Simulation of plastic fatigue crack growth by a two scale eXtended Finite Element Method

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Abstract :

A new technique for the finite element modelling of elastic-plastic fatigue crack growth with frictional contact on the crack faces is presented. The eXtended Finite Element Method (XFEM) is used to discretize the equations, allowing for the modelling of arbitrary cracks whose geometry are independent of the finite element mesh. This paper presents an augmented Lagrangian formulation in the X-FEM framework in order to simulate elastic-plastic crack growth with treatment of contact and friction. The original formulation, which takes advantages of a coarse and a fine discretization close to the crack tip, is presented. In a second time the numerical issues and numerical integration are adressed, and finally numerical examples are shown to validate the method.

Résumé :

Cet article présente une nouvelle technique de modélisation éléments finis de la propagation de fissure en élasto-plasticité avec prise en compte du contact-frottement le long des lèvres de la fissure. La méthode des éléments finis étendus X-FEM est utilisée pour la discrétisation, ce qui permet de modéliser des géométries de fissures quelconques et ce indépendamment du maillage. Une formulation de type Lagrangien augmenté adaptée au formalisme X-FEM est proposée et permet de simuler la propagation avec prise en compte de la plasticité et du contact-frottement. Cette formulation originale tire profit d'une modélisation à deux échelles : l'une grossière pour décrire l'ensemble de la structure et l'autre fine à proximité de la pointe de fissure. Après avoir présenté celle-ci, on s'intéressera aux aspects numériques tels que l'intégration numérique et on présentera des exemples pour valider la méthode.

Key-words :

Non-linear fracture mechanics; X-FEM; Fatigue Crack growth.

1 Introduction

The X-FEM uses the partition of unity in two ways: first to take into account the displacement jump across the discontinuity far from the crack tip, and second to enrich the approximation close to the front by considering the appropriate asymptotic fields Moës *et al*. (1999). Several LEFM issues were treated with the X-FEM such as elastic fatigue crack growth, crack growth with friction, arbitrary 3D fatigue crack growth with level set methods and dynamic crack growth. In recent works the method has been applied to non-linear issues such as phase transformation, large strain analysis of rubber like materials, and finally it has been modified by the authors to deal with plasticity Elguedj *et al*. (2006). The main purpose of this contribution is, in the framework of the eXtended Finite Element Method, to treat the case of multiple non-linearities with a two scale approach: a coarse description of the structure is assumed (the X-FEM approach allows us to obtain an accurate solution without meshing the crack) and a fine discretization close to the crack tip in order to accurately describe the plastic and contact zone. The presented method will focus on the case of plasticity combined with frictional contact and is applied to fatigue crack growth analysis. In the case of material non-linearities, several issues have to be addressed.

2 The Coarse Scale: the X-FEM

In this paper, the coarse scale is linked to length scales of the order of the structure or the crack (geometrical aspects). In this respect, one uses the eXtended Finite Element Method, first introduced in Moës *et al*. (1999), in which an enrichment basis is added to the classical finite element basis approximation. This is done using the Partition of Unity Method Babuška *et al*. (1997). The enriched basis shape functions are associated to new degrees of freedom and the displacement field can be written:

$$U = \sum_{i \in \mathcal{N}} N_i(x) U_i + \sum_{i \in \mathcal{N}_{cut}} N_i(x) H(x) a_i + \sum_{i \in \mathcal{N}_{front}} \sum_{\alpha} N_i(x) B_\alpha(x) b_{i,\alpha} \tag{1}$$

$\mathcal{N}$ is the set of the standard finite element nodes, $\mathcal{N}_{cut}$ the set of nodes whose support is completely cut by the crack and $\mathcal{N}_{front}$ the set of nodes whose support contains a crack front. N_i are the standard finite element shape functions.

