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Stratified turbulence in the atmosphere and the oceans

Erik Lindborg

Linné Flow center
Department of Mechanics, KTH
Stockholm Sweden
erikl@mech.kth.se

Abstract :

Some recent progress in the field of stratified turbulence is reported. The progress has been made by combining recent theoretical development with simulations and data analysis. On the basis of scaling arguments, numerical simulations and observational analysis it is argued that the dynamics of the mesoscale range, that is wave lengths from one to five hundred kilometres, in the upper troposphere and lower stratosphere is controlled by a special type of forward energy cascade which is typical for strongly stratified turbulence. In contrast to the classical Kolmogorov cascade, the stratified turbulence cascade is characterised by two radically different length scales in the horizontal and the vertical and is a cascade of both kinetic and potential energy. Kinetic energy is observed to be approximately equipartitioned between vortical and divergent modes. It is also suggested that this type of dynamics can be found in the interior of the ocean at horizontal scales between 10 metres and 10 kilometres. A number of observational studies previously reported in the literature will be referenced in support to this.

Mots-clefs :

stratified turbulence

1 Introduction

It is a remarkable fact that the mesoscale (wave lengths $\sim 1 - 500$ km) energy wave number spectra of the free atmosphere exhibit a $k_h^{-5/3}$ -dependence (Vinnichenko 1970 ; Nastrom *et al.* 1984) just as the spectrum of three-dimensional isotropic turbulence. The classical Nastrom-Gage spectra are reproduced from Nastrom and Gage (1985) in figure 1. Two conflicting hypotheses have been put forward to explain the $k_h^{-5/3}$ -spectra. It has either been assumed (Gage 1979 ; Lilly 1983) that they arise as the result of an inverse energy cascade of the same type as can be observed in two-dimensional turbulence, or it has been assumed (Dewan 1979 ; 1997) that they arise as the result of a forward energy cascade of nonlinearly interacting gravity waves. A forward energy cascade is a process in which large structures of motion break down to smaller structures in a chain process. In this way, energy is transferred from small to large wave numbers through the energy spectrum of the system undergoing the cascade. An inverse cascade, on the other hand, is a process in which small structures merge to form larger structures and energy is transferred in the opposite direction through the spectrum. Lindborg (2006) argued that one of the two hypotheses is, on the whole correct, namely the forward cascade hypothesis. It was also argued that the mesoscale cascade motion is of a special type which arises in the presence of strong stratification, that is in the limit of small horizontal Froude number, $F_h = \epsilon^{1/3}/Nl_h^{2/3}$. Here, N is the Brunt-Väisälä frequency and ϵ is the energy dissipation rate. A characteristic feature of the motion is layer formation. In the presence of strong stratification two-dimensional vertically oriented vortices with horizontal length scale l_h will split up into layers with vertical length scale $l_v \sim F_h l_h$, a result which is also found in recent experimental (Billant & Chomaz 2000) and theoretical (Billant & Chomaz 2001) investigations. The layers will split up into thinner layers which in turn will split up into even thinner layers and so on. As the layers become

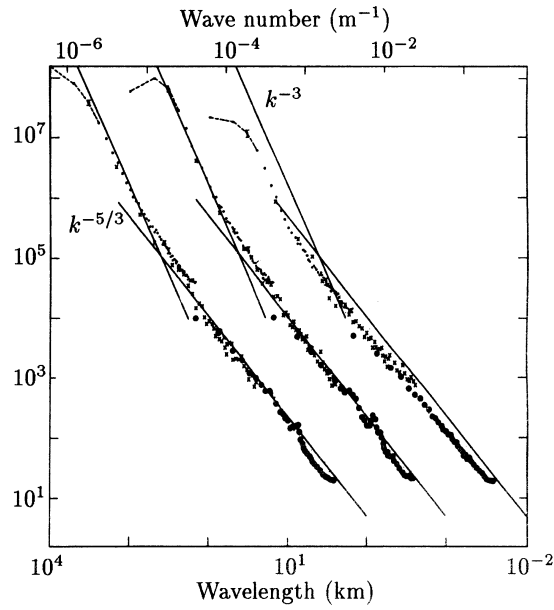


FIG. 1 – Horizontal energy spectra of zonal and meridional wind and temperature, reproduce from Nastrom & Gage (1985). The spectra of meridional wind and temperature are shifted one and two decades to the right respectively.

