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To cite this version:

Hirsh Majid, Daniel Jolly, Hassane Abouaïssa, Gildas Morvan. Advanced control for real-time dynamic traffic routing problem. IESM 2013, Oct 2013, Agdal, Morocco. hal-03353685

HAL Id: hal-03353685

https://hal.science/hal-03353685v1

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Advanced Control for Real-Time Dynamic Traffic Routing Problem

Hirsh Majid, Hassane Abouaïssa, Daniel Jolly, Gildas Morvan

***Abstract*—The aim of this paper is to present a new algorithm to deal with the real-time Dynamic Traffic Routing (DTR) problem using a second order traffic flow model (METANET). The proposed approach rests on the use of the concept of differential flatness for which the structure of the trajectories of the dynamics can be completely characterized, without integration of any differential equations. The objective is to achieve user-equilibrium on alternate routes in a network setting. The relevance of the algorithm is demonstrated via a set of numerical simulations.**

Keywords: Traffic modelling, Dynamic traffic assignment, Flatness-based control.

I. INTRODUCTION

Traffic control represents the main way to improve the freeway throughput and to ensure an efficient, safe and less polluting transportation of goods and persons [8]. It also contributes to a large reduction of direct and indirect costs.

Freeway traffic control can be achieved via a set of actions and measurements such as: dynamic speed limits, route guidance, dynamic assignment and dynamic traffic routing , ramp metering, ... etc.

Dynamic Traffic Assignment (**DTA**) and Dynamic Traffic Routing (**DTR**) represents one of the most efficient solutions to steer the congestion problems especially the no-recurrent one. DTR/DTA or control of traffic diversion rests on the determination of a time-dependent split variables at the diversion point in order to achieve a user-equilibrium traffic pattern [6]. Several algorithms have been developed for this problem. Most of them rest on optimization approaches such those proposed by [22]. As stated in [7], the proposed optimization algorithm attempts to solve the DTR problem by optimizing the objective functions for the nominal model over the planning horizon and it is not adapted for on-line DTR control. Papageorgiou [19] and Messmer & Papageorgiou [16] have proposed algorithms for Dynamic Traffic Assignment (DTA) based on the linear quadratic regulator, and nonlinear optimization techniques. Other techniques use expert systems to deal with the diversion problem [14]. Liu & al. [9] have proposed a strategy based on a model reference adaptive control in order to guide the real-world traffic flow to evolve towards the desired states especially under emergency evacuation. Kachroo & Özbay [5] have designed feedback and fuzzy control laws for an on-line diversion problem. Another algorithm for DTR and DTA problems based on the nonlinear H_∞ was proposed in [4]. These algorithms are based on a simple conservation law and a linear fundamental diagram proposed by [2] that do not reflect all the traffic flow phenomena.

In this paper, we use a specific class of complex systems called "Differentially flat systems" to solve the user-equilibrium DTA/DTR. The main characteristics of flatness is particularly well tuned for allowing one to solve inverse dynamics problems and one builds off of that fundamental solution in using the structure of flatness to solve more general control problems [18]. A system is flat if it does exist a set of variables called "*flat outputs*", such that the system is (non-differentially) algebraic over the differential field generated by the set of flat outputs. In other words, a system is flat if we can find a set of outputs (equal in number to the number of inputs) such that all states and inputs can be determined from these outputs without integration. Moreover, in this paper the non-destination oriented METANET model [26] [1] is used which is a second-order, deterministic, macroscopic traffic flow model that consists of the Payne model [21] discretized in both space and time with some model enhancements proposed by Papageorgiou added.

The paper is organized as follows:

- Section II, presents the main principles of the DTR problem and its mathematical formulation.
- Section III, recalls the main definitions of flat systems.
- Section IV shows the control design methodology using the concept of flatness.
- Section V provides some numerical simulations for a sample network.
- Section VI concludes the paper and outlines some tracks for future developments.

II. DYNAMIC TRAFFIC ROUTING FORMULATION

In this section, we first present a mathematical formulation which is usable for the design of DTA/DTR flat controller. We expose a second order traffic flow model (METANET). Notice that there are two ways to describe the system dynamics for DTA/DTR problems: the link-based model and the route-based model. In this paper we focus on the route-based model.

A. DTR model

For the sake of simplicity, we consider the case of two alternate routes divided into n_1 and n_2 sections respectively,

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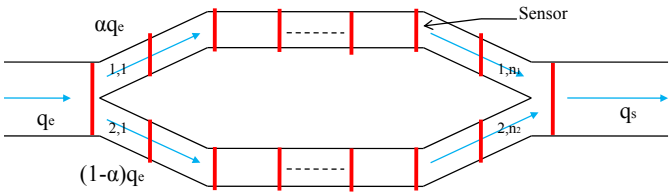

Figure 1. Sample example of two alternate routes

depicted in Figure 1.

The METANET model is used to design the studied system; The equation of dynamic density is:

$$\dot{\rho}_{i,j}(t) = \frac{1}{L_{i,j}}\left[q_{i,j-1}(t) - q_{i,j}(t)\right] \tag{1}$$

where;
$(i,j) = ((1,1),(1,2),\ldots,(1,n_1),(2,1),(2,2),\ldots,(2,n_2))$.
$L_{i,j}$ is the section length and $\rho_{i,j}$ represents the traffic density at section j of the route i.

