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A new procedure for the determination of structural characteristics of sandwich plates in medium frequencies

Christophe Droz^{a,1}, Olivier Bareille^a, Mohamed N. Ichchou^a

^a*École Centrale de Lyon, 36 Avenue Guy de Collongue, 69134 Écully Cedex, France*

Abstract

This paper presents a new wave-based material identification strategy for sandwich panels, able to estimate their tensile and equivalent shear moduli from single shot measurements. The proposed algorithm is based on the determination of the maximal wave speed in the bending-to-shear transition bandwidth. A definition is proposed for the transition phenomenon resulting from the conversion of flexural waves in the composite waveguide, and the existence of a local energy velocity maxima is demonstrated using the 4th-order sandwich theory. Analytical expressions are derived for the equivalent bending and shear parameters in terms of the transition frequency and the maximal group velocity. An practical iterative procedure is also proposed for the experimental identification of the transition bandwidth, while a Hilbert Transform magnitude is employed to measure time of arrivals of the flexural wave pulses. The robustness of the method is highlighted using a numerical case study and an experimental validation is finally conducted on a sandwich panel with composite faces. Results are compared with static measurements and a wave-based technique (Inhomogeneous Wave Correlation) and show significant accuracy improvements while ensuring reduced post-processing times.

Keywords: Honeycomb, sandwich panel, inverse identification, group velocity, wave

1. Introduction

Composite assemblies have become a crucial component in transportation and aerospace industries due to their exceptional strength-to-weight ratios. Sandwich panels have the most widespread utilization due to their wide range of possible configurations. They are usually composed of a soft and lightweight core made of foam or honeycomb construction, surrounded by thin fibre-reinforced skins. However, the design and optimization of these structures in vibroacoustic environments remain challenging, especially due to the difficulty to perform an accurate modelling of the composite assemblies in the medium frequency range [1], too high for deterministic finite element models to apply while the main assumptions of energy distribution underlying the Statistical Energy Analysis (SEA) are not met [2]. Yet, although the design of structurally advanced

Email address: christophe.droz@gmail.com (Christophe Droz)

¹Current address: KU Leuven, Department of Mechanical Engineering, Division PMA, Celestijnenlaan 300 - box 2420, Leuven, Belgium

composites does not usually requires models of the local scale's components (such as the aramid fibers of a honeycomb construction, or the resin of its skins), accurate predictions of the overall dynamical behaviour at the global scale are fundamental.

Although a large number of mechanical characteristics of sandwich's components can be found in the literature, the increasing research effort on the development of composite skins and structurally advanced core's constructions yields a challenge in terms of prediction of the equivalent core's and skin's material parameters [3]. Besides, the same nominal sandwich panel (same core size, materials and stacking sequence) can exhibit different mechanical characteristics, depending on the manufacturing process or epoxy resin density, which can reach more than 30% of the total mass. Additionally, the use of material tables defined using static experiments for the modelling of sandwich panels in the vibroacoustic range can become an issue. Indeed, although frequency-independence of the mechanical parameters is a fair assumption in a given bandwidth, static and frequency identifications often show great discrepancies.

Various methods were devoted to the determination of effective core properties of sandwich panels [4, 5], Karakoç and Freund [6] proposed an experimental procedure to determine the compliance matrix of Nomex structures. For dynamical identification, modal methods are often employed, Matter et al. [7] proposed an identification technique for evaluating the elastic and damping properties of sandwich laminates with soft cores. Other methods based on free modes and comparisons with analytical and FE methods were also developed [8, 9, 10] to determine elastic properties of sandwich structures. Recently, wave-based methods were developed for the determination of these equivalent properties. They advantageously avoid the uncertainties produced by the boundaries on the modal responses and enable effective characterization at higher frequencies, where modal overlap becomes too important to use modal data (see [11] for an example of high-frequency characterization of polypropylene composites).

The propagation of guided waves in sandwich structures has indeed been the subject of intense research in the recent years. Many studies are concerned with exploiting wave dispersion characteristics (also called dispersion curves) to understand wave's physics in heterogeneous structures and design composite waveguides with advanced capabilities [12, 13]. One can cite, among other: the design of auxetic pyramidal cores [14], the vibroacoustic optimization of sandwich's geometries [15, 16], or the inclusion of internal resonators to improve damping properties [17] and a very large number of applications [18, 19] can also be found for defect detection in sandwich structures.

For these new types of composites, studies were conducted on experimental wave-based methods for the identification of effective elastic properties of sandwich constructions: an inverse wave approach for estimating structural wave parameters was proposed by McDaniels and Shepard [20]. The Inhomogeneous Wave Correlation (IWC) method was proposed in [21] for the identification of the dispersion curves in two dimensions, also called k -space. Least mean square algorithm is then applied to sandwich plate theory to obtain the homogenized panel properties. Despite a rather good accuracy, these methods require to measure and post-process the entire displacement field to extract the wavenumber space, leading to expensive and time-consuming characterizations. One of the consequences is a possible inaccuracy when using standard sandwich material tables provided in the literature.

This paper addresses the genuine need of wave-based mid-frequency identification methods for sandwich panels, involving simple experimental set-ups. A strategy is investigated for the

elastic parameter identification of sandwich panels using Time-of-Flight (ToF) measurements [22, 23], since measuring the energy velocity in plates is a mature technique extensively used in the literature (interested reader may refer to [24, 25, 26] or Sikdar et al. [27] for a recent application to honeycomb composites). The proposed method focuses on a phenomenon called *transition*, which is a specific type of wave conversion in waveguides [28, 29], occurring in sandwich structures subjected to flexural vibrations.

