding campaign, [3] shows that a longer duration of the campaign reduces the probability of success, while [4] observes a positive correlation between duration and success.

This illustrates the need for a theoretical model that is able to give insights on the impact of different parameters on the funders' choices and our aim in this paper is to fill this gap by developing a theoretical model for the choice of funders among several projects of interest. In order to be realistic, we base our model on the empirical observations about funders' behavior that have been reported in several papers. For instance [4] observes that CF has reinforcement characteristic in the sense that contributions generate additional contributions.

[6] studies the choice of donors toward funding a project early or wait until the end of the funding window and shows that if the donor decides to fund immediately, he can signal to the crowd the potential of the project; he can also decide to wait in order to see whether other donors fund the project. [7] show that donors mostly fund projects at the beginning or the end of a campaign. This tendency can be explained by the observations of other empirical works such as [8] who have identified friend and family (F&F) as a source of early funding for projects that account for 15 to 20% of raised funds for equity-based campaigns and 30 to 40% for reward-based campaigns. Other potential funders tend to procrastinate and the late funding peak can be explained by the well-known deadline effect [9] that pushes them to support the project near the campaign end.

The contribution of this work resides in its game theoretical formulation of the funder's strategies that includes the behavioral observations. We conduct the equilibrium analysis of the proposed game, which includes solving the issues of existence, uniqueness, and determination of a Nash equilibrium (NE).

The rest of the paper is organized as follows. Sec. II formulates the problem under study and introduces the main notation and concepts necessary for the model description. The game-theoretic analysis of the resulting model for two projects is presented in Sec. III and closed-form expressions for the Nash Equilibrium are derived. The general case of  $K$  projects is analyzed in section IV as a stochastic game and subgame perfect equilibria are derived using dynamic programming. Section V models the behaviour of founders who see their projects threatened by other projects on the platform and try to influence their campaign by injecting some money in it. The paper ends with some concluding

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remarks.

## II. PROBLEM STATEMENT

### A. Platform and funders

We consider a crowdfunding platform based on the threshold pledge model (or the all-or-nothing principle(AON)) [10], meaning that if the project is not 100% funded, the funding will be returned to investors without penalty. This platform proposes a set of projects  $\mathcal{K} = \{1, 2, \dots, K\}$  that are of interest to a set of potential investors, henceforth referred to as *agents*,  $\mathcal{A} = \{1, 2, \dots, N\}$ , with  $N \in \mathbb{Z}_{\geq 0}$ . We assume that each investor desires to invest in one project at most. At any discrete-time instant  $t \in \mathbb{Z}_{\geq 0}$ , each project has an accumulated promised investment denoted by  $x_k(t) \in \mathbb{R}_{\geq 0}$  for all  $k \in \mathcal{K}$ , which starts the crowdfunding campaign an initialization time  $t_k \in \mathbb{Z}_{\geq 0}$  and ends at some  $T_k \in \mathbb{Z}_{\geq 0}$ , with  $t_k < T_k$ . Without any loss in generality, we order the projects based on the end times, i.e., we consider  $T_1 \leq T_2 \leq \dots \leq T_P$ .

Each project  $k \in \mathcal{K}$  requires a certain threshold of investors  $\tau_k \in \mathbb{Z}_{\geq 0}$  in order to succeed by the end time, i.e. project  $k$  succeeds if and only if  $x_k(T_k) \geq \tau_k$ . Additionally, each agent makes an investment of  $\delta_k$  while investing in the project. For the ease of reading and without any loss of generality, we normalize  $\delta_k$ 's to 1. When successful, each agent that invested in  $k$  has a net gain denoted by  $R_k \in \mathbb{R}_{> 0}$ . If an agent invests in project  $k$  at time  $t_k \leq t \leq T_k$  and the project fails, the agents the amount he invested is returned to him, but the agent still loses due to the discount factor  $\gamma < 1$ .

