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# Efficient computation of arbitrary beam scattering on a sphere: comments and rebuttal, with a review on the angular spectrum decomposition.

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## Abstract

The present paper may be viewed as built from comments concerning a recently published paper in *Journal of Quantitative Spectroscopy and Radiative Transfer*, and actually constitutes as well a rebuttal of some of the claims it contains. The issue considered concerns the computation of scattering properties of the interaction between an arbitrary electromagnetic shaped beam and a homogeneous sphere. Our discussion of the issue may be easily extended as well to more general cases, beyond the simplest case of a homogeneous spherical scatterer, up to the case of arbitrary shaped particles in the framework of T-matrix formulation. Finally, we take the opportunity of the present paper to review the use of the angular spectrum decomposition in light scattering.

Keywords: Generalized Lorenz-Mie theories; beam shape coefficients; angular spectrum decomposition; T-matrix formulation.

## 1 Introduction.

During the last four decades, a vigorous effort has been devoted to the development of generalized Lorenz-Mie theories (GLMTs) describing the interaction between an arbitrary shaped (structured) beam and a set of particles when Maxwell's equations can be solved by using a method of separation of variables, e.g. [1] for a textbook, [2], [3], [4] for recent reviews, and references therein. Among the set of GLMTs, the GLMT *stricto sensu* deals with the case when the scatterer is a homogeneous spherical particle defined by its diameter and its complex refractive index, e.g. [5], [6], and references therein, dating back to 1982 [7].

A recent paper [8] dealt with the same problem as the one handled by the GLMT, but using a brute force computation relying on the angular spectrum decomposition (ASD). This ASD is indeed an efficient way to deal with the evaluation of certain coefficients, called beam shape coefficients (BSCs), which are a specific feature of GLMT. However, using the ASD through what is here called a brute force approach has long been known to be less efficient and accurate. Yet, the authors of [8] claim that their method "provides a significant computational advantage to address any incident waveform in a fast and efficient way". This statement is misleading insofar as it could encourage newcomers to pursue unfruitful directions. The present paper therefore provides comments to [8] and a rebuttal of some of its claims.

The paper is organized as follows. Section 2 provides a discussion of basic features related to the GLMT, specifically concerning the encoding role of the BSCs, and the different classical ways allowing one to evaluate them. Section 3 deals with the ASD, reviews the literature using the ASD in connection with the evaluation of the BSCs, and introduces the brute force approach discussed in Section 4. Section 5 discusses benchmark data challenging the use of brute force approaches. Section 6 is a conclusion.

## 2 Beam shape coefficients.

In GLMT, the structure of the illuminating beam is encoded in a set of BSCs, traditionally denoted as  $g_{n,TM}^m$  and  $g_{n,TE}^m$  with  $n$  ranging from 1 to  $\infty$ ,  $-n \leq m \leq +n$ ,  $TM$  standing for "Transverse Magnetic" and  $TE$  standing for "Transverse Electric". There are several basic ways to evaluate the BSCs. The original "natural" method is by using double or triple quadratures, e.g. [9], [10]. Because this method is computationally prohibitive when the quadratures have to be evaluated numerically, which is for instance the case for Gaussian beams, other computationally more efficient methods have been developed, namely (i) the finite series method [11], [12] which, after having been essentially given up for a long time, became timely again recently, e.g. [13], [14], and references

therein, and (ii) localized models. The history of localized models is reviewed in [15], to be complemented with [16] and [17]. These models have been for a long time the most popular and efficient methods but, recently, their limits in the case of nondiffracting beams, e.g. [18], [19] (among others) and of helical beams [20], [21] (among others) have been pointed out, leading to the renewal of the finite series techniques, e.g. [14] and references therein. In the case of the classical Lorenz-Mie theory, the BSCs become trivial. First, the double set of bi-index BSCs  $g_{n, TM}^m$  and  $g_{n, TE}^m$  reduces to a single set of uni-index BSCs  $g_n$  which are all equal to a  $n$ -independent phase factor which may be taken equal to 1, e.g. [1], pages 152-155.

BSCs were introduced in spherical coordinates to deal with the GLMT *stricto sensu*. Variants are available in circular cylindrical coordinates, e.g. [22], [23], [24], in elliptical cylindrical coordinates, e.g. [25], and references therein, or in spheroidal coordinates, e.g. [26] for a review. In all these cases, BSCs are obtained from the expressions of the electric and magnetic fields expressed in terms of coordinates. In particular, in the case of spherical coordinates, BSCs are more specifically obtained from the radial components of the electric field (for the  $TM$ -coefficients) and of the magnetic field (for the  $TE$ -coefficients).