In section 2, the practical consequences of the transition on flexural waves' velocities are studied and an definition based on 4th-order sandwich theory is proposed for the transition frequency. The existence of a local velocity maximum around this frequency is exploited to dissociate the shear and bending contributions to the wave motion in the composite. An identification procedure based on transient wave pulses velocities is proposed in section 3 and applied to a sandwich waveguide. The numerical results are compared with analytical results and the accuracy of the identification method is discussed. In section 4, an experimental case study is proposed. The mechanical characteristics of a sandwich panel involving a Nomex honeycomb core surrounded with carbon-reinforced skins are estimated using the proposed method and compared with the IWC results and manufacturer's data.

2. Transition frequency in symmetric sandwich

2.1. Theoretical background

The governing equations for the anti-symmetric motion of sandwich waveguides are described hereby [30, 31]. Consider a sandwich plate whose normal deflection is assumed connected to the core and shear rotations β and γ . The total deflection w in the direction x produced by β and γ is:

$$\frac{\partial w}{\partial x} = \beta + \gamma \quad (1)$$

Considering a linear Lagrangian $H = T - U$, where U and T stand for the potential and kinetic energies, Hamilton's principle yields the following variational expression for a conservative system :

$$\delta \int H dt = 0 \quad (2)$$

Combining Eqs.(1) and (2) gives the differential equations for β and w :

$$2D_2 \left(\frac{\partial^4 w}{\partial x^4} - \frac{\partial^3 \beta}{\partial x^3} \right) - G_c h_c \left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \beta}{\partial x} \right) + \mu \frac{\partial^2 w}{\partial t^2} = 0 \quad (3)$$

$$2D_2 \left(\frac{\partial^3 w}{\partial x^3} - \frac{\partial^2 \beta}{\partial x^2} \right) - D_1 \frac{\partial^2 \beta}{\partial x^2} + I_p \frac{\partial^2 \beta}{\partial t^2} - G_c h_c \left(\frac{\partial w}{\partial x} - \beta \right) = 0 \quad (4)$$

where G_c is the core's shear modulus, h_c and h_s are the core's and skin's thickness. The overall flexural stiffness D_1 , the layer's flexural stiffness D_2 and the mass moment of inertia per unit width

I_p and mass per unit area μ are written:

$$D_1 = E_c \frac{h_c^3}{12} + E_s h_s \left(\frac{h_c^2}{2} + h_c h_s + \frac{2h_s^2}{3} \right), \quad D_2 = E_s \frac{h_s^3}{12}, \quad (5)$$

$$I_p = \rho_c \frac{h_c^3}{12} + \rho_s h_s \left(\frac{h_c^2}{2} + h_c h_s + \frac{2h_s^2}{3} \right), \quad \mu = 2h_s \rho_s + h_c \rho_c. \quad (6)$$

where E_s and ρ_s are the skin's Young modulus and density, and ρ_c is the equivalent core's density.

Eliminating the angular displacement β in Eqs.(5) and (6) and assuming plane waves propagate along the x-direction : $w(x, t) = w_0 e^{-j(\omega t - kx)}$, with k the wavenumber and ω the circular frequency yields:

$$2D_1 D_2 k^6 - 2D_2 I_p k^4 \omega^2 - (\mu + 2D_2 \mu + I_p G_c h_c) k^2 \omega^2 + G_c h_c D_1 (k^2 - \mu \omega^2) + I_p \mu \omega^4 = 0 \quad (7)$$

Two main assumptions are introduced: the rotation inertia is negligible compared with the shear effects $I_p \omega^2 \ll G h_c$, and the bending stiffness per unit width of the skins are small in comparison with the one of the plate: $D_2 \ll D_1$. Therefore the 4th-order dispersion relation in the sandwich becomes:

$$S k^4 - \mu \omega^2 k^2 - \mu \omega^2 S / D = 0 \quad (8)$$

where $S = h_c G_c \left(1 + \frac{h_s}{h_c} \right)^2$ and $D \simeq E_s h_s \left(\frac{h_c^2}{2} + h_c h_s + \frac{2h_s^2}{3} \right)$ are the transverse shear rigidity and bending stiffness of the plate. Note that Eqs.(1-8) are derived for propagation in x -direction. In the case of a honeycomb sandwich with composite face sheets, the equivalent bending and shear coefficients depend on the considered direction $k_\theta = f(\mu, S_\theta, D_\theta)$.

2.2. Asymptotic solutions for shear and bending modulus identification

The solution of Eq.(8) gives the flexural wavenumber:

$$k^2 = \frac{\mu}{2S} \left(\omega^2 + \omega \sqrt{\omega^2 + \frac{4S^2}{\mu D}} \right) \quad (9)$$

When D , S and m are fixed, two asymptotic behaviours can be distinguished: at low frequency, the sandwich plate is ruled by the skins' Young modulus and has classical plate behaviour. The 'bending' wavenumber and group velocity are written:

$$k_{bending} = k(\omega \rightarrow 0) = \sqrt[4]{\frac{\mu}{D}} \sqrt{\omega}, \quad ; \quad c_g(\omega \rightarrow 0) = 2 \sqrt[4]{\frac{D}{\mu}} \sqrt{\omega} \quad (10)$$

while in higher frequencies the sandwich's motion is mainly governed by the shear coefficient S . The asymptotic wavenumber and group velocity become:

$$k_{shear} = k(\omega \rightarrow \infty) = \sqrt{\frac{\mu}{S}} \omega, \quad ; \quad c_g(\omega \rightarrow \infty) = \sqrt{\frac{S}{\mu}}. \quad (11)$$

Between these two frequency domains, both the shear and bending modulus contribute to the flexural motion. In sandwich structures, the *transition frequency* refers to the passage from a flexural wave governed by the skins' stiffness to one governed by the core's transverse shear. The asymptotic expressions given in Eqs.(10) and (11) were used by Ghinet and Atalla [32] and more recently in [15, 33, 34] to give interpretations of the transition in sandwich panels. In [33], the transition frequency ω_T was defined by the geometric intersection between the asymptotic wavenumbers, $k_{bending}(\omega_T) = k_{shear}(\omega_T)$, leading to a transition frequency $\omega_T = \frac{S}{\sqrt{\mu D}}$. It should be noted that a similar definition based on the asymptotic modal densities [32] (or group velocities) yields: $\omega_T = \frac{1}{2} \frac{S}{\sqrt{\mu D}}$.