We use  $y_n(t) \in \mathcal{K} \cup \{0\}$  to denote the investment choice of agent  $n \in \mathcal{A}$  at time  $t \in \mathbb{R}_{\geq 0}$ .  $y_n(t) = 0$  indicates that the agent has not invested in any project at time  $t$  and we initialize this variable with  $y_n(0) = 0$  for all  $n \in \mathcal{A}$ . At any time  $t$ , if  $y_n(t) = 0$ , the agent is free to change is choice to any  $k \in \mathcal{K}$  such that  $t \leq T_k$ . However, once a project is chosen, this choice can not be changed unless the project fails. We can therefore describe the constraints on investment choices as follows

$$y_n(t+1) \in \begin{cases} \{0\} \cup \{k \in \mathcal{K} | T_k \geq t+1\} & \text{if } y_n(t) = 0 \\ \{y_n(t)\} & \text{if } y_n(t) = k \in \mathcal{K} \ \& \ t < T_k \\ \{0\} & \text{if } y_n(t) = k \in \mathcal{K}, t = T_k, x_k(T_k) < \tau_k \\ \{y_n(t)\} & \text{if } y_n(t) = k \in \mathcal{K} \ \& \ x_k(T_k) \geq \tau_k \end{cases} \quad (1)$$

and any investment profile must satisfy (1).

The population of agents is divided into several classes, as indicated by the empirical literature [11], denoted by the following sets

- 1) A set  $C \subset \mathcal{A}$  of *casual* investors who aim at maximizing their utility and observe the states of funding  $x_k(t)$ , but are not well informed on  $\{R_k, k \in \mathcal{K}\}$ .
- 2) A set of agents  $P \subset \mathcal{A}$  who comprise the *players* set, they are aware of all the project parameters and therefore can estimate the net gain  $R_k$  they generate in case of success. They try to maximize their expected gain based on the current state of investments  $x(t)$ .

- 3) Sets  $F_k \subset \mathcal{A}$  of Friends and Family (F&F) who have personal interest in specific projects  $k$ . While the first two sets of agents make rational decisions in order to maximize their gain, the funders in  $F_k$  set invest only in project  $k$ . They will invest in the project as soon as possible to encourage further investment [7].
- 4) Set  $\mathcal{N}$  of naive investors, also known in the literature as fools to complete the 3F group, i.e., Friends, Family and Fools [12]. They randomly invest in projects at random times.

The presence of the last two sets results in  $x_k(t) > 0$  for all  $k \in \mathcal{K}$  and for all  $t$ . The latter results in an increment of the project investment at each stage modeled by a Gaussian variable at each step with mean  $\lambda$  and variance  $\sigma_0^2$  (which can be interpreted as the sum of a large number of Bernoulli variables). Finally, we can write the accumulated investment for any project  $k \in \mathcal{K}$  as

$$x_k(t) \sim x_k(0) + \sum_{n \in C \cup P} \mathbf{1}_{\{k\}}(y_n(t)) + \mathbf{N}(\lambda(t-t_k), \sigma_0^2(t-t_k)) \quad (2)$$

where  $\mathbf{1}_S(y)$  is the indicator function which takes the value 1 when  $y \in S$  and 0 otherwise, and  $\mathbf{N}(\mu, \sigma^2)$  denotes the normal distribution with mean  $\mu$  and variance  $\sigma^2$ .

The primary objective of this work is to characterize the behavior of the agents in classes  $C$  and  $P$  using Game theory.

### B. Temporal strategy

We denote by  $u_n \in \mathbb{R}$ , the net utility or gain of agent  $n \in \mathcal{A}$ .

We first study the behavior of the agents in  $C$ . Indeed the casual set agents only need to decide on when to invest in a project and not on which project since they are not aware/not interested in the precise values of  $R_k$ . We assume that any agent  $n \in C$  invests in project  $k$  with some probability  $\pi_k$ , i.e.  $\Pr(y_n(T_k) = k) = \pi_k$ . If the agent  $n \in C$  decides to invest in project  $k$  at time  $t' \leq T_k$ , then its net utility is

$$u_n(t') = R_k \Pr(x_k(T_k) \geq \tau_k) \gamma^{T_k-t'} - \delta_k (1 - \gamma^{T_k-t'}) \Pr(x_k(T_k) < \tau_k) \quad (3)$$

with  $R_k > 0$  and  $\Pr(x_k(T_k) < \tau_k) \in [0, 1]$  unknown to the casual agent. However, it is clear that for any value of these parameters, the utility is maximized by picking  $t' = T_k$  as the loss term will vanish due to the discount factor. Therefore, we can predict that the set of casual investors invest in this project at  $T_k$ :

