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Plastic limit analysis modelling with contact interfaces

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Abstract:

The mechanical behaviour of the unilateral contact with Coulomb dry friction is a non linear dissipative process that does not satisfy the normality law [1]. In this work, we consider structures made by rigid perfectly plastic material or isotropic hardening one. The hardening character is considered by using the sequential limit analysis procedure [4]. The aim of this communication is to develop a numerical algorithm for kinematical limit analysis with contact and Coulomb's dry friction. The kinematical approach of limit analysis that consists to minimize the total dissipation of plasticity and friction in interfaces, involves static variables (normal contact reaction) in the objective function. In order to solve this question, a successive approximation algorithm is adopted. Contact conditions will be treated by the use of a classical penalisation procedure. The finite element displacement method and mathematical programming algorithms are used to perform numerical solutions.

Keywords:

Plastic limit analysis, bipotential theory, finite element method.

1 Introduction

In plasticity area, limit analysis is one of the most fundamental methods. In fact, they are used for a direct prediction of plastic limit state. Nevertheless, rigorous upper or lower bounds can not be obtained by using the classical kinematic or static theorems in presence of friction in interfaces, only for a few simple and particular cases. This is caused by the important effect of loss of the classical normality law. In fact, the mechanical behaviour of the unilateral contact with Coulomb dry friction is a non linear dissipative process that does not satisfy the normality law [1]. It belongs to the set of non associated laws [1].

On the other hand, by adopting the bipotential concept, the upper and lower limit , in limit analysis theorems, are extended to the case of presence of unilateral contact with Coulomb's dry friction [1,2]. For simplicity, in this work, only rigid perfectly plastic structures or isotropic hardening ones are considered. We can introduce the hardening character basing on the sequential limit analysis procedure, presented in [3], where, we can actualise the geometry and material properties by solving sequences of limit analysis problems. The solid can have interfaces with other solids assuming unilateral contact with Coulomb's dry friction. Contact conditions, involving the three statuses, such as, no contact, sticking and sliding are taken into account by the use of the contact bipotential. The application of this point of view using the bipotential is successfully applied in the plasticity of metals [5]. The aim of this communication is to develop a numerical algorithm for kinematical limit analysis with contact and Coulomb's dry friction. The kinematical approach of limit analysis consists of minimizing the sum of the total dissipation, produced by the plasticity within the body and by friction in contact interfaces, under boundary conditions, with respect to the velocity displacement. Because the dissipation produced by friction involves static variables (normal contact reaction) the problem of minimization may be solved by a successive approximation algorithm [4]. Contact conditions will be treated by the use of a classical penalisation procedure.

The finite element displacement method and mathematical programming algorithms are used to perform numerical solutions.

2 Contact interfaces and bipotential

To model contact interfaces between deformable bodies, let Ω_1 and Ω_2 be two rigid plastic bodies in contact at a point M . The instantaneous velocities of the particles of Ω_1 and Ω_2 at the point M are denoted $\dot{\mathbf{u}}_1$ and $\dot{\mathbf{u}}_2$, then the relative velocity $\dot{\mathbf{u}}$ is defined as the difference $\dot{\mathbf{u}} = \dot{\mathbf{u}}_1 - \dot{\mathbf{u}}_2$. The contact traction acting at M from Ω_2 to Ω_1 is denoted \mathbf{t} . Let \mathbf{n} be defined as the normal unit vector in the direction of Ω_1 . The dual variables $\dot{\mathbf{u}}$ and \mathbf{t} may uniquely be decomposed in their normal ($\dot{\mathbf{u}}_n$ and t_n) and tangential ($\dot{\mathbf{u}}_t$ and t_t) components as follows:

$$\dot{\mathbf{u}} = \dot{\mathbf{u}}_t + \dot{\mathbf{u}}_n \mathbf{n} \text{ and } \mathbf{t} = t_t + t_n \mathbf{n} \quad (1)$$

The usual Coulomb's law of unilateral contact with dry friction (coefficient m), adopted here, describing the three well – known statues, non contact, contact with sticking and contact with sliding is:

$$\left\{ \begin{array}{ll} \text{If } \dot{\mathbf{u}}_n > 0 & \text{then } t = 0, \quad \text{no contact,} \\ \text{else if } \dot{\mathbf{u}} = 0 & \text{then } |t_t| < m t_n \quad \text{contact with sticking} \\ \text{else } \dot{\mathbf{u}} \neq 0 \text{ and } \dot{\mathbf{u}}_n = 0 & \text{then } t_n > 0 \\ & \text{and } t_t = -m t_n \frac{\dot{\mathbf{u}}_t}{\|\dot{\mathbf{u}}_t\|}, \quad \text{contact with sliding} \end{array} \right. \quad (2)$$