Following the work done in Hutchinson (1968); Rice *et al*. (1968) one has shown in Elguedj *et al*. (2006) that, in the case of elastic plastic conditions, the tip enrichment basis given in Moës *et al*. (1999) is not the most appropriate. It may be replaced by the one given in Equation 2 deriving from the Hutchinson-Rice-Rosengren elastic-plastic asymptotic fields, where a power-law hardening material $\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0}\right)^n$ is considered.

$$[B_\alpha] = r^{\frac{1}{n+1}} \left[\sin\frac{\theta}{2},\ \cos\frac{\theta}{2},\ \sin\frac{\theta}{2}\sin\theta,\ \cos\frac{\theta}{2}\sin\theta,\ \sin\frac{\theta}{2}\sin 3\theta,\ \cos\frac{\theta}{2}\sin 3\theta \right] \tag{2}$$

In Elguedj *et al*. (2006) the authors have shown that this plastic enrichment basis, coupled with a Newton-Raphson iteration algorithm to compute equilibrium in conjunction with a radial return mapping scheme to compute plastic irreversible strains, is able to model the cyclic behavior of a cracked structure.

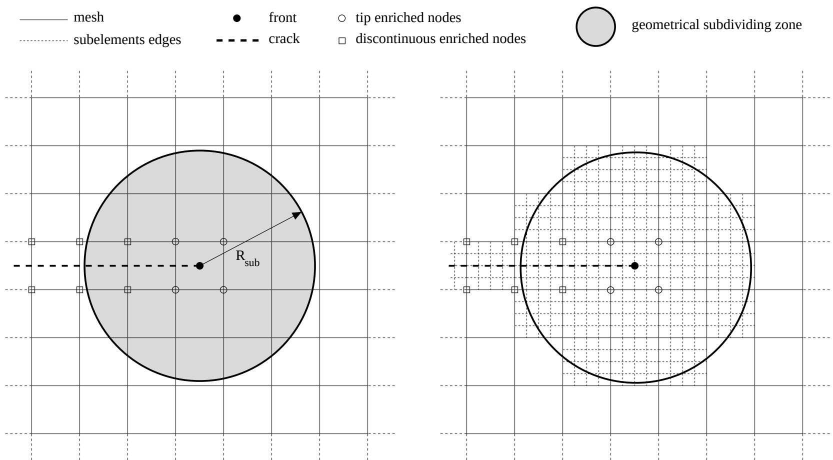

Figure 1: Geometrical subdividing of the elements around the tip element. The elements whose center is inside the disk are subdivided.

3 The Fine Scale

3.1 Integration of the plastic model

The fine scale is devoted to material aspects: confined plasticity and contact behaviour. The fatigue crack growth is highly dependent on the plastic and contact behaviour close to the crack tip, and in particular during the propagation. In this respect, a fine representation of the plastic behaviour is implemented, and it appears quite useful to have a fine knowledge of the stress state in an element that will contain the crack tip at the next computational step. One proposes to subdivide several elements (that are not cut by the crack at the step considered) around the element containing the crack tip. This idea will be referred subsequently as geometrical subdividing (Figure 1).

As one would expect this technique to be independant of the mesh density, the size of the geometrical subdivided zone must be physically based. One can see on Figure 1 that the elements whose center is inside a disk centered on the crack tip are subdivided with the same subquadrangles as proposed in Elguedj *et al.* (2006). Following the hypothesis of confined plasticity, the radius R_{sub} of the disk can be defined using the elastic estimations of the radius of the plastic zone. Using the Dugdale-Barenblatt formula for the radius of the plastic zone, Rsub is defined by the following equation in mode I:

$$R_{sub} = \beta \times \frac{\pi}{16} \frac{K_I^2}{\sigma_y^2} \tag{3}$$

Where K_I is the mode I stress intensity factor and σ_y is the yield stress and β a proportionality coefficient. With this technique, considering an element that is not subdivided at step n and that is subdivided at step $n+1$, the internal variables defined at the Gauss quadrature points of its subquadrangles are initialized easily (since this element is not yet in plastic at step n). Therefore, no projection is needed as the internal variables are null in elastic conditions.

3.2 Contact treatment

To be able to take into account contact and friction between the crack faces, it is necessary to discretize the interface. Due to the fact that the crack does not conform a priori to the finite element mesh, the interface is subdivided into one-dimensional segments. For each segment,