$$\mathbf{1}_k(y_n(t)) = 0, \quad (4)$$

for all  $t < T_k$ ,  $n \in C$ . At  $T_k$ , each casual player decision can be modeled as a Bernoulli variable (invest in project  $k$  with probability  $\pi_k$  and wait otherwise). We can therefore use the central limit theorem when  $|C|$  is large to write

$$\sum_{n \in C} \mathbf{1}_k(y_n(T_k)) \rightarrow_d \mathbf{N}(\pi_k |C|, \sigma_k^2 \sqrt{|C|}) \quad (5)$$

where  $\rightarrow_d$  indicates convergence in distribution and  $\sigma_k^2 = \frac{(1-\pi_k)^2 + \pi_k^2}{2}$ .

The agents  $n \in P$  are aware of the behavior of casual users, i.e., (5), and also of the project parameters. They make their decision at  $t$  based on  $x(t)$ . Due to the lack of signaling or communication channel between the players, there is no way that player  $n \in P$  can observe the specific action of another player  $m \in P$ . In fact, the players must base their decision solely on the state vector  $x(t)$ . However, the actions of the players affect the transition probability of  $x(t)$ , with the actions constrained based on (1).

In general, the interaction can be seen as a stochastic game if we quantize  $x(t)$  and consider a discrete process instead of a Gaussian distribution to model the random investments. From (3) and the condition  $x_k(T_k) \geq \tau_k$ , we can model the state and utility as a stochastic process depending on the investment profile  $y(t)$ . Indeed, in this case, the existence of a Nash equilibrium (mixed or pure) can be shown as the set of actions and states are both finite [13]. However, it is very hard to obtain useful insights in this case due to the complexity of the problem and this case is left as an extension.

Each player at time  $t$  will make a decision solely based on  $x(t)$  due to the lack of signaling. As a first simplification, using the same reasoning as in (3), we suppose that these players will only invest in project  $k$  at  $T_k$  (if it picks project  $k$ ) formally stated in the following assumption, inspired by the behavior of the agents in the set  $C$ .

*Assumption 1:* Any player  $n \in P$ , will invest in a project  $k$  only at  $T_k$ , i.e.  $y_n(t) \neq k, \forall t < T_k$ .

### III. PLAYER BEHAVIOR WHEN $K = 2$

#### A. Static game formulation

Under Assumption 1, when  $K = 2$ , each player  $n \in P$  only needs to decide if they invest project 1 or wait for 2, at time  $T_1$ . This problem can be seen as a static one-shot game. There are exactly two investment strategies (or actions)  $a_n \in \{0, 1\}$  available for  $n \in P$ ,

- 1 : Invest in project 1 at  $T_1$ . If it succeeds, all future opportunity in project 2 is lost and if it fails, invest in project 2 at  $T_2$ .
- 0 : Ignore project 1 and invest only in project 2 at  $T_2$ .

Therefore, the strategy of any player for the game in the case of  $K = 2$  is decided once and for all at  $T_1$  because if it invests in 1,  $a_n = 1$  and  $a_n = 0$  otherwise. This simplifies the stochastic game structure and the interactions can therefore be modeled as a static game. We can write the strategic form of the resulting static game as

$$\mathcal{G}_S := (P, \{0, 1\}^{|P|}, \{u_n\}_{n \in P}),$$

where  $\{0, 1\}^{|P|}$  is the set of strategies for all players, and  $u_n(a_n, a_{-n})$  is the expected gain for  $n \in P$  calculated as

$$\begin{aligned} u_i(a_i, a_{-i} | x(T_1 - 1)) = \\ R_1 \text{ if } x_1(T_1) \geq \tau_1 \text{ \& } a_i = 1, \\ \gamma^{T_2 - T_1} R_2 \Pr(x_2(T_2) \geq \tau_2 | x_2(T_1 - 1), a_{-i}) \text{ otherwise.} \end{aligned} \quad (6)$$

Here,  $a_{-n}$  is a vector of the strategies by all players other than  $n$ , i.e., for all  $i \in P \setminus \{n\}$ . In the following, we explicitly find the Nash equilibrium when  $K = 2$ .

As a first step, we will define the concept of a pure Nash equilibrium (NE).

*Definition 1:* A profile of actions  $a^*$  is a pure NE if

$$\forall a_i \in \mathcal{A}_i, \quad u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*) \quad (7)$$

for all  $i \in P$ .