Basing on the concept of the bipotential, it's shown that the previous constitutive law may be considered as one of an Implicit standard Materials by introducing the function [1]:

$$\forall \dot{\mathbf{u}}_n \geq 0 \text{ and } |t_t| \leq m t_n, \quad b_c(-\dot{\mathbf{u}}, t) = m t_n \|\dot{\mathbf{u}}_t\| \quad (3)$$

Otherwise, $b_c(-\dot{\mathbf{u}}, t) = +\infty$

Then, the complete frictional contact law and its inverse may be written as subdifferential of the contact bipotential [1]:

$$-\dot{\mathbf{u}} \in \partial_t b_c(-\dot{\mathbf{u}}, t), \quad t \in \partial_{-\dot{\mathbf{u}}} b_c(-\dot{\mathbf{u}}, t) \quad (4)$$

3 Limit analysis in presence of frictional contact interfaces

Limit analysis is a fundamental method among the direct ones used to compute plastic limit state of a structure without performing the full tedious elastoplastic computation. Taking into account the notation, introduced before in the previous section, let $\Omega = \Omega_1 \cup \Omega_2$ be a structure of boundary S , subjected to imposed surface tractions $\bar{\mathbf{t}}$ on the part S_1 of S and vanishing imposed velocities on the part S_0 . The remaining part $S_2 = S - S_1 - S_0$ may be the contact surface. As usual in limit analysis procedure, surface tractions $\bar{\mathbf{t}}$ on S_1 are proportional to a fixed reference loading $\bar{\mathbf{t}}^0$ such that $\bar{\mathbf{t}} = a \bar{\mathbf{t}}^0$, where a is the load factor. If a plastic collapse mechanism is characterized by a field couple $(\dot{\mathbf{u}}, S)$, the limit load factor is defined by:

$$\int_{\Omega} D(\dot{\mathbf{u}}) d\Omega + \int_{S_2} b_c(-\dot{\mathbf{u}}, t(S)) dS = a^l \left[\int_{S_1} \dot{\mathbf{u}} \cdot \bar{\mathbf{t}}^0 dS \right] \quad (5)$$

where the first term represents the total plastic dissipation depending to the used criterion. Here, the Von Mises Criterion for what $D(\mathfrak{d}) = \sqrt{2/3} S_Y \|\mathfrak{d}\|$ is adopted, where S_Y is the yield stress and $\|\mathfrak{d}\|$ is the Euclidian norm of the strain deviator. By analogy with the definition of the limit load factor, the kinematic one, denoted \mathfrak{a}^k , associated to a velocity field \mathfrak{v}^k , characterizing an admissible mechanism is defined by:

$$\mathfrak{a}^k = \frac{\int_{\Omega} D(\mathfrak{d}(\mathfrak{v}^k)) d\Omega + \int_{S_2} b_c(-\mathfrak{v}^k, t(S)) dS}{\int_{S_1} \mathfrak{v}^k \cdot \bar{\mathbf{t}}^0 dS} \quad (6)$$

Taking into account of the normalization condition $\int_{S_1} \mathfrak{v}^k \cdot \bar{\mathbf{t}}^0 dS = 1$ and basing on the kinematic theorem [1], the kinematical approach of limit analysis consists of the following mathematical programming minimization problem:

$$\text{Inf} \quad \int_{\Omega} D(\mathfrak{d}(\mathfrak{v}^k)) d\Omega + \int_{S_2} b_c(-\mathfrak{v}^k, t(S)) dS \quad (7)$$

Subject to :

$$\mathfrak{d}_m(\mathfrak{v}) = 0 \text{ in } \Omega; \quad \mathfrak{v}_n \geq 0 \text{ on } S_2$$

Boundary conditions on S_0

$$\int_{S_1} \mathfrak{v}^k \cdot \bar{\mathbf{t}}^0 dS = 1$$

In this mathematical program, the stress field S (and the contact reaction) is kept fixed to an optimal value corresponding to the limit state when the velocity field \mathfrak{v}^k represents the unknowns of the problem.

4 Solving algorithms

The solving of the above mathematical programming problem requires treating two questions, the presence of statical variables in the kinematic approach and the no smooth nature of the objective function formulated by the plastic and friction dissipations.

4.1 Coupling question

Naturally, the use of the kinematic approach leads automatically to the use of kinematically admissible approximations as finite element method. The contact reaction can be computed approximately when the velocity field is known but only by using the constitutive relations. So, this question of coupling between reactions and velocities, on the contact interface S_2 , may be solved iteratively by a successive approximation algorithm as, explained by the following way.

