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Absolute and convective instabilities in an inviscid compressible mixing layer

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Abstract :

This study aims to examine the effects of compressibility on shear flow instabilities. The modal theory predicts compressible-flow instability whatever the Mach number. We improve the description of the latter by distinguish the absolute and convective instabilities of a wavepacket. We study an inviscid and two-dimensional mixing layer at an arbitrary Mach number subject to three-dimensional disturbances. The eigenvalue problem is solved with the help of a spectral method. We ascertain the effects of the distribution of temperature and velocity in the mixing layer on the transition between convective and absolute instabilities. It appears that, in most cases, the most unstable situation arises when the Mach number is infinite, which leads us to think that a new type of instability emerges in the supersonic range.

Résumé :

Cette étude a pour objectif d'examiner les effets de la compressibilité sur les instabilités d'écoulements cisaillés. La théorie modale prédit une instabilité des écoulements compressibles quelque soit le nombre de Mach. Nous améliorons la description de cette dernière en distinguant les instabilités absolues et convectives d'un paquet d'ondes. Nous étudions une couche de mélange bidimensionnelle et non-visqueuse perturbée à un nombre de Mach arbitraire. Le problème aux valeurs propres est résolu à l'aide d'une méthode spectrale. Nous évaluons les effets de la distribution de vitesse et température dans la couche de mélange sur la transition entre les instabilités convectives et absolues. Il apparaît que, dans la plupart des cas, la situation la plus instable survient quand le nombre de Mach est infini, ce qui nous amène à penser qu'un nouveau type d'instabilité émerge dans le domaine supersonique.

Key-words :

compressible shear flows; absolute and convective instabilities; transition to turbulence

1   Introduction

The theory of stability of shear flows has been the object of an intense study for the past century due to its numerous applications in engineering, geophysics, and astrophysics. Velocity shear generates a dynamical instability and the most common and fastest one is the Kelvin-Helmholtz (KH) instability which occurs in fluid and plasma inviscid shear layers. It intervenes in many phenomena in fluid dynamics (Jackson and Grosch 1989; Caillol 2005; Kelley, Chen, Beland et al. 2005), in space, astrophysical, and laboratory plasmas (Mills, Longbottom, Wright et al. 2000; Hasegawa 2004) when steep velocity gradients emerge. The most relevant examples are the generation of water waves by wind blowing over the surface of the water and the solar wind plasma transport across the terrestrial magnetopause at the interface between the solar wind and the Earth's magnetosphere. In aircraft engines and, space and astrophysical plasmas, the total velocity jump across the velocity shear layer has large sonic Mach numbers and taking into account compressibility is essential. KH instabilities create turbulence that enhances mixing and generates large-scale vortical structures (Miura 1997). At supersonic speeds, instability growth