2.3. Transition and maximum wave velocity

The definitions given above for the transition can be used for example to define the bandwidth on which regression models based on Eqs.(10) and (11) can be applied. It avoids the use of non-linear regression models based on Eq.(8) but requires a priori knowledge of the transition to ensure the accuracy. Although the effective bending modulus can be approximated from the dispersion curves below the transition, this is not the case for the shear modulus. Indeed, the shear velocity cannot be measured in sandwich plates due to the presence of the second transition, shown in figure 1a. This transition is associated with a localization of the flexural wave in the skins and occurs once the hypothesis $D_2 \ll D_1$ is no longer applicable. Consequently the asymptotic velocities cannot be measured and used to determine a core's shear coefficient.

However, it can be seen that the group velocity reaches a local maximum at 1 kHz. Since the group velocity describes the average local velocity of energy transport in the wave diffusion process, the physical meaning of this value can be seen as the maximum energy flux for a normalized energy density [35].

Since the shear velocity is not reachable, this maximum of energy velocity is the only point of the dispersion curves which is independent of the bending stiffness within the transition bandwidth (see figure 1b.), hence can be considered a pure core's shear effect. In the following, the transition is defined by the frequency at which the wave velocity reaches this local maximum. With this definition, the group velocity can be written from Eq.(9):

$$c_g = \frac{\partial \omega}{\partial k} = \frac{2\Omega \sqrt{\frac{\mu}{2S}(\omega\Omega + \omega^2)}}{\frac{\mu}{2S}(2\omega\Omega + \Omega^2 + \omega^2)}, \quad (12)$$

where $\Omega = \sqrt{\omega^2 + \omega_0^2}$ and $\omega_0 = \frac{2S}{\sqrt{\mu D}}$. The transition frequency defined above requires the frequency derivative of the velocity, written:

$$\frac{\partial c_g}{\partial \omega} = \frac{\omega^2 \Omega - \Omega^3 + 2\omega_0^2 \omega}{(\omega^2 \Omega + \Omega^3 + 2\omega_0 \omega^2 + 2\omega^3) \sqrt{\frac{\mu}{2S}(\omega\Omega + \omega^2)}}$$

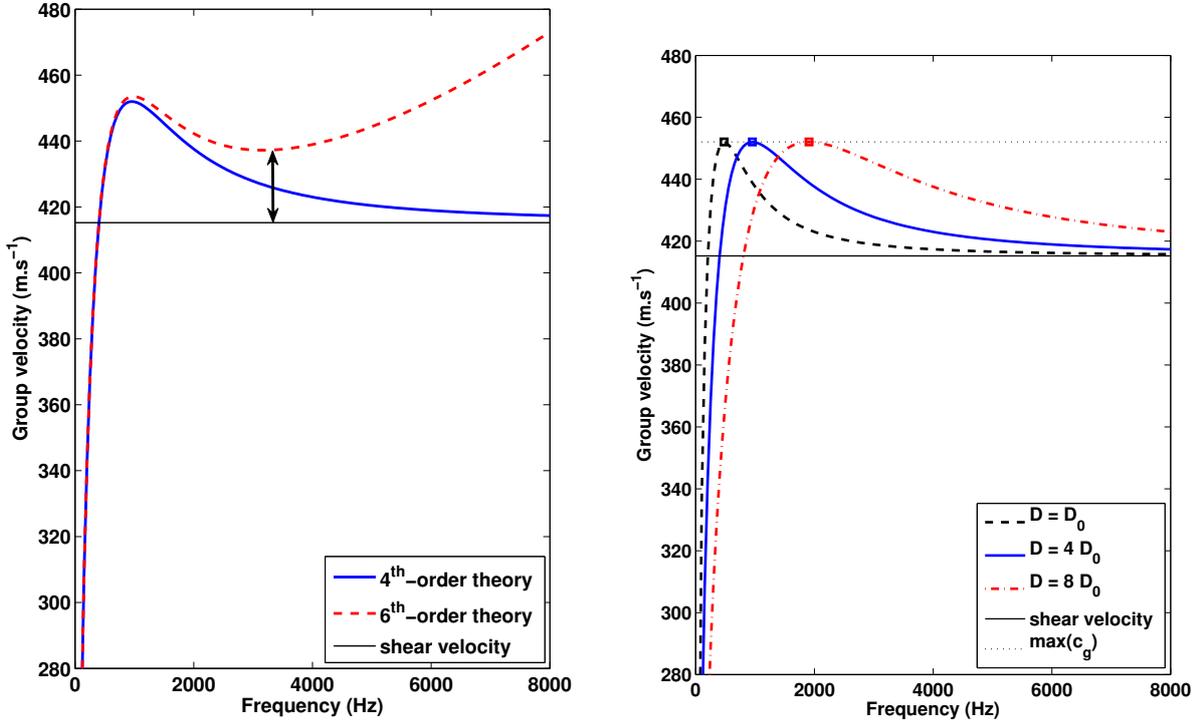


Figure 1: (a) Comparison of the group velocities obtained through 4th- (—) and 6th-order theory (- - -). Sandwich parameters: $D/\mu = 1100m^4.s^{-4}$, $S/\mu = 172420m^2.s^{-2}$, $h_c = 10mm$, $h_s = 0.6mm$. (b) Influence of the bending modulus ($D_0 = 2653Nm$) on the frequency and value of the maximal group velocity. Shear velocity: $c_{shear} = 415.2m.s^{-1}$, maximum velocity $c_{max} = 452m.s^{-1}$.