We assume that we start from an initial state characterized by the couple (\mathfrak{v}, t) with null value. Let (\mathfrak{v}^i, t^i) denoted the value of contact reactions and velocities at the i^{th} iteration. It is assumed to be known at the current iteration. The value of the couple (\mathfrak{v}, t) at the next iteration will be determined in two times as follows:

- In this first step, the stress field appearing in the objective function to minimise is taken as a simple parameter which is kept constant during this step and having t^i as a value. In this time, the value of velocity \mathfrak{v}^{i+1} is obtained by solving the kinematical problem (7). Package software

of optimization computation is used for solving the non-linear mathematical programming problem for what the displacement velocity constitutes the variables of the mathematical program.

- In the second time, keeping the velocity field constant and equal to $\dot{\mathbf{u}}^{i+1}$ obtained in the previous step, it is the contact reaction that will be computed by means of the constitutive relations (4).

This iterative process will be repeated as far as convergence using one of usual criteria. This algorithm concerned with the solving of the coupling question between stresses and displacement velocities is applied in the case of an analytical determination of the plastic limit load and it's corresponding to collapse mechanism of plane frames with the presence of frictional contact support [2].

4.1 Regularization procedure of plastic and frictional dissipations

To use correctly package software of optimization computation that uses deterministic algorithms, the non linear objective function must be differentiable. In fact, package software requires to provide the gradients of the objective function. For this reason, we need to regularize the objective function, composed by plastic dissipation and frictional one that are non smooth. The regularized model used here consists to replace the rigid perfectly plastic behaviour by a fictitious linear viscous perfectly plastic one considering that the total strain rate $\dot{\mathbf{e}}$ is divided in two parts [3]. Then the regularized dissipation is determined from the inf-convolution concept, defined by [3]:

$$d(\dot{\mathbf{e}}) = \inf_{\dot{\mathbf{e}}^p} \left\{ m \|\dot{\mathbf{e}} - \dot{\mathbf{e}}^p\|^2 + \frac{K_c}{2} (\dot{\mathbf{e}}_m)^2 + D(\dot{\mathbf{e}}^p) \right\} \quad (8)$$

The parameters K_c and m are fictitious constants and must be estimated for each problem. Adopting the von Mises model, the stationary condition of the inf-convolution () gives the algorithm:

$$\text{If } \|\dot{\mathbf{e}}\| \geq e_d \frac{k}{\sqrt{2}}, \quad d(\dot{\mathbf{e}}, S) = \frac{K_c}{2} (\dot{\mathbf{e}}_m)^2 + m \left\{ \|\dot{\mathbf{e}}\|^2 - (\|\dot{\mathbf{e}}\| - e_d \frac{k}{\sqrt{2}})^2 \right\}, \quad (9)$$

$$\text{Else,} \quad d(\dot{\mathbf{e}}, S) = \frac{K_c}{2} (\dot{\mathbf{e}}_m)^2 + m \|\dot{\mathbf{e}}\|^2 \quad (10)$$

This regularization concept allows expressing approximately stresses as the gradient of the regularized dissipation such that:

$$\mathbf{S} = \frac{\partial d}{\partial \dot{\mathbf{e}}} \quad (11)$$

By a similar way of the previous case of plasticity, the contact bipotential needs to be regularized. On the contact interface S_2 , the regularization procedure assumes:

$$\dot{\mathbf{u}} = \dot{\mathbf{u}}^e + \dot{\mathbf{u}}^f \quad (12)$$

Where, $\dot{\mathbf{u}}^f$ is the frictional tangential velocity and $\dot{\mathbf{u}}^e$ is the fictitious velocity part, used to regularize the contact bipotential. By considering the decomposition of the velocity and the contact reaction into their normal and tangential components, the regularized contact bipotential is obtained by the inf-convolution concept:

$$b(-\dot{\mathbf{u}}, t) = \inf_{-\dot{\mathbf{u}}_t^f} \left\{ \frac{1}{2} K_n (\dot{\mathbf{u}}_n - \dot{\mathbf{u}}_n^f)^2 + \frac{1}{2} K_t (\dot{\mathbf{u}}_t - \dot{\mathbf{u}}_t^f)^2 + b_c(-u, t) \right\} \quad (13)$$

By following the same way as used for the plasticity regularization (previous paragraph), we find the algorithm:

$$\text{If } \dot{\mathbf{u}}_n^f > 0, \text{ then } \begin{cases} \dot{\mathbf{u}}_n^f = \dot{\mathbf{u}}_n \\ t_n = 0 \end{cases}, \quad \text{no contact}$$

$$\text{and } b(-\dot{\mathbf{u}}, t) = 0 \quad (14)$$

$$\text{If } \dot{\mathbf{u}}_n^f = 0, \text{ then } \begin{cases} \dot{\mathbf{u}}_n^f = 0 \\ t_n = -K_n \dot{\mathbf{u}}_n \end{cases}, \quad \text{contact}$$

$$\text{Putting } a = \|\dot{\mathbf{u}}_t\| - \frac{m}{K_t} t_n \quad \text{and } v = \frac{-\dot{\mathbf{u}}_t}{\|\dot{\mathbf{u}}_t\|}$$

$$\text{If } a > 0 \text{ then } \begin{cases} \dot{\mathbf{u}}_t^f = -av \\ t_t = m t_n v \end{cases}$$

$$\text{and } b(-\dot{\mathbf{u}}, t) = \frac{1}{2} K_n \dot{\mathbf{u}}_n^2 + \frac{1}{2} K_t (\dot{\mathbf{u}}_t - \dot{\mathbf{u}}_t^f)^2 + m t_n \|\dot{\mathbf{u}}_t\| \quad (15)$$