The relationships between the traffic flow $q_{i,j}$ and the mean speed $v_{i,j}$ is:

$$q_{i,j}(t) = \rho_{i,j}(t)v_{i,j}(t)\lambda_{i,j} \tag{2}$$

$\lambda_{i,j}$ is the number of lanes. $v_{i,j}$ is a dynamic mean speed of section j on the route i.
The equation of dynamic mean speed is:

$$\begin{aligned}\dot{v}_{i,j}(t) &= \tfrac{1}{\tau}(V_{\rho_{i,j}}(t) - v_{i,j}(t)) + \tfrac{1}{L_{i,j}}v_{i,j}(t)(v_{i,j-1}(t) \\ &\quad -v_{i,j}(t)) - \tfrac{\nu}{\tau L_{i,j}}\tfrac{\rho_{i,j+1}(t)-\rho_{i,j}(t)}{\rho_{i,j}(t)+\kappa}\end{aligned} \tag{3}$$

$V_{\rho_{i,j}}$ is defined as a nonlinear expression (See [15]):

$$V_{\rho_{i,j}}(t) = vf_{i,j}\exp\left[\left(-\frac{1}{a}\right)\left(\frac{\rho_{i,j}(t)}{\rho_{c_{i,j}}}\right)^a\right] \tag{4}$$

$\rho_{c_{i,j}}$ represents the critical density and a, a model parameter. $vf_{i,j}$, the free-flow speed.

The control input $\alpha(t) \in [0,\ 1]$ is defined as an exogenous variable of the system [20], it is a split rate which allows to reach a user equilibrium traffic pattern: For the case of two routes, the equations of entry flow in each route are:

$$\begin{cases} \alpha(t)q_e(t) = & q_{1,in}(t) \\ (1-\alpha)q_e(t) = & q_{2,in}(t) \end{cases} \tag{5}$$

$q_e(t)$ is the incoming measured flow. The control objective is to find the split rates α in order to minimize the total travel time TT.
Kachroo and Özbay [7] have formulates the DTR problem as follow: find α_0, the optimal $\alpha(t)$, which mininizes

$$J(\alpha) = \int_0^{t_f}\left[\sum_{i=1}^{m} TT(\rho_i) - \sum_{m+1}^{m+p} TT(\rho_i)\right]^2 dt \tag{6}$$

where, TT is the travel time function and t_f the final time.

In the general case, consider a traffic flow system with n alternate routes. The system is described by the same ordinary differential equation (1) where, $(i,j) = ((1,1),\ldots,(1,n_1),(2,1),\ldots,(2,n_2),(n,1),\ldots,(n,n_n))$. The problem consists then on finding a set of split variables, α_{n-1}, where, $\sum_{i=1}^{n-1}\alpha^i = 1$ which minimize the total travel time TT, (See [7], for optimal formulation of the DTR problem for a general case)

$$\begin{cases} \alpha_1(t)q_e(t) = & q_{1,in}(t) \\ \alpha_2(t)q_e(t) = & q_{2,in}(t) \\ \vdots & \\ (1-\alpha_1-\alpha_2-,\ldots,-\alpha_{n-1})q_e(t) = & q_{n,in}(t) \end{cases} \tag{7}$$

Note that for such complex problem, an open loop control structure is not sufficient [4], it calls for a more robust feedback control one. In this context, we highlight the interest of an attractive approach based on the concept of differential flatness [11] [10]. The following section, recalls the main definitions and principles of differentially flat systems.

III. Differentially flat systems

The DTR problem can be solved by a typical use of control theory which consists the system dynamics inversion to compute the required inputs to achieve a user-equilibrium traffic pattern. As stated in [18], This inversion may involve finding appropriate inputs to steer a control system from one state to another or may imply to find inputs to follow a desired trajectory for some or all of the state variables of the system. Generally, if it does exist, the solution to such problems will not be unique. In this case a trade off between the system performance for the stability and actuation effort must be done. Frequently, this trade off is described as in [6], as a cost function balancing the desired performance objectives with stability and actuation effort, resulting in an optimal control problem as solved in [16].

The basic paradigm used in most, if not all, control techniques is to exploit the mathematical structure of the system to obtain solutions to the inverse dynamics and feedback regulation problems. The most common structure to exploit is linear structure, where one approximates the given system by its linearization and then uses properties of linear control systems combined with appropriate cost function to give closed form (or at least numerically computable) solutions. By using different linearizations around different operating points, it is even possible to obtain good results when the system is nonlinear by "scheduling" the gains depending on the operating point. As the systems that we seek to control become more complex, the use of linear structure alone is often not sufficient to solve the control problems that are arising in applications. This is especially true of inverse dynamics problems, where the desired task may span multiple operating regions and hence the use of a single linear system is inappropriate.

In this paper, we concentrate on a specific class of systems, called "*differentially flat* systems", for which the structure of

the trajectories of the (nonlinear) dynamics can be completely characterized.

The concept of Flat systems was first introduced by Fliess and coworkers ([11] [10]). more than 10 years ago using the formalism of differential algebra (See [13]) for a slightly different approach of differentially flat systems). This special class of non-linear control systems described by ordinary differential equations: differentially flat systems form a special class of nonlinear control systems for which systematic control methods are available once a flat-output is explicitly known. The flatness-based concept was developed in a differentially algebraic context and was later expressed using Lie-Bäcklund transformation [12].