Another line of investigation relies on the fact that an arbitrary shaped beam may be viewed as a collection of plane waves traveling in different directions, leading to what is usually called the Angular Spectrum Decomposition (ASD) which is the result of using a Fourier transform, e.g. Sections 4 and 5 in [27], and also [28] which extensively discussed the relationship between BSCs and plane wave spectra of ASD. The ASD may lead to deep insights concerning the structure of laser light and scattering properties. For instance, it allows one to gain a deep understanding of the failure of the optical theorem in the case of structured illumination, or the fact that laser light propagates slower than the speed of light, even in vacuum, as described in [29]. Also, it is worth here mentioning that the ASD of a simple approximation to a Gaussian beam (with an elliptical cross section) was derived by Carter [30]. But, more importantly, the ASD may also be useful as a complementary method allowing one to evaluate BSCs as we shall review in the next section, see as well subsection 2.6 in [4].

There also exist a semi-analytical approach to light scattering, valid for arbitrary shaped particles, called Extended Boundary Condition Method (EBCM), e.g. [31], [32]. Most of the applications of EBCM do not deal with arbitrary shaped beams, for instance when studying the scattering properties of airborne particles in atmospheres. Nevertheless, when EBCM is used under the case of structured beam illumination, BSCs may have to be used as well, e.g. subsection 8.1 in [3] quoting papers by Mackowsky and Mishchenko [33], Chen *et al.* [34], [35], Wang *et al.* [36], Zheng *et al.* [37], [38], Briard *et al.* [39], Mishchenko and Dlugach [40], Phillips *et al.* [41], Brzobohaty *et al.* [42], Simpson *et al.* [43], Li *et al.* [44], and Wu *et al.* [45], to be completed by a more recent paper, namely by Vennes *et al.* [46] in which BSCs for Gaussian beams are evaluated by using a localized approximation.

Traditionally, the EBCM has been named the T-matrix method. However, the very definition of what is a T-matrix method implies that the GLMT

as well is a T-matrix method as discussed in [47], leading possibly to confusions when people stating that they are using a T-matrix method actually used a GLMT. In particular, in [47], it is recalled that, for scattering by a sphere, the T-matrix elements are the partial wave scattering amplitudes  $a_n$  and  $b_n$ , i.e. the usual Mie coefficients of the classical Lorenz-Mie theory. This being said, a so-called T-matrix formulation for spherical particles has been published by Khaled *et al.* [48] in 1972. The signature of the fact that they used an EBCM is visible in a sentence saying that "the submatrix elements are integrated over the surface of the scatterer" ([48], page 299). However, because these elements reduce to the Mie coefficients (without needing the use of actual integrals, e.g. their Eq.(31)), it is fair to say that they actually "*implicitly*" used a version of GLMT (restricted to Gaussian beams). For the use of the ASD in the context of EBCM and other related contexts, the reader may refer to subsection 6.1 in [27] with earlier papers by Barber and Yeh [49], Colak *et al.* [50] and Yeh *et al.* [51].

### 3 ASD, structured beams and BSCs.

A background concerning the ASD may be found in [52], [53] and in Goodman, Section 3.7 of [54], and Section 3.10 of [55]. Then, having discussed references related to the EBCM in the previous section, the first paper using "*explicitly*" the ASD for the evaluation of BSCs in the GLMT-framework is, as far as we are aware of, by Doicu and Wriedt in 1997 [56] who started from a vector potential in the framework of the Davis procedure to the formulation of linearly polarized Gaussian beam [57]. The electric field is thereafter obtained as a double quadrature of plane wave components expressed in terms of the Fourier transform of the vector potential. This expression then allows one to return to the classical approach to the evaluation of BSCs (evaluating BSCs from fields) and to express them in terms of single quadratures over a polar angle (which is faster to evaluate than usual double or triple quadratures of the original quadrature method). A fairly significant number of papers were afterward devoted to the ASD in the context of light scattering, particularly concerning the use of the ASD to evaluate BSCs. The literature, likely to be fairly exhaustive, is reported below using a chronological order.

In 2010, Chen *et al.* [58] used an ASD of Bessel beams to obtain analytical expressions of BSCs for Bessel beams of arbitrary order and polarization.

In 2011, Marston derived uni-index BSCs  $g_n$  for quasi-Gaussian Bessel beam superposition described by an ASD and applied them to the scattering of focused beams by spheres [59], [60], while Pawliuk and Yedlin used a 2D-ASD to study the scattering by a family of parallel circular cylinders [61].