Solving $\frac{\partial c_g}{\partial \omega} = 0$ gives the solution for the transition frequency:

$$\omega_T = \frac{2S}{\sqrt{3\mu D}}. \quad (13)$$

Therefore the maximum group velocity is written:

$$c_g(\omega_T) = \frac{4}{3} \sqrt{\frac{2S}{3\mu}}. \quad (14)$$

Noteworthy, the velocity at the transition is independent of the bending stiffness D , as observed in figure 1b. The shear coefficient S can therefore be determined without the use of regression models requiring the entire dispersion curves in the considered bandwidth. Besides, the asymptotic and transition velocities are related by the coefficient $\frac{c_g(\omega_T)}{c_s} = \left(\frac{4}{3} \sqrt{\frac{2}{3}}\right)$ which is independent of the sandwich material parameters. This standard exceedance of approximately 8.87% of the shear

velocity can lead to a significant underestimation of the time-of-flight in the transition bandwidth. As the maximal energy velocity of a wave pulse can be easily determined in a given frequency range, the shear coefficient can be derived using the following expression:

$$S = \frac{27\mu}{32} \max(c_g)^2. \quad (15)$$

The bending modulus can then be determined using Eqs.(13) and (15):

$$D = \frac{4S^2}{3\mu\omega_T^2}. \quad (16)$$

Both S and D can therefore be determined in the direction of propagation using the maximal velocity and its frequency in a sandwich panel. In the following, a numerical validation is proposed for a sandwich waveguide whose material characteristics are known.

3. Numerical validation

In this section, the group velocity and transition frequency are determined in a sandwich structure to identify its equivalent mechanical characteristics. Consider the waveguide defined in table 1. Vertical motion is applied at $x = X_0$ to produce a flexural motion while the displacements are acquired at $x = X_1$ where $X_1 - X_0 = 3$ m. The excitation signal is a tone burst of amplitude $U_0 = 10^{-6}$ m involving n_0 cycles at the frequency f_0 with Hanning window defined by:

$$u(t) = \begin{cases} U_0 \sin\left(\frac{\pi f_0 t}{n_0}\right) \sin(2\pi f_0 t), & t \leq \frac{n_0}{f_0} \\ 0, & t > \frac{n_0}{f_0} \end{cases} \quad (17)$$

Material	Thickness	Density (kg.m ⁻¹)	Young modulus (GPa)	Shear modulus (MPa)
Skin	0.3 mm	1451	$E_s = 81$	$G_s = 2780$
Core	25 mm	53	$E_c = 5.23$	$G_c = 50$

Table 1: Dimensions and material characteristics of the sandwich beam. Note that the core is described as an homogeneous medium. The beam's width is $l_y = 10$ mm and the damping is not considered in this model.

Note that the value of n_0 has a significant influence on the bandwidth of the excitation spectrum, hence on the possible spatial dispersion of the wave pulses. An illustration of the acquisition procedure is given in figure 2. The propagation of wave pulses with different frequencies and durations is computed using FEA on a distance of 3 m. The Hilbert Transform $H[x(t)]$ of a time signal $x(t)$ is commonly used to calculate the main attributes of $x(t)$. It is defined by:

$$H[x(t)] = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau} d\tau. \quad (18)$$



Figure 2: Illustration of the numerical procedure for measuring the group velocities in the beam. Simulations are conducted using the FEA package Abaqus CAE V6.13 with Abaqus/Explicit solver. Linear C3D8 elements are used and a minimum mesh density of 15 elements per wavelength is applied, leading to a 120000 elements model at 13.2 kHz. Note that the choice of a beam is motivated by the elevated computational times associated with 2D transient simulations.