The flatness-based control methods may be expected to play a very significant role in high technology applications in the next few years, similar to what happened for nonlinear control in the last decade [25]. The main property of flat systems is that all the state and input variables can be expressed directly, without integration of any differential equation, in term of the set of so-called "flat output" and a number of its time derivatives. More precisely, the entire system behavior is determined by the trajectory of a finite collection of quantities: flat outputs. This leads to a simple and elegant trajectory design. For a given system, the number of flat outputs is equal to the number of the system inputs. The flatness concept is closely related to the state feedback linearization.

A. Flat Systems Definition

In this section, We just sketch a tutorial definition of flatness for state-space control system. Consider the smooth system defined using the following equation:

$$\dot{x} = f(x, u); \quad x \in R^n, \quad u \in R^m \tag{8}$$

where $x = (x_1, x_2, ..., x_n)$ vector of state variables and $u = (u_1, u_2, ..., u_m)$, m scalar control. The system (8) is flat if and only if, there exist m real smooth functions $h = (h_1, h_2, ..., h_m)$ depending on x and a finite number of u derivatives, says β, such that, generically, the solution (x, u) of the square differential-algebraic system ($t \mapsto y(t)$ is given)

$$\dot{x} = f(x, u), \quad y(t) = h(x, u, \dot{u}, ..., u^{(\beta)}) \tag{9}$$

does not involve any differential equation and thus is of the form:

$$x = \Phi(y, \dot{y}, ..., y^{(\beta)}), \quad u = \Psi(y, \dot{y}, ..., y^{(\beta+1)}) \tag{10}$$

where, Φ and Ψ are smooth functions, and β is some finite number [23]. The quantity y is of fundamental importance: it is called "*flat output*" or linearizing output. In the control language, the flat output y is such that, the inverse of $\dot{x} = f(x, u), y = h(x, u, \dot{u}, ..., u^{(\beta)})$ has no dynamics ([17] [23]). Differentially flat systems are very useful in situations where the explicit generation of trajectories is required. Since the behavior of flat systems is determined by the flat outputs, one can plan the trajectories in the outputs space and then connect these to appropriate inputs. More precisely, from the trajectories of the flat outputs y , we can deduce immediately the trajectories of the state x and the input u variables. Applications of the flatness concept to engineering problems field have grown steadily in recent years and a variety of case studies have been shown to be flat and flatness based controllers based on trajectories generation by polynomial interpolation and then closing the loop on the obtained trajectories have been developed.

B. Trajectories planning for Flat systems

The main advantage of flat systems is there ability for solving control problems. The synthesis of control laws is divided into: "*trajectory planning*" corresponding to the open-loop control synthesis and "*trajectory tracking*" which corresponds to the closed-loop control design.

For flat systems, there is a systematic method for trajectory planning (See [24]). Often these trajectories are required to parameterize a finite time transition between two stationary regimes. These equilibria can be computed (locally) from relations such that:

$$u = \Psi(y, \dot{y}, ..., y^{(\beta+1)})$$

by substituting $y^{(i)} = 0, i > 0$. Thus, they are, specified by constant values of the flat output.

The transition between two states is determined by choosing a reference trajectory for the flat output, and y being differentially independent. The trajectories of its components can be chosen independently, and freely. In principle, any sufficiently smooth curve $t \mapsto y_i^*(t)$ may be used. A particularly simple method uses polynomials to define reference trajectories. Then the coefficients of the polynomials, follow from the initial $(t = 0)$ and final values $(t = T)$ of y by solving a linear system of equations (See [24] for the exact method of coefficients calculation).

IV. Differential flatness for the DTR problem

In [3], authors introduce flatness-based control for dynamic traffic routing for two alternate routes with a single section using first order traffic flow model. Here we propose the same type of control using the METANET model. Moreover, in order to illustrate the relevance of the proposed algorithm, we extend it to a case of two alternate routes with two sections. The simulated freeway portion is shown in figure (2).

A. DTR control for two alternate routes with two sections

The space discretized equations of the system is

$$\dot{\rho}_{11}(t) = \frac{1}{L_{11}} \left[\alpha q_e - \rho_{11}(t) v_{11}(t)\right] \tag{11}$$

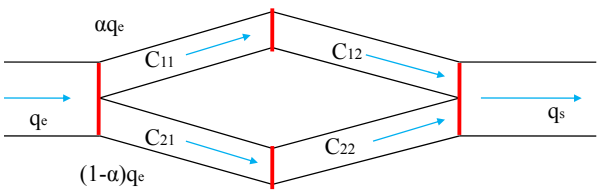

Figure 2. Simulated freeway section

$$\dot{v}_{11}(t) = \tfrac{1}{\tau}(V_{\rho_{11}}(t) - v_{11}(t)) + \tfrac{1}{L_{11}} v_{11}(t)(v_{10}(t) - v_{11}(t)) - \tfrac{\nu}{\tau L_{11}} \tfrac{\rho_{12}(t)-\rho_{11}(t)}{\rho_{11}(t)+\kappa} \tag{12}$$