In 2013, Lock [62] studied the ASD of a focused Gaussian beam expressed in the Davis fifth-order formulation [63], [64], and examined as well the cases of a transversely confined electromagnetic beam with arbitrary profile in the focal

plane and of a general zero-order Bessel beam. He clarified the significance of the ASD for the evaluation of BSCs, the BSCs of a beam being a superposition of the BSCs of the individual plane wave components in the spectrum. The same year, Hyde *et al.* [65] used an ASD to study the partially-coherent wave scattering from cylinders. The reader interested by partially-coherent wave scattering may refer to a complementary paper on the same issue published in 2015 [66].

The same year (2015), Lock [67] pointed out that the ASD spectrum in general contains both oscillatory and evanescent components, and paid a particular attention to the scattering of the evanescent components of the ASD spectrum of a tightly focused EM beam by a spherical particle. BSCs of the entire beam are evaluated by summing the BSCs of individual plane wave components weighted by their respective amplitude and phase. The same year again, Mitri used an ASD to study the scattering of linear, circular, radial, azimuthal polarized Bessel beams and unpolarized beams by a sphere [68].

In 2016, the same author (Mitri) similarly used the ASD to study the optical pulling force on a magneto-dielectric Rayleigh sphere in zeroth and first-order Bessel tractor polarized beams [69], and to deal with cylindrical particle manipulation using a nonparaxial Hermite-Gaussian light-sheet beam. Moore and Alonso [70] used an ASD to provide an analytical approach to describe the interaction of a sphere with of a monochromatic scalar continuous field, elaborating on a previous work presented in a conference [71].

In 2017, Mitri relied on the use of the ASD to study radiation pressure cross-sections exerted by optical tractor Bessel polarized beams [72], to describe nonparaxial fractional Bessel and Bessel-Gauss auto-focusing light-sheet pincers [73] and to study optical radiation forces and spin torques exerted by such beams on subwavelength prolate and oblate spheroids [74], to investigate nonparaxial scalar Airy light-sheets and their higher-order spatial derivatives with BSCs evaluated by a numerical integration procedure [75] and to study pulling and spinning reversal forces exerted by such beams on a subwavelength absorptive sphere [76]. Also, radiation forces and torques exerted by various light-sheet beams on absorptive circular cylinders have been evaluated in a GLMT framework with BSCs being derived using the ASD [77], and a study has been devoted to the description of self-bending scalar and vector bottle sheets [78]. These are to be completed with Li *et al.* [79] who studied the reflection and transmission of Laguerre-Gauss beams in a dielectric homogeneous slab, relying on the ASD to obtain analytical expressions of the total reflected and transmitted field intensities, Qin *et al.* [80] who carried out a Debye series analysis of internal and near-surface fields for a homogeneous sphere illuminated by an axicon-generated vector Bessel beam in the GLMT framework, with an analytical expression of BSCs being derived using the ASD method, Li *et al.* [81] who used again a GLMT framework to study optical spin torques induced by vector Bessel beams on a light-absorptive sphere of arbitrary size, with BSCs calculated using the ASD method, and Lu *et al.* [82] dealing with the calculation of optical forces exerted on dielectric and metallic microparticles illuminated by a vector Airy beam, with BSCs evaluated by using an ASD.

In 2018, Wen *et al.* [83] investigated the interaction of a vector Bessel-

Gaussian beam (VBGB) with a dielectric sphere in the GLMT framework, derived the electric field of the VBGB using an ASD and obtained analytical expressions of the BSCs. Li *et al.* [84] used an ASD to study the characteristics of reflected and transmitted fields of a vector Bessel beam through multilayered isotropic media. Gong *et al.* [85] studied the scattering of a vector Bessel beam by a charged sphere, with BSCs expressed by analytical expressions using the ASD. Qiu and Shen [86] reviewed different methods to evaluate the BSCs of a Gaussian beam and, in particular, demonstrated that BSCs expressions can be simplified to 1D-quadratures using the ASD.

In 2019, Lu *et al.* [87] employed the ASD to derive the BSCs of circular Airy beams with different polarizations, and to study the scattering of such beams by a spherical particle in the GLMT framework, and Alinezhad and Reihani [88] evaluated BSCs using the ASD to discuss optical forces and trapping efficiency in optical tweezers,

In 2020, Yang *et al.* [89] studied the interaction between an axicon-generated vector Bessel beam and a charged sphere in the GLMT framework, using analytical expressions for the BSCs derived from the ASD method, Wang *et al.* [90] dealt with the ASD applied to evanescent waves and to an exponential behavior of BSCs with respect to the increase of the partial wave number, while Song *et al.* [91] investigated the optical torque exerted on an absorptive dielectric sphere of arbitrary size illuminated by a linearly-polarized Airy light-sheet, relying on the GLMT associated with an ASD approach.

