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# Numerical modeling of interfacial soil erosion

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## Résumé :

*Le but de cette étude est de développer un modèle numérique de simulation de l'érosion de la surface d'un sol sous l'effet d'un écoulement. Nous utilisons des équations de conservation avec relation de sauts. Un modèle à pénalisation est utilisé pour calculer les équations de Stokes autour d'un obstacle, avec une méthode aux domaines fictif pour s'affranchir d'un maillage dépendant de la géométrie et pouvoir utiliser des schémas rapides de volumes finis sur maillage cartésien. La capacité du modèle à prédire l'érosion de surface du sol est confirmée par la présentation de plusieurs résultats de simulation 2D.*

## Abstract :

*The aim of this study was to develop a numerical model for simulating the surface erosion occurring at a fluid/soil interface undergoing a flow process. The balance equations with jump relations are used. A penalization procedure is used to compute Stokes equations around obstacles, with a fictitious domain method, in order to avoid body-fitted unstructured meshes and to use fast and efficient finite volumes approximations on Cartesian meshes. The water/soil interface evolution is described with a Level Set function. The ability of the model to predict the interfacial erosion of soils is confirmed by presenting several 2-D simulations.*

**Mots clefs :** Erosion, Fictitious domains, Level Set

## 1 INTRODUCTION

Overtopping, overflowing, piping, and the internal erosion of soil resulting from seepage flow, are the main cause of serious failure at hydraulic works (dykes, levees, dams), in terms of the risk of downstream areas being flooded. These processes are also liable to occur in natural soils. In the petroleum (oil and gas) industry, the sand production processes occurring in wellbores can be said to be interfacial erosion processes. There exists a large body of literature on the modeling of soil erosion in the field of sediment transport and the hydraulics of free-surface flows, but most of these studies have taken a global approach to the subject. Water flow is often expressed in terms of its average velocity, in the framework of shallow water equations. Erosion is viewed as a spatial transport capacity gradient rather than a process of particle removal caused by the flow, and the question as to how fluid/soil interfaces behave has been rather neglected.

Several models for surface sand erosion under axial and radial flow conditions have been proposed in the framework of continuum mixture theory [10]. The process of erosion is assumed in these studies to involve a smooth transition from solid-like to fluid-like behaviour. This transition is described with a three-phase model (comprising solid, fluid, and fluidized phases). These three phases interact while being constrained by balance equations. A sink term is introduced into the mass balance equations to describe the detachment of sand particles via an erosion law. These models have been mainly applied to granular materials and piping backward erosion. The philosophy underlying the present approach differs from that in which previous theories of surface erosion were based in that: 1) our description deals with singular fluid/soil interfaces; 2) our description deals with the surface erosion occurring at a fluid/soil interface undergoing a flow process which is tangential to this interface, and which is driven by the tangential shear stress.

The basic balance equations and erosion constitutive law were developed by Brivois et al. [3] and Bonelli et al. [2]. For the sake of simplification, sedimentation and deposition processes are neglected here, and the flow is assumed to be dilute [6]. A penalization procedure is used to compute Stokes equations around obstacles, with a fictitious domains method [1]. The water/soil interface evolution is described with a Level Set function [9]. Spatial derivatives are computed with a fifth order WENO (weighted essentially non-oscillatory) scheme [5], while time derivatives are computed with a fourth order Runge-Kutta (RK4) scheme. A finite volume formulation is developed on a Cartesian MAC mesh [7]: the physical domain is given by the pressure mesh, while the velocity is given at the middle of each face of the cell. The aim was to avoid body-fitted unstructured meshes in order to use fast and efficient finite volumes approximations on Cartesian meshes [4].

## 2 METHODOLOGY

### 2.1 Governing Equations

Consider the surface erosion of a soil subjected to a flow running parallel to the fluid/soil interface. The soil is eroded by the flow, which then carries away the detached particles. The basic balance equations and erosion constitutive law were developed by Brivois et al. [3]. The quantity of particles present in the fluid is taken here to be small enough to be able to assume that the flow is dilute. For the sake of simplification, the sedimentation and deposition processes, as well as the gravitational forces, are not considered here.