Before looking at the NE of the game  $\mathcal{G}_1$ , let us define the function

$$\Phi(f) = \text{Erf} \left( \sqrt{|C|} \sigma_2 (x_2(T_1 - 1) + f + \pi_2 |C| - \tau_2) \right). \quad (8)$$

for  $f \in \mathbb{Z}_{\geq 0}$ , where Erf is the Gaussian error function. This function is increasing in  $f$  and is sigmoidal. We also define the two thresholds on the values of collected funds by project 2:

$$\bar{x}_2 = \frac{1}{\sqrt{|C|}} \text{Erf}^{-1} \left( \frac{R_1}{R_2} \gamma^{T_1 - T_2} \right) - 1 - \pi_2 |C| + \tau_2 \quad (9)$$

and

$$\underline{x}_2 = \frac{1}{\sqrt{|C|}} \text{Erf}^{-1} \left( \frac{R_1}{R_2} \gamma^{T_1 - T_2} \right) - |P| - \pi_2 |C| + \tau_2 \quad (10)$$

*Theorem 1:* The game  $\mathcal{G}_S$  has at least one and at most 2 pure Nash equilibria, specifically,

- If  $R_2 \Phi(1) \gamma^{T_2 - T_1} > R_1$ , i.e. if  $x_2(T_1 - 1) > \bar{x}_2$ , the only pure NE is when  $a_n^* = 0$  for all  $n \in P$ , i.e. all follower agents will wait for the second project.
- If  $R_2 \Phi(|P|) \gamma^{T_2 - T_1} < R_1$ , i.e. if  $x_2(T_1 - 1) < \underline{x}_2$ , the only pure NE is when  $a_n^* = 1$  for all  $n \in P$ , i.e. all follower agents will invest in the first project at  $T_1$ .
- Otherwise, i.e., if  $\underline{x}_2 \leq x_2(T_1 - 1) \leq \bar{x}_2$ , there are two pure NE at  $a_n^* = 1 \forall n \in F$  and  $a_n^* = 0 \forall n \in F$ .

*Proof:* At time  $T_1$ , for a given  $a$ , a player  $i \in P$  will estimate the probability that project 2 will succeed, given project 1 succeeded, as

$$\begin{aligned} \Pr(x_2(T_2) \geq \tau_2 | x_1(T_1) > \tau_1, x_2(T_1 - 1)) \\ = \text{Erf} \left( \sqrt{|C|} \sigma_2 (x_2(T_1 - 1) + |P| \right. \\ \left. - \sum_{n \in P \setminus \{i\}} a_n + \pi_2 |C| - \tau_2) \right) \\ = \Phi(f) \end{aligned} \quad (11)$$

where  $f = |P| - \sum_{n \in P \setminus \{i\}} a_n$ .

Let us call  $\alpha(a)$  to denote the probability that project 1 succeeds and  $\beta$  to denote the probability that project 2 will succeed given project 1 failed (with all players investing and therefore independent of  $a$ ). We can now calculate the expected utility for player  $i$  corresponding to actions 0 and 1 if project 1 succeeded as

$$u_i(1, a_{-i} | x_1(T_1) \geq \tau_1) = R_1 \quad (12)$$

and

$$u_i(0, a_{-i} | x_1(T_1) \geq \tau_1) = R_2 \gamma^{T_2 - T_1} \Phi \left( |P| - \sum_{n \in P \setminus \{i\}} a_{-i} \right) \quad (13)$$

On the other hand, if project 1 failed, both actions will lead to the same payoff as all players will invest in project 2. Therefore, we can calculate the gain by picking action 0 over action 1 as

$$(R_2 \gamma^{T_2 - T_1} \Phi(f) - R_1),$$

where  $f$  is as before. Since  $\alpha(0, a_{-i}) > 0$  as the probability of project 1 succeeding is never 0, this gain is positive when  $R_2 \gamma^{T_2 - T_1} \Phi(f) \geq R_1$  and negative otherwise. Now, we observe that