In 2021, Zhang *et al.* [92] studied the scattering of nonparaxial Bessel pincer light-sheets in a GLMT framework, using the ASD to expand the electric field of the beam and expressing the BSCs in terms of 1D-integrals, Cui *et al.* [93] used the ASD to deal with the dynamical characteristics and scattering of Bessel beams upon reflection near the Brewster angle, and Zhang and Shen [94] used the ASD to study circular Bessel beams (CBB) and elliptical Bessel beams (EBB), either standard or produced by the refraction of circular Bessel beams at oblique incidence on an interface. Depending on the case considered, BSCs are obtained as closed forms (CBB) or as 1D-integrals (EBBs).

This section has given evidence that a significant number of papers have used the ASD in connection with the description of structured beams, and with the evaluation of BSCs. These papers used what we may call hybrid approaches. These approaches have to be opposed to other approaches that we shall now consider.

## 4 ASD brute force approach.

Because the ASD provides a decomposition of structured beams in terms of plane waves, there is an "obvious" and conceptually simple way to describe the scattering of structured beams, namely calculate the scattering of each plane wave in the spectrum and then add the contributions of all the plane waves to

obtain the contribution of the structured beam. There are basically two ways to implement such an approach. To describe this dichotomy, let us distinguish between a local (in the Fourier space) Cartesian system  $Oxyz$  attached to a plane wave in the spectrum, with the plane wave propagating along the  $z$ -direction toward positive  $z$ 's, and a global  $OXYZ$  Cartesian system attached to the beam as a whole, with the beam "as a whole" propagating along the  $Z$ -direction toward positive  $Z$ 's.

The first approach is as follows. Let us consider a plane wave component in the spectrum. We may describe the scattering of this plane wave in the global system, and directly obtain the scattered fields generated by this plane wave component in this global system. However, because this plane wave is generically tilted with respect to the direction of propagation of the beam, the BSCs for this plane wave are not the trivial BSCs of the classical Lorenz-Mie theory. They have to be evaluated using transformations of BSCs through rotations of coordinate systems [95], [96], [97], [98], [99]. The BSCs of the whole field are then obtained by the summation of the BSCs of the plane wave components in the spectrum, and the scattering problem may then be solved in a GLMT framework. In principle, this approach might be carried out relying on an analytical approach up to the moment when the summation of the individual BSCs has to be carried out numerically. Kashter *et al.* [8], quoting the above papers describing the BSCs transformations through rotations, criticized the use of such tilted plane waves with tilted BSCs, stating that "the mathematical expression for the transformation is complicated and requires additional calculations [...] which might cause the data processing to be less effective, in terms of duration of processing, compared to the simple case of the Mie calculation". But it has never been claimed that the use of tilted waves in this first approach is recommended. In any case, it does not represent the most convenient way of using GLMT with global BSCs  $g_{n,TM}^m$  and  $g_{n,TE}^m$ , so that the aforementioned statement by Kashter *et al.* [8] is deeply misleading, and gives an implicit, and erroneous disqualification of GLMT. This misleading presentation of the GLMT is repeated in the conclusion in which it is stated that, in GLMT, "the scattering is calculated by the beam shape coefficient for each phase pattern (including tilted plane wave)". This misses the point that the GLMT actually uses global BSCs allowing one to solve the scattering problem in a single shot, in particular when the two-dimensional integral of the BSCs of the individual plane waves multiplied by the beam weighting coefficients can be evaluated analytically for the Davis model of a focused beam, and for Bessel beams [62], [100], [101], and, more generally, when the global BSCs are evaluated by using one of the techniques listed in Section 2 (quadratures in cases where they can be analytically carried out, finite series, localized models).

The second approach is as follows. We may solve the scattering problem for each plane wave component in its local coordinate system, that is to say in a configuration which is exactly the one of the classical Lorenz-Mie theory. The scattered fields from each component are afterward obtained in the global system by a rotational transformation and, eventually, all the rotated fields are summed up to obtain the fields scattered by the structured beam as a whole.



As we are now going to discuss, it may be viewed as a brute force approach, and is used in the paper by Kashter *et al.* [8], which is the motivation of the present paper.