Let us take a computation domain  $\Omega$  with fluid domain  $\Omega_f(t)$  and soil domain  $\Omega_s(t)$ . The fictitious domain approach [1] allows to describe the behavior of the two sub domains with a set of equations defined on the whole domain  $\Omega$ :

$$\begin{cases} \operatorname{div} \mathbf{u} = 0 \\ \rho \left[ \frac{\partial \mathbf{u}}{\partial t} + \operatorname{div}(\mathbf{u} \otimes \mathbf{u}) \right] = \operatorname{div} \mathbf{T} - \frac{\mu I_s}{K_s} \mathbf{u} \end{cases} \quad (1)$$

where  $\rho$  is the water density,  $\mathbf{u}$  is the velocity,  $\mathbf{T}$  is the stress tensor,  $\mu$  is the water viscosity,  $K_s$  is a penalization coefficient (having the dimension of a geometric permeability) and  $I_s$  is the existence function of the soil domain, which is unity within  $\Omega_s(t)$  and zero elsewhere. The stress tensor takes the following form:  $\mathbf{T} = -p\mathbf{I} + 2\mu\mathbf{D}(\mathbf{u})$  where  $p$  is the pressure,  $\mu$  is the water viscosity, and  $\mathbf{D}(\mathbf{u}) = \frac{1}{2}(\operatorname{grad} \mathbf{u} + (\operatorname{grad} \mathbf{u})^T)$  is the symmetric part of the velocity gradient.

As the last term of the right-hand side of Eq. (1)<sub>2</sub> is zero in the fluid domain, Eq. (1) corresponds to the Navier-Stokes equation. In the soil domain, the penalization leads to very small values of  $\mathbf{u}$ , and the left-hand side of Eq. (1) is negligible, as well as the viscous part of  $\mathbf{T}$ . This time, Eq. (1) gives a Darcy-like equation, namely  $\mathbf{u} = -(K_s / \mu) \operatorname{grad} p$  with a permeability nearly equal to zero, which imposes the velocity to tends towards zero in the soil domain. From the global model Eq. (1), we define therefore asymptotically by penalty the local equations related to the fluid and to the soil.

We take  $\Gamma(t)$  to denote the fluid/soil interface. As there is a process of erosion, a mass flux crosses this interface. As a result,  $\Gamma(t)$  is a moving interface. The soil is assumed to be rigid, and the frame is attached to the soil in question ( $\mathbf{u}$  can be considered as uniform in the soil, and equal to zero). Since this soil is saturated and devoid of seepage, the mass flux crossing the interface is an erosion flux. Lastly, the soil is homogeneous. Balance equations can be transformed into jump conditions across  $\Gamma(t)$  [2]. Let us take  $\mathbf{n}$  to denote the unit vector normal to  $\Gamma(t)$  oriented from the flow to the soil, and  $\mathbf{v}_\Gamma$  to denote the velocity of  $\Gamma(t)$ . We take  $\dot{m}$  to denote the total flux of material (both particles and water) crossing the interface, defined by:

$$\dot{m} = \rho(\mathbf{v}_\Gamma - \mathbf{u}) \cdot \mathbf{n} \quad (2)$$

The mass and momentum jump equations on  $\Gamma(t)$  are as follows:

$$\begin{cases} \llbracket \dot{m} \rrbracket = 0 \\ \dot{m} \llbracket \mathbf{u} \rrbracket + \llbracket \mathbf{T} \rrbracket \cdot \mathbf{n} = 0 \end{cases} \quad (3)$$

where  $\llbracket \cdot \rrbracket$  is the jump across the interface.