$$\dot{\rho}_{12}(t) = \frac{1}{L_{12}} \left[\rho_{11}(t) v_{11}(t) - \rho_{12}(t) v_{12}(t)\right] \tag{13}$$

$$\dot{v}_{12}(t) = \tfrac{1}{\tau}(V_{\rho_{12}}(t) - v_{12}(t)) + \tfrac{1}{L_{12}} v_{12}(t)(v_{11}(t) - v_{12}(t)) - \tfrac{\nu}{\tau L_{12}} \tfrac{\rho_{13}(t)-\rho_{12}(t)}{\rho_{12}(t)+\kappa} \tag{14}$$

$$\dot{\rho}_{21}(t) = \frac{1}{L_{21}} \left[(1-\alpha) q_e - \rho_{21}(t) v_{21}(t)\right] \tag{15}$$

$$\dot{v}_{21}(t) = \tfrac{1}{\tau}(V_{\rho_{21}}(t) - v_{21}(t)) + \tfrac{1}{L_{21}} v_{21}(t)(v_{20}(t) - v_{21}(t)) - \tfrac{\nu}{\tau L_{21}} \tfrac{\rho_{22}(t)-\rho_{21}(t)}{\rho_{21}(t)+\kappa} \tag{16}$$

$$\dot{\rho}_{22}(t) = \frac{1}{L_{22}} \left[\rho_{21}(t) v_{21}(t) - \rho_{22}(t) v_{22}(t)\right] \tag{17}$$

$$\dot{v}_{22}(t) = \tfrac{1}{\tau}(V_{\rho_{22}}(t) - v_{22}(t)) + \tfrac{1}{L_{22}} v_{22}(t)(v_{21}(t) - v_{22}(t)) - \tfrac{\nu}{\tau L_{22}} \tfrac{\rho_{23}(t)-\rho_{22}(t)}{\rho_{22}(t)+\kappa} \tag{18}$$

$$V_{\rho_{11}}(t) = v f_{11} \exp\left[\left(-\tfrac{1}{a}\right)\left(\tfrac{\rho_{11}(t)}{\rho_{c_{11}}}\right)^a\right]$$

$$V_{\rho_{12}}(t) = v f_{12} \exp\left[\left(-\tfrac{1}{a}\right)\left(\tfrac{\rho_{12}(t)}{\rho_{c_{12}}}\right)^a\right]$$

$$V_{\rho_{21}}(t) = v f_{21} \exp\left[\left(-\tfrac{1}{a}\right)\left(\tfrac{\rho_{21}(t)}{\rho_{c_{21}}}\right)^a\right]$$

$$V_{\rho_{22}}(t) = v f_{22} \exp\left[\left(-\tfrac{1}{a}\right)\left(\tfrac{\rho_{22}(t)}{\rho_{c_{22}}}\right)^a\right]$$

The travel time function is obtained as follow:

$$y_1(t) = \frac{L_{11}}{v_{11}(t)} + \frac{L_{12}}{v_{12}(t)} \tag{19}$$

$$y_2(t) = \frac{L_{21}}{v_{21}(t)} + \frac{L_{22}}{v_{22}(t)} \tag{20}$$

The variable y is equal to the difference in the travel time on the two sections.

$$y(t) = \left[\left(\frac{L_{11}}{v_{11}(t)} + \frac{L_{12}}{v_{12}(t)}\right) - \left(\frac{L_{21}}{v_{21}(t)} + \frac{L_{22}}{v_{22}(t)}\right)\right] \tag{21}$$

The first time derivative leads to:

$$\dot{y}(t) = A + B + C + D + \alpha E \tag{22}$$

The terms A, B, C, and D ... represents derivatives of sections 21, 11, 12 and 22 respectively. And the term E consists of all the terms in which α does exist. It comes from sections 11 and 21. The term A is composed of three terms (A_1, A_2 and A_3);

$$A_1 = \frac{L_{21}}{(v_{21})^2} \left[\frac{\frac{v_{21} - \frac{v f_{21}}{\sigma_4}}{\tau} - \frac{\nu \sigma_6}{L_{21}\tau(\kappa+\rho_{21})}}{\tau} - \frac{\frac{\nu(\rho_{21} v_{21} - \rho_{22} v_{22})}{L_{22}}}{L_{21}\tau(\kappa+\rho_{21})} \right]$$

$$A_2 = \frac{L_{21}}{v_{21}} \rho_{21} \left[\frac{\frac{v f_{21}\sigma_8}{L_{21}\rho_{c_{21}}\sigma_4}}{\tau} - \frac{\frac{\nu}{L_{21}}}{L_{21}\tau(\kappa+\rho_{21})} + \frac{\nu\sigma_6}{(L_{21})^2\tau\sigma_9} \right]$$

$$A_3 = \frac{L_{21} q_e}{(v_{21})^2} \left[\frac{-\frac{v f_{21}\sigma_8}{L_{21}\rho_{c_{21}}\sigma_4}}{\tau} + \frac{\frac{\nu}{L_{21}}}{L_{21}\tau(\kappa+\rho_{21})} - \frac{\nu\sigma_6}{(L_{21})^2\tau\sigma_9} \right]$$