There are *a priori* two complications in the brute force approach. The first one is related to the fact that, in general, the spectrum contains an infinite number of plane wave components. Therefore, in numerical evaluation, we must be content with a limited number of computations, using a discrete finite sample of components taken from the infinite number actually present in the spectrum. We must therefore expect a problem of accuracy. Because the scattering patterns are more and more complicated when the parameter size of the scatterer increases (the number of lobes in the pattern is roughly equal to the size parameter of the scattering sphere), we must expect an increasing loss of accuracy when the size of the scatterer increases.

For the second complication, we have to remark that many scattering calculations have to be carried out, depending on the number of plane waves sampled from the spectrum. This is in contrast with the GLMT where, because the structure of the illuminating beam is encoded in the BSCs, the scattering problem can be solved in a single numerical shot. In other words, the brute force approach is a many-steps approach. A first step requires a sampling of plane waves. The second step requires many Mie computations. And a third step requires the combination of the results of the many Mie computations carried out in the previous step. Therefore, it is not difficult to conjecture that the brute force approach will be computationally more intensive than the GLMT-approach where the BSCs of the shaped beam are analytically known or may be evaluated by using efficient techniques, e.g. Section 2 again.

These two complications find their origin in the fact that there is no formal effort to summarize and sum up the effects of all the components in the plane wave spectrum into a simple and single formulation. This comment justifies the descriptive name of "brute force approach" to the numerical integration of the spectrum over plane wave components. We are now going to check *a posteriori* (after numerical experiments) the *a priori* statements concerning lacks of computational efficiency and of accuracy.

To illustrate the computational inefficiencies, we may return to computations carried out by one of us in 1995 [102] concerning comparisons between different ways of achieving scattering calculations, using a Compaq 386-33 MHz personal computer equipped with a Weitek numerical processor. For certain typical parameters (in particular for a sphere diameter  $d = 100 \mu\text{m}$ , and an incident wavelength  $\lambda = 0.6328 \mu\text{m}$ ), and 361 values of the scattering angle  $\theta$ , GLMT-computations for off-axis Gaussian beams with the BSCs evaluated analytically for a Davis model of a focused Gaussian beam required 195 s. A corresponding classical Lorenz-Mie calculation required only 3 s, that is to say the off-axis Gaussian beam program runs almost 70 times slower than Mie theory (at least for the studied parameters). Note that these timing figures were valid 26 years ago and that to-day they would be much smaller. Nevertheless, from this, we might already infer that a brute force approach using a sample of more than 70 plane waves will run slower than GLMT. We may refine this

evaluation by recalling that the number of lobes in the scattering pattern is roughly equal to the size parameter. For a diameter  $d = 100 \mu\text{m}$ , and an incident wavelength  $\lambda = 0.6328 \mu\text{m}$ , the size parameter is  $\alpha = \pi d/\lambda \simeq 500$ . If, as a rule of thumb (admittedly unsecure), we assume that we need 10 plane waves to correctly sample one lobe, we would need 5000 Mie calculations, so that the brute force approach would roughly run  $5000/70 \simeq 70$  time slower than the GLMT. The computational intensiveness of the use of a brute force approach is also mentioned by Moore and Alonso [103].

Concerning accuracy, we may consider the paper by Kashter *et al.* [8]. This paper reports on what could be seen as an improvement with respect to computational times, namely the authors generated a database of Mie solutions which are retrieved before summing up all the sampled solutions with a "careful consideration of amplitude, phase, polarization, and propagation direction in the initial plane wave decomposition", although no comparison of computational times with respect to GLMT are provided. But the accuracy problem remains a significant one. The authors indeed acknowledged that "the effect on the overall numerical error due to a limited number of samples [...] has not been accurately studied and will be shown to be important in nonparaxial cases". They also commented that they "can estimate the impact of the sampling densities which will be shown to play a significant role in the case of large tilting angles". More specifically, errors may occur for at least two reasons.

