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Assyrian Merchants meet Nuclear Physicists: History of the Early Contributions from Social Sciences to Computer Science. The Case of Automatic Pattern Detection in Graphs (1950s–1970s)

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Abstract

Community detection is a major issue in network analysis. This paper combines a socio-historical approach with an experimental reconstruction of programs to investigate the early automation of clique detection algorithms, which remains one of the unsolved NP-complete problems today. The research led by the archaeologist Jean-Claude Gardin from the 1950s on non-numerical information and graph analysis is retraced to demonstrate the early contributions of social sciences and humanities. The limited recognition and reception of Gardin's innovative computer application to the humanities are addressed through two factors, in addition to the effects of historiography and bibliographies on the recording, discoverability, and reuse of scientific productions: 1) funding policies, evidenced by the transfer of research effort on graph applications from temporary interdisciplinary spaces to disciplinary organizations related to the then-emerging field of computer science; and 2) the erratic careers of algorithms, in which efficiency, flaws, corrections, and authors' status, were determining factors.

Keywords: history of computing, clique detection, community detection, social networks, sociometry, graph analysis, algorithm

Over the last 20 years, the application of methods based on graph theory has become widespread in many scientific fields, including the social sciences and humanities (hereafter SSH). The major proponents of these methods are physicists and computer scientists (e.g. Albert-László Barabási, Theodore Lewis), who recognize a specific scientific domain they term “network science”. However, in contrast to contemporary narratives, the use of graphs (hereafter a synonym of networks) and more generally of algorithms are far from being new in SSH. The current trend in network science is to dismiss the early applications and improvements of the graph analysis methods that were promoted by SSH researchers from the 1950s. This paper addresses how these early applications occurred and the reasons why they might have been forgotten or dismissed. Clique detection, a method to detect connected subsets of nodes in a graph, is an example of such early algorithmic innovation. The collective research conducted on this subject by the French linguist, archaeologist and documentation specialist Jean-Claude Gardin (1925–2013) is explored. Recent research has started to emphasise the major role Gardin played in the second half of the 20th century in the applications of information retrieval, automatic discourse analysis, and expert systems (Dallas 2015) in SSH, and more generally in epistemology (Plutniak 2017).

1 The clique problem

When a phenomenon is modelled by a graph (e.g. friendship relationships between individuals, relationships between proteins, interactions among animals) detecting particularly dense parts of the graph can be of interest (Figure 1). Many of the modern methods to achieve this are based on detecting classes of nodes, by determining a higher density of edges (a class of methods nowadays commonly called “community detection”¹) or the identification of elementary patterns, such as triadic configurations (Wasserman and Faust 1994). A clique is another example of such specific patterns, which is of particular interest due to its early definition in the development of graph theory and applications.

The concept was first formally defined in a paper published by two mathematician psychologists in the *Psychometrika* journal (Luce and Perry 1949). Later, the French psychologist and graph theorist specialist Claude Flament (1930–2019) reminded that a clique “is a systematization of a current notion in *sociometry*: all the individuals of a clique choose each other.” (Flament 1965, p. 37). Nevertheless, Luce, Perry, and Flament were all mathematicians committed to applying mathematics to psychological and sociological phenomena. The research fields of psychometry and sociometry occasionally overlapped and were particularly invested in by mathematicians (Freeman 2004). Sociometry refers to a field of inquiry originally coined in the early 1930s by the Romanian-American psychologist Jacob Levy Moreno (1889–1974). It grew from his 1934 major publication *Who shall Survive? A New Approach to the Problem of Human Interrelations* illustrating the use of graphs to represent and analyse empirically observed relationships and then from the *Sociometry* journal (published 1937–1977).

From a mathematical perspective, the common definition of a clique is a “maximal complete subgraph of a graph” (Moon and Moser 1965); or, formally:

$G' = (X'; V')$, a subgraph of $G = (X; V)$, is a clique if $(x, y) \in V' \Leftrightarrow x$ and $y \in X'$, that is, if all the possible arcs exist in G' .
(Flament 1965, p. 37)

As a clique can contain a clique, Flament also distinguished the concept of a maximal clique, “a clique which is not a subgraph of a clique”. Note that the publication of formal definitions did not limit variations in the use and semantics of the clique concept in the following decades; for example, defining

¹Many of these methods rely on the optimization of modularity (Newman and Girvan 2004).

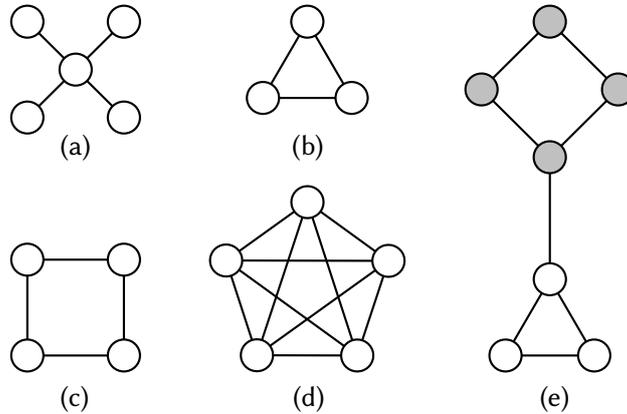


Figure 1: Examples of graph motifs discussed in the text: star (a), cycles (b, c), cliques (b, d), a graph with two communities (e).

a clique as a “subset of members who are more closely identified with one another than they are with the remaining members of their group” (Hubbell 1965, p. 377), i.e. as a synonym of the current concept of “community”².

Despite the simplicity of the definition of cliques, automatizing their detection through an algorithm raises serious computational difficulties, mainly due to combinatorial explosion. Clique detection was included among the 21 NP-complete problems identified by Richard Karp (1935–), namely problems which are not computationally solvable in a deterministic time (Karp 1972). In this context, the research conducted from the late 1950s by Gardin and his collaborators was one of the early efforts to automatize clique detection.