thinner they will become sensitive to instabilities causing them to break up in the horizontal direction. In this way, a cascade is set up which carries energy from large to small scales. Lindborg (2005 ; 2006) carried out a series of box-simulations of the Boussinesq equations with and without system rotation. The ratio between the vertical side, L_z , and a horizontal side, L_x , of the box was set so that $L_z/L_x \sim F_h$, which means that the boxes are highly elongated. A special type of forcing was designed to test the forward cascade hypothesis. Energy was randomly injected into Fourier modes for which the vertical wave number is equal to zero and the horizontal wave number is equal to four. These modes correspond to large horizontal motions lacking all vertical variation. Another weak forcing was added to trigger the development of vertical gradients. A stationary state was reached after some time, in which layers are constantly formed. By performing a simulation in which the box height L_z has been doubled in comparison with another simulation, leaving everything else unchanged, it was also demonstrated that the results are independent of the chosen L_z , *i. e.* the thickest layers which develop are thinner than the height of the box. In the simulations, a stable forward cascade of kinetic and potential energy was observed and the mesoscale $k_h^{-5/3}$ energy spectra were reproduced. In figure 2 we have reproduced the spectra kinetic and potential energy spectrum from simulations at very strong stratification, carried out in a very elongated box, with and without system rotation. The spectra have the form

$$E_K(k_h) = C_K \epsilon_K^{2/3} k_h^{-5/3}, \quad (1)$$

$$E_P(k_h) = C_P \epsilon_K^{-1/3} \epsilon_P k_h^{-5/3}, \quad (2)$$

where ϵ_K is the average dissipation rate of the horizontal kinetic energy, ϵ_P is the average dissipation rate of potential energy, and C_K and C_P are universal constants. Somewhat surprisingly it was found that the two constants take the the same value : $C_K \approx C_P \approx 0.51$.

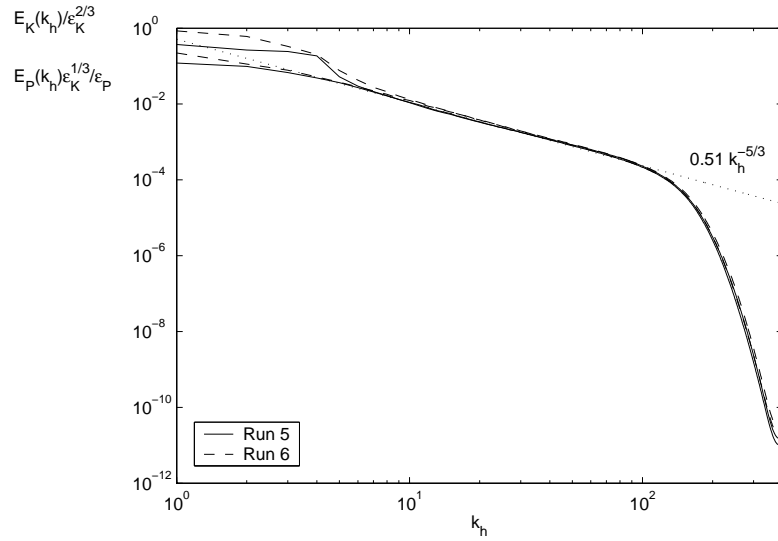


FIG. 2 – Horizontal kinetic and potential energy spectra from box-simulations with $F_h = 9.6 \times 10^{-4}$ and $L_z/L_x = 1/138$. Solid lines : no system rotation. Dashed lines : rather strong system rotation ; Rossby-number = 0.14.

2 Some recent developments

A question which has been debated is whether the mesoscale spectrum is generated by vortical motions or waves. Kinematically, wave motions are predominantly generated in modes which carry horizontal divergence, $\nabla_h \cdot \mathbf{u}_h$, of horizontal velocity (divergent modes) while vortical motions are divergence free and are predominantly generated in modes which are carrying vertical vorticity (rotational modes). In a new theoretical development (Lindborg 2007) it is demonstrated that the two-point correlation functions, Q_{zz} and P , of vertical vorticity and horizontal divergence can be written in terms of velocity structure functions as,

$$Q_{zz}(\rho, z) = -\frac{1}{2} \left[\frac{1}{\rho} \frac{\partial D_{\rho\rho}}{\partial \rho} - \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left(\rho^2 \frac{\partial D_{\phi\phi}}{\partial \rho} \right) \right], \quad (3)$$

$$P(\rho, z) = -\frac{1}{2} \left[\frac{1}{\rho} \frac{\partial D_{\phi\phi}}{\partial \rho} - \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left(\rho^2 \frac{\partial D_{\rho\rho}}{\partial \rho} \right) \right], \quad (4)$$

where ρ is the horizontal distance between the two points, z is the vertical coordinate of the vector \mathbf{r} , separating the two points, $D_{\rho\rho}$ is the structure function including the radial velocity component and $D_{\phi\phi}$ the structure function including the azimuthal velocity component. Q_{zz} and P are calculated on a horizontal plane ($z = 0$), using structure functions calculated from aircraft data by Lindborg (1999). It is shown that Q_{zz} is much larger than P at synoptic scales while at mesoscales Q_{zz} and P are of equal order of magnitude. The vorticity and divergence spectra are also calculated from the two-point correlation functions. In the mesoscale range the vorticity and divergence spectra are of the same order. It is argued that both spectra arise as a result of the same type of motion : stratified turbulence. These results are compared with a new series of numerical simulations (Lindborg & Brethouwer 2007) of stratified turbulence forced in rotational and divergent modes. In these simulations it is found that stratified turbulence can be generated by each type of forcing and that kinetic energy is approximately equipartioned between rotational and divergent modes in the $k_h^{-5/3}$ -range. In order to investigate to what extent internal waves are present in the simulations, frequency spectra of divergent, buoyant