rates become smaller and mixing rates lower in unbounded flows. Large-scale structures are nevertheless observed in supersonic mixing layers. KH instability weakens as the convective Mach number increases while oblique disturbances become more and more unstable and dominant. Wall-confined layers are more unstable than free shear layers in supersonic motions; the reflection of acoustic waves from the walls generates a new type of instability (Lu and Wu 1991). The study of compressible hydrodynamic shear flows has a long history. Blumen, Drazin and Billings (1975) showed that the inviscid mixing layer $U(y) = \tanh(y)$ was temporally unstable with respect to two-dimensional disturbances whatever M. Ragab and Wu (1989) and Jackson and Grosh (1989) examined the spatial instabilities of a free and hyperbolic-tangent evolving mixing layer with arbitrary stream velocities. They observed two modes; the first exists in the subsonic régime when the conditions at infinity are an exponential decay of the mode amplitude. This mode is acoustic. As Mach number is higher, two exponential decays are no longer possible: one or two conditions at infinity are radiative. As M becomes larger than the threshold value M^* limiting these domains with different boundary conditions, a second mode appears. The so-called fast mode has the larger wave speed; it decays exponentially in the fast stream and radiates in the slow stream. The slow mode has a contrary behaviour at infinity. They are two vortex modes; the sonic mode turns into a vortex mode as $M > M^*$. No unstable modes exist in the domain where both boundary conditions are radiative. In both studies, cooling the slow stream results in an increase of the growth rate for $M < M^*$, the process is inversed for higher Mach numbers. An increase of the Mach number leads to a decrease of the growth rates by a factor of 5 to 10 up to M^*. Three-dimensional instabilities yield the highest growth rates. The introduction of viscosity by Ragab and Wu (1989) had a stabilizing effect at all frequencies. We here use again a normal-mode approach but we focus on the spatial development of disturbances while they are advected by their group velocity. We therefore highlight the important distinction between convective and absolute instabilities as initially emphasized in shear flow instabilities by Huerre and Monkewitz (1985). This distinction is clearly dependent upon the reference frame. A flow is said to be *absolutely* unstable if the response to a disturbance in space and time is unbounded everywhere for large time. A flow is said to be *convectively* unstable if the response decays to zero everywhere, the response is a wavepacket propagating downstream from the source, the waves forming the packet growing. Pavithran and Redekopp (1989) analysed the convective and absolute instabilities in a free mixing layer when velocity and temperature fields have hyperbolic-tangent profiles. The parameters are the Mach number and the ratios of velocity and temperature of each stream. Jackson and Grosch (1990) carried out a similar study in a boundary layer flow with an identical velocity field, the temperature being linked to the velocity by the Crocco relation. Both papers restricted to the subsonic range. They noticed that the mixing layer became more convectively unstable as the Mach number increased. On the other hand, cooling the slow stream extends the domain of absolute instability. The shear layer can even become absolutely unstable when both streams are coflowing.

Our objective is to examine the influence of the velocity and temperature ratios on the absolute/convective transition in an inviscid free mixing layer where the Mach number can attain supersonic values.

2 Formulation

We consider a two-dimensional compressible mixing layer which separates two streams of different speeds and temperatures. The flow is unbounded, its direction is following the x-axis, the shear is y-orientated and z is normal to the plane of the flow. The mean velocity is approx-

imated by a hyperbolic tangent. We here assume that viscosity is negligible. The temperature and velocity fields are then uncoupled but we choose an identical profile for the temperature:

$$U = \frac{1}{2}[1 + \beta_U + (1 - \beta_U)\tanh(\eta)], \qquad T = \frac{1}{2}[1 + \beta_T + (1 - \beta_T)\tanh(\eta)], \qquad (1)$$

where η is the cross-stream variable in the Howarth-Dorodnitzyn transformation (Jackson and Grosch 1989). All quantities are nondimensionalized by the values of the variables in the fast stream, taken to be at $\eta = +\infty$. β_U is the ratio of the speed of the slow stream to that of the fast stream. $\hat{M}$ is the Mach number of the fast stream. We always have $-1 < \beta_U < 1$ and $\beta_T > 0$. The flow field is perturbed by introducing wave disturbances whose amplitude is function of η; for example, the pressure perturbation is

$$p = \Pi(\eta) \exp[i(\hat{\alpha}x + \hat{\beta}z - \hat{\omega}t)],$$

with $\hat{\alpha}$ and $\hat{\beta}$ wavenumbers and $\hat{\omega}$ the frequency. The equation governing Π is

$$\Pi'' - \frac{2U'\Pi'}{U - \hat{c}} - T\left[(\hat{\alpha}^2 + \hat{\beta}^2)T - \hat{\alpha}^2\hat{M}^2(U - \hat{c})^2\right]\Pi = 0, \qquad (2)$$

where $\hat{c} = \hat{\omega}/\hat{\alpha}$ (Jackson and Grosh 1989). Equation (2) is easily turned into an analogous two-dimensional disturbance equation by Squire transform

$$\alpha^2 = \hat{\alpha}^2 + \hat{\beta}^2, \quad \hat{\alpha} = \alpha\cos\theta, \quad \hat{\beta} = \alpha\sin\theta, \quad \alpha M = \hat{\alpha}\hat{M}, \quad \hat{c} = c, \quad \text{and} \quad \hat{\omega} = \omega\cos\theta :$$

$$\Pi'' - \frac{2U'\Pi'}{U - c} - \alpha^2 T[T - M^2(U - c)^2]\Pi = 0. \qquad (3)$$