The term B is composed of two terms (B_1 and B_2);

$$B_1 = \frac{-L_{11}}{(v_{11})^2} \left[\frac{\frac{v_{11} - \frac{v f_{11}}{\sigma_3}}{\tau} - \frac{\nu\sigma_5}{L_{11}\tau(\kappa+\rho_{11})}}{\tau} - \frac{\frac{\nu(\rho_{11} v_{11} - \rho_{12} v_{12})}{L_{12}}}{L_{11}\tau(\kappa+\rho_{11})} \right]$$

$$B_2 = \frac{-L_{11}}{v_{11}} \rho_{11} \left[\frac{\frac{v f_{11}\sigma_7}{L_{11}\rho_{c_{11}}\sigma_3}}{\tau} - \frac{\frac{\nu}{L_{11}}}{L_{11}\tau(\kappa+\rho_{11})} + \frac{\nu\sigma_5}{(L_{11})^2\tau\sigma_{10}} \right]$$

The term C is composed of three terms (C_1, C_2 and C_3);

$$C_1 = \frac{-L_{12}}{(v_{12})^2} \left[\frac{-\sigma_{13} + \frac{v_{12} - \frac{v f_{12}}{\sigma_{11}}}{\tau} - \frac{v f_{12}\sigma_{15}(\rho_{11} v_{11} - \rho_{12} v_{12})}{L_{12}\rho_{c_{12}}\sigma_{11}}}{\tau} \right]$$

$$C_2 = \frac{1}{v_{12}} \left[\frac{v_{11} - \frac{v f_{11}}{\sigma_{19}}}{\tau} - \frac{v_{12} - \frac{v f_{12}}{\sigma_{11}}}{\tau} + \sigma_{13} - \frac{\nu(\rho_{11} - \rho_{12})}{L_{11}\tau(\kappa+\rho_{11})} \right]$$

$$C_3 = \frac{\sigma_{17}}{(v_{12})^2} \left[\frac{v_{12} - \frac{v f_{12}}{\sigma_{11}}}{\tau} - \sigma_{13} \right]$$

The term D is composed of three terms (D_1, D_2 and D_3);

$$D_1 = \frac{L_{22}}{(v_{22})^2} \left[\frac{-\sigma_{14} + \frac{v_{22} - \frac{v f_{22}}{\sigma_{12}}}{\tau} - \frac{v f_{22}\sigma_{16}(\rho_{21} v_{21} - \rho_{22} v_{22})}{L_{22}\rho_{c_{22}}\sigma_{12}}}{\tau} \right]$$

$$D_2 = \frac{-1}{v_{22}} \left[\frac{v_{21} - \frac{v f_{21}}{\sigma_{20}}}{\tau} - \frac{v_{22} - \frac{v f_{22}}{\sigma_{12}}}{\tau} + \sigma_{14} - \frac{\nu(\rho_{21} - \rho_{22})}{L_{21}\tau(\kappa+\rho_{21})} \right]$$

$$D_3 = -\frac{\sigma_{18}}{(v_{22})^2} \left[\frac{v_{22} - \frac{v f_{22}}{\sigma_{12}}}{\tau} - \sigma_{14} \right]$$

The term E is composed of two terms one concerns section 11 and the other concerns section 21;

$$E_{11} = \frac{L_{11}q_e}{(v_{11})^2}\left[\frac{\frac{vf_{11}\sigma_7}{L_{11}\rho_{c_{11}}\sigma_3}}{\tau} - \frac{\frac{\nu}{L_{11}}}{L_{11}\tau(\kappa+\rho_{11})} + \frac{\nu\sigma_5}{(L_{11})^2\tau\sigma_{10}}\right]$$

$$E_{21} = \frac{L_{21}q_e}{(v_{21})^2}\left[\frac{\frac{vf_{21}\sigma_8}{L_{21}\rho_{c_{21}}\sigma_4}}{\tau} - \frac{\frac{\nu}{L_{21}}}{L_{21}\tau(\kappa+\rho_{21})} + \frac{\nu\sigma_6}{(L_{21})^2\tau\sigma_9}\right]$$

$\sigma_3 = exp\left(\frac{1}{a}(\frac{\rho_{11}}{\rho_{c_{11}}})^a\right)$, $\sigma_4 = exp\left(\frac{1}{a}(\frac{\rho_{21}}{\rho_{c_{21}}})^a\right)$, $\sigma_5 = \rho_{11} - \rho_{12}$, $\sigma_6 = \rho_{21} - \rho_{22}$, $\sigma_7 = (\frac{\rho_{11}}{\rho_{c_{11}}})^{(a-1)}$, $\sigma_8 = (\frac{\rho_{21}}{\rho_{c_{21}}})^{(a-1)}$, $\sigma_9 = (\kappa+\rho_{21})^2$, $\sigma_{10} = (\kappa+\rho_{11})^2$, $\sigma_{11} = exp\left(\frac{1}{a}(\frac{\rho_{12}}{\rho_{c_{12}}})^a\right)$, $\sigma_{12} = exp\left(\frac{1}{a}(\frac{\rho_{22}}{\rho_{c_{22}}})^a\right)$, $\sigma_{13} = \frac{v_{12}\sigma_{17}}{L_{12}}$, $\sigma_{14} = \frac{v_{22}\sigma_{18}}{L_{22}}$, $\sigma_{15} = (\frac{\rho_{12}}{\rho_{c_{12}}})^{(a-1)}$, $\sigma_{16} = (\frac{\rho_{22}}{\rho_{c_{22}}})^{(a-1)}$, $\sigma_{17} = v_{11} - v_{12}$, $\sigma_{18} = v_{21} - v_{22}$, $\sigma_{19} = exp(\frac{1}{a}(\frac{\rho_{11}}{\rho_{c_{11}}})^a)$ and $\sigma_{20} = exp(\frac{1}{a}(\frac{\rho_{21}}{\rho_{c_{21}}})^a)$.