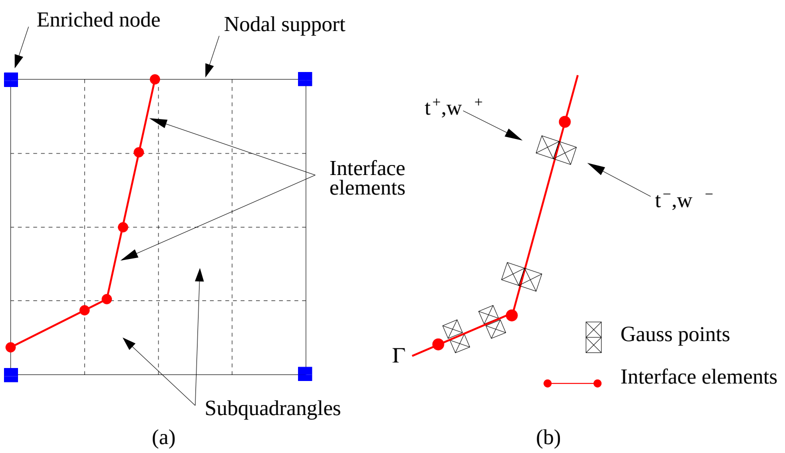

Figure 2: (a) zoom on a finite element with interface elements (b) definition of the pairs (t^+,w^+) and (t^-,w^-) associated with Gauss quadrature points on each side of the crack (Γ^+ and Γ^-).

one determines the intersection of these segments with the sub-element mesh, which results in a set of one-dimmensional sub-elements also called interface elements (see Figure 2). In order to numerically integrate the different terms on the interface, Gauss quadrature points are used along each of the one-dimensionnal interface elements. The interface displacement field w and traction field t are then discretely interpolated on each side of the crack as shown on Figure 2(b).

4 Global Formulation

Consider the body $\Omega \subset R^2$ containing an internal boundary Γ representing a crack. The crack faces are denoted by Γ^+ and Γ^- such that $\Gamma = \Gamma^+ \cup \Gamma^-$.. The boundary of Ω is denoted by $\partial\Omega$ and can be split in two sets: $\partial_1\Omega$ on which the displacement field u_d is enforced (Dirichlet boundary conditions) and $\partial_2\Omega$ on which the surface traction F_d is enforced (Neumann boundary conditions). One assumes quasi-static loading of the body and absence of body forces. The stress and displacement fields are denoted by σ and u respectively. One also introduces the equivalent quantities on the crack faces: traction t and displacement w denoted by $+$ on Γ^+ (t^+,w^+) and $-$ on Γ^- (t^-,w^-). For convenience the notations w (respectively t) will be used when referring to both w^+ and w^- (respectively t^+ and t^-).

Due to the part associated with contact, and since the material can be non-linear as well, an iterative strategy is necessary to solve the problem. A mixed formulation is obtained from the principle of virtual power where the superscript (i) corresponds to the i-th iteration and the subscript n corresponds to the $n-th$ computational time step. Following the work done in Simo *et al*. (1992), a penalty regularization of the contact problem is done, leading to the following equation:

$$\begin{aligned} 0 \quad = \quad & -\int_\Omega \sigma_n^{(i)} : \varepsilon(u^\star) d\Omega + \int_{\partial_2\Omega} F_{dn}.u^\star dS + \int_\Gamma \Lambda_n^{(i)}.u^\star \mid_\Gamma dS \\ & + \int_\Gamma (t_n^{(i-1)} + \alpha w_n^{(i-1)}).w^\star dS - \int_\Gamma (\Lambda_n^{(i)} + \alpha w_n^{(i)}).w^\star dS \\ & + \int_\Gamma \Lambda^\star.(u_n^{(i)} \mid_\Gamma - w_n^{(i)}) dS \quad \forall (u^\star, w^\star) \in \mathcal{U}_0, \ \forall \Lambda^\star \in \mathcal{L}_0 \end{aligned} \tag{4}$$

5 Propagation Strategy

In Moës *et al*. (1999), the authors use the X-FEM to simulate fatigue crack growth in linear elastic bodies. Within this condition each computational step (i.e. each time the crack is growing) is a calculation independent of the previous ones (except the crack update). In the presented method, as the material non-linearities are taken into account, the history of the quantities, especially the stresses, contact and internal variables, have not to be forgotten between two computational steps. Following these observations the strategy used in the presented method is comparable to the one proposed in Réthoré *et al*. (2005) in the framework of linear elastic dynamic crack propagation. One considers what happens during a “pseudo time step” from computational time n to $n+1$.