From now on, $\Re$ and $\Im$ respectively denote the real and imaginary parts of a complex number while the subscripts r and i respectively define the real and imaginary parts of the mode features: frequency, wavenumber and wave speed. Whatever $\Pi(\alpha, c)$ solution of (3), $\Pi(-\alpha, c)$ is also a solution. Whatever $\Pi(\alpha, c)$ solution of (3), $\Pi^*(\alpha^*, c^*)$ is also a solution, so we can restrict the studied (α, c)-domain to $\alpha_r \geq 0$ and $c_i \geq 0$.
The boundary conditions for Π are obtained by considering the limiting forms of (3) as $\eta \to \pm\infty$. The required solutions are exponentials of the form

$$\eta \to \pm\infty, \qquad \Pi \to \exp(\mp\Omega_\pm\eta), \qquad (4)$$

$$\text{where} \quad \Omega_+^2 = \alpha^2[1 - M^2(1 - c)^2], \qquad \Omega_-^2 = \alpha^2\beta_T[\beta_T - M^2(\beta_U - c)^2].$$

The non-radiation condition for unstable oscillations comes down to $\Re(\Omega_+) > 0$ and $\Re(\Omega_-) > 0$. As the profiles (U, T) have constant "tails", the unbounded-flow problem can come down to a finite-boundary problem in the η-range $[\eta_-, \eta_+]$ such as $|U - U(\pm\infty)| \ll 1$ as $|\eta| \geq \min[-\eta_-, \eta_+]$ (Keller's boundaries). Conditions (4) therefore become

$$\left(\frac{d}{d\eta} + \Omega_+\right)\Pi(\eta_+) = 0, \qquad \left(\frac{d}{d\eta} - \Omega_-\right)\Pi(\eta_-) = 0. \qquad (5)$$

3   Spectral method

We use a new cross-stream variable Z such as

$$\frac{\eta}{\eta_+} = \mu Z + (1 - \mu)Z^3, \qquad -1 \leq Z \leq 1$$

where $\eta_- = -\eta_+$ and $0 < \mu < 1$. Equation (3) is rewritten with this new variable and the related boundary conditions become

$$\left(\frac{d}{dZ} \pm \alpha\kappa\varpi_\pm\right)\Pi(\pm 1) = 0\,, \quad \Omega_\pm = \alpha\varpi_\pm\,, \quad \kappa = (3 - 2\mu)\eta_+\,. \tag{6}$$

The problem comes down to the search for the eigenvalues $\lambda = \alpha\kappa$ ($\Re(\lambda) \geq 0$) and eigenfunctions Π, the wave speed c being given. An approximate solution is obtained by expanding Π in a finite Chebyshev series. A Chebyshev collocation method is used to discretize the problem. The collocation points in the interval $[-1, 1]$ are chosen to be

$$Z_j = \cos\frac{\pi j}{N + 1}\,, \qquad j = 0, 1, \ldots, N + 1\,;$$

Π_{N+1} is the interpolation polynomial of Π of degree $N + 1$ and we call $\Pi_j = \Pi_{N+1}(Z_j)$. By introducing the boundary conditions (6), the problem reduces to N linear relations that are represented in a matricial form by a standard eigenvalue system

$$(A - \lambda I)X = 0, \tag{7}$$

$$\text{with} \quad X^t = [\lambda^3\bar{\Pi}, \lambda^2\bar{\Pi}, \lambda\bar{\Pi}, \bar{\Pi}] \qquad \text{and} \qquad \bar{\Pi}^t = [\Pi_1, \Pi_2, \ldots, \Pi_N].$$

4 Convective and Absolute Instabilities Computation

Double α-roots of the dispersion relation determine the large-time behaviour of the initial-value problem. These roots satisfy the equations

$$D = \text{Det}[\mathcal{M}(\lambda)] = 0 \text{ and } \frac{\partial D}{\partial \lambda} = 0\,.$$

The related root α_0 is a saddle-point in the complex α plane, meeting point of upstream and downstream branches and ω_0 is a branch point in the complex ω plane.