First, the Mie solutions for individual plane wave components of the ASD are stored in the database by using a discrete representation, with a limited angular sampling density. Relative errors in total scattering for Mie computations are then evaluated by comparing scattering cross-sections rigorously evaluated in terms of a summation involving Mie coefficients (Eq.(4) in [8]) and evaluated using the stored discrete representation (Eq.(7) in [8]). The sampling density is characterized by the number  $L = 2L_1 = L_2$  in which  $L_1$  and  $L_2$  are the number of discrete samples along spherical angular coordinates  $\theta$  and  $\varphi$  respectively. For a size parameter equal to 100 (size diameter of about  $16 \mu\text{m}$  for a wavelength of  $0.5 \mu\text{m}$ ), i.e. for the largest size parameter considered in the figures of [8] displaying the numerical error estimations, the errors are found to reach up to 16% for  $L = 128$  and require to use  $L = 512$  to decrease to 1%. By comparison, standard Mie scattering codes are numerically accurate to  $\sim 10^{-6}\%$ . We may anticipate that these errors would go on increasing significantly for larger size parameters. Furthermore, the procedure used by Kashter *et al.* [8] requires a rotational transformation from the local Mie coordinate system associated with the direction of the plane wave component to a global coordinate system associated with the direction of the incoming beam viewed as a whole. The errors mentioned above evaluated in the Mie coordinate system then propagate to the global system and are displayed in Fig.(6) of [8] where they can become significant depending on the value of an Euler angle  $\beta$  associated with the rotation between the local and the global coordinate systems. For instance, they can reach 50% for  $\beta = 45^\circ$  and a size parameter equal to 10, corresponding to a particle diameter equal to about  $1.6 \mu\text{m}$  and to the largest size parameter considered in Fig.6) of [8]. Concerning the results for the scattering

on an arbitrary incident waveform in their section 4, the authors mention that the maximal error involved, corresponding to a maximal value of  $\beta = 4.05^\circ$ , is approximately 1.4%. Again, this is far more important than the standard accuracy of Mie codes.

Furthermore, it does incorporate the effect of using a limited number of plane waves extracted from the sampling of the infinite number of plane wave components present in the ASD, i.e. of the necessity of sampling the ASD. If one does this in the context of Discrete Fourier Transforms instead of a Continuous Fourier Transform, so as to make use of the speeding up of the process by the Fast Fourier Transform algorithm, an additional source of error is due to the discretization of the angular spectrum. This source of error is not convincingly discussed in [8].

Actually, for a better evaluation of the quality of the results, we would need systematic comparisons between GLMT-computations and Kashter's method computations, both from the point of view of computational times and from the point of view of accuracy. Such comparisons are not available. It is furthermore to be noted that, in the visible range, a size parameter equal to 10 (the largest value considered in Fig.6b) corresponds to a particle diameter equal to  $1.6 \mu\text{m}$ . This may be viewed as a small particle, too small for many applications such as for the measurements of spray droplets by phase-Doppler anemometry, e.g. [104], [105], when particle diameters equal to  $100 \mu\text{m}$  (size parameter equal to about 600) or more may have to be considered for accurate measurements. In the case of Gaussian beam illumination, the authors of [8] stated that the largest waist radius considered in their Fig.8 (i.e.  $w_0 = 10 \mu\text{m}$ ) is 20 times larger than the sphere radius, corresponding to a sphere diameter equal to  $1 \mu\text{m}$  which is indeed very small for many applications, although it may be sufficient for some other applications. Nevertheless, even for the small size parameters considered, the error calculation of the author's method appears to be quite crude, and is significantly higher than that of standard GLMT computations.

Already, in 1988, fast and accurate GLMT-computations for particle diameters up to  $100 \mu\text{m}$  could be achieved on microcomputers when the BSCs of the incident beam were able to be analytically evaluated. In view of these remarks, the comments of Kashter *et al.* according to which they provided a method with "a significant computational advantage to address any incident waveform in a fast and efficient way" is at least premature, to say the least. Another example of a brute force approach is by Zakowicz [106] without any comparison with GLMT in terms of computational times and accuracy being provided, but the author did not make any unsubstantiated claim concerning the efficiency of his approach.

## 5 Benchmark data.

Any proposal for a brute force approach should be compared with GLMT-results which would provide benchmark data, in terms of computational times

and of accuracy, before being acceptable, and/or considered as an approach superior to the GLMT-approach or even as an alternative. Such benchmark data may be obtained from GLMT-computer programs which are available as an additional material to [1] and which can be downloaded from <http://extras.springer.com>. Capabilities of GLMT are illustrated below using figures which have been produced with an independent GLMT-code written with Matlab from Xidian University, and exhibiting scattering diagrams (more specifically displaying  $|\mathbf{E}|^2$ ) in arbitrary units for particles illuminated on-axis by Gaussian beams. We strongly believe that such results could hardly be obtained in a competitive way by using a brute force approach. In particular all these figures are for a particle diameter  $d$  equal to 2 mm, for an incident wave-length  $\lambda$  equal to 0.6328  $\mu\text{m}$ , corresponding to a size parameter  $\pi d/\lambda$  about equal to 10 000. Furthermore, the beam waist radii are  $w_0 = 5$  mm and 0.75 mm, the first one corresponding to a slightly converging beam with a ratio  $(2w_0)/d = 5$  larger than 1, and a small beam confinement factor equal to  $s = 1/(kw_0) \approx 2.10^{-5}$  close to the one of a plane wave, while the second one corresponds to a more tightly converging beam with a ratio  $(2w_0)/d = 0.75$  smaller than 1, and a larger beam confinement factor equal to  $s = 1/(kw_0) \approx 10^{-4}$ . Figs 1 and 2 are for a homogeneous sphere, with a refractive index of 1.33, for  $w_0 = 5$  mm and 0.75 mm respectively. Since GLMT-approaches and brute force approaches are not limited to homogeneous particles, these figures are complemented with Figs 3 and 4 which are for a multilayered sphere with 500 layers, a refractive index decreasing linearly from 1.36 at the surface down to 1.33 at the core, again for  $w_0 = 5$  mm and 0.75 mm respectively.