- 1) This gain is always strictly positive if  $R_2 \gamma^{T_2 - T_1} \Phi(1) > R_1$  as  $\phi(f)$  is monotonically increasing. Therefore, the action  $a_i = 0$  for all  $i \in P$  is a pure NE. Additionally, for any player  $i \in P$ ,  $u_i(1, a_{-i}) < u_i(0, a_{-i})$ . Therefore, this is also the only pure NE.
- 2) Using similar arguments, if  $R_2 \gamma^{T_2 - T_1} \Phi(|P|) < R_1$ , then the action 1 is always dominant implying that  $a_i^* = 1$  is the only pure NE.
- 3) In the final case, when  $R_2 \Phi(0) \gamma^{T_2 - T_1} \leq R_1 \leq R_2 \Phi(|P| - 1)$ , we can easily verify that both  $a_i^* = 0$  and  $a_i^* = 1$  are pure NE as when  $\sum_{n \in P \setminus \{i\}} a_n = 0$  or  $\sum_{n \in P \setminus \{i\}} a_n = |P| - 1$ , the strategy of  $a_i = 0$  or  $a_i = 1$  satisfies (7).

Moreover, no other pure NE exists. This can be proven by contradiction by assuming that there exists a NE  $a^*$  such that  $\sum_{n \in P \setminus \{i\}} a_n^* = f$ , with  $f \in \{1, \dots, |P| - 2\}$ . In this case, there is at least one player  $i \in P$  such that  $a_i^* = 1$  which implies  $R_2 \gamma^{T_2 - T_1} \Phi(f) \leq R_1$ . There also exists at least one player  $j$  such that  $a_j^* = 0$ , which implies that  $R_2 \gamma^{T_2 - T_1} \Phi(f - 1) \geq R_1$ . However  $\Phi(\cdot)$  is a strictly increasing function, leading to a contradiction. ■

Theorem 1 characterizes the possible NE for the static game  $\mathcal{G}_S$ . Note that in the third case of the theorem, both equilibria of waiting for project 2 or not are possible. In this case, the equilibrium where all players wait for the second project offers a higher expected payoff, but unless the players can cooperate and communicate with each other, there is no way of guaranteeing that this equilibrium will be reached. Player behavior in games with multiple NE are in general unpredictable and depends on the existence of communication channels. In practice, investors who are particularly interested in project 2 might encourage others to join them using public forums or other methods of communication.

### B. Numerical illustrations

To illustrate how  $R_2$  and  $x_2(T_1 - 1)$  impact the NE profiles, we provide Figure 1 which marks the regions where

there are unique pure NE and the region with two pure NE as stated in the three cases of Theorem 1. The other parameters are held constant with values  $R_1 = 1$ ,  $\gamma = 0.999$ ,  $T_2 - T_1 = 50$ ,  $|C| = 100$ ,  $\pi_2 = 0.20$ ,  $\sigma_2^2 = 10$ ,  $\lambda = 1$ ,  $\sigma_0^2 = 0.2$  and  $|P| = 50$ .

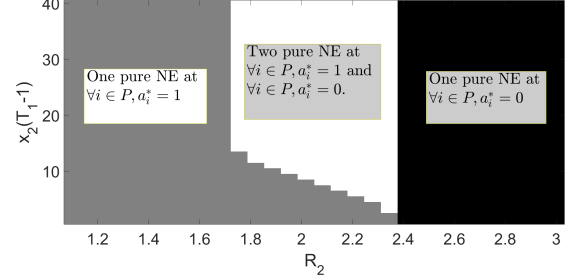


Fig. 1. Regions corresponding to the three cases mentioned in Theorem 1 formed by varying  $R_2$  and  $x_2(T_1 - 1)$ .

We now turn to the evaluation of the NE for different sets of parameters of project 2, namely the amount of funds to be collected  $\tau_2$  and the amount of funds already collected at decision time  $T_1 - 1$ , while fixing  $R_2$  to  $1.5 * R_1$ . It can be observed in Figure 2 that the only possible NE when the amount of remaining funds to be collected is low is to wait for project 2. When the requested funds increases with respect to the already collected amount, there are two possible Nash equilibria (wait or invest in 1). When the objective is too ambitious, all users invest in project 1 without waiting. Figure 3 illustrates the impact of the remaining duration and the discount factor on the equilibrium. We can observe that a high  $\gamma$  (a low discount rate) encourages waiting. For a fixed discount factor, a longer remaining campaign duration increases the probability that project 2 succeeds to collect its required amount and pushes agents to wait. This ambiguity about the role of the campaign duration matches empirical observations. For instance, while [3] reports a negative correlation between the campaign duration and its success, [4] describes the inverse tendency with a positive correlation. The game theoretical results indicate that the design of the campaign duration is not an easy task, as a too short duration prevents from collecting sufficient funds and a too long one may lead to negative impacts due to funders' impatience.