Finally, by considering of these developments on the regularization procedure, the mathematical programming problem (7) representing the kinematics approach of limit analysis is modified as follows:

$$\text{Inf } \int_{\Omega} d(\mathfrak{E}(\dot{\mathbf{u}}^k)) d\Omega + \int_{S_2} b(-\dot{\mathbf{u}}^k, t(\mathfrak{S})) dS$$

$$\text{Subject to :} \quad (16)$$

$$\text{Boundary conditions on } S_0$$

$$\int_{S_1} \dot{\mathbf{u}}^k \cdot \vec{t}^0 dS = 1$$

It may be remarked, when we compare the non regularized dissipations and regularized ones, that the plastic incompressibility condition and the contact one are absent in the later problem.

6 Finite element discretisation

To perform practical problems, the finite element method is adopted. As usual in kinematic approaches, the discretisation of the displacement field is defined by the relation, $\dot{\mathbf{u}}(x) = N(x)\dot{\mathbf{U}}$, where $\dot{\mathbf{U}}$ is the unknown nodal velocity vector and $N(x)$ is the matrix of polynomial shape function. By considering the compatibility conditions, the strain tensor rate field is defined by: $\mathfrak{E}(x) = B(x)\dot{\mathbf{U}}$, where $B(x)$ is the classic matrix composed by the first derivatives of shape functions with respect to space variables: $B(x) = \text{grad}_s N(x)$. In the following, we suppose that the structure Ω is discretized in a number n^e of finite elements and n_c^e contact finite element. The regularized bipotential, is given by the following discretisation form:

$$B(\dot{\mathbf{U}}, \mathfrak{S}) = \int_{\Omega} d(\mathfrak{E}) d\Omega + \int_{S_2} b(-\dot{\mathbf{u}}, t(\mathfrak{S})) dS = \sum_{i=1}^{n^e} \int_{\Omega_e} d(B(x)\dot{\mathbf{U}}) d\Omega + \sum_{i=1}^{n_c^e} \int_{S_2^e} b(-N\dot{\mathbf{U}}, t) dS \quad (17)$$

On the other hand, the discretized form of the normalization condition in terms of nodal velocity vector is:

$$F_0^T \cdot \mathcal{U} = \int_{S_1} \bar{t}^0 \cdot \mathcal{M} dS = \sum_{i=1}^{n_{se}} \int_{S_{se}} \bar{t}^0 \cdot \mathcal{M} dS = 1 \quad (18)$$

With n_{se} and n_c^e are respectively the number of finite elements of the boundary where forces are imposed and the number of contact elements; S_{se} and S_2^e represent respectively the length of a finite element on the boundary S_1 and S_2 . F_0 is a reference imposed nodal forces on the part S_1 of the boundary, it is defined by:

$$F_0 = \int_{S_1} N^T \bar{t}^0 dS \quad (19)$$

As mentioned before in the section 4, the couple (\mathcal{U}, S) characterizing the limit state will be computed by an iterative procedure in two steps. Starting with the fact that the couple (\mathcal{U}^i, S^i) is known and after, we are looking for $(\mathcal{U}^{i+1}, S^{i+1})$:

1. S^i is fixed and a new approximation \mathcal{U}^{i+1} is computed by solving the following minimization problem by a non linear mathematical programming algorithm. To deal with, Minos software [28] is used:

$$\begin{aligned} & \text{Inf} \int_{\Omega} b(B(x)\mathcal{U}, S) d\Omega \\ & \text{Subjected to : } F_0^T \mathcal{U} = 1 \end{aligned} \quad (20)$$

2. \mathcal{U}^{i+1} is fixed and a new approximation S^{i+1} is computed by using constitutive relations. Taking into account that the regularized bipotential is differentiable:

$$S^{i+1} = \frac{\partial d(\mathcal{U}^{i+1})}{\partial \mathcal{M}} \text{ in } \Omega \text{ and } t^{i+1} = \frac{\partial b(-N\mathcal{U}^{i+1}, t^i)}{\partial (-\mathcal{M})} \text{ on } S_2 \quad (21)$$

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