2 Gardin and Garelli’s study of the Ancient Assyrian trade network

In 1961, Gardin and the Assyriologist Paul Garelli (1924–2006) published probably the first automated application of graph theory to analyse historical materials, in the *Annales* journal (Gardin and Garelli 1961). They aimed to reconstruct a commercial network that was active in the 19th century BC in present-day Cappadocia and, to do so, to: 1) automatize the deduction of the geographical locations of the merchants mentioned; 2) identify the general structure of the network of commercial relationships and the merchant groups;

²See also Lankford 1974 for a review of detection methods where “clique” means “community”.

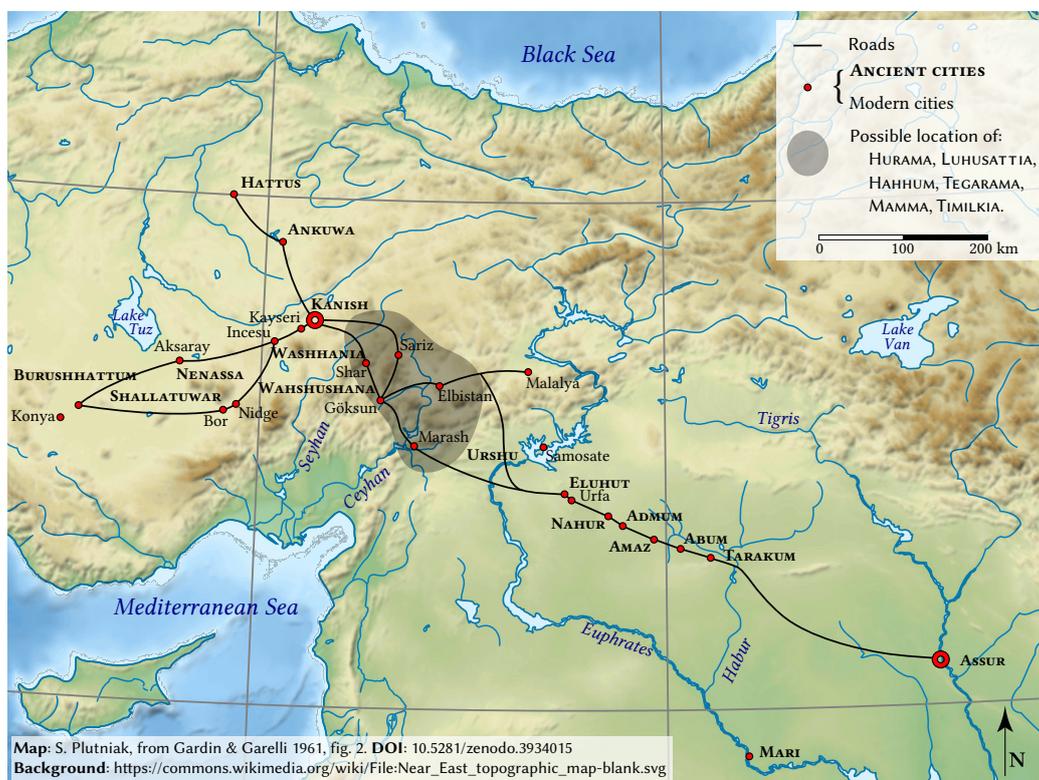


Figure 2: Assyrian commercial expansion during the 19th century BC (redrawn and adapted from Figure 2 from Gardin and Garelli 1961).

3) discuss the interpretation of the synchronic and diachronic aspects of the resulting graph; 4) propose readable graphic representations of this graph; and 5) determine the degree and type of specialization of each merchant. The objectives, covering most of today's major aspects of empirical graphs analysis, and the dataset used in this study, were unusual at the time, due to their ambition and size respectively.

The reconstruction of the Assyrian trade network was based on the corpus of cuneiform tablets found at the Kültepe site (near Kayseri, see Figure 2). These tablets have two characteristics: first, they document with text very ancient human activities (the 2nd millennium BC), and second, there is a large amount of information available because these tablets have been collected, translated and published since the 19th century, first by orientalist and then by specialized Assyriologists. In the early 1920s, about 500 tablets had been published; by the end of the 1950s, this number had risen to about 2600. This

overabundance of material was an obstacle for research³.

Gardin considered overcoming it by data sampling and the use of computer. Sampling was carried out on two levels: first the tablets (the texts) and then the names of the identified merchants. At a meeting held in March 1961, Gardin indicated that the completion of the project presupposed the coding of the 2000 tablets studied by Garelli. The results which had been obtained so far were based on about 200 of them⁴. Nothing was mentioned about the criteria used to select the tablets to be analysed. In a presentation in Paris at the “*Séminaire sur les modèles mathématiques dans les sciences sociales*”, Gardin announced the number of “approximately” 1000 tablets (Gardin 1961, p. 23). The number of merchants’ names stated in the tablets was estimated at 1500 in the 1961 article, although the authors pointed out the possible problems of homonymy⁵. A given merchant was defined by the number of transactions in which they were involved, since the authors assumed that it was a robust approximation of their relative importance (Gardin and Garelli 1961, p. 854). In the 1961 article, the analysis focused on the thirty most important merchants⁶ and the early clique detection algorithm was applied to this subset.

Paradoxically, despite the significance of the data analysed and its method, this study is rarely cited in the historical and archaeological literature on the ancient Orient, and never cited for its methodological dimension⁷. It is also absent from the few publications available on the history of graph algorithms (Lankford 1974, Freeman 1988, Barthélemy and Guénoche 1988, pp. 201–205, Pardalos and Xue 1994). Moreover, the available reports on this history are limited to the chronology of the different methods, without consideration of their social and intellectual contexts. The rare citations of this research in subsequent studies contributed to its invisibility and other factors must be considered.

From the perspective of the social and intellectual history⁸ of computing

³For a detailed account of these data, see Plutniak 2018, pp. 7–11.