## IV. GENERAL CASE OF $K \geq 2$

Under Assumption 1, the general case of  $K$  projects can be seen as a stochastic game of  $K - 1$  stages. At each stage  $k \in \{1, \dots, K - 1\}$ , player  $n \in P$  must decide at  $T_k$ , if he invests in project  $k$  or wait for a future project. This decision is based on the last seen state of the vector of collected funds, i.e.  $x_k(T_k - 1), x_{k+1}(T_k - 1), \dots, x_K(T_k - 1)$ . The action at each stage  $T_k$  is to invest in project  $k$  or to wait for a future project. Indeed, some of these actions may not be possible if a previous project succeeded or the player had invested in an ongoing project. Nevertheless, at each stage, we can

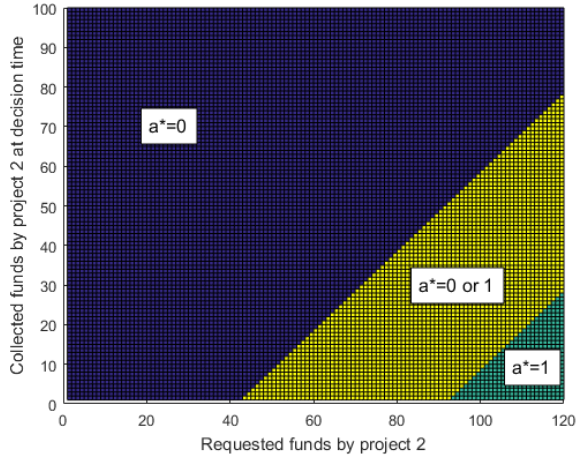


Fig. 2. Impact of the parameters of project 2 on the NE.

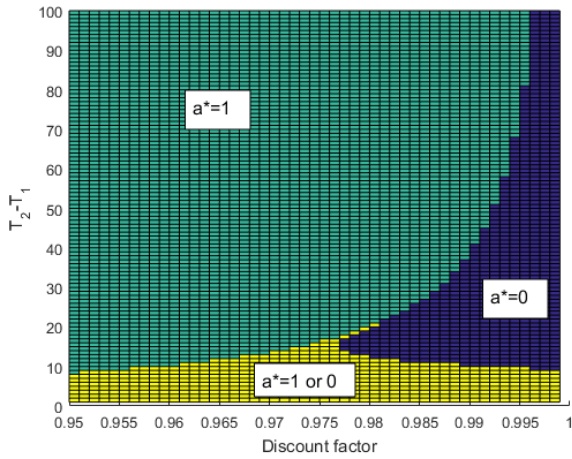


Fig. 3. Impact of the discount factor and the remaining campaign duration.

denote the action set of a player as being represented by  $a_n(k) \in \{0, 1\}$  for  $n \in P$ .

First, we use  $P_k$  to denote the set of remaining players at stage  $k$  with  $P_1 = P$ , following which, the set is updated as per

$$P_{k+1} = \begin{cases} \{n \in P | a_n(k) = 0\} & \text{if } x_k(T_k) \geq \tau_k \\ P_k & \text{otherwise} \end{cases} \quad (14)$$

Correspondingly, we can define the utility at stage  $k$  as

$$u_{n,k}(a_n(k), a_{-n}(k), P_k | x(T_k - 1)) = \begin{cases} R_k & \text{if } x_k(T_k) \geq \tau_k \ \& \ a_n(k) = 1, \\ \gamma^{T_{k+1} - T_k} \mathbb{E}_{|x(T_k - 1)} [u_{n,k+1}(a(k+1), P_{k+1} | x(T_{k+1} - 1))] & \text{otherwise.} \end{cases} \quad (15)$$

for all  $n \in P$  and  $k < K - 1$ . Note that the utility at the following stage depends on  $P_{k+1}$  determined as a function of  $a_{-n}(k)$  and  $P_k$ . At the final stage, we can explicitly include

