Real-time dynamic traffic routing using variable structure control

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Hirsh Majid [1] [2], Hassane Abouaïssa [1], Daniel Jolly [1], Gildas Morvan [1]

***Abstract*— In this paper, we present a new algorithm to deal with the real-time dynamic traffic routing (DTR) problem using a second order traffic flow model (METANET). We investigate variable structure control (VSC) as a high-speed switched feedback control resulting in sliding mode. The control objective is to minimize the difference of the travel times on alternate routes in a network setting. Accordingly, the congestion could be reduced/eliminated. The relevance of the proposed algorithm is demonstrated via a set of numerical simulations in the case of the same as well as different geometric conditions.**

Keywords: Macroscopic model, variable structure control, sliding mode control, dynamic traffic routing.

I. INTRODUCTION

Traffic control represents an efficient way to improve the freeway throughput and to ensure an efficient, safe and less polluting transportation of goods and persons [1]. It also contributes to a high reduction of direct and indirect costs.

Freeway traffic control can be achieved via a set of actions and measurements such as: dynamic speed limits, route guidance, dynamic assignment and dynamic traffic routing, ramp metering, etc.

Dynamic traffic assignment (**DTA**) and dynamic traffic routing (**DTR**) represent some of the most efficient solutions to steer congestion problems especially the no-recurrent ones. DTR/DTA or control of traffic diversion rests on the determination of time-dependent split variables at the diversion point in order to achieve a user-equilibrium traffic pattern [2] in a way that one route not be overloaded and the others underused. Accordingly, the congestion could be alleviated/eliminated. Several algorithms have been developed to solve this problem. Most of them rely on optimization approaches such as those proposed by [3]. As stated in [4], the proposed optimization algorithm attempts to solve the DTR problem by optimizing the objective functions for the nominal model over the planning horizon and it is not adapted for on-line DTR control. Papageorgiou [5] and Messmer & Papageorgiou [6] have proposed algorithms for dynamic traffic assignment (DTA) based on the linear quadratic regulator, and nonlinear optimization techniques. Other techniques use expert systems to deal with the diversion problem [7]. Liu & al. [8] have proposed a strategy based on a model reference adaptive control in order to guide the real-world traffic flow to evolve towards the desired states

[1] Univ. Lille Nord France, F-59000 Lille, France. U-Artois, LGI2A, EA. 3926 Technoparc Futura, F-62400 Béthune, France. (hassane.abouaissa, daniel.jolly, gildas.morvan)@univ-artois.fr.

[2] (Corresponding author). University of Sulaimani, Faculty of Engineering Sciences, Department of Civil Engineering, Sulaimani, Iraq. hirshmajid@yahoo.com.

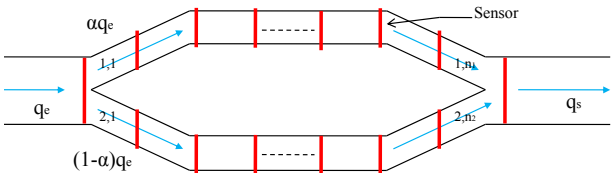

Fig. 1. Sample example of two alternate routes

especially under emergency evacuation. Kachroo & Özbay [9] have designed feedback and fuzzy control laws for an on-line diversion problem. Another algorithm for DTR and DTA problems based on the nonlinear H_∞ was proposed in [10]. See also [11] and [12] for more information about route guidance strategy.

In this paper, we exploit the variable structure control theory (VSC). The control algorithm consists of two parts; trajectory planning (open loop control) which is achieved using a specific class of complex systems called '*Differentially flat systems*'. The trajectory tracking (closed loop control) is ensured by a high-speed switched feedback control resulting in sliding mode. In contrast of [13], the non-destination oriented METANET model [14] [15] is used in the design of the control algorithm.

The paper is organized as follows: Section II presents the main principles of the DTR problem and its mathematical formulation. Section III recalls the main definitions of the control system. Section IV shows the control design methodology using the concept of flatness and sliding mode control. Section V provides some numerical simulations for a sample network. Section VI concludes the paper and outlines some tracks for further developments.