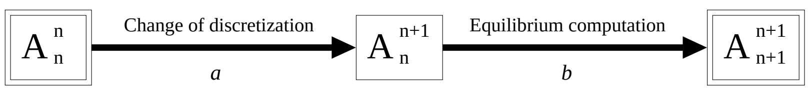

Figure 3: Calculation strategy

As presented on Figure 3, one has to perform two steps: change the discretization (denoted by *a* on Figure 3) and compute the equilibrium (denoted by *b* on Figure 3). Changing the discretization (step *a*) requires the determination of A_n^{n+1} (the vector of unknown at computational time n written on the set of shape functions/subelements Gauss points at computational time $n+1$). In the step *b* the equilibrium is computed in order to obtain A_{n+1}^{n+1} with the method presented in the previous sections.

6 Numerical Examples

A mode I SE(T) specimen was submitted to a cyclic tension compression loading with a ratio $R = -1$. The contact consitutive law on the crack faces is chosen to be frictionless for this example. The aim of this example is therefore to show the influence of the plasticity at the crack tip on the contact behaviour under a compressive state. One can notice on Figure 4 that the crack remains open near the tip due to plasticity while the lips are closed far from the tip due to the global compressive state.

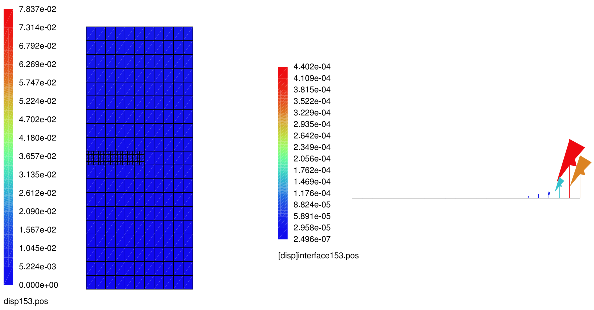

Figure 4: Amplified deformed configuration of the specimen (a) Displacement jump across the crack faces (b).

A mode I CT specimen was submitted to a cyclic tension loading with loading ratios from $R = 0.1$ to $R = 0.7$. It is well known from Elber (1970) that crack closure appears under global cyclic tension for loading ratios smaller than 0.7 due to the plasticity at the crack tip. One can see on Figure 5 the deformed configuration of the specimen at the maximum load. One can notice the bumps on the crack faces after propagation due to the stress redistribution at the back of the succeding tips. Depending on the load ratio, these bumps will be responsible for possible crack closure as presented in Elber (1970), and will define the well known opening stress intensity factor that governs Paris law based fatigue crack growth.

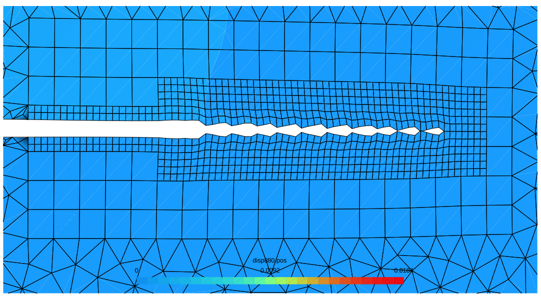

Figure 5: Norm of the displacement field plotted on the amplified deformed configuration at $F = F_{min}$ after several growing steps.

7 Conclusions

A method has been presented to improve finite elements approximations in the framework of Elastic Plastic with contact and friction Fracture Mechanics. The main hypothesis of this study is that one only considers the case of confined plasticity *i.e.* one only enriches the element containing the crack front. A plastic enrichment basis, that captures well the Hutchinson-Rice-Rosengren plastic singularities, is used in the framework of the eXtended Finite Element Method, coupled with an augmented Lagrangian Newton like iterative algorithm and a radial return mapping scheme to compute contact / friction and plastic flow. In this approach, the formulation takes advantage of a global coarse mesh linked to the X-FEM strategy, and a fine discretization which allows to account for confined plasticity and contact between the crack faces. Indeed, the fine model is linked to the fact that the fatigue crack growth is highly dependent on the plastic and contact behaviour close to the crack tip. The new partitioning integration strategy is developed to avoid the projection of internal variables as the crack propagates. The main idea is to partition a bigger set of elements that surround the crack tip, in order to initialize internal variables while in elastic conditions. As a consequence, this two scale strategy allows to obtain a fine description for fatigue plastic crack growth: as an example, one can model the bumps on the crack faces after propagation due to the stress redistribution at the back of the succeeding tips. Finally, depending on the load ratio, closure effects can be observed with a high influence on fatigue crack growth.

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