$x_K(T_{K-1} - 1)$  as a parameter of the function  $\Phi$ :

$$u_{n,K-1}(a_n(K-1) | x(T_{K-1} - 1)) = \begin{cases} R_{K-1} & \text{if } x_{K-1}(T_{K-1}) \geq \tau_{K-1} \ \& \ a_n(K-1) = 1, \\ \gamma^{T_K - T_{K-1}} R_K \Phi(|P_K|, x_K(T_{K-1} - 1)) & \text{otherwise.} \end{cases} \quad (16)$$

These set of utilities, transition probabilities of  $x_k(t)$  and actions  $a_n(k)$  define a stochastic game  $\mathcal{G}$ . We can define a sub-game  $\mathcal{G}_k$  for  $\mathcal{G}$  with the utility at stage  $k$  and the action taken at that stage once the strategies of future stages are fixed, as this will let us evaluate the expected utility of future stages. This lets us define

*Definition 2:* A NE of  $\mathcal{G}$ , denoted as  $a(1)^*, \dots, a(K-1)^*$  is said to be sub-game perfect if  $a^*(k)$  is a NE of the sub-game  $\mathcal{G}_k$  for all  $k \in \{1, \dots, K-1\}$ .

At the last state, for a given  $x_K(T_{K-1} - 1)$ , the sub-game  $\mathcal{G}_{K-1}$  has at least one pure NE as shown in the previous section as the sub-game  $\mathcal{G}_{K-1}$  is mathematically equivalent to the static game  $\mathcal{G}_S$  defined previously. We can therefore find a sub-game perfect NE for the stochastic game  $\mathcal{G}$  using a dynamic programming based approach. This involves taking the expectation over  $x_{k+1}(T_{k+1})$  given  $x_{k+1}(T_k - 1)$  while calculating the expected utility for  $k+1$  while at stage  $k$ , and assuming that a pure NE is played when at stage  $k+1$ . Since the NE at stage  $K-1$  is known, we can find the NE at stage  $K-2$  by taking an expectation of future  $x(T_{K-1} - 1)$  and assuming the NE is played at  $K-1$  for all of these realizations.

Since the expectation of the future utility  $u_{k+1}$  will be monotonically increasing w.r.t  $P_{k+1}$ , we can use similar arguments as in Theorem 1 to conclude that the pure NE of any subgame  $\mathcal{G}_k$  will be that when *all players* either invest in the project  $k$  or wait. However, this result is not mathematically proven in this version of the paper and is left for future extensions. This is nevertheless shown in the following numerical examples illustrating the sub-game perfect NE.

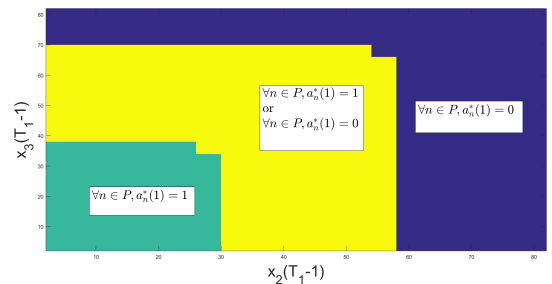


Fig. 4. NE profile based on  $x_2(T_1 - 1)$  and  $x_3(T_1 - 1)$  when  $K = 3$ .

We now take the parameters  $\lambda_0 = 1$ ,  $|C| = 100$ ,  $\sigma_0^2 = 0.2$ ,  $\gamma = 0.999$ ,  $\pi_2 = \pi_3 = 20$ ,  $\sigma_2^2 = \sigma_3^2 = 10$ ,  $\tau_2 = \tau_3 = 100$ ,  $R_1 = 1$ ,  $R_2 = 1.5$ ,  $R_3 = 2$ ,  $T_k = 10k$  for all  $k \in \{1, 2, 3\}$ .

