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Electric Bus Charging Scheduling Strategy with Stochastic Arrival Time and State of Charge ^{*}

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Abstract. To alleviate the range anxiety of drivers and time-consuming charging for electric buses (eBuses), opportunity fast-charging has gradually been utilized. Considering that eBuses have operational tasks, identifying an optimal charging scheduling will be needed. However, in the real world, arrival time and state of charge (SOC) of eBuses are uncertain. Therefore, it is challenging for the charging station to efficiently schedule charging tasks. To solve the problem, this paper develops a two-stage stochastic eBus charging scheduling model. In the first stage, eBuses are assigned to designated chargers. After the arrival time and SOC are realized, the second stage determines the charging sequence of eBuses on each charger. The objective is to minimize the penalty cost of tardiness by determining the charging start time and the corresponding charging duration time. Then, a sample average approximation (SAA) algorithm is applied. Additional numerical experiments are performed to verify the efficiency of the stochastic programming model and algorithm.

Keywords: eBus charging scheduling problem · Uncertain arrival time and SOC · Stochastic two-stage programming · Sample average approximation

1 Introduction

The reduction of fossil and the increase of carbon emissions force people to turn their attention to renewable energy. Electric buses are widely welcomed for their smooth driving process, low noise, and zero emission [8, 13]. However, the limited range is a key challenge in operating eBuses. The range of an eBus is about 265 miles, while diesel buses can run for nearly 2.5 times more than eBuses, which may cause severe range anxiety among drivers [12]. Another challenge is charging. Though there is a relatively wide-scale adoption of eBuses, the charging supply is in shortage for the limited space of charging stations. Therefore, efficient charging scheduling to meet the charging needs is necessary.

Overnight charging is the most popular charging strategy, which needs sufficient capacity of the battery. However, increased battery size may reduce the passenger capacity. Thereby opportunity

^{*} Supported by the National Natural Science Foundation of China (NSFC) under Grants 72021002, 71771048, 71432007, 71832001, 71972146, and 72071144.

charging is gradually received more and more attention, which not only makes smaller, lighter batteries possible, but also keeps eBuses stay in service without driving range restrictions. With charging stations at the end point of each route, eBuses are charged at higher power – a level of around 300 to 450kW – during their layover time of 15 to 25 minutes. Then the bus can run the route again several times. Taking Fig. 1 as an example, the eBus departs according to the timetable at the bus terminal, and the initial SOC should meet the energy required for at least one run of the driving route. After finishing a driving route, it returns to the original departure station, and makes an appointment for charging during layovers. Then the charging station schedules charging tasks when the eBus arrives at the station. Finally, the eBus will drive back to the bus terminal after completing the charging task and wait for the next departure. Since eBuses are susceptible to weather, road conditions, and human flow factors, they may arrive at the charging station before or after the reservation time point, and the SOC is also unknown before the actual arrival.

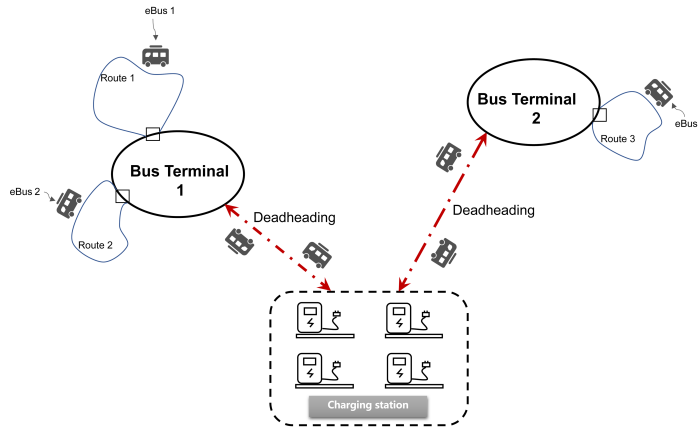


Fig. 1. Charging structure of two bus terminals and single charging station

Considering that charging times should be less than layovers, this can be challenging to schedule charging tasks during peak periods, which may require either partial charging or risk a tardiness. Previous studies have analyzed the impact of different charging scheduling on the power grid and operational costs with deterministic settings [3, 6, 7, 11, 13]. However, it is hard for the charging station to receive full information about eBuses for the complexity of road conditions. Therefore, our work investigates how to allocate chargers to the eBuses during their layovers with uncertain arrival times and SOC in order to minimize the total cost.

To the best of our knowledge, we are the first to introduce a two-stage stochastic programming model in this domain. The main contributions of this work include (1) a two-stage stochastic charging scheduling problem is studied for the first time, (2) SAA algorithm is proposed to deal with the considered problem.

The rest of this paper is organized as follows. Section 2 describes the problem and a two-stage stochastic programming model is developed. In Sect. 3, the SAA method is applied to solve the problem. Computational experiments are presented in Sect. 4 and conclusion is given in Sect. 5.

2 Problem Description and Formulation

In this section, the problem is described in detail and a two-stage stochastic programming formulation is proposed.

2.1 Problem Description

In this work, we focus on investigating a stochastic eBus charging scheduling problem, where the eBus arrives at a random time with a random arrival SOC, to minimize the penalty cost of eBuses' tardiness. For the addressed problem, given a set of chargers K , the set of eBuses to be charged is denoted by $B = \{1, 2, \dots, n\}$. The eBuses run from two different bus terminals to the same charging station for charging. Therefore, the transportation time of eBus $j \in B$ to the charging station is related to the distance Dis_j between them and calculated as Dis_j/v_j , where v_j denotes the average speed of eBuses. We allow for tardiness of the operation plan if the eBus does not have enough energy before the charging deadline for the next trip [4]. To reduce battery consumption, the eBus must stop charging when the charge level reaches the upper SOC limit. The remaining work is based on the following assumptions:

- (1) Adopt intermittent charging mode. Reference proved that when the battery always in the “half-and-half-discharge” state has a long cycle life [13];
- (2) Each eBus $j \in B$ can only be allocated to one charger $k \in K$;
- (3) One charger $k \in K$ can only charge one eBus $j \in B$ at the same time;
- (4) The chargers are of the same type, using constant current charging, and the relationship of charging capacity and charging time is linear, i.e., the charging time is calculated by dividing the expected charging capacity by the charging power p ;
- (5) The charging capability of the charging station is sufficient;
- (6) Once an eBus starts charging, it must be continuously charged until the charging task is completed;
- (7) To protect the batteries and ensure the service life of eBuses, each residual capacity of the battery should be greater than SOC_{min} (i.e., 30%) of its full capacity for the departure of next trip, and less than SOC_{max} (i.e., 70%).

2.2 Two-stage Stochastic Mathematical Model

In this section, input parameters and decision variables are presented, then the problem is formulated into a two-stage stochastic mathematical model [P1].

Input Parameters

Ω : Set of scenarios, indexed by ω ;

B : Set of eBuses, $B = \{1, 2, \dots, n\}$, indexed by i, j ;

K : Set of chargers, indexed by k ;

$a_j(\omega)$: Uncertain arrival time of eBus $j \in B$ under scenario $\omega \in \Omega$;

$\theta_j(\omega)$: Uncertain SOC of bus $j \in B$ under scenario $\omega \in \Omega$;

$\xi(\omega)$: Vector of uncertain arrival time and SOC under scenario $\omega \in \Omega$,
and $\xi(\omega) = [a_1(\omega), a_2(\omega), \dots, a_n(\omega), \theta_1(\omega), \theta_2(\omega), \dots, \theta_n(\omega)]^T$;

$P(\omega)$: Probability of scenario $\omega \in \Omega$;

- Dis_j : The distance of eBus $j \in B$ from each bus terminal to the charging station;
 v_j : Average driving speed of eBus $j \in B$;
 d_j : Charging deadline of eBus $j \in B$;
 SOC_{max} : Upper limit SOC of each eBus;
 SOC_{min} : Lowest allowed SOC of each eBus;
 Q_j : Energy consumed for performing a trip by eBus $j \in B$;
 p : Charging power of each charger;
 Cap_j : Battery energy capacity of eBus $j \in B$;
 e_j : Energy consumption per kilometer of eBus $j \in B$;
 L_j^{\max} : The maximum charging time allowed for eBus $j \in B$;
 L_j^{\min} : The minimum charging time that eBus $j \in B$ must meet;
 c_j : eBus j 's per-unit-time tardiness penalty cost;
 \mathcal{M} : A large enough number.

Decision Variables

- x_{jk} : Binary variable, equal to 1 if eBus $j \in B$ is assigned to charger $k \in K$, and 0 otherwise;
 $y_{ij}^k(\omega)$: Binary variable, equal to 1 if eBus $i \in B$ is charged immediately before eBus $j \in B$ on charger $k \in K$ under scenario $\omega \in \Omega$, and 0 otherwise;
 $S_j(\omega)$: Start charging time of eBus $j \in B$ under event $\omega \in \Omega$;
 $E_j(\omega)$: Charging end time of eBus $j \in B$, i.e., the time when eBus $j \in B$ is completed charging under scenario $\omega \in \Omega$;
 $C_j(\omega)$: Completion time of eBus $j \in B$, i.e., the time when eBus $j \in B$ is completed charging and back to the bus terminal under scenario $\omega \in \Omega$;
 $T_j(\omega)$: Tardiness of eBus $j \in B$ under scenario $\omega \in \Omega$.

The two-stage stochastic programming model **[P1]** can be constructed as follows:
[P1]:

$$\min f = \sum_{\omega \in \Omega} \sum_{j \in B} c_j \cdot T_j(\omega) \cdot P(\omega) \quad (1)$$

$$s.t. \quad \sum_{k \in K} x_{jk} = 1, \quad \forall j \in B \quad (2)$$

$$y_{ij}^k(\omega) + y_{ji}^k(\omega) \geq 1 - \mathcal{M} \cdot (2 - x_{ik} - x_{jk}), \quad \{i \neq j\} \in B, k \in K, \omega \in \Omega \quad (3)$$

$$y_{ij}^k(\omega) + y_{ji}^k(\omega) \leq 1 + \mathcal{M} \cdot (2 - x_{ik} - x_{jk}), \quad \{i \neq j\} \in B, k \in K, \omega \in \Omega \quad (4)$$

$$S_j(\omega) \geq a_j(\omega) + \sum_{k \in K} \frac{Dis_j}{v_j} \cdot x_{jk}, \quad \forall j \in B, \omega \in \Omega \quad (5)$$

$$S_j(\omega) \geq E_i(\omega) - \mathcal{M} \cdot (1 - y_{ij}^k(\omega)), \quad \{i \neq j\} \in B, k \in K, \omega \in \Omega \quad (6)$$

$$E_j(\omega) \leq S_j(\omega) + L_j^{\max}, \quad \forall j \in B, \omega \in \Omega \quad (7)$$

$$E_j(\omega) \geq S_j(\omega) + L_j^{\min}, \quad \forall j \in B, \omega \in \Omega \quad (8)$$

$$C_j(\omega) = E_j(\omega) + \sum_{k \in K} \frac{Dis_j}{v_j} \cdot x_{jk}, \quad \forall j \in B, \omega \in \Omega \quad (9)$$

$$T_j(\omega) \geq C_j(\omega) - d_j, \quad \forall j \in B, \omega \in \Omega \quad (10)$$

$$x_{jk} \in \{0, 1\}, \quad \forall j \in B, k \in K, \omega \in \Omega \quad (11)$$

$$y_{ij}^k(\omega) \in \{0, 1\}, \quad \forall i, j \in B, i \neq j, k \in K, \omega \in \Omega \quad (12)$$

$$S_j(\omega), C_j(\omega), T_j(\omega) \in \mathbb{R}, \quad \forall j \in B, k \in K, \omega \in \Omega \quad (13)$$

The objective function (1) is to minimize the penalty cost of eBuses' tardiness. Constraint (2) implies that each eBus $j \in B$ can only be allocated to one charger. Constraints (3) - (4) mean if both eBuses i and j are charged on the same charger k (i.e., $x_{ik} = 1$ and $x_{jk} = 1$), there must exist a precedence relation, i.e., $y_{ij}^k(\omega) + y_{ji}^k(\omega) = 1$. Constraint (5) implies that each eBus $j \in B$ should be charged after the arrival time at the bus terminal plus the travel time to the charging station under scenario $\omega \in \Omega$. Constraint (6) restricts that no eBus can be charged before the charging end time of its predecessor under each scenario $\omega \in \Omega$, which is linearised by the big-M method. Constraint (7) represents the maximum time when eBus j is completed charging under scenario $\omega \in \Omega$, where $L_j^{\max} = 60 \cdot (SOC_{\max} - \theta_j(\omega)) \cdot Cap_j / p$. Constraint (8) represents the minimum time when eBus j is completed charging under scenario $\omega \in \Omega$, where $L_j^{\min} = 60 \cdot [e_j \cdot Dis_j + Q_j + Cap_j \cdot (SOC_{\min} - \theta_j(\omega))] / p$. Constraint (9) defines the completion time of eBus $j \in B$ under scenario $\omega \in \Omega$. Constraint (10) defines tardiness of eBus $j \in B$ under scenario $\omega \in \Omega$. Constraints (11) - (13) give the domains of decision variables.

3 Solution Approach

SAA is a well known approach for solving stochastic programs with a set of smaller and tractable scenarios [1, 5, 9, 10]. When the number of given scenarios $|\Omega|$ is sufficiently large and $|\Omega| \rightarrow +\infty$, the optimal objective value of **P2** almost certainly converges to the true optimal target value [2]. For our problem, a set Ω of scenarios is randomly generated by Monte Carlo simulation, corresponding to Ω realisations of uncertain vector ξ , denoted by $\xi(1), \xi(2), \dots, \xi(\Omega)$. The SAA approach approximates problem **P1** by the following problem **P2**:

[P2]:

$$\min \quad \hat{f} = \frac{1}{\Omega} \sum_{\omega \in \Omega} \sum_{j \in B} c_j \cdot T_j(\omega) \quad (14)$$

$$s.t. \quad (2) - (13) \quad \omega = 1, 2, \dots, \Omega \quad (15)$$

4 Numerical Experiments

In this section, numerical experiments are conducted to evaluate the performance and efficiency of the solution approach we propose. The stochastic model is implemented by the off-the-shelf commercial optimisation solver CPLEX 12.8. And all the numerical experiments are run on a PC with Core I7 3.4GHz processor and 8GB RAM under Windows 10 Operating System. The computation time of the SAA is limited to 3600s.

4.1 Scenario Generation and Data Description

The input data involve $B = 5$ eBuses from two different bus terminals and $k = 2$ chargers in a charging station. In Table 1, it is noted that (1) the first column denotes eBus indexes; (2) column 2 denotes the average speed of each eBus; (3) column 3 reports the charging deadline of each eBus; (4) column 4 provides the energy consumption per kilometer of each eBus; (5) column 5 provides the distance between the bus terminals and charging station; (6) the 6th column provides the penalty costs; (7) column 7 provides battery energy capacity of each eBus.

Table 1. Input parameter data about eBuses

j	v_j	d_j	e_j	Dis_j	c_j	Cap_j
1	25	20	1.2	2	2	250
2	22	25	1.5	3	1	300
3	28	20	1.5	3	3	300
4	23	30	1.2	2	3	250
5	27	30	1.5	3	2	300

4.2 The Performance of SAA Algorithm

In the computational experiments, the arrival time and SOC are randomly generated. And two probability distribution, i.e., uniform and normal distributions are tested. We assume the mean values of arrival time and SOC are $\mathbb{E}[a_j] = 3$ and $\mathbb{E}[\theta_j] = 0.3$, respectively. The corresponding standard deviations are set to 1 and 0.1. Let sample size $\Omega = 10$, and each instance is tested $M = 10$ times to obtain its average results. Additionally, mean values, coefficient of variation (CV), 85th percentiles and 99th percentiles of the total costs over all the scenarios are employed to evaluate the proposed SAA algorithm. Computational results are reported in Table 2. We can observe that (1) the objective value under uniform distribution is 21.34, which is 8.25% lower than that under normal distribution; (2) the coefficient of variation under uniform distribution is 47.04%, about 31.93% lower than under normal distribution; (3) the 85th percentile under uniform distribution is about 5.49% larger than under normal distribution. This may be because the random numbers generated by normal distribution are closer to the mean values. However, the 99th percentile obtained under uniform distribution is 39.09, about 28.06% smaller than that under normal distribution. This may be explained that the extreme case under normal distribution may be worse; (4) the average computation time under uniform distribution is 14.01, about 63.1% larger than that under normal distribution.

Table 2. Computational results under uniform and normal distributions

$M = 5$	Uniform distribution	Normal distribution
Objective	21.34	23.26
CV(%)	47.04	69.11
85th percentile	31.51	29.87
99th percentile	39.09	54.34
Time(s)	14.01	8.59

Considering that SAA has unstable objective values in small sample experiments. Therefore, we conduct the experiment to illustrate the role of sample size Ω in the SAA algorithm. The arrival time and SOC are randomly generated under normal distribution. The results are detailed in Table 3. With the increase of the sample size Ω , the optimal value decreases continuously, but at the cost of increasing calculation time greatly.

Table 3. Quality of solutions with various sample size Ω ($M=10$)

Ω	Objective	85th percentile	99th percentile	Time(s)
5	121.67	142.95	153.54	1.72
10	23.40	29.45	34.29	8.97
15	22.33	39.46	44.78	64.47
20	18.8	25.76	32.16	417.56
25	-	-	-	>3600

4.3 Sensitivity Analysis

In this section, sensitivity analysis results are presented based on the illustrative example. We examine the impact of standard deviation for arrival time. The standard deviations of a_j for any $j \in B$ is among $0.1 \dots 1$ (fixed $M = 10, \Omega = 10$). The results are shown in Table 4. We can find that large standard deviations of arrival time under normal distribution lead to smaller objective values.

5 Conclusion

In this work, we investigate a two-stage stochastic eBus charging scheduling model, in which arrival time and SOC are assumed to be uncertain. The objective is to minimize the penalty cost of tardiness by determining the charging start time and the corresponding charging duration time. Then, SAA algorithm is applied to solve the problem. Additional numerical experiments are carried out to illustrate the effectiveness of SAA algorithm. For further research, one of the directions is to develop more efficient heuristic algorithms to shorten the computation time.

Table 4. The impact of standard deviation of arrival time

Standard deviation	Objective	85th percentile	99th percentile	Time(s)
0.1	21.21	30.24	40.13	17.06
0.2	23.09	38.56	58.35	21.83
0.3	24.27	32.04	36.09	18.39
0.4	24.19	30.64	37.78	17.86
0.5	21.77	33.66	44.10	19.58
0.6	22.30	38.99	41.22	16.91
0.7	22.85	35.34	40.03	17.24
0.8	19.56	32.41	33.56	12.98
0.9	18.72	31.96	39.43	14.22
1.0	21.02	37.34	39.91	15.38

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