The studied system is characterized by one input (control) variable, we have then one flat output $F = y$, which represents the difference in travel time function between the two routes. Thus the equation (22) can be rewritten as:

$$\dot{F}(t) = A + B + C + D + \alpha E \tag{23}$$

from which

$$\alpha(t) = \frac{\dot{F}(t) - (A+B+C+D)}{E} \tag{24}$$

The expression of the state variable allows to choose a suitable trajectory of the travel time (the flat output). The equation of the control (input) variable allows to add additional constraints to this travel time trajectory. This means that all important properties of the system (22) are contained in such a differential parametrization.

B. Trajectory planning:

The equation (24), corresponds to an open loop control algorithm. In order to define the trajectory planning, a suitable desired trajectory F^* has to be defined. According to the expression of the control variable in (24), this trajectory must have smooth derivatives up to order two. In order to reduce computational effort in real time situation, one can build this reference trajectory for the travel time (flat output) using a polynomial interpolation [25] from the initial and final conditions of the travel time $(F(t_i) = F_i,\ \dot{F}(t_i) = 0)$ and $(F(t_f) = F_f,\ \dot{F}(t_f)) = 0)$. This is accomplished by prescribing the following desired trajectory for the flat output F:

$$F^*(t) = \begin{cases} F_{t_i} & \text{for } t < t_i \\ F_{t_i} + (F_{t_f} - F_{t_i})\varphi(t, t_i, t_f) & \text{for } t_i \le t \le t_f \\ F_{t_f} & \text{for } t > t_f \end{cases} \tag{25}$$

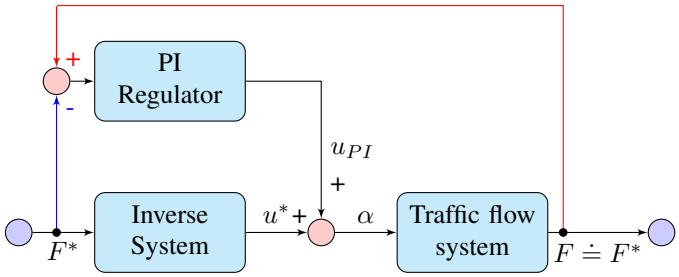

Figure 3. flow chart (Reduced-order estimator)

where $\varphi(t, t_i, t_f)$ is a polynomial function of time, exhibiting a sufficient number of zero derivatives at times t_i and t_f. (For the polynomial calculation see e.g. [24] [25]):

$$\varphi(t) = \begin{cases} 0 & \text{if } t < t_i \\ 3\left(\frac{t-t_i}{t_f-t_i}\right)^2 - 2\left(\frac{t-t_i}{t_f-t_i}\right)^3 & \text{if } t_i \le t \le t_f \\ 1 & \text{if } t > t_f \end{cases} \tag{26}$$

Thus, by replacing the term $\dot{F}(t)$ in equation (24) by the term of desired trajectory $\dot{F}^*(t)$, we obtain the nominal open loop control:

$$\alpha^*(t) = \frac{\dot{F}^*(t) - (A+B+C+D)}{E} \tag{27}$$

C. Trajectory tracking:

Due to the parameter variations and disturbances in traffic flow, and changes in the traffic conditions, the open loop control is not sufficient to control the traffic flow. To ensure a steady state and reduce the influence of parameter variations, the traffic flow has to be operated in closed loop.

Using the flatness-based open loop presented in algorithm equation (24), an additional feedback can be determined in order to achieve desired dynamic behaviour and to compensate the external disturbances. Here, a PI regulator is used to feedback the system (See Fig. 3). Thus, the closed-loop control schema including the flatness-based loop can be obtained as follow:

$$\nu(t) = \dot{F}^*(t) - k_1(F(t) - F^*(t)) - k_2\int(F(t) - F^*(t))dt \tag{28}$$

where k_1 and k_2 are parameters that must be selected so as to satisfy the desired performances of the closed loop system and to ensure asymptotically stabilization of the input variable. The control law α now reads:

$$\alpha(t) = \left[\frac{\dot{F}^*(t) - k_1(F(t) - F^*(t)) - k_2\int(F(t) - F^*(t))dt}{E} - \frac{A+B+C+D}{E}\right] \tag{29}$$

V. Numerical simulations

Figure (2) is a schematic diagram of the freeway portion used in the simulation. It shows two routes with the same

Table I
THE MODEL AND SIMULATION PARAMETERS

Parameter	value	Parameter	value
a	2.34	ρ_c	36 veh/km.
τ	18 sec	ρ_{max}	180 veh/km.
ν	60 $\frac{km^2}{h}$	vf	90$\frac{km}{h}$.
κ	40 $\frac{veh}{km}$	q_{max}	$\rho_c v f exp(\frac{-1}{a})$.
Time step	5 sec	Cell length	500m