⁴“Comité de direction. Réunion du 16 mars 1961”, 16-03-1961, JCG 1, MAE.

⁵Gardin and Garelli 1961, p. 847. However, in the “*Séminaire sur les modèles mathématiques dans les sciences sociales*”, Gardin mentioned “about” 3000 names (Gardin 1961, p. 23) and, in the later republication of the *Annales* article in English, the number of 20,000 is given (Gardin 1965a, p. 380).

⁶Gardin 1961, p. 25. The authors expected that subsequent analyses would have to cover about 100 names (Gardin and Garelli 1961, p. 876, Gardin 1962a, p. 457). In a later commentary, Gardin mentions calculations made on a square matrix corresponding to the relationships among 200 merchants (Gardin 1965c, p. 389).

⁷For a detailed bibliographic study of its reception, see Plutniak 2018, pp. 30–37.

⁸Archives consulted: Jean-Claude Gardin’s files at the Maison Archéologie-Ethnologie, Paris X University, Nanterre, France (hereafter abbreviated MAE); EURATOM files of the

(De Mol and Primiero 2014), the first part of this paper reports on the scientific significance of Gardin’s research. Two factors which limited the subsequent reception and development of these applications in the SSH are then addressed: 1) an organizational factor, related to the lack of interest of historians and philologists in this approach and, consequently, the transfer of Gardin’s research program on graphs from a SSH research institution to applied mathematics and physics institutions; 2) an epistemic factor, related to the performances and potential error of the different algorithms, which also determine whether they are used by researchers.

3 An organizational move from an interdisciplinary to a computing science space

The analysis of the Assyrian trade network was the result of cooperation between two new research organizations focused on methods and interdisciplinarity, the *Centre d’analyse documentaire pour l’archéologie* (CADA) led by Gardin in Paris, and the European Scientific Information Processing Center (CETIS), a service created in 1961 as part of the Joint Research Centre of the European Atomic Energy Community (EURATOM). After presenting these organizations and their collaboration, this section shows 1) how the development of non-numerical data processing was the common background and motivation for this cooperation and 2) how this unusual interdisciplinary effort terminated, to be pursued by organizations dedicated to computer science.

3.1 A precarious interdisciplinary methodological space

3.1.1 The *Centre d’analyse documentaire pour l’archéologie*

The CADA was a laboratory of the French National Centre for Scientific Research (CNRS) created on the initiative of Gardin, located in Paris and Marseille. Despite its name, it was not restricted to archaeology but focused on methodological research in information retrieval and mathematics applied to a wide range of disciplines, including archaeology, history, literature, philology, linguistics, and medicine.

The composition of its scientific direction board reflects the uncommon nature of this research organization, in terms of its scientific themes and

Historical archives of the European Commission, in Fiesole, Italy (HAEU) ; CADA files at the CEPAM archaeology laboratory, Nice, France (CEPAM); files of the Institut Blaise Pascal, private files of Pierre-Éric Mounier-Kuhn (IBP).

position in the institutional organization of scientific research in France⁹. It gathered leading figures in French science policy, famous orientalists (e.g. Claude Schaeffer), open-minded methodological innovators (e.g. Henri Seyrig, Claude Lévi-Strauss). These actors were politically influential and shared Gardin's ambitious methodological projects; they also explain the possibility and the funding of the CADA. In addition, Gardin benefited from privileged relations with some members of the 5th and 6th sections of the *École pratique des hautes études* (EPHE), in particular with some of the creators of the *Maison des sciences de l'homme* (MSH), including Clemens Heller, a close collaborator of Fernand Braudel. The MSH was then in its early stages of development, in the form of an association called *Association Marc Bloch*, what facilitated Gardin to obtain a research contract between this association and the EURATOM Joint Research Centre in 1959, allowing him to develop his work on computers.

3.1.2 The research contract between the Marc Bloch association and Euratom

In July 1959, the EURATOM created a *Groupe de Recherche sur l'information scientifique automatique* (GRISA), under the direction of the mathematician Paul Braffort (1923–2018). A policy of funding research by contract was immediately developed¹⁰. Various scientific fields were involved, including “sociology”¹¹: a project on the “*Traitement automatique de l'information dans les sciences humaines*”¹² was funded, under the direction of Gardin. The initial budget was 227,400 francs¹³, (388,948 € in 2019¹⁴). The purpose of the research contract was to develop an automatic documentation system based on the premise that “the language of the social sciences differs little from natural language, which is used in all exact sciences to communicate results”¹⁵, thus justifying the support of EURATOM. The research group led

⁹For a detailed description, see Plutniak 2018, pp. 11–14.

¹⁰As testified by the 22 contracts listed in MEYER-UHLENRIED, Karl-Heinrich, “organization de la collaboration entre la Commission et les institutions scientifiques des pays de la Communauté pour l'élaboration d'un langage documentaire (principe des contrats)”, 15 January 1960, BAC-059-1980 0209, HAEU.

¹¹Broadly understood to include physio-psychology.

¹²Contract 001-60-1 CETF from the 10 March 1960.

¹³“Contrat de recherche entre la Communauté européenne de l'énergie atomique et l'Association Marc Bloch”, 21 December 1959, BAC-118-1986 1442, HAEU.

¹⁴Equivalent purchasing power in euros in 2017, taking into account monetary erosion due to inflation, calculated using the INSEE converter (<https://insee.fr/fr/information/2417794>).

¹⁵Gardin J.-C., “Programme d'études sémiologiques et documentaire (1961–1965)”, October 1960, box Gardin 6, CEPAM, p. 3.

by Braffort aimed to develop a universal documentation language. Its main applications would be in the priority fields of EURATOM, namely physics and mathematics, fields where scientific expression is highly standardized. Braffort and Gardin believed that if it was possible to develop a documentation language for scientific fields where expression is much less standardized and more dependent on natural language (such as sociology), then this language would be *a fortiori* applicable to the less complex sciences whose expression is more standardized.

The calculations relating to the Assyrian tablets did not directly concern this project, which created the SYNTOL documentation language. However, the calculations were possible due to the relational and financial resources obtained through this contract since the cost of the computer calculation hours had to be financed (in this case, 21.52 machine hours on IBM 650 computers)¹⁶.

3.1.3 Accessing cutting-edge computational resources

Gardin's approach consisted in systematically analysing the entities mentioned in the Akkadian texts, then coding them onto punched cards. There was a large number of descriptors, as in a similar repository previously planned and about which Gardin wrote:

[...] by the free combinatorial game of about five hundred terms, referring to the fundamental elements of the natural environment and human actions –concrete (e.g. technology) or abstract (e.g. functions, institutions)– the researcher can discover, in a vast and dispersed literature, all the relevant passages that shed light on the question he formulated: the transport of wood between the Syrian coast and Mesopotamian countries, penalties incurred by a slave and a free man for the same crimes and, disturbances caused by the demobilization of troops, etc. (Gardin 1960, p. 14.)

Concerning the relationships between the Assyrian merchants, the information in each “economic affairs” documented in the Akkadian texts was coded using thirteen variables coded into the 80 columns of the punch cards¹⁷. The commercial relationships were then represented as a matrix and the purpose of the analysis was to automate the detection of particular sub-graphs, including cycles, stars, and cliques. Despite the relatively few numbers of affairs and merchants, this type of analysis quickly raised combinatorial explosion,

¹⁶“Travaux du CETIS”, p. 8-9, 28 November 1960, BAC-118-1986 1431, HAEU.

¹⁷See the description of the variables in Gardin and Garelli 1961, pp. 848-850.

requiring the use of a computer. The calculations were carried out by André Debroux (1932–) and Peter Ihm (1926–2014) (Gardin and Garelli 1961, p. 875). Debroux was working for IBM Belgium, while collaborating with the GRISA, and Ihm, a German statistician, was a member of the GRISA. In 1960, the GRISA was located in Brussels and did not yet have its own computers. Hours of calculations were rented or loaned by other organizations (Braffort and Gazzano 1961, p. 56), including the Free University of Brussels (ULB), which had an IBM 650 computer¹⁸. The computer used to analyse the Assyrian trade relations was therefore either that of ULB or IBM Belgium.

After the 1961 publication, Gardin envisaged continuing the experiment by integrating more merchants, but he pointed out the limits due to the insufficient computer power in Belgium:

The calculation of these groups, based on the matrix of relationships observed between a hundred individuals taken two by two, is, however, a complicated operation, which already exceeds the capabilities of an average computer such as the IBM 650. A new program is about to be completed, for a more powerful machine (IBM 7090), with the collaboration of the European Scientific Information Processing Center. (Gardin 1962a, p. 457.)

The mathematical and computational problem was thus “transmitted” to the GRISA researchers. In 1961, this research group had become a component of the CETIS, which was established on the EURATOM complex at Ispra, near Varese in the north of Italy, under the direction of Braffort. Thus, the research contract between the Marc Bloch association and the EURATOM facilitated Gardin’s access to the computers used by the researchers at GRISA and to mathematicians able to define and program the calculation methods.

3.2 The rise of non-numerical data processing

3.2.1 Early developments in Europe

In addition to his activities in SSH, Gardin became involved in the then emerging scientific community at the intersection of mathematics, logic, linguistics, and documentation. Gardin was a graduate in linguistics and conducted his early research in automatic documentation. From the end of the 1950s, he participated in the first European research groups devoted to information retrieval and, more generally, to non-numerical information. The “Leibniz Seminars” brought together the most advanced scholars in this new

¹⁸Acquired in 1957 (Halleux and Xhayet 2007, p. 159).

field, with three main lines of research: automation of proof demonstrations, games simulation, and automation of documentation and translation.

In his 1961 paper on the Assyrian trade network, Gardin wrote that the processes executed by the computer were calculations, in the broad sense given to this word by logicians, but different from those relating to numerical operations (Gardin and Garelli 1961, p. 838). A few years later, Braffort gave a more detailed distinction between numerical and non-numerical computing problems in his book on cybernetics:

In numerical problems, the algorithm contains many formulas, arithmetic or logical and, in the machine, one can take advantage of the existence of registers linked to arithmetic and logical structures (accumulators, index registers).

[...]

In non-numerical problems, operations to be performed on the data use the information taken from the basic documentation, and express properties of the problem, but not those directly related to those of arithmetic and algebraic logic (e.g. commutativity, associativity). (Braffort 1968, p. 135.)

Braffort, as the director of the CETIS, was one of the initiators of the Leibniz seminars. In 1960, Gardin participated in the meeting entitled “*Enseignement préparatoire aux techniques de la documentation automatique*”, organised in Brussels by a linguist from the CETIS, André Leroy¹⁹. The next year, Gardin presented a conference entitled “*L’analyse sémantique dans les langues naturelles et formalisées*” at the *Non-Numerical data-processing Symposium*, held at the IBM centre in Blaricum (Netherlands), again organised by the CETIS²⁰. In 1961, he was recruited as a *directeur d’études* at the EPHE and entitled his teaching “*Automatique non-numérique*”, thus referring to:

the other and more recent branch [which] covers a less well-defined field: “non-numerical data processing”, “information storage and retrieval”), automation of work on scientific texts (translations, documentation, etc.), so-called “heuristic” studies in the United States, etc.²¹.

¹⁹This teaching took place from 15 to 22 February, 1960. Non-numerical information was one of the topic discussed, as illustrated by J. Poyen’s presentation “*Quelques problèmes posés par le traitement de l’information non numérique*”.

²⁰The symposium took place from 24 to 27 April, 1961, AAC 117, MAE.

²¹Gardin, J.-C., “Propositions pour un enseignement sur l’automatique non-numérique (1962)”, 1961, box Gardin 6, CEPAM.

Through this work, Gardin became familiar with the researchers in this emerging field and with the scientific perspectives they promoted. Some of these researchers were among the first proponents of innovative work –Bourbaki in mathematics, Yehoshua Bar-Hillel and Noam Chomsky in language theory, and Claude Lévi-Strauss in anthropology– but also among the first and most informed critics of this work. For example, Marcel-Paul Schützenberger (1920–1996) distanced himself from the Bourbaki group²², Braffort and Leroy opposed the theory of the linguist Lucien Tesnière (1893–1954) and that of Noam Chomsky (1928–), and Bernard Jaulin (1934–2010) and Jacques Roubaud (1932–) criticized the use of mathematics by Lévi-Strauss.

Graph analysis was considered as one of the problems raised by non-numerical information. For example, Jacques Arzac (1929–2014), director of the *Institut de programmation* in Paris, reminded that under the “non-numerical information” title, they “mainly taught graph theory” (Arsac 1988). Therefore, for the researchers committed to this field, such as Braffort and Jaulin, the analysis of the Assyrian trade network, for which Gardin had requested their assistance, was a case study directly related to their research concerns about non-numerical information.

3.2.2 The Assyrian trade network from the mathematicians’ perspective

The GRISA conducted research on numerical (in particular, the calculation of nuclear reactors) and non-numerical information (in particular information retrieval). In the 1960 organization chart, Peter Ihm is associated with the working team on “applied mathematics”²³. The team’s report specifies the work on the “Problem of the Assyrian merchants (within the framework of the Gardin contract)”:

The results of this study will be used as pilot examples for the more general problem of determining the degree of dependence, the point of gravity, for example, in a set of arbitrary objects which are related by quantifiable relationships.

We are looking for an automatic information extraction algorithm based on the possible relationship between the elements of each pair of objects.²⁴

²²During WWII, Schützenberger participated in the French Resistance as Gardin, joining London in 1943.

²³“Rapport GRISA”, no. 2, May 1960, CEAB12-640, HAEU.

²⁴From the chapter on Ihm’s activities in “Travaux du CETIS”, 28 November 1960, BAC-118-1986 1431, HAEU.

The problem raised by the case of the Assyrian commercial network is reformulated into a more general mathematical problem by Ihm. Research on graphs was an important axis of investigation at the GRISA/CETIS. In a 1961 document entitled “*Progrès de l’automatique appliquée au domaine du traitement des informations*”, the aims of this research were detailed in the section “*L’étude des ensembles de graphes*”:

Diagrams, stemma, correlograms, flowcharts of arithmetic machines, analog diagrams, for example, . . . are all productions of these algebraic structures recently studied under the name of “graphs”. It was interesting to study these algebraic structures whose importance was growing steadily.

- to represent and process them in the machine;
- to define the distance of two graphs, whose applications are obvious in the numerical domain (approximate calculation) as in the non-numerical domain (synonymy);
- finally to estimate the statistical properties of a population of graphs.²⁵

In summary, when the analysis of the Assyrian commercial network was conducted, Gardin believed that it demonstrated the interest of automating the processing of non-numerical information in SSH. Graph analysis as such was a scientific objective, but only from the point of view of mathematicians. This division grew in the following years.

3.3 Graph research projects move towards computer science organizations

Projects following the 1961 Assyrian trade network analysis illustrate a progressive move of the research projects on graphs to research organizations dedicated to applied mathematics and computing.

3.3.1 A complementary study: social networks in the New Hebrides

The methods developed for the Assyrian trade network were immediately reused in the CADA for another project on contemporary social networks in the New Hebrides (South Pacific Ocean, then a French condominium).

²⁵“Réunion du comité scientifique et technique du 14-3-1961. Progrès de l’automatique appliquée au domaine du traitement des informations”, 20 February 1961, BAC-118-1986 1431, HAEU.

The ethnographer Jean Guiart (1925–2019) conducted research on the local social structure, focusing on the system by which individuals could receive “titles” of prestige which were also related to places. Guiart collected a dataset of about 1200 titles and 500 places, related to individuals with about 2700 relations (Espirat et al. 1973, p. 370). This research is first mentioned in relation to the CADA in 1962, after Guiart asked for support in running an experimental computer-based analysis of this large dataset (Gardin 1962b)²⁶. Two of Gardin’s colleagues supervised this collaboration: Marie-Salomé Lagrange (1935–2011), a member of the CADA, and Monique Renaud, a member of another research group led by Gardin, the *Section d’automatique documentaire* (SAD)²⁷ at the *Institut Blaise Pascal*, then the main place for computing research in Paris. A person was recruited to code the data on punched cards, Mrs A.-M. Nougaret²⁸, preliminary tabulations were performed by the *Service mécanographique* of the CNRS, and the main calculations were executed in 1963 on the IBM 1401 and 704 computers of the *Institut Blaise Pascal* (Espirat et al. 1973, p. 387). Lagrange and Renaud wrote an internal report in 1965²⁹ and the final results were only published in 1973. In addition, the analysis of the graphs includes detecting chains of different lengths, “arborescent” structures, and cycles of different lengths, but cliques were not studied (Figure 3). As illustrated, research organizations dedicated to computing started to play a major role, considering both the actors and the instruments.

3.3.2 The Sad’s programming language for graph analysis

Based on the Assyrian and Polynesian projects, Gardin was keen to push forward their research on graphs. However, he favoured the SAD, rather the CADA for this purpose, although he still led these two research organizations. In 1964, he obtained a grant from the *Délégation générale à la recherche scientifique et technique* (DGRST, a public scientific funding body) to conduct research on graphs at the SAD³⁰. In a paper presenting the research conducted at the SAD, Gardin emphasized the reasons motivating the development of such a language:

²⁶“Orientation des travaux à partir du 2e semestre 1962”, June 1962, box Gardin 6, CEPAM, p. 3.

²⁷NSF 1959, p. 121.