In Figure 4, we plot the possible NE compared to  $x_2(T_1 - 1)$  and  $x_3(T_1 - 1)$  when  $K = 3$ . These equilibria were calculated using Monte-Carlo simulations for realizing the value of  $x_3(T_2 - 1)$  in the future for all possible values

of  $P_2$ , which is used to calculate the NE of the sub-game  $\mathcal{G}_2$  using Theorem 1. These equilibria and a realization of  $x_2(T_2)$  are then plugged in to obtain the future decisions and the utility of not picking project 1. By repeating this evaluation over  $10^5$  realizations of  $x_2(T_2)$  and  $x_3(T_2 - 1)$ , we are able to compute the pure NE of the sub-game  $\mathcal{G}_1$ . For any realization, playing this NE for  $\mathcal{G}_1$  and the NE for  $\mathcal{G}_1$  constitutes a sub-game perfect equilibrium of  $\mathcal{G}$ .

## V. FOUNDER GAME

In the previous section, the funders in  $P$  set were playing a game between in order to choose the project that maximizes their utilities. While the project founders (holders) were considered as passive, they have been reported in the literature as taking some actions that aim at influencing the funders or to change the issue of the funding campaign. In practice, some founders inject some amount of money in their project, especially when it is close to success, using other classical sources of funding. As this practice is forbidden on most of platforms [5], they may activate their network of funders, or may coordinate amount founders of projects in other platforms in order to exchange support [14]. Let  $y_{max}$  be the maximum amount of funding the project older is able to place urgently.  $y_{max}$  is usually limited to a small amount. At  $T_k - 1$ , in order to incite funders to skip project  $k$ , founders of project  $k + 1, \dots, K$  may place an amount of money, limited to  $y_{max}$ , in their respective project. This corresponds to a Stackelberg game that we detail in the following for the case  $K = 2$ .

Let the expected utility of founder of project  $k$  be  $U_k > 0$  if he gets its project fully funded on the platform. If the founder has to complement the funding by an amount  $y$ , we assume that his utility become  $U_k - y$ . We also assume the  $U_k - y_{max} > 0$ . We have the following result:

*Theorem 2:* The Stackelberg equilibrium for founder of project 2 that corresponds to a subgame perfect equilibrium with respect to subsequent funders actions is the following:

- 1) if  $\bar{x}_2 - y_{max} < x_2 < \bar{x}_2$ , invest an amount  $y = \bar{x}_2 - x_2(T_1 - 1) + 1$
- 2) otherwise, if  $\underline{x}_2 - y_{max} < x_2 < \underline{x}_2$ , invest an amount  $y = \underline{x}_2 - x_2(T_1 - 1) + 1$ .
- 3) if any of the above conditions is not verified, do not invest any amount.

*Proof:*

We do not provide a complete proof due to lack of space but only some hints. For proving item 1, consider the case where the threshold  $\bar{x}_2$  is not reached. There is either one NE for funders to invest in project 1 or two NE that consist in funding project 1 or waiting. The expected utility of project 1 founder is thus either equal to 0 or to an unpredictable value. If he invests an amount of money equal to  $y$  so that  $x_2(T_1 - 1) > \bar{x}_2$  and as he makes the first move, funders in  $P$  who take their decisions at  $T_1$  will observe an amount of collected funds larger than  $\bar{x}_2$  and their NE will be to wait for project 2. The utility of project 2 becomes equal to  $(U_k - y)\Phi(y + |P|) > 0$ . Any positive value of  $y$  that makes  $x_2 + y > \bar{x}_2$  increases the utility of the founder compared to a

situation where he does not intervene. To complete the proof, we can show that the resulting utility is decreasing with  $y$ , for small value of  $y$ . Item 2 is proven similarly, having in mind that an unpredictable utility (0 or  $(U_k - y)\Phi(y + |P|)$ ) is better than 0. ■

Note that the founder of project 1 may also invest at  $T_1 - 1$  in his project in order to increase its probability of success<sup>1</sup>, but this does not change the outcome of the game as the NE does not depend on  $x_1$ .

## VI. CONCLUSION

In this paper, we introduced a novel game theoretical framework to model investor behavior in crowdfunding platforms. We considered a series of  $K$  projects that are of interest to common audience sets that aim at maximizing their utilities. We modeled the behaviour of well-informed agents using game theory. We defined a stochastic game in the general case and provide explicit expressions for the Nash equilibria in the special case of 2 projects of interest. For more than 2 projects, we define sub-game perfect equilibria and compute them using dynamic programming. We then considered a Stackelberg equilibrium where project founders try to influence funders decisions.

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<sup>1</sup>it has been reported in [3] that many projects collect exactly 100% of their requested amounts and this may be due to the founders making up the difference at later stages