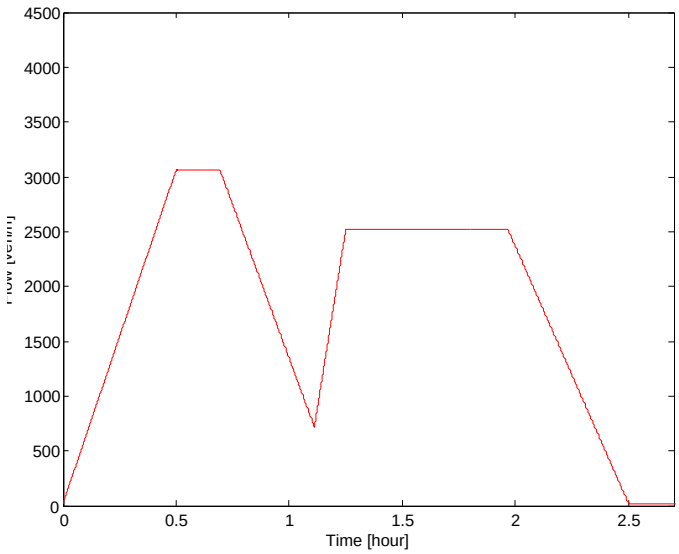

Figure 4. Traffic income flow

length and geometric conditions. Each route is partitioned into 2 identical cells (each cell = 500m). At the end of each cell, one loop detector (sensor) is installed. The model parameters are depicted in table (I). The theoretical results are illustrated by some simulations.

The macroscopic simulation has been done during 165 minutes. Traffic incoming flow for the simulation is shown in figure (4). In figures (5, 6, 7 and 8) densities and mean speeds of the two sections and the two routes are shown respectively under control algorithm (see section IV-C). From figures, it is clear that both density and mean speed of the system in each reciprocal cell are equal, it is mean that the travel time difference between these two routes are zero. In figure (9) the control value for each route is given. In figure 10 the travel time of each routes and their difference is shown after applying the control algorithm.

VI. CONCLUSION

In this paper, a closed loop control algorithm was simulated in order to control the incoming flow of two routes that have the same length and geometry. The control algorithm is designed to minimize the travel time and optimize the flow on the freeway portion. The control was applied on the METANET model which is a second order traffic flow model discretized in time and in space. The results show the relevance of the control by which the density and mean speed of the two routes equalized. These results inspire us to extend this work to deal with more complicated network in the future.