II. DYNAMIC TRAFFIC ROUTING FORMULATION

In this section, we first present a mathematical formulation which is used for the design of DTA/DTR flat controller. We expose a second order traffic flow model (METANET).

For the sake of simplicity, we consider the case of two alternate routes divided into n_1 and n_2 sections, respectively, as depicted in Fig. 1.

The proposed control approach rests on the use of METANET model. The dynamic equation of the traffic density reads:

$$\dot{\rho}_{i,j}(t) = \frac{1}{L_{i,j}} \left[q_{i,j-1}(t) - q_{i,j}(t) \right] \tag{1}$$

where (i,j)=$(1,1),(1,2),\ldots,(1,n_1),(2,1),\ldots,(2,n_2)$. $L_{i,j}$ is the section length and $\rho_{i,j}$ represents the traffic

density at section j of the route i.

The relation between the traffic flow $q_{i,j}$ and the mean speed $v_{i,j}$ is:

$$q_{i,j}(t) = \rho_{i,j}(t) v_{i,j}(t) \lambda_{i,j} \tag{2}$$

where $\lambda_{i,j}$ is the number of lanes. $v_{i,j}$ is the dynamic mean speed of section j on the route i.

The equation of dynamic mean speed is:

$$\begin{aligned} \dot{v}_{i,j}(t) &= \tfrac{1}{\tau}(V_{\rho_{i,j}}(t) - v_{i,j}(t)) + \tfrac{1}{L_{i,j}} v_{i,j}(t)(v_{i,j-1}(t) \\ &\quad - v_{i,j}(t)) - \tfrac{\nu}{\tau L_{i,j}} \tfrac{\rho_{i,j+1}(t) - \rho_{i,j}(t)}{\rho_{i,j}(t) + \kappa} \end{aligned} \tag{3}$$

$V_{\rho_{i,j}}$ is defined as a nonlinear expression called fundamental diagram mean speed (see [16]):

$$V_{\rho_{i,j}}(t) = v f_{i,j} exp\left[\left(-\frac{1}{a}\right)\left(\frac{\rho_{i,j}(t)}{\rho_{c_{i,j}}}\right)^a\right] \tag{4}$$

$\rho_{c_{i,j}}$, $v f_{i,j}$, a, ν, κ and τ are constant parameters which reflect particular characteristics of a given traffic system [14].

The control input $\alpha(t) \in [0,\ 1]$ is defined as an exogenous variable of the system [12], it is a split rate that allows to reach a user equilibrium traffic pattern. For the case of two alternate routes, the equations of entry flow in each route are:

$$\begin{cases} \alpha(t) q_e(t) = & q_{1,in}(t) \\ (1-\alpha) q_e(t) = & q_{2,in}(t) \end{cases} \tag{5}$$

where $q_e(t)$ is the traffic demand. The control objective is to find the optimal split rate α in order to minimize the differences $J(\alpha)$ between the travel time TT of the two alternate routes.

Kachroo and Özbay [4] have formulated the DTR problem as follows: finding α_0 and the optimal $\alpha(t)$, which minimizes

$$J(\alpha) = \int_0^{t_f} \left[\sum_{j=1}^{n_1} TT(\rho_{1,j}) - \sum_{j=1}^{n_2} TT(\rho_{2,j})\right]^2 dt \tag{6}$$

where $TT(\rho_{1,j})$ and $TT(\rho_{2,j})$ are the travel times function of section j on the route 1 and 2 respectively. t_f is the final time.

In the general case, consider a traffic flow system with n alternate routes. The system is described by the same ordinary differential equation (1) where, $(i,j) = ((1,1),\ldots,(1,n_1),(2,1),\ldots,(2,n_2),(n,1),\ldots,(n,n_n))$.