²⁸As is often the case, very little information is available about subaltern research workers.

²⁹Lagrange and Renaud. 1965. *Étude d’un réseau sociologique aux Nouvelles-Hébrides, sur calculateur*, 55 p., mentioned in Gardin 1965b.

³⁰Contract between the SAD and the DGRST n° 64 FR 175. See also: SAD, “Rapport d’activités situation 1963–1964”, 6 March 1964, box Gardin 6, CEPAM.

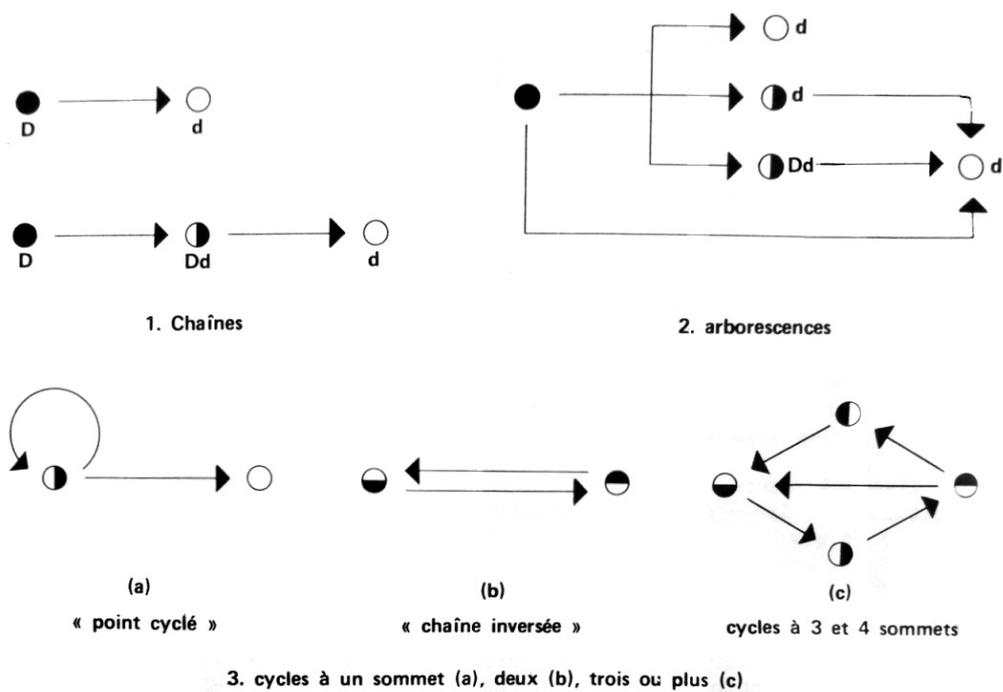


Figure 3: Patterns of interest in the study of the New Hebrides social network. (from Espirat et al. 1973, p. 390). “D” stands for dominating titles and “d” for dominated titles.

However, programming the corresponding algorithms sometimes suffers from certain insufficiencies specific to the common symbolic languages, designed for numerical computation; then came the idea to define a programming language to specifically treat graph problems, our experience showed that they formed a large enough family to justify such a project. (Gardin 1965b.)

Dirk Muysers, then an engineer-programmer at the SAD, was in charge of this research, with the support of a young Greek mathematician recruited in 1965 for this purpose, Ion Arghiridis (1944–). Muysers succeeded in creating the language, that he presented in a first report communicated to the DGRST in 1966³¹. The report ended with two examples in which the language was used to implement a method to enumerate all the cycles in an oriented graph, and a method to find the shortest path between two nodes. However, this promising research quickly ended, for at least two identified reasons: Muysers stopped his research activities and left the CNRS in 1968, and the DGRST funded another project on graphs at the Paris *Institut de programmation*.

3.3.3 The “graph contract” of the *Institut de programmation*

The *Institut de programmation* was founded in 1963 in Paris and placed under the direction of Jacques Arsac. It was intended to be a counterpart to the *Institut Blaise-Pascal* to develop the teaching of programming and computer science (Mounier-Kuhn 2012, p. 428). However, research was also conducted at the *Institut*, in particular on non-numerical information and graphs. In 1964, the *Institut* benefited from a contract on graphs funded by the DGRST. This research was carried out by Jean Berstel (1941–) and Jean-François Perrot (1941–), two former Schützenberger’s students, who co-signed the final report³².

Arsac, Perrot and Berstel were trained in mathematics and dedicated to the different aspects of computer science. This contrasted with CADA’s perspective where research in computing was a consequence of the data applications from the SSH. Besides its strictly mathematical interest, research on graphs had practical consequences in the domains where they can be applied, i.e. to optimize processes in operational research. Computer science was becoming a key domain for national development, as illustrated by the *Plan Calcul* launched by De Gaulle’s government in 1966. This general

³¹Muysers D., *Projet d’un langage de programmation numérique et non numérique, adapté plus particulièrement au traitement des données structurées par réseau*. Report to the DGRST, April 1966, JCG 156, MAE.

³²Perrot J., Berstel, J.-F. 1967. *Rapport final de la Convention de Recherche DGRST 65 FR 002*, “Contrat Graphes”, mimeograph, Institut de Programmation.

context, and the reorientation of the DGRST financial effort on specialized research organizations, help to understand why Gardin and his collaborators abandoned their research on graphs. Subsequently, Gardin focused his research on information retrieval. He no longer referred to the analysis of the Assyrian trade network in his writings. He only reconsidered this study in the early 1970s when, turning his research towards the simulation of reasoning, he reinterpreted his early experiments on graphs as a prefiguration of his new interest in artificial intelligence³³.

4 The efficiency, flaw, and career of algorithms

After summarizing the chronology of clique detection methods, this section presents Bernard Jaulin and his algorithm. Then a report of our attempt in reconstructing and comparing this method to other algorithms is presented. The section ends with a discussion on flawed algorithms.

4.1 A chronology of clique detection methods

The concept of a clique had already been used in the sociometric literature (e.g. Forsyth and Katz 1946) when it was first formally expressed in 1949 by Robert Duncan Luce (1925–2012), an American mathematician psychologist and games theorist (Luce and Perry 1949)³⁴. From the early 1950s, procedures of matrix re-ordination were used to prove the structure of a graph, but not cliques in particular (Beum and Brundage 1950). The first algorithm was based on matrix algebra (Harary and Ross 1957, hereafter Harary’s method) and, two years later, a different method based on the construction of graphs in succession was proposed (Paull and Unger 1959). However, these methods were theoretical and few or no implementations were published before the presentation of a program for cluster detection³⁵ (i.e. community detection), written for a UNIVAC 1 machine by Duncan MacRae and co-published with the sociologist James Coleman (1926–1995) (Coleman and MacRae 1960).

This highlights the state-of-the-art when Gardin and Jaulin worked on the Assyrian trade network, for which the dataset size required a computer:

The mathematical problem of clique detection has been approached

³³See Plutniak 2018, pp. 26–30.

³⁴The same year a programmatic use of algebra to determine cliques was also published, although with no detailed procedure (Festinger 1949, p. 156).

³⁵For a review, see Lankford 1974.

by many authors, from varying angles. The main issue, here, is in the size of the matrix which has to be analysed to that end (*ca.* 200 rows x 200 columns); for any given detection method, it is unrealistic to consider its application on so large a matrix without the help of a computer. (Gardin 1965c, p. 389)

Harary's method was known by the authors, but they believed it could be improved in terms of efficiency and speed (Jaulin 1961, p. 51, Gardin and Garelli 1961, p. 865). So Jaulin developed his own algorithm.

4.2 Bernard Jaulin and his algorithm for clique detection

In 1960, after becoming an engineer at the *École des Arts et Métiers* and obtaining a bachelor of science degree, Jaulin joined the “*Bureau d'études pour le traitement automatique de l'information dans les sciences humaines*”, created and led by Gardin in the framework of the contract between the Marc Bloch association and the EURATOM. Gardin being a social scientist interested in formal approaches, and Jaulin being a mathematician interested in social sciences, they started a close collaboration also based on friendship. Together they organised in 1959 the first training program on non-numerical information processing at the *Institut Blaise Pascal*, and Gardin asked Jaulin to contribute to the study of the Assyrian trade network. In 1964, Jaulin became Director of the Centre for Applied Mathematics of the *Maison des sciences de l'homme* (former Marc Bloch association). In July 1966, Jaulin and Gardin organised a conference entitled “*Les applications du calcul dans les sciences de l'homme*”³⁶, in Rome. This conference was supported by the UNESCO International Computing Centre (ICC)³⁷, also located in Rome and directed by the French mathematician Claude Berge (1926–2002). Berge was then a leading figure in graph theory and also participated, as Gardin, in the “Non-numerical data processing symposium” in Blaricum in 1961. Jaulin and Gardin's conference in Rome was immediately followed by an “International Seminar on Graph Theory and its Applications”, organised by Berge and the ICC, demonstrating the real interest for graph theory at the time.

Jaulin presented his algorithm for clique detection in 1961 at the “*Séminaire sur les modèles mathématiques en sciences humaines*” of the EPHE in Paris. The working papers of this seminar are the only source describing this algorithm (Jaulin 1961). A CADA internal report entitled “*Sur une méthode de détermination des cliques dans un graphe symétrique*” existed, but it has not

³⁶Later published as Gardin and Jaulin 1968.

³⁷For a history of this centre, see Nofre 2014.

been conserved in the archive repositories³⁸. Jaulin’s method was presented as an improvement of Harary’s algorithm, using a different approach, namely boolean algebra, and included the notion of uniclinal points (reduction of the graph to treat nodes that belong to only one clique). It was suitable for symmetric and unweighted graphs. Gardin pointed out the last feature as a possible flaw, because it did not consider different relationship intensities between merchants. However, he argued that 1) they lacked a scale to decide the values used to weight the edges; and 2) they wanted to focus on clique and cycle detection, without considering the weighting of the graph (Gardin and Garelli 1961, p. 864). Jaulin’s algorithm was implemented twice, but neither programming code of implementation has been recovered. The first was in 1960, by André Debroux (1932–?) from IBM Belgium and Otto Hermann from the CETIS³⁹. It was written in SOAP (Symbolic Optimal Assembly Program), the assembly language for the IBM 650 computer, and executed on this type of machine at IBM Belgium. The second was in 1962, for a IBM 704 computer⁴⁰, although it was probably abandoned. Consequently, we relied on the working paper to reconstruct Jaulin’s method.

4.3 Algorithm archaeology

As archaeologists practising experimental archaeology to reproduce objects and their applications, computing historians have recently attempted to reconstitute computers and programs (De Mol and Bullynck 2008, Haigh, Priestley, and Rope 2014). Sometimes, computer scientists also perform these exercises. For example, French mathematicians Jean-Pierre Barthélemy (1945–2010) and Alain Guénoche (1947–) implemented the Paull and Unger’s algorithm for clique detection thirty years after it was published in 1959 (Barthélemy and Guénoche 1988, pp. 201–205). Note that from 1971, Guénoche was a member of the CADA and collaborated for about ten years in other research groups associated to Gardin.

An attempt to implement Jaulin’s algorithm was carried out with the support of Guénoche. However, the description in the 1961 working paper was so incomplete and muddled that it failed. A workaround was to extract the data sample presented in the paper on the Assyrian trade network to illustrate the detection of cliques and process it with other methods to compare the results. Some of these results are contradictory in this paper, e.g. the

³⁸It is indexed but absent in the MAE archives, and not conserved in the CEPAM and HAEU archives

³⁹CETIS, *Travaux du CETIS*, 28 November 1960, BAC-118-1986 1431, HAEU.

⁴⁰CNRS, “Centre d’analyse documentaire pour l’archéologie” in *Rapport d’activité. Octobre 1961–Octobre 1962*, p. 417.

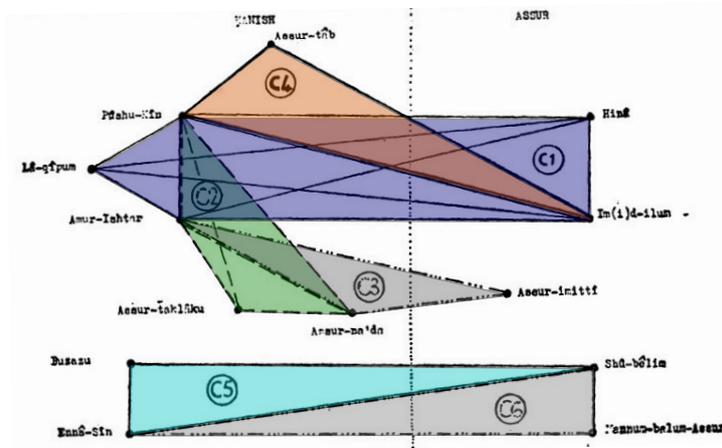


Figure 4: Illustration of clique detection in the Assyrian trade network (Figure 10 in Gardin and Garelli 1961, p. 868, adapted to highlight the cliques).

adjacency matrix presented in Figure 7 contains six cliques whereas the legend indicates three cliques and a singleton, which is actually part of a clique. Consequently, the data was extracted from a network plot (Figure 4). Five methods were applied, including Harary's algorithm, which was implemented in R language for this research⁴¹. All the methods identified the same six cliques as reported in Gardin and Garelli's paper (Table 1). However, Harary's method surprisingly found two more cliques. This result and the contradictory legend of Gardin and Garelli's paper drew our attention to errors in the use of formal methods.

4.4 Flawed algorithms

The different result generated by Harary's method led to suspect a flaw in the algorithm. Retracing the history of this method confirms this hypothesis. Frank Harary (1921–2005) was an American mathematician specialised in graph theory, with a great interest in developing applications for a wide range of domains, including social anthropology (Hage and Harary 1983). After the publication of his method for clique detection in 1957 (Harary and Ross

⁴¹The algorithms used included: Bron and Kerbosch 1973, implemented in the `maxClique()` function from the *RBGL* package; Östergård 2001, `qpGetCliques()`, *qpgraph* package; Makino and Uno 2004: `clique.census()` function, *sna* package; and Eppstein, Löffler, and Strash 2010, `max_cliques()` function, *igraph* package. See supplementary materials.

Table 1: Identifier and size of the six cliques reported in Gardin and Garelli 1961, compared with the cliques detected by different methods.

Id	Size (nodes)	Gardin & Garelli 1961	Harary 1957	Bron 1973	Makino 2004	Osertgard 2001	Eppstein 2010
1	5	x	x	x	x	x	x
2	4	x	x	x	x	x	x
3	3	x	x	x	x	x	x
4	3	x	x	x	x	x	x
5	3	x	x	x	x	x	x
6	3	x	x	x	x	x	x
–	3	–	x	–	–	–	–
–	3	–	x	–	–	–	–

1957), the method was extended to weighted graphs about 12 years later (Doreian 1969). During that time, nobody saw that the 1957 algorithm was flawed before Harary himself noted it:

To set the record straight, that algorithm determines not only all the cliques of a graph, but sometimes a few other subgraphs as well. Correct algorithms for clique detection have subsequently been derived independently by several experts in computer programming. (Harary 1970, p. 6.)

Consequently, the method was then modified and implemented in the PL/1 language (Dixon 1973). This case illustrates how an algorithmic error can persist over a long time, without limiting its scientific spread (in January 2021, *Google Scholars* reported 236 citations of the 1957 paper).

As we saw previously, clique detection was not applied in the study of the New Hebrides social network and was virtually forgotten by Gardin and his colleagues in their subsequent research. Similarly, Jaulin’s method is virtually absent from specialised literature on graphs, except a brief mention in Flament’s book on the *Applications of Graph Theory to Group Structure* (Flament 1963, p. 37)⁴². Contrary to the widespread use of Harary’s method, the flawed nature and the limited efficiency of Jaulin’s method might have finally prevented its author from publishing it. We have already mentioned how the only available description of the algorithm was muddled. In addition,

⁴²Later literature reviews do not mention Jaulin’s method, even those published in French, *e.g.* Schneider 1973.

Jaulin himself pointed out a limitation of his method, indicating that in this case Harary's method must be used (Jaulin 1961, p. 56). The contrasting careers of these two flawed algorithms highlight the significant importance of external factors shaping the adoption and use of formal methods.

5 Conclusion

By combining a socio-historical approach with an experimental reconstruction this paper offers several contributions to the current development of computer science history. It brings to light forgotten early automated applications of graphs in the SSH developed in Europe, while emphasizing the role these disciplines played in this respect. The contribution of Gardin and his colleagues were particularly innovative. However, as illustrated by the case of the Assyrian trade network study, their work received a very limited reception and little recognition in proportion to their originality and methodological value. In addition to the effects of literature bibliographies on the discoverability and reuse of scientific productions, this paper discussed two possible factors for this under-valuation:

- the effects of research funding policies, demonstrated by the transfer of research effort on non-numerical information processing and graph applications from temporary interdisciplinary spaces (the CADA and the CETIS) to disciplinary organizations related to the then-emerging computer science;
- the erratic careers of algorithms, in which efficiency, flaws, corrections, and authors' status, were determining, but unpredictable factors.

These research case studies involved undeniably cutting-edge innovations for their time. Detailed analyses show that their long-term scientific value and effects are less controllable than what is sometimes assumed by science policy-makers who attempt to organise science as technological innovations are managed.

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Supplementary material

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