We now explain the procedure to evaluate the double roots of our dispersion relation. In a reference frame moving with the constant speed V, the frequency becomes $\tilde{\omega} = \omega - \alpha V$ and the double root satisfies: $\frac{\partial\tilde{\omega}}{\partial\alpha}(\alpha_0) = 0$. Determining $\alpha_{0,r}$ for a given $\alpha_{0,i} = \alpha_i$ by solving $\frac{\partial\omega_i}{\partial\alpha_r}(\alpha_0) = 0$, we deduce $V = \frac{\partial\omega_r}{\partial\alpha_r}(\alpha_0)$. The double-root in the laboratory frame is found out when $V = 0$ while incrementing α_i. The approach must be here modified. Indeed, c is the variable and α is the unknown function. The reference frame saddle-point is then found out while varying c_r and fixing c_i by computing the root (α_r, α_i) of

$$c_i\left[\left(\frac{\partial\alpha_r}{\partial c_r}\right)^2 + \left(\frac{\partial\alpha_i}{\partial c_r}\right)^2\right] + \alpha_i\frac{\partial\alpha_r}{\partial c_r} - \alpha_r\frac{\partial\alpha_i}{\partial c_r} = 0\,.$$

All partial derivatives are calculated at constant c_i by finite-difference approximations. According to Briggs-Bers criterion, the moving frame is absolutely unstable if $\tilde{\omega}_i > 0$ and convectively unstable if $\tilde{\omega}_i < 0$.

We now describe the branch point separating the regions of absolute and convective instabilities as M varies from 0 up to the hypersonic range. Figure 1 (a) shows the values of β_U. A constant trend is apparent whatever β_T. There exists a critical Mach number M_{cr} whose value depends on β_T for which, when $M < M_{cr}$, increasing the Mach number increases the amount of back-flow necessary to cause absolute instability. When $M > M_{cr}$, the process is inversed. Cooling the slow stream extends the domain of absolute instability at any Mach number. For β_T sufficiently small, the shear layer may become absolutely unstable in the subsonic range even when
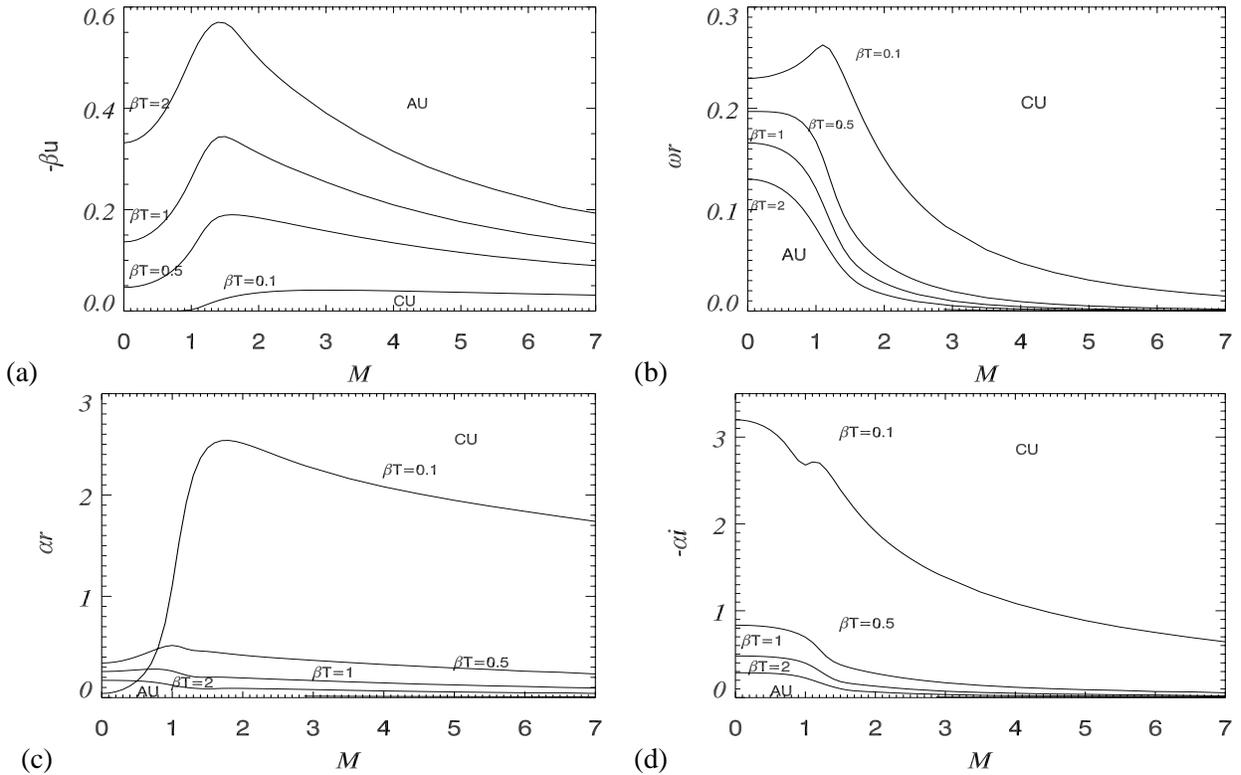
Figure 1: Absolute/convective instabilities transition as a function of M: (a) the velocity ratio $-\beta_U$, (b) the frequency ω_r, (c) the wavenumber α_r and (d) the spatial growth rate $-\alpha_i$ for various values of β_T. *AU* stands for absolutely unstable and *CU* for convectively unstable.