We believe that carrying out efficiently similar computations with a brute force approach would represent a challenge. Another issue is that GLMT-computations do not only deal with the computation of scattering fields, but also with the computations of internal fields (including resonances), of scattering, absorption, and extinction cross-sections, and of radiation forces and torques. The efficiency of brute force approaches for the evaluation of such quantities should be examined as well.

## 6 Conclusion.

The recent paper by Kaster *et al.* [8] provides an opportunity to discuss two different approaches to the scattering of structured beams by regular particles, either (i) by using BSCs which summarize all the information contained in the infinite number of plane waves present in the ASD or (ii) dealing individually with a number of individual scattering computations for a sample of

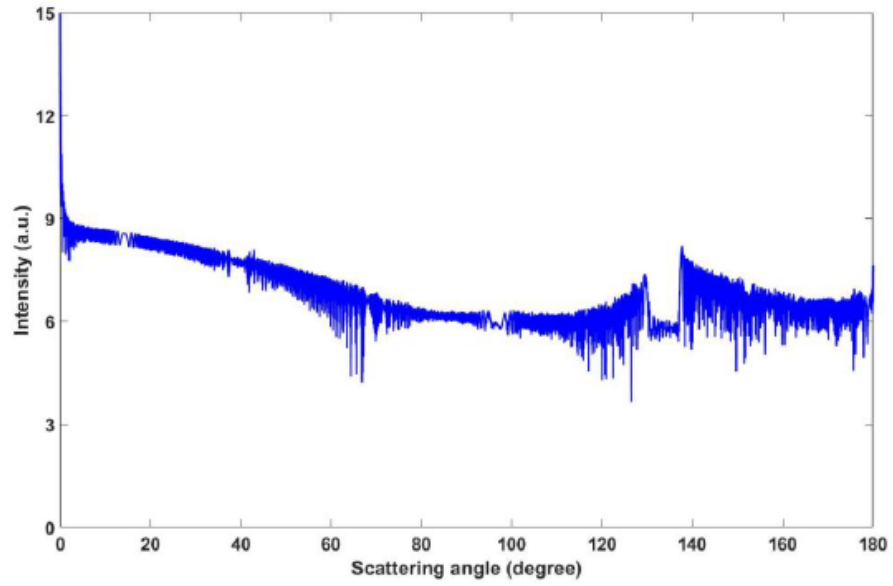