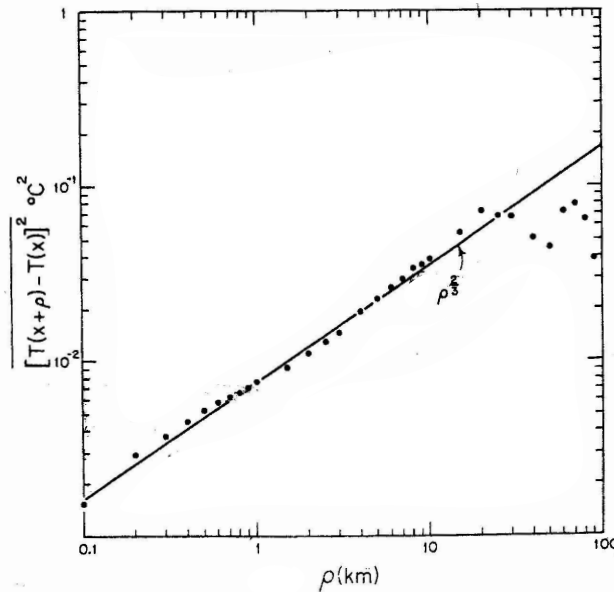


FIG. 3 – Horizontal temperature structure function reproduced from Voorhis & Perkins (1966).

and rotational energy from individual Fourier modes are calculated. It is found that waves are present in the largest scales in the computational domain but become less and less prevalent with decreasing scale. In the inertial range there are few signs of internal waves.

3 Observational data from the ocean

The theory of stratified turbulence as developed by Billant & Chomaz (2001) and Lindborg (2006) is based on similarity and scaling arguments which is supposed to have general validity. If the the $k_h^{-5/3}$ -spectrum at atmospheric mesoscales can be explained by this theory, we must expect that a similar inertial range should exist also in the interior of the ocean, since the same general conditions for such a dynamics are also fulfilled in this case. Dimensional reasoning and scaling arguments suggest that stratified turbulence may exist in the range of vertical scales from about one to one hundred meters and in the range of horizontal scales from about ten metres to ten kilometres. This range of scales has traditionally been thought to be dominated by internal gravity waves (Garret & Munk 1972). We have search the literature in order to investigate whether there are cases where observational data can give support for the alternative hypothesis of stratified turbulence, rather than gravity waves. As a matter fact, we have found no cases which are inconsistent with the stratified turbulence interpretation and a large number of cases which are fully consistent. We will present a large number of reported cases of a measured $k_h^{-5/3}$ -range at the relevant horizontal scales in the interior of the ocean. There are also a number of temperature structure function measurements reported in the literature which also give strong support for the stratified turbulence interpretation.

The number of cases is actually so large that we cannot review them within the space given to us in this article. Here, we will restrict ourselves to reference the measurement by Voorhis & Perkins (1966) who measured temperature wave number spectra and structure functions in the near-surface thermocline in a region 200 km northwest of Bermuda. A thermistor mounted on a cable was towed at mean depth of 99 m around a star-shaped pattern, so that spectra

could be measured along tracks with different orientation. Garret & Munk (1972) reference their measurements as giving support for their suggestion that the spectrum should fall off as k_h^{-2} . Voorhis & Perkins' own interpretation, however, is that they have measured a $k_h^{-5/3}$ -spectrum. Moreover, they also report measurements of the horizontal temperature structure function, and these give even stronger support for the $k_h^{-5/3}$ -interpretation. If the temperature spectrum falls off as $k_h^{-5/3}$ in a certain wave number range, then the horizontal temperature structure function should grow as $\rho^{2/3}$ in the corresponding range of separations ρ (Monin and Yaglom, 1975). The horizontal temperature structure function, $\langle \delta T \delta T \rangle$, is the mean of the square of the difference, δT , between the temperatures at two points at the same depth whose relative horizontal separation distance is ρ . The temperature spectrum is supposed to be proportional to the potential energy spectrum in the ocean. Thus, a horizontal potential energy spectrum of the form $k_h^{-5/3}$ should correspond to a temperature structure function growing as $\rho^{2/3}$. In figure 3 we have reproduced the temperature structure function given by Voorhis & Perkins. As can be seen, the data points follow the curve $\rho^{2/3}$ very closely in a range of separations between 100 m to 10 km.

4 Conclusions

In this contribution we will present some recent advancement of the concept of stratified turbulence. Scaling arguments, simulations and data analysis suggest that the mesoscale motions above the boundary layer in the troposphere as well as in the stratosphere are dominated by stratified turbulence with an associated forward energy cascade. We present a new data analysis which enables us to decompose the observed mesoscale structure functions and spectra into rotational and divergent parts. These two parts are of about equal magnitude in the mesoscale range. This is consistent with results from numerical simulations in which kinetic energy is approximately equipartitioned between rotational and divergent modes. We also present the results of a literature search indicating that stratified turbulence with an associated $k_h^{-5/3}$ inertial range may also be frequently prevalent in the ocean at horizontal scales between ten meters and ten kilometres.

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