The problem consists then in finding a set of split variables, α_{n-1}, where, $\sum_{i=1}^{n-1} \alpha^i = 1$, that minimizes total travel time TT, (see [4], for optimal formulation of the DTR problem for a general case)

$$\begin{cases} \alpha_1(t) q_e(t) = & q_{1,in}(t) \\ \alpha_2(t) q_e(t) = & q_{2,in}(t) \\ \vdots & \\ (1 - \alpha_1 - \alpha_2 -, \ldots, -\alpha_{n-1}) q_e(t) = & q_{n,in}(t) \end{cases} \tag{7}$$

Note that for such a complex problem, an open loop control structure is not sufficient [10], it calls for a more robust feedback control one. In this context, we highlight the interest of an attractive approach based on the concept of differential flatness [17] [18]. The following section, recalls the main definitions and principles of differentially flat systems.

III. Sliding mode flatness-based control systems

In this paper, we exploited one of the most attractive methods that can be applied to a broad class of nonlinear systems resulting in controllers that are robust to modeling errors and unknown disturbances. We investigated variable structure control (VSC) as a high-speed switched feedback control resulting in sliding mode. The gains in each feedback path switch between two values according to a rule that depends on the value of the state at each instant. The purpose of the switching control law is to derive the non-linear system's state trajectory onto a prespecified surface in the state space and to maintain the system's state trajectory on this surface (switching surface) for subsequent time.

In standard sliding mode control, or first order sliding mode control (FOSMC) [19] and [20] , the sliding surface is chosen so that it has a relative degree of one with respect to the control input. In such a case, the control input u is, for example, of the form;
$u(t) = u_{eq}(t) + u_d(t)$, where u_{eq} is the continuous function and $u_d(t) = -K_d sign(s(t)) - K_p s(t)$ is the discontinuous function. K_d specifies the speed of convergence of the closed-loop system in order to $s(t) = 0$. To combine a small switching gain with fast convergence, the discontinuous control term could be extended with a proportional feedback term $K_p s(t)$ [21].

u_{eq} ensures that $\dot{s} = 0$. It is the equivalent control, it depends only on the switching surface $s(t)$ and not on the control function u_d [20]. Thus, it can be said that u_{eq} introduces the trajectory planning (open loop control) of the output. In this context, the term u_d should introduce the trajectory tracking (closed loop control) of the output.

In this paper, instead of u_{eq}, the trajectory is made using a specific class of control system called '*differential flatness*'
.

The concept of flat systems was first introduced by Fliess and al [17] [18] more than a decade ago using the formalism of differential algebra (see [22] for a slightly different approach of differentially flat systems). This special class of non-linear control systems described by ordinary differential equations: differentially flat systems form a special class of nonlinear control systems for which systematic control methods are available once a flat-output is explicitly known.

The flatness-based control methods may be expected to play a very significant role in high technology applications in the next few years, similar to what happened for nonlinear control in the last decade [23]. The main property of flat systems is that all the state and input variables can be expressed directly, without integration of any differential equation, in terms of the set of so-called "flat output" and a number of its time derivatives. More precisely, the entire system behavior is determined by the trajectory of a finite collection of quantities: flat outputs. This leads to a simple and elegant trajectory design. For a given system, the number

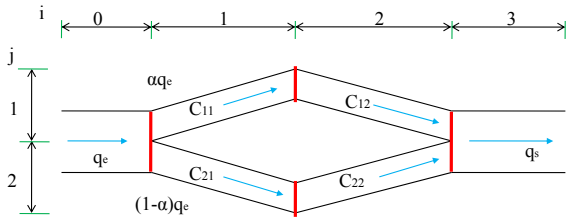

Fig. 2. Simulated freeway section

of flat outputs is equal to the number of the system inputs. For more details on flatness see also [18], [23], [24], [25], [26] and [27].

IV. Sliding mode flatness-based control for the DTR problem

Here, we introduce a first order sliding mode flatness-based control using the METANET model for dynamic traffic routing. In the freeway portion, there are two alternate routes with two sections in each route, see Fig. 2.