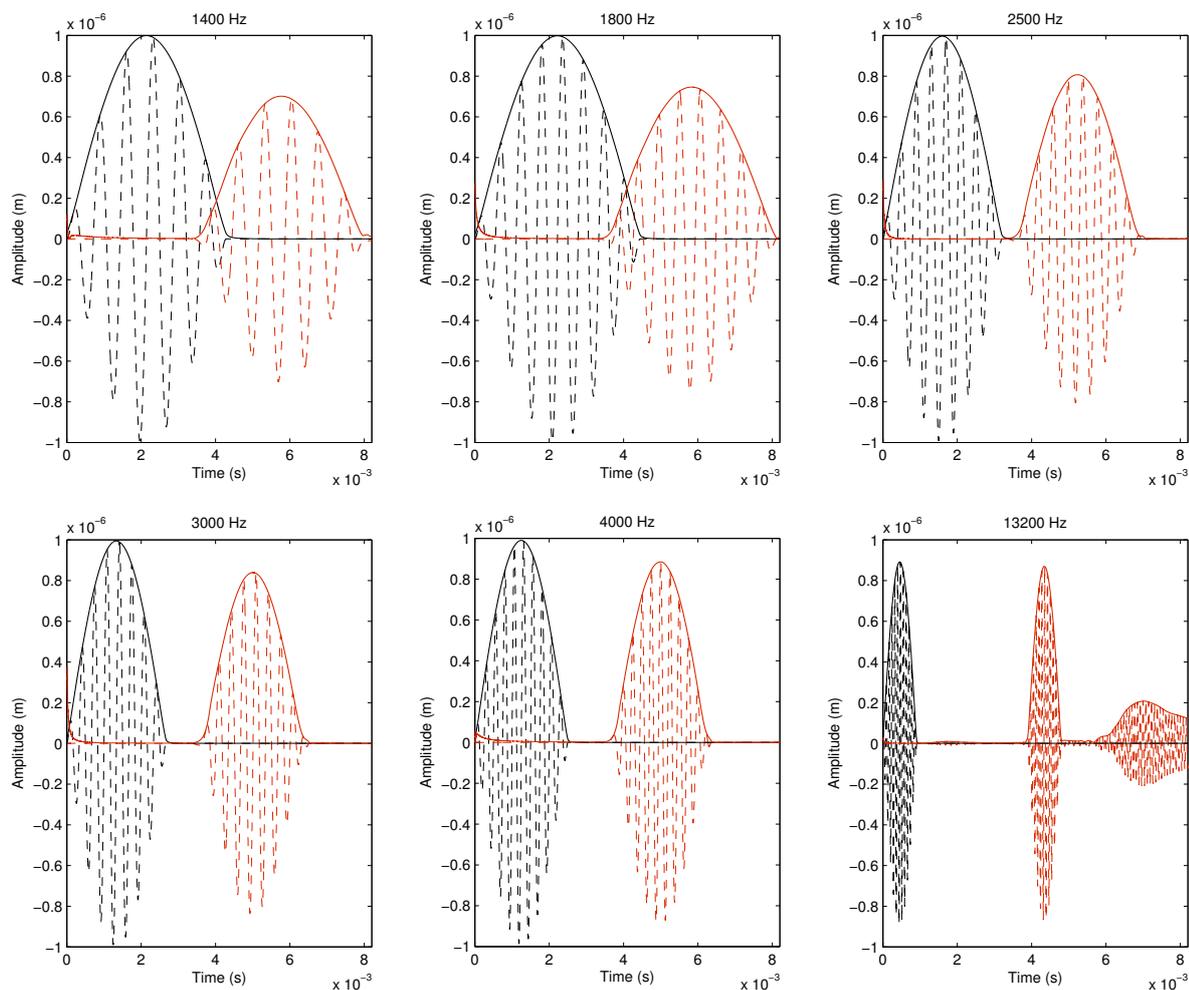


Figure 3: Time acquisition at $x = 0$ (- -) and $x = 3m$ (- · -) in the sandwich waveguide. The number of cycles is: 6 at 1400 Hz, 8 between 1.8 kHz and 3 kHz, 10 at 4 kHz and 12 at 13.2 kHz. The time of flight is measured at the maximal amplitude of Hilbert transform (solid lines) signal.

The signal envelope is determined at emission, X_0 and arrival, X_1 using Hilbert transform while the time delay is defined by the time difference between the maximal amplitudes of the envelopes.

Results are shown in figure 3 for six wave pulses with different frequencies. At 1.4 kHz the pulse exhibits higher dispersion due to the reduced number of cycles, while negligible dispersion occurs above 4 kHz. A converted wave is observed at 13.2 kHz, due to the application of a vertical motion at the free edge, which is not associated with the shear wave motion. The accuracy obtained on the wave's speed clearly increases with frequency, although the absence of reflected and converted waves at low frequencies allows a reliable determination of the envelope's maximum. Exact values for the equivalent bending and shear coefficients can be obtained from Eqs.(5) and (6): $D = 7,778 \text{ Nm}$, $S = 1.2805 \times 10^6 \text{ N/m}$ and the mass per unit area is $\mu = 2.196 \text{ kg.m}^{-2}$. The group velocities obtained through 4th-order theory are compared with the transient simulations in figure 4.

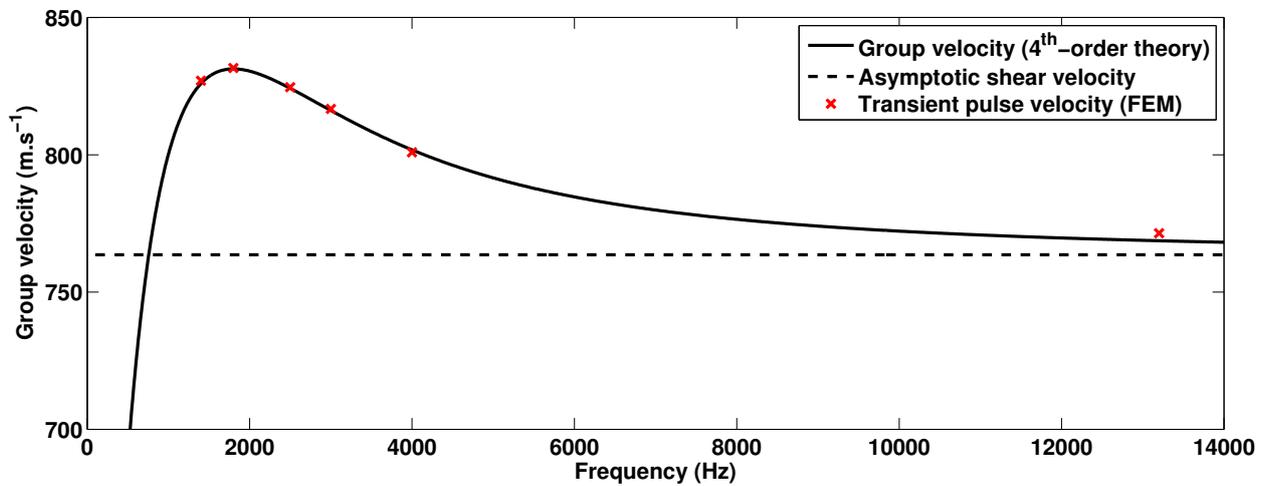


Figure 4: Comparison between theoretical dispersion curves and the velocities obtained by ToF with transient pulse simulations at 1.4 kHz, 1.8 kHz, 2.5 kHz, 3 kHz, 4 kHz and 13200 kHz. The half-amplitude bandwidth of the excitation spectrum varies from 396 Hz for the first pulse (1.4 kHz) to 1870 Hz for the last pulse (13.2 kHz).

Noteworthy, the velocities obtained through transient response are in very good agreement with the analytical results. The accuracy is good, considering the frequency spectrum bandwidth of the excitation signals (only 6 cycles at 1.4 kHz). A 1% error is observed at 13.2 kHz, which is consistent with the appearance of a second transition frequency (around 14 kHz) resulting in a dissociation from the asymptotic shear velocity. The analytical shear velocity is $c_s = 763.6 \text{ m.s}^{-1}$. Using this approximated velocity and Eq.(11) for the determination of S yields a 2.1% overestimation of the shear coefficient.

The transition occurs at 1.8 kHz and the maximum group velocity is $c_{max} = 831.6 \text{ m.s}^{-1}$. Using the relations derived in the previous section yields $D = 7,792.4 \text{ Nm}$ and $S = 1.2811 \times 10^6 \text{ N/m}$ which reduces the error to 0.05%. The mechanical characteristics of the panel obtained through transition analysis are therefore :

$$E_{\text{trans}} = 81.16 \text{ GPa} \quad \text{and} \quad G_{\text{trans}} = 50.04 \text{ MPa} \quad (19)$$

which is a very close from the the exact values for E_s and G_c .

Based on these simulations, the maximal velocity can be used more effectively than the shear velocity to determine equivalent material properties of the considered sandwich panel. It can be noted that the second transition occurs in high frequencies in the considered example: a reduced core's thickness and thicker skins would result in an even more advantageous use of the transition compared with the shear velocity. It should also be mentioned that the results of the simulations presented in figure 4 could be obtained from a fewer number of pulse acquisitions involving broadband spectrum excitations, combined with Fourier Transform to determine the frequency components. Although this approach can produce faster results in simulations, a better accuracy is expected (see section 4) from tunable quasi-monochromatic wave pulses for the identification of the transition frequency.

4. Experimental case study

4.1. Experimental set-up and mechanical characteristics

The experiment is carried out on a rectangular sandwich panel measuring $60\text{ cm} \times 288\text{ cm}$, placed in a vertical position. The constitutive materials are a 10 mm-thick Nomex honeycomb core involving a 3.2 mm cell size, while propagation is considered in the W -direction. The core is surrounded by 0.6 mm-thick Hexforce skins with multi-axial carbon-reinforced fibres mixed with SR1700 epoxy resin. The density of the skins is $\rho_s = 1451\text{ kg.m}^{-3}$ and the core's density is measured using the overall panel's weight $\rho_c = 99\text{ kg.m}^{-3}$ (manufacturer: 96 kg.m^{-3}). A picture of the components is shown in figure 5 and a description of the experimental set-up is shown in figure 6.



Figure 5: Picture of a section of the sandwich panel, the width is the W -direction.

The shaker is placed at the edge of the plate while a laser vibrometer is used to measure normal displacements at the surface. The IWC method [21] is employed to estimate the equivalent shear and bending modulus in the plate. The shaker is used to produce the harmonic excitation while the displacement field is measured using 66 points (referred as *IWC measurements* in figure 6), distributed along 50 cm, providing a spatial resolution of 7.5 mm and the wavenumbers between 0 and 4000 Hz.

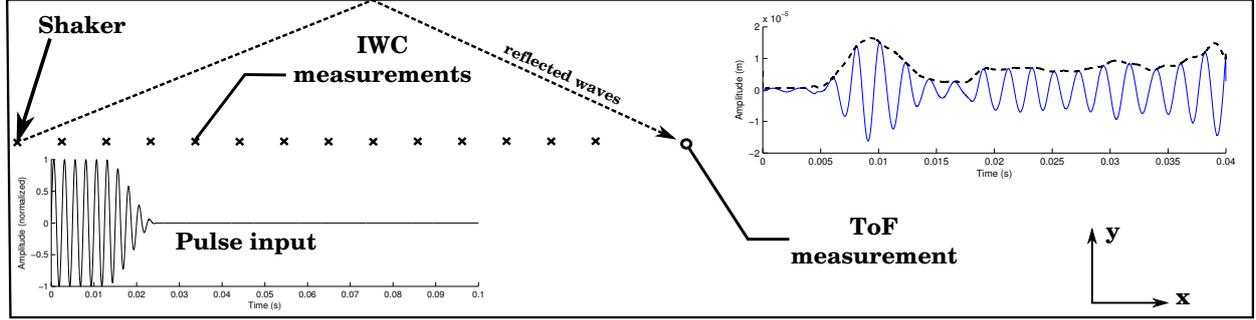


Figure 6: Experimental set-up for the IWC and ToF measurements. The pulse input is generated using the shaker while the time delay is measured at the *ToF measurement* point. An illustration of the time signal and its Hilbert Transform is shown. Note that due to wave reflection at the edges, it is preferable to use the first phase arrival (FPA) to compute the group velocity, rather than the time delay of the mean wave pulse.