Figure 1: Homogeneous sphere. Beam waist radius: 5 mm

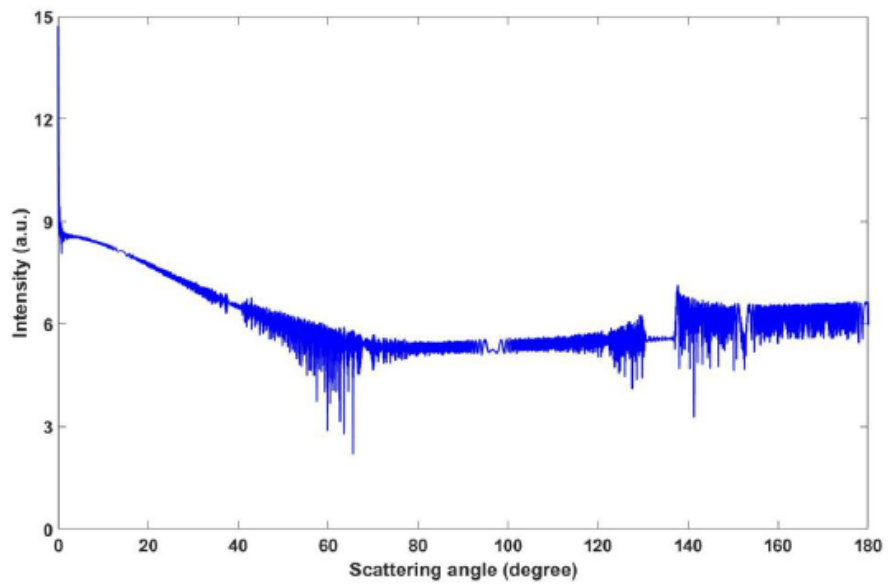


Figure 2: Homogeneous sphere. Beam waist radius: 0.75 mm

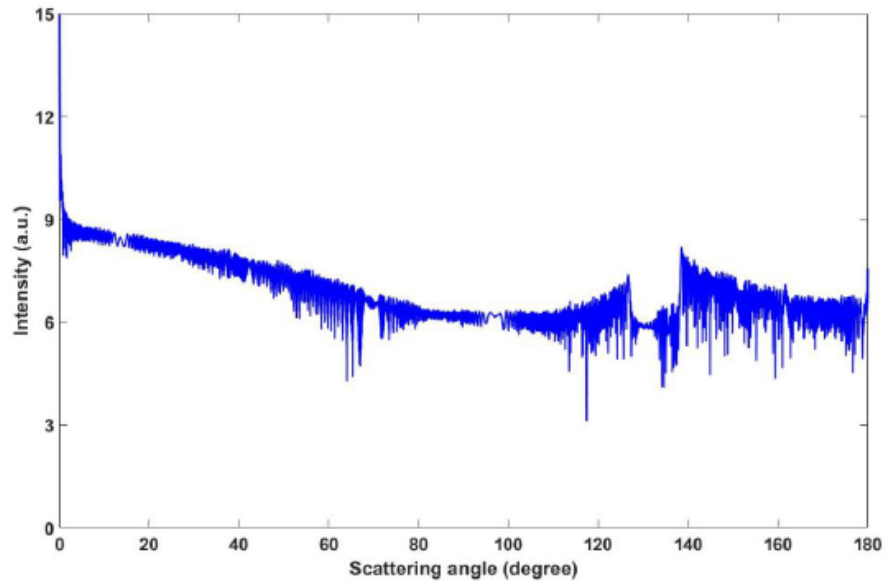


Figure 3: Multilayered sphere. Beam waist radius: 5mm

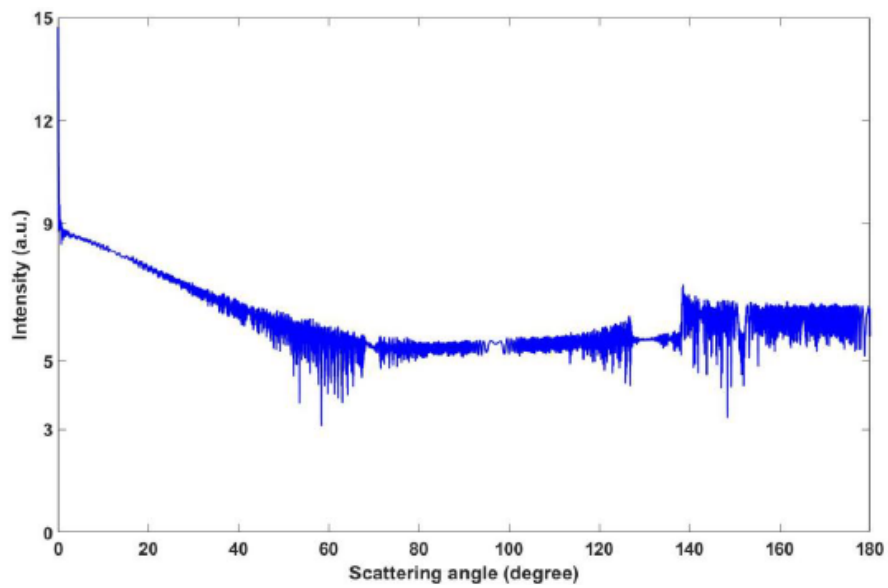


Figure 4: Multilayered sphere. Beam waist radius: 0.75 mm

plane wave components before summing up the results of these individual computations. Contrarily to what is strongly suggested in [8], there is no evidence that the brute force approach is definitely superior, or even is simply a competitive alternative to the use of a GLMT. As far as the contrary is not clearly demonstrated, we believe that a brute force approach is not incorrect, but is a relatively computationally inefficient and inaccurate approach. Finally, we have taken the opportunity of the present paper to review the use of the ASD in light scattering.

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