A. DTR Control for Two Alternate Routes With Two Sections

The equations of the system discretized in space are

$$\dot{\rho}_{11}(t) = \frac{1}{L_{11}}\left[\alpha q_e - \rho_{11}(t)v_{11}(t)\right] \tag{8}$$

$$\begin{aligned}\dot{v}_{ji}(t) \quad = \quad & \tfrac{1}{\tau}(V_{\rho_{ji}}(t) - v_{ji}(t)) + \tfrac{1}{L_{ji}}v_{ji}(t)(v_{ji-1}(t) \\ & -v_{ji}(t)) - \tfrac{\nu}{\tau L_{ji}}\tfrac{\rho_{ji+1}(t)-\rho_{ji}(t)}{\rho_{ji}(t)+\kappa}\end{aligned} \tag{9}$$

$$\dot{\rho}_{21}(t) = \frac{1}{L_{21}}\left[(1-\alpha)q_e - \rho_{21}(t)v_{21}(t)\right] \tag{10}$$

$$\dot{\rho}_{j2}(t) = \frac{1}{L_{j2}}\left[\rho_{j1}(t)v_{j1}(t) - \rho_{j2}(t)v_{j2}(t)\right] \tag{11}$$

$$V_{\rho_{ji}}(t) = vf_{ji}exp\left[\left(-\frac{1}{a}\right)\left(\frac{\rho_{ji}(t)}{\rho_{c_{ji}}}\right)^a\right]$$

The travel time function is obtained as follows:

$$y_1(t) = \frac{L_{11}}{v_{11}(t)} + \frac{L_{12}}{v_{12}(t)} \tag{12}$$

$$y_2(t) = \frac{L_{21}}{v_{21}(t)} + \frac{L_{22}}{v_{22}(t)} \tag{13}$$

The variable y is equal to the difference in the travel times the two sections.

$$y(t) = \left[\left(\frac{L_{11}}{v_{11}(t)} + \frac{L_{12}}{v_{12}(t)}\right) - \left(\frac{L_{21}}{v_{21}(t)} + \frac{L_{22}}{v_{22}(t)}\right)\right] \tag{14}$$

The first time derivative leads to:

$$\dot{y}(t) = A + B + C + D + \alpha E \tag{15}$$

The terms A, B, C, and D represent derivatives of sections 21, 11, 12 and 22 respectively. The term E consists of all the terms in which α does exist. It comes from sections 11 and 21. A is composed of three terms (A_1, A_2 and A_3);

$$A_1 = \frac{L_{21}}{(v_{21})^2}\left[\frac{v_{21} - \frac{vf_{21}}{\sigma_4}}{\tau} - \frac{\nu\sigma_6}{L_{21}\tau(\kappa+\rho_{21})} - \frac{\nu(\rho_{21}v_{21}-\rho_{22}v_{22})}{L_{22}\tau(\kappa+\rho_{21})}\right]$$

$$A_2 = \frac{L_{21}}{v_{21}}\rho_{21}\left[\frac{\frac{vf_{21}\sigma_8}{L_{21}\rho_{c_{21}}\sigma_4}}{\tau} - \frac{\frac{\nu}{L_{21}}}{L_{21}\tau(\kappa+\rho_{21})} + \frac{\nu\sigma_6}{(L_{21})^2\tau\sigma_9}\right]$$

$$A_3 = \frac{L_{21}q_e}{(v_{21})^2}\left[\frac{-\frac{vf_{21}\sigma_8}{L_{21}\rho_{c_{21}}\sigma_4}}{\tau} + \frac{\frac{\nu}{L_{21}}}{L_{21}\tau(\kappa+\rho_{21})} - \frac{\nu\sigma_6}{(L_{21})^2\tau\sigma_9}\right]$$

B is composed of two terms (B_1 and B_2);

$$B_1 = \frac{-L_{11}}{(v_{11})^2}\left[\frac{v_{11} - \frac{vf_{11}}{\sigma_3}}{\tau} - \frac{\nu\sigma_5}{L_{11}\tau(\kappa+\rho_{11})} - \frac{\nu(\rho_{11}v_{11}-\rho_{12}v_{12})}{L_{12}\tau(\kappa+\rho_{11})}\right]$$

$$B_2 = \frac{-L_{11}}{v_{11}}\rho_{11}\left[\frac{\frac{vf_{11}\sigma_7}{L_{11}\rho_{c_{11}}\sigma_3}}{\tau} - \frac{\frac{\nu}{L_{11}}}{L_{11}\tau(\kappa+\rho_{11})} + \frac{\nu\sigma_5}{(L_{11})^2\tau\sigma_{10}}\right]$$