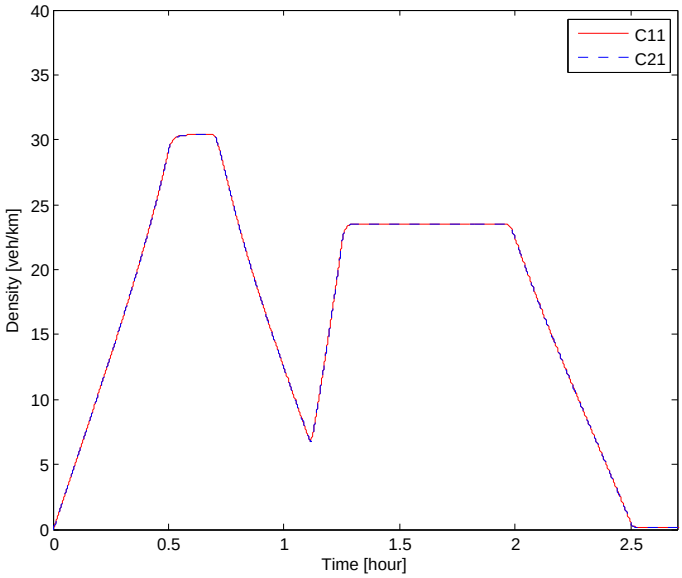

Figure 5. Traffic density of section 1

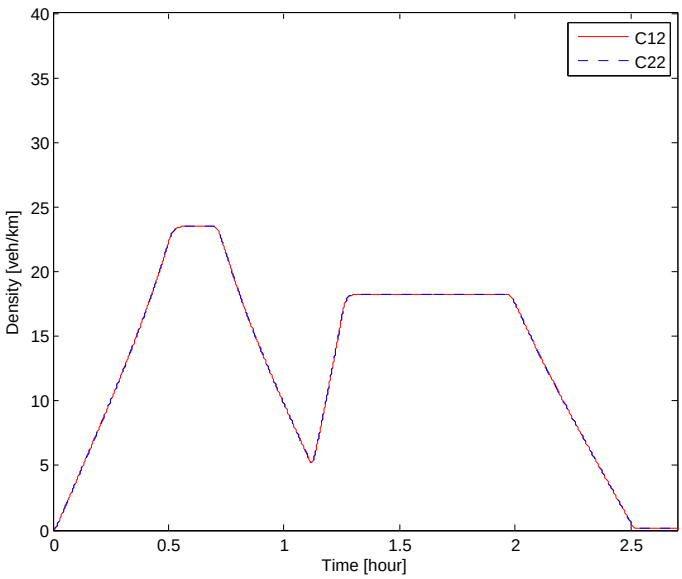

Figure 6. Traffic density of section 2

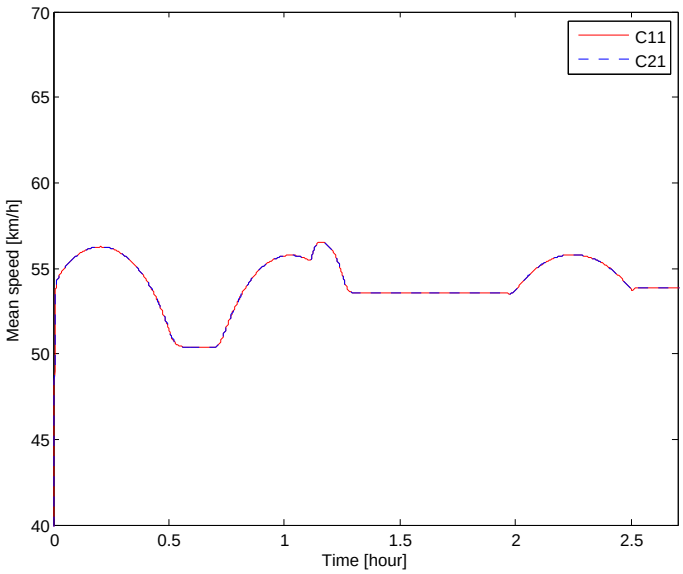

Figure 7. Traffic mean speed of section 1

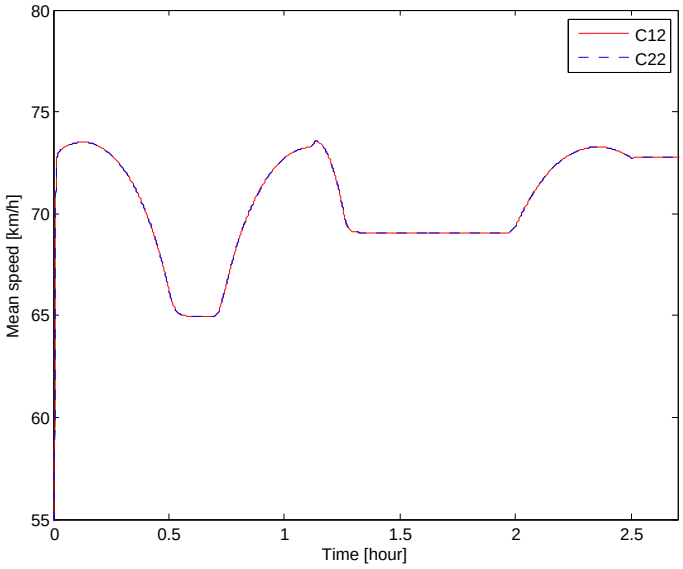

Figure 8. Traffic mean speed of section 2

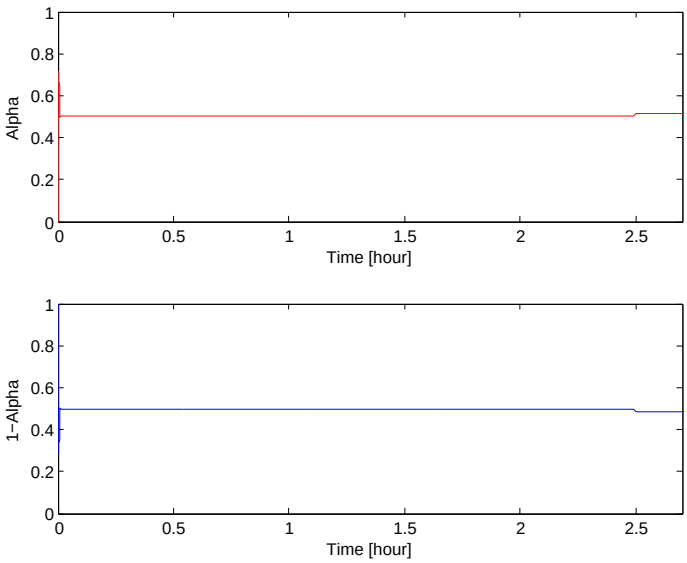

Figure 9. Traffic control value

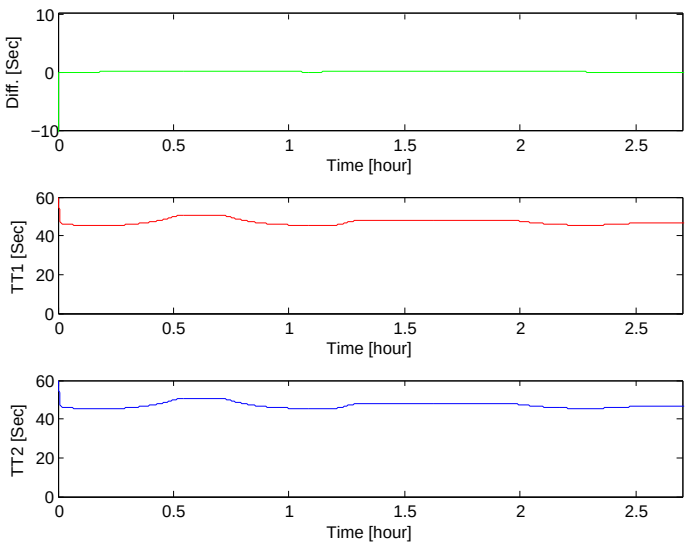

Figure 10. Travel time of the two routes and their difference

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