## 2.2 Erosion Law

The motion of the interface is unknown and is a part of the solution of the problem. From (2), it emerges that the interface velocity can be expressed as a function of the mass flux, and the latter therefore becomes the main unknown, for which a constitutive law is required: the erosion law. Erosion laws dealing with soil surface erosion by a tangential flow are often written in the form of threshold laws such as:

$$\dot{m} = \begin{cases} k_{er}(\tau - \tau_c) & \text{if } \tau > \tau_c \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

where  $\tau = \sqrt{(\mathbf{T}\mathbf{n})^2 - (\mathbf{n}\mathbf{T}\mathbf{n})^2}$  is the tangential (shear) stress exerted on the soil. This erosion law dates back a long way. It was first used to in studies on free-surface flows. We used the same constitutive law here to model more general situations. The threshold  $\tau_c$  can be referred to as the critical shear stress, and  $k_{er}$  is the coefficient of soil erosion.

The tangential velocities are taken to be continuous across  $\Gamma(t)$  (no-slip condition at the interface): this is the only assumption about  $\Gamma(t)$  mainly because of the lack of experimental data. The tangential stress  $\tau$  is therefore continuous across  $\Gamma(t)$ . Based on above assumption, the interface velocity can be related to the flux of eroded material as follows:

$$\mathbf{v}_\Gamma = \frac{\dot{m}}{\rho_s} \mathbf{n} \quad (5)$$

Some simplifications, both physically acceptable and expedient, were used in order to obtain this model. We take  $V_{ref}$  to denote the reference water velocity. The quantity  $k_{er}V_{ref}$ , defined as “erosion kinetic number”, is a key dimensionless number [2]. Careful examination of the whole system shows that, if  $k_{er}V_{ref} \ll 1$  the erosion velocity is much lower than the flow velocity, and the amount of eroded material is much lower than the amount of water. As a consequence, the characteristic volume concentration of eroded particles in the flow is very small. This finding is consistent with our assumption of a dilute suspension flow, which is not affected by the presence of eroded particles.

## 2.3 The Level Set Method

The Level Set method [9] accounts for the interface, by means of a function  $\psi$ , negative in the fluid and positive in the soil. The interface  $\Gamma(t)$  is represented by the zero level set of  $\psi$ . A further development of the method [4] indicates that a relevant approach is to choose this function as the signed distance to the interface. The displacement of the interface is driven by a transport equation:

$$\frac{\partial \psi}{\partial t} + \mathbf{v}_\Gamma^{ext} \cdot \text{grad} \psi = 0 \quad (6)$$

where  $\mathbf{v}_\Gamma^{ext}$  is an extended definition of the interface velocity. Indeed,  $\mathbf{v}_\Gamma$  is defined only on the interface, and we solve a transport equation along the normal of the interface to extend this quantity

in the whole domain. In order to avoid computation oscillation near the interface, this transport equation is solved in two steps: first in the soil domain and then in the fluid domain.

The normal to the interface is given by the gradient of the level set function:

$$\mathbf{n} = \frac{\text{grad } \psi}{\|\text{grad } \psi\|} \quad (7)$$

The level set function  $\psi$  is defined in the whole domain: the vector  $\mathbf{n}$  is therefore defined in the whole domain. This property allows to extend the interface velocity  $\mathbf{v}_\Gamma$  along the normal vector  $\mathbf{n}$ .

## 2.4 Numerical Aspects

Eq. (1) is achieved by the use of the finite volume method on staggered Cartesian grid, the MAC method introduced by Harlow & Welch [7]. The coupling velocity-pressure is classically treated by an augmented Lagrangian scheme. The pressure is linked to the mass balance equation as follows:

$$p^{m+1} = p^m - r_1 \text{div}(\mathbf{u}^{m+1}) \quad (8)$$

where  $r_1$  is a numerical parameter. The momentum balance equation Eq. (1)<sub>2</sub> is solved with the iterative Stabilized Bi-Conjugate Gradient method (BiCGStab). After inserting Eq. (8) this equation, with a different numerical parameter  $r_2$  in order to increase the numerical convergence, Eq. (1)<sub>2</sub> becomes:

$$\rho \frac{\partial \mathbf{u}^{m+1}}{\partial t} - \text{div} [2\mu \mathbf{D}(\mathbf{u}^{m+1})] - \frac{\mu I_s}{K_s} \mathbf{u}^{m+1} - r_2 \text{grad} [\text{div}(\mathbf{u}^{m+1})] = -\text{grad } p^m - \rho \text{div}(\mathbf{u}^m \otimes \mathbf{u}^m) \quad (9)$$

In Eqs. (8) and (9),  $m$  denotes the iteration number of the iterative solver. The solver iterates with a tolerance  $\varepsilon_1$  on the velocity and a tolerance  $\varepsilon_2$  on the residual momentum. The time derivative is approached with an explicit Euler discretization. The time step is determined by the Courant-Friderichs-Levy (CFL) stability condition. The steady Stokes solution computed without erosion defines the initial condition. In Eq. (6), the spatial gradients are approached with a high order Weighted Essentially Not-Oscillating (WENO5) scheme introduced by Jiang & Peng [5], and the time discretization use a fourth order Runge-Kutta scheme (RK4). The erosion velocity is much lower than the flow velocity. Experimental evidence shows that the ratio flow velocity/erosion velocity range from  $10^{-2}$  to  $10^{-5}$  [2]. This is the very reason to solve separately Eqs. (1) and (6) with different time steps.

## 3 RESULTS

### 3.1 Erosion of a circle

We consider a 2-D circle of soil located in the center of a water channel (height  $H=6$  cm; length  $L=15$  cm). A wall condition is imposed on the horizontal sides ( $\mathbf{u} = 0$ ), while a pressure gradient of 0.4 Pa is imposed between the two vertical sides. The mesh was composed of 9000 cells. The numerical parameters were chosen as follows:  $\varepsilon_1 = 10^{-3}$ ,  $\varepsilon_2 = 10^{-7}$ ,  $r_1 = r_2 = 10^3$ . Table 1 lists the others parameter values.

Table 1. Parameters values.

$k_{er}$ (s/m)	$\tau_c$ (Pa)	$K_s$ (m <sup>2</sup> )	$\rho_s$ (kg/m <sup>3</sup> )	$\rho$ (kg/m <sup>3</sup> )	$\mu$ (Pa.s)
0.1	0.01	$10^{-7}$	2700	1000	0.001

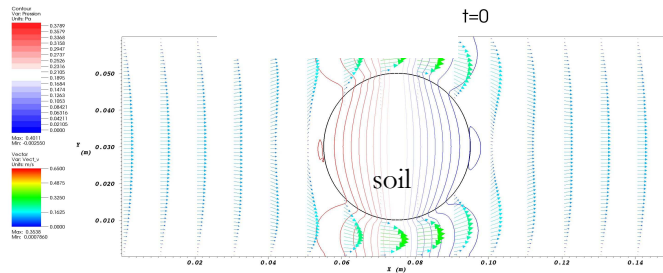


Fig. 1. Initial condition of the circle erosion test case. Contours values are pressure. Vectors are the velocity field. The black line is the interface ( $\psi = 0$ ).

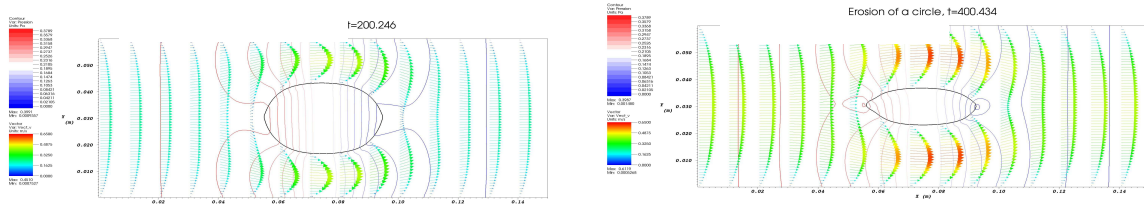


Fig. 2. Shape evolution of the circle during the erosion phase.

The initial steady flow is represented in Fig. 1. This very simple example helps to understand qualitatively the influence of erosion on the shape evolution of the soil domain during the erosion phase, which is represented in Fig. 2.

### 3.3 Erosion of a channel

We consider a 2-D channel with a non erodible domain at the inlet. The pressure drop is set to 4 Pa. The whole domain is filled by soil except a horizontal channel in the middle (initial width 6 mm). A non erodible domain is set at the inlet (length 2 cm). Indeed, we intended to check that the numerical model was able to deal with the singularity between erodible and non erodible soil during the erosion process. The mesh was composed of 7900 cells. Table 1 lists the parameter values.

The initial steady flow, which is actually the Poiseuille solution, is represented in Fig. 5. The shape evolution of the soil domain during the erosion phase is represented in Fig. 6. The only possible source of inhomogeneity is the spatial gradient of the channel width. Fig. 6 confirms that this model successfully accounted in qualitative terms for the surface erosion of a soil subjected to a flow running parallel to the fluid/soil interface.

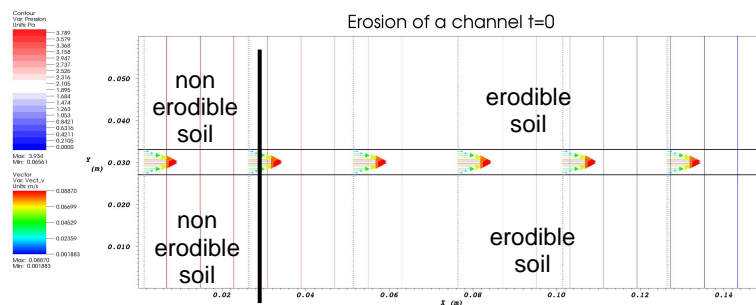


Fig. 6. Initial condition of the channel erosion test case. Contours values are pressure. Vectors are the velocity field. The black line is the interface ( $\psi = 0$ ).

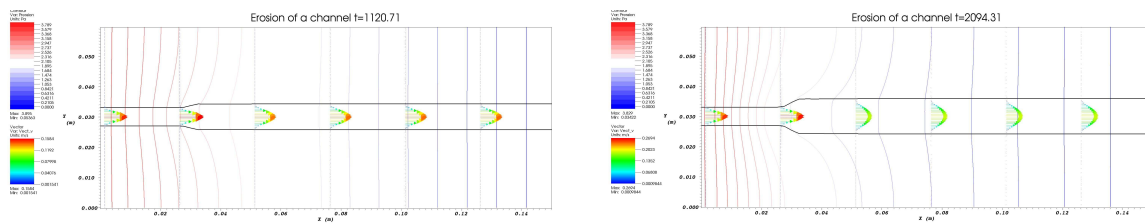


Fig. 7. Shape evolution of the channel during the erosion phase.

## 4 CONCLUSION

The aim of this study was to draw up a numerical model for simulating the surface erosion of a soil subjected to a flow running parallel to the fluid/soil interface. We have presented an erosion model combining an erosion law and Navier-Stokes flow. The water/soil interface evolution was described with a Level Set function. A finite volume formulation was developed on a Cartesian MAC mesh: the physical domain is given by the pressure mesh, while the velocity is given at the middle of each face of the cell. The aim was to avoid body-fitted unstructured meshes in order to use fast and efficient finite volumes approximations on Cartesian meshes. The ability of the model to predict the interfacial erosion of soils was confirmed by presenting several 2-D simulations. Of course, other important phenomena should be considered in a more comprehensive modelling of erosion, like turbulence (for the flow) or unsaturated state and seepage (for the soil).

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