C is composed of three terms (C_1, C_2 and C_3);

$$C_1 = \frac{-L_{12}}{(v_{12})^2}\left[\frac{-\sigma_{13} + \frac{v_{12} - \frac{vf_{12}}{\sigma_{11}}}{\tau} - \frac{vf_{12}\sigma_{15}(\rho_{11}v_{11}-\rho_{12}v_{12})}{L_{12}\rho_{c_{12}}\sigma_{11}}}{\tau}\right]$$

$$C_2 = \frac{1}{v_{12}}\left[\frac{v_{11} - \frac{vf_{11}}{\sigma_{19}}}{\tau} - \frac{v_{12} - \frac{vf_{12}}{\sigma_{11}}}{\tau} + \sigma_{13} - \frac{\nu(\rho_{11}-\rho_{12})}{L_{11}\tau(\kappa+\rho_{11})}\right]$$

$$C_3 = \frac{\sigma_{17}}{(v_{12})^2}\left[\frac{v_{12} - \frac{vf_{12}}{\sigma_{11}}}{\tau} - \sigma_{13}\right]$$

D is composed of three terms (D_1, D_2 and D_3);

$$D_1 = \frac{L_{22}}{(v_{22})^2}\left[\frac{-\sigma_{14} + \frac{v_{22} - \frac{vf_{22}}{\sigma_{12}}}{\tau} - \frac{vf_{22}\sigma_{16}(\rho_{21}v_{21}-\rho_{22}v_{22})}{L_{22}\rho_{c_{22}}\sigma_{12}}}{\tau}\right]$$

$$D_2 = \frac{-1}{v_{22}}\left[\frac{v_{21} - \frac{vf_{21}}{\sigma_{20}}}{\tau} - \frac{v_{22} - \frac{vf_{22}}{\sigma_{12}}}{\tau} + \sigma_{14} - \frac{\nu(\rho_{21}-\rho_{22})}{L_{21}\tau(\kappa+\rho_{21})}\right]$$

$$D_3 = -\frac{\sigma_{18}}{(v_{22})^2}\left[\frac{v_{22} - \frac{vf_{22}}{\sigma_{12}}}{\tau} - \sigma_{14}\right]$$

E is composed of two terms. The first one concerns section 11 and the second belongs to section 21;

$$E_{11} = \frac{L_{11}q_e}{(v_{11})^2}\left[\frac{\frac{vf_{11}\sigma_7}{L_{11}\rho_{c_{11}}\sigma_3}}{\tau} - \frac{\frac{\nu}{L_{11}}}{L_{11}\tau(\kappa+\rho_{11})} + \frac{\nu\sigma_5}{(L_{11})^2\tau\sigma_{10}}\right]$$

$$E_{21} = \frac{L_{21}q_e}{(v_{21})^2}\left[\frac{\frac{vf_{21}\sigma_8}{L_{21}\rho_{c_{21}}\sigma_4}}{\tau} - \frac{\frac{\nu}{L_{21}}}{L_{21}\tau(\kappa+\rho_{21})} + \frac{\nu\sigma_6}{(L_{21})^2\tau\sigma_9}\right]$$

$\sigma_3 = exp\left(\frac{1}{a}(\frac{\rho_{11}}{\rho_{c_{11}}})^a\right)$, $\sigma_4 = exp\left(\frac{1}{a}(\frac{\rho_{21}}{\rho_{c_{21}}})^a\right)$, $\sigma_5 = \rho_{11} - \rho_{12}$, $\sigma_6 = \rho_{21} - \rho_{22}$, $\sigma_7 = (\frac{\rho_{11}}{\rho_{c_{11}}})^{(a-1)}$, $\sigma_8 = (\frac{\rho_{21}}{\rho_{c_{21}}})^{(a-1)}$, $\sigma_9 = (\kappa + