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# The Internal Wave field Produced by a Mountain and its Wake in the Atmosphere

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## Abstract :

*A forecast model is developed for the internal wave field generated by stratified flow over three-dimensional topography. The model is based on high-resolution nonlinear numerical simulation of the near-field flow around the topography combined with a ray-tracing of the far-field propagation of the internal waves generated by the near-field flow. The rays are traced in a Fourier-transform domain, and the resulting ray solution is mapped into a spatial solution by inverse Fourier transform. Examples of the wave field predicted by this model are compared with fully nonlinear numerical simulations using mesoscale forecast models.*

## Résumé :

*Un modèle de prédiction est développé pour le champ des ondes internes générées par un flux stratifié par-dessus d'une topographie trois-dimensionnelle. Le modèle se base en simulations numériques non-linéaires de haute-résolution du flux du champ le proche au tour de la topographie, combinée avec le traçage des rayons du champ le lointain de propagation des ondes internes générées par le flux du champ proche. Les ondes sont tracées avec une domaine de transformation Fourier, et la solution des rayons est transformée par une transformation Fourier inverse. Exemples du champ d'onde prévu par ce modèle sont comparées avec des simulations numériques non-linéaires complètes en utilisant des modèles de prédiction de mesoscale.*

## Key-words :

### **internal waves ; ray theory ; topography**

Mountain waves are internal gravity waves generated by stratified flow over topography. They carry momentum vertically and are an important source of mixing in the stratosphere and the upper parts of the troposphere. The associated mountain-wave turbulence can also be hazardous to aviation. Here we discuss a new possibility for forecasting mountain waves in a height dependent background. A mesoscale model is used to simulate the flow at low levels near the mountain. This flow can be substantially nonlinear, with vortex shedding, flow separation, and wavebreaking. Such nonlinearity is the norm for mountains more than a couple kilometers high. The wavefield that emerges from the low-level flow is more nearly linear and is modeled by a Fourier method, which is initialized by the mesoscale model results. The Fourier method can be run to higher altitudes and at much higher resolution than is practical with a mesoscale model. The higher resolution is vital for identifying locations of mountain wave breaking and turbulence.

The present paper is an initial investigation of this approach. We keep things simple and consider an example with uniform background wind and stratification. This approach is easily generalized for an arbitrary height-dependent background wind and stratification.

## 1 The method

We work in terms of the vertical velocity  $w(x, y, z)$  and the vertical eigenfunction  $\tilde{w}(k, l, z)$ . The spatial coordinates are  $(x, y, z)$ . The horizontal wavenumbers are  $k, l$  in the directions  $x, y$  respectively, and  $w$  is obtained from  $\tilde{w}$  by the inverse Fourier transform

$$w(x, y, z) = \iint_{-\infty}^{\infty} \tilde{w}(k, l, z) e^{i(kx+ly)} dk dl . \quad (1)$$

This is the general form for the Fourier solution.

Since we limit ourselves to a uniform background in this preliminary study, the vertical eigenfunctions are exponentials, scaled in the anelastic approximation by the inverse square root of the mean density  $\rho$ :

$$\tilde{w}(k, l, z) = [\rho(z_i)/\rho(z)]^{1/2} \tilde{w}_0(k, l) e^{-im(z-z_i)}. \quad (2)$$

In a height dependent background we would replace the above expression with a ray solution expressed in Fourier coordinates, as described, for example, by Broutman et al. (2003). The vertical wavenumber  $m$  satisfies the gravity wave dispersion relation

$$m = (k^2 + l^2)^{1/2} (N^2/\hat{\omega}^2 - 1)^{1/2}, \quad (3)$$

where  $N$  is the buoyancy frequency,  $\hat{\omega} = -kU$  is the intrinsic frequency of the mountain waves, and  $U$  is the background wind speed, taken to be in the  $x$ -direction.

The lower boundary condition is imposed at the height  $z_i$ , which we call the initialization height. In the standard Fourier method, a linearized lower boundary condition is imposed. This can be expressed as  $\eta(x, y, z_i) = h(x, y)$ , where  $\eta$  is the vertical displacement of the mountain waves and  $h(x, y)$  is the mountain height. Converting to  $w$ , this becomes  $w(x, y, z_i) = -Uh_x(x, y)$ , or for the vertical eigenfunction  $\tilde{w}$ ,

$$\tilde{w}(k, l, z_i) = -i\hat{\omega}\tilde{h}(k, l), \quad (4)$$

where  $\tilde{h}(k, l)$  is the Fourier transform of  $h(x, y)$ .

The idea in this paper is to replace the linearized lower boundary condition (4) with

$$\tilde{w}(k, l, z_i) = \tilde{w}_M(k, l, z_i). \quad (5)$$

The vertical velocity  $w_M$  is computed by a mesoscale model, and  $\tilde{w}_M$  is the horizontal Fourier transform of  $w_M$ .

The mesoscale model simulation below was produced with NCAR's Weather Research and Forecast model (WRF), described Skamarock et al. (2005). WRF is non-hydrostatic, compressible, and discretized with high-order finite differences. The domain is 300 km by 300 km in the horizontal, with 1km grid spacing, and 30km in the vertical with 100m grid spacing. The results are run for a model time of 6 hours.

## 2 Results

The mountain has the form

$$h(x, y) = h_m a^3 / (x^2 + y^2 + a^2)^{3/2}, \quad (6)$$

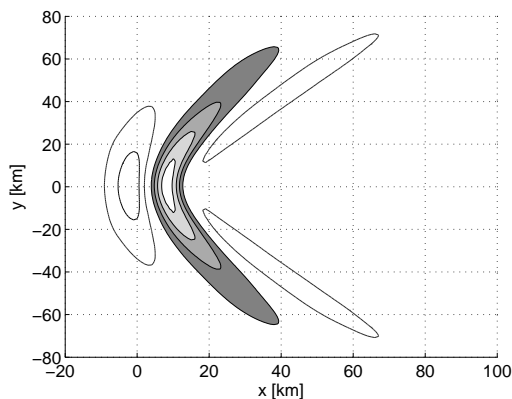


Figure 1: The solution for  $w$  at  $z = 15$  km computed by the Fourier method when initialized by the linearized lower boundary condition (4). Values range from  $-0.77$  to  $1.30 \text{ ms}^{-1}$ . Here and in subsequent figures, shaded contours have positive values, the zero contour is omitted, and the contour interval is  $0.3 \text{ ms}^{-1}$ .

where the mountain width is  $a = 10$  km and the mountain height is  $h_m = 1.5$  km. We set  $U = 10 \text{ ms}^{-1}$  and  $N = 0.01 \text{ s}^{-1}$ , giving a Froude number  $Fr = U/Nh_m = 2/3$ .

We first present in Figure 1 the standard Fourier solution for  $w$ , as initialized with the linearized lower boundary condition (4) imposed at  $z_i = 0$ . This is plotted at a height of 15 km. A commonly used modification of (4) for the dividing streamline parameterization leads to a similar result.

Next we replace (4) with the WRF solution for  $w$  at  $z_i = 5$  km height, plotted in Figure 2. This height was chosen because the WRF results shown significant wavebreaking below about 4 km. The WRF solution and the Fourier solution at 15 km are plotted in Figure 3. Vertical cross sections along the centerline of the mountain at  $y = 0$  are shown in Figure 4. These results reveal good agreement, above  $z_i$ , between the WRF model and the Fourier method when the Fourier method is initialized by WRF. For example, looking along the top of both plots in Figure 4, at  $z = 20$  km, we see similar positions for the phases and similar horizontal wavelengths. The differences near the edges of the wavefield are probably due at least in part to our simple treatment in the Fourier method of the transience due to the sudden startup of the flow.

The Fourier solution below  $z_i$  can be useful for testing parameterizations of wave generating topography, such as the dividing streamline parameterization.

### 3 Comments

This study demonstrates that at least for a simple case it is possible to obtain reasonable results with the Fourier method by initializing it with a mesoscale model at a height just above the nonlinear low-level flow. The results are quite different from what would have been predicted by the Fourier method with the linearized lower boundary condition. Our test case is the same one studied numerically by Schar and Durran (1997). We have also tested (not reported here) other cases in Schar and Durran (1997), with similar success.

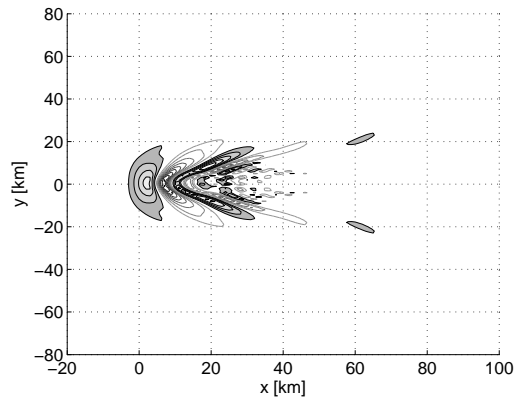


Figure 2: The WRF solution for  $w$  at  $z = 5$  km, used as the lower boundary condition for the Fourier method. Values range from  $-2.76$  to  $1.78$   $\text{ms}^{-1}$ .

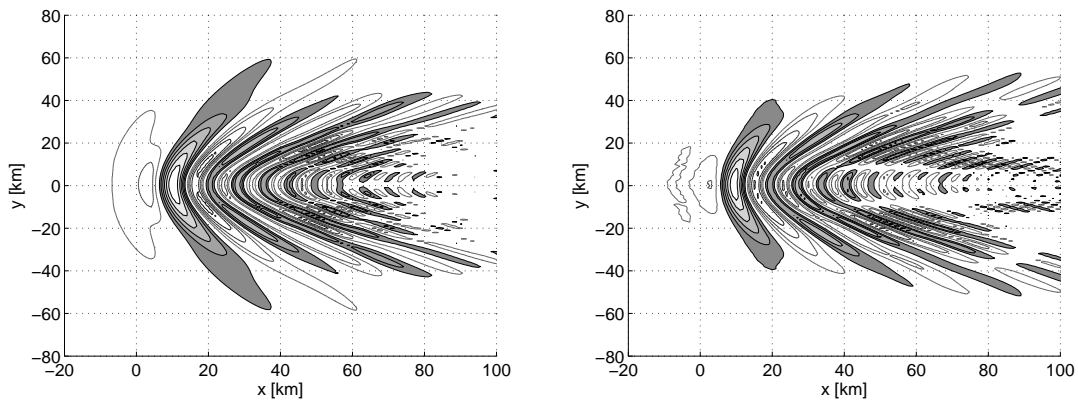


Figure 3: The solution for  $w$  at  $z = 15$  km computed by the WRF model (left panel) and by the Fourier method (right panel). The Fourier solution is initialized with the WRF solution of Figure 2. Values range from  $-1.55$  to  $2.00$   $\text{ms}^{-1}$  for the Fourier method, and from  $-1.60$  to  $2.01$   $\text{ms}^{-1}$  for the WRF model.

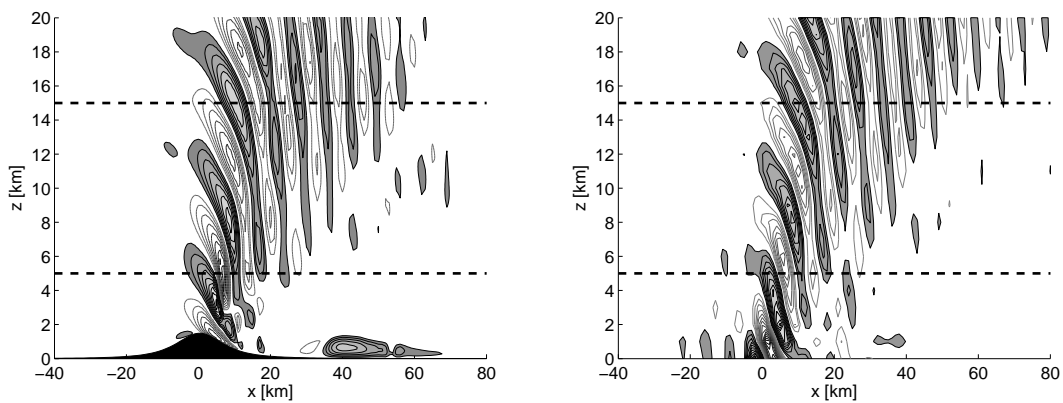


Figure 4: Left panel: the WRF solution for  $w$  at  $y = 0$ , along the centerline of the mountain. Dashed lines indicate the initialization height for the Fourier method ( $z_i = 5$  km) and the height at which the WRF and Fourier solutions are compared in the Figure 3 ( $z = 15$  km). Right panel: The corresponding Fourier solution.

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