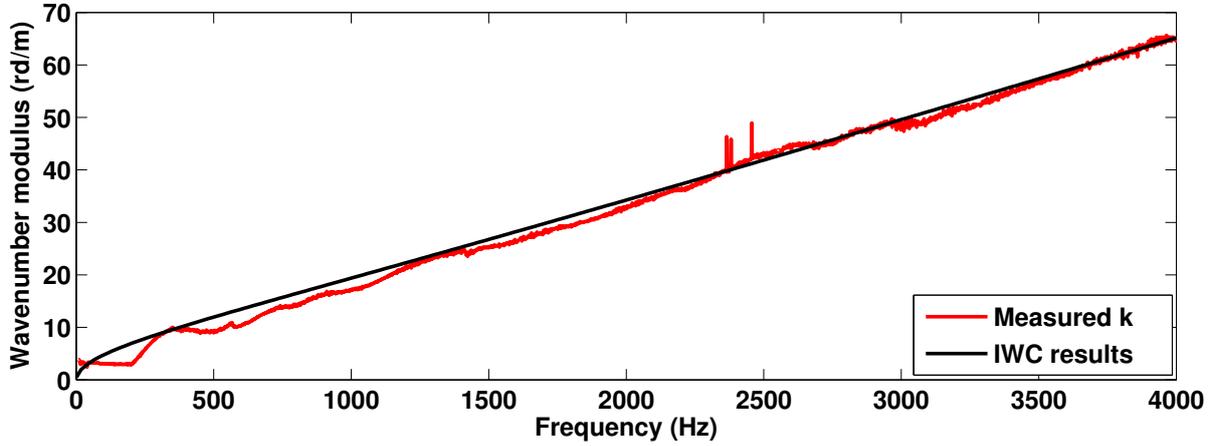


Figure 7: Real wavenumber in the main direction measured in the sandwich plate. A significant error is expected in the low frequencies, due to the limited number of periods for wavelengths determination. The convergence increases in higher frequencies, providing equivalent material characteristics and a good correlation with analytical results.

The wavenumbers measured by the IWC method are shown in figure 7. The resulting material characteristics are:

$$E_{iwc} = 62 \text{ GPa} \quad \text{and} \quad G_{iwc} = 37.8 \text{ MPa} \quad (20)$$

while the manufacturer provides the following mechanical characteristics:

$$E_{stat.} = 70 \text{ GPa} \quad \text{and} \quad G_{stat.} \in [30 - 38] \text{ MPa} \quad (21)$$

which correspond to an 11% error on E_s while G_c is located in the upper range of the prediction interval.

4.2. ToF estimation of the shear and bending coefficients

The method proposed in section 2 is hereby applied to estimate the Young modulus and shear coefficient from the group velocity at the transition. First, the shaker is used to generate wave

pulses in the plate at $x = 0$ (as shown in figure 6). The duration of the pulse is chosen above 10 cycles to reduce dispersion.

The procedure is conducted as follows: Pulse signals of frequency ω are generated in the structure and controlled by trigger. The velocity is measured for each pulse at 1 m from the source using scanning laser vibrometer. The pulse frequency is increased by steps of 20 Hz between 500 Hz and 1000 Hz and 50 Hz between 1000 Hz and 1500 Hz in order to approximate the transition frequency. It is characterized by a decreased of group velocity with frequency. Each pulse is averaged between 30 and 50 times to reduce experimental noise. The proposed procedure is summarized in figure 8. Note that an increased propagation distance d is required for sandwich structures whose transition frequency is low compared with their group velocity.

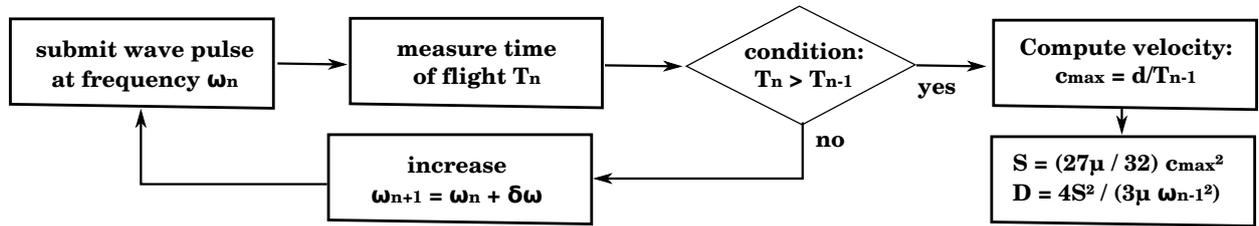


Figure 8: Experimental procedure for the determination of c_{max} and ω_T summarized as a flow diagram. Note that the comparison between T_n and T_{n-1} can be achieved on the first arrival of the pulse (FPA), without requiring signal processing and the computation of the group velocity.

Although a first estimation of the transition frequency can be performed without signal post-processing, it is required for determining the velocity. This step allows an approximation of the transition frequency between 800 Hz and 950 Hz. A refined sweep is then conducted between 830 Hz and 900 Hz to obtain a more accurate value. The time signal is then extracted and post-processed using Hilbert Transform and compared with the excitation signal to compute the time delay. A comparison between the Hilbert Transform of the time signals for various pulses between 500 Hz and 1200 Hz is presented in figure 9.a. It can be seen that the signal amplitude is influenced by the pulse frequency. Above 5 ms, a gap can also be observed between two pulses in the time signal. These effects are produced by the reflections at the edges ($y = 0$ and $y = 600$ mm) and at the free end of the plate ($x = 2880$ mm). It is therefore clear from figure 9.a that an extensive signal processing work would be required to measure the mean energy velocity while taking into account these reflections.

The main advantage of the proposed method lies in the absence of dispersion of the wave pulse in the forward direction, resulting from the fact that the transition is associated with the maximum group velocity. Indeed, the spectrum contributions of a finite pulse centred on the transition frequency (see for example figure 4) will exhibit velocities below or equal to the maximum group velocity. A practical consequence is a negligible dispersion of the first arriving components of the pulse. It means that the time of flight can be estimated using the difference between the FPA of the input and measured signals.

The Hilbert transform of the FPA is presented in figure 9.b. Noteworthy, a refined frequency sampling for the pulses frequencies and a zoom on the first arrival allows to locate the transition at 880 Hz with a 10 Hz accuracy. Further accuracy would be achievable with a better frequency

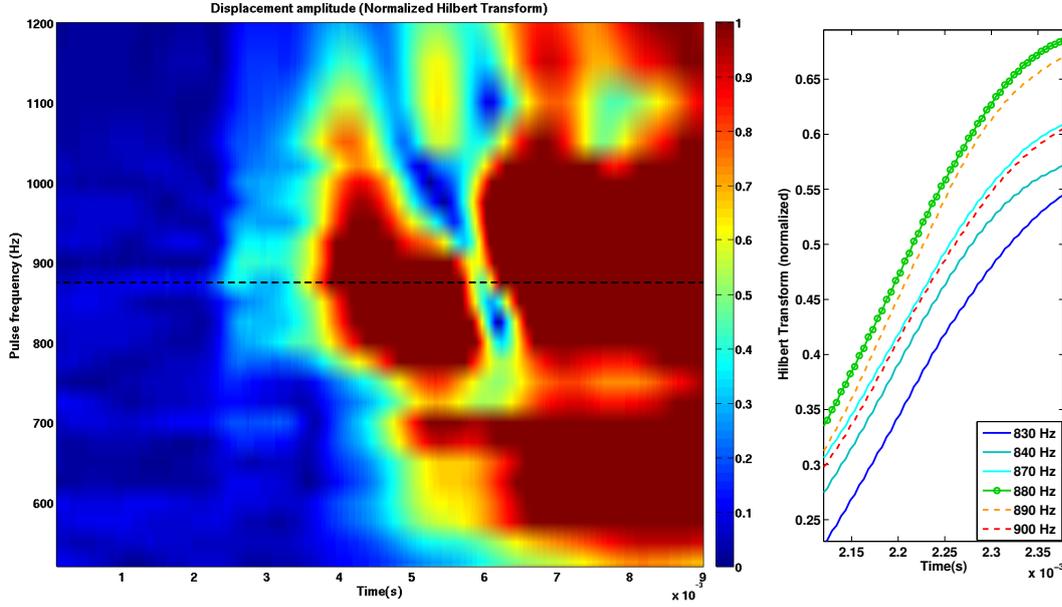


Figure 9: (a): Displacement amplitudes (Hilbert transform) at a distance $x_0 = 1$ m from the excitation source. The amplitude is normalized at 4 ms to highlight the FPA of the pulse. The first incoming wave pulse is observed at 880 Hz (- - -) while the time of arrival is $t = 2.373$ ms. Note that additional filtering procedures can significantly increase the accuracy of the measured velocity (see for example [36, 37]). (b): Comparison of the different measured pulses arrivals between 800 Hz and 900 Hz. A 10 Hz resolution is sufficient to obtain a good evaluation of the transition frequency.

resolution, but it will not be needed in the considered case study. Indeed, the second consequence of the dispersion characteristics at the transition is that the maximal velocity will not be affected by inaccuracies on the transition frequency, provided the excitation spectrum bandwidth covers the exact transition. The time of arrival therefore reaches its minimum at 880 Hz for $t = 2.373$ ms after a 1 m propagation so the maximal velocity is: $c_{max} = 421.4 \text{ m.s}^{-1}$. Considering the surface density $\mu = 2.7402 \text{ kg.m}^{-2}$ and using Eqs.(15) and (16), the bending and shear coefficients become: $D = 2,683 \text{ Nm}$ and $S = 4.1058 \times 10^5 \text{ N/m}$. The equivalent Young modulus of the skins and shear coefficient of the honeycomb core in the main direction are therefore:

$$E_{\text{ToF}} = 69.8 \text{ GPa} \quad \text{and} \quad G_{\text{ToF}} = 36.5 \text{ MPa} \quad (22)$$

which is a significant improvement compared with the IWC results since E_{ToF} and G_{ToF} are both in very good agreement with the manufacturer's values. The authors emphasize that in the case of an orthotropic panel, this procedure only provides the directional values for E and G in the propagating direction.

5. Concluding remarks

This paper has explored a new characterization technique based on the wave dispersion characteristics of guided waves in sandwich plates. The method provides the tensile and shear modulus of sandwich structures in the medium-frequency range by taking advantage of the singular

wave dispersion characteristics of flexural waves within the so-called transition bandwidth. The bending-to-shear transition phenomenon was studied using the WFEM and compared with theory. It was shown that this transition is associated with a local maximum of the group velocity and analytical expressions were derived for the maximal velocity and its frequency. It has also been observed that this maximum wave speed is independent of the shear coefficient.

It is emphasized that the proposed wave-based method has significant advantages: the accuracy of the structural parameters is not altered by the presence of dissipative or uncertain boundaries since the data is obtained locally, through single-shot measurements. It is a considerable advantage compared with a number of stationary or other existing methods, since it can then be applied in situ, without requiring additional sample of the structure. The use of the transition bandwidth results in the absence of wave pulse dispersion in the forward direction of propagation, hence enables the use of an FPA measurement. The consequence is a significant increase of the ToF robustness in low frequency while the post-processing effort becomes almost negligible.

To summarize, the paper resulted in the following findings:

- A definition for the transition phenomenon using the existence of a local energy velocity maxima,
- A FPA strategy for determining this velocity from single shot measurements at low frequencies,
- An in situ procedure for measuring effective mechanical parameters of sandwich plates in the vibroacoustic range.

It can be noted that this strategy however requires the existence of a transition which may not always occur when the skins' thickness is large compared with the core's. The flexural waves in the sandwich are therefore converted into flexural waves in the skins while the core's shear effect is negligible. In general the width does not have any influence on the ToF results. Yet, in some highly directive structures the first incoming phase can be produced by reflections on the edges with higher velocities than in the main direction. A reduction of the distance between the acquisition points is then recommended. Furthermore, note that the proposed method may also apply to other wave types, since it is reminded that the bending-to-shear transition in sandwich panels is a specific type of wave conversion.

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