the streams are coflowing. The velocity ratio β_U becomes positive for $\beta_T = 0.1$ as $M < 0.8$ and reaches values of order 10^{-3}. On the other hand, we expect absolute instability to vanish for a certain value of $\beta_T > 1$ from the subsonic range. Infinite Mach number is the most unstable case as all curves tends to zero as $M \to \infty$. Figure 1 (b) shows the transition branch point frequency. A similar separation of the evolution of ω_r with M happens according to whether the regime is subsonic or supersonic. In the first range, ω_r is quasi-constant, then it decays rapidly for $M \sim 1$. When the motion is truly supersonic, the decrease is smoother. When $\beta_T = 0.1$ however, the behaviour of ω_r is different, it is increasing up to $M = 1$ and then decreasing. The hotter the slow stream is, the smaller the frequency is. The asymptotic frequency for infinite Mach number is zero. Figure 1 (c) displays the evolution of the wavenumber. This does not vary substantially in the subsonic range when β_T is of order one, but increases steeply when β_T is small; α_r reaches a maximal value of 2.539 at $M = 1.8$ for $\beta_T = 0.1$ whereas all other maxima are below $\alpha_r = 0.5$. The wavenumber α_r increases as β_T decreases. We expect the high instability of the cooled slow stream mixing layers to be attenuated by viscosity. In the supersonic range, α_r diminishes slowly toward zero. Figure 1 (d) gives the spatial growth rate of the transition branch point. In the subsonic range, it decreases slighly for order-one β_T and then more rapidly in the supersonic range. The spatial instability is getting stronger when the slow stream is cooled. The case $\beta_T = 0.1$ is singular again; α_i decreases strongly as $M < 1$ and tends to zero as M tends to infinity.

5

5   Conclusions

This study confirms Pavithran and Redekopp (1989) and, Jackson and Grosch (1990)'s studies undertaken at subsonic régime. Cooling the slow stream of a mixing layer has a destabilising effect whatever M may be. However, the supersonic instability has different properties from those of subsonic instability. Indeed, for $M > 1$ the domain of absolute instability starts to increase with M in the $(-\beta, M)$ plane and the most unstable state would be when M is infinite. When the slow stream is very cold, the incompressible state becomes the most unstable with a positive-β_U transition in a subsonic range. Both these states correspond to a mixing layer whose slow stream reaches ($\beta_T \ll 1$) or both streams reach ($M \to \infty$) the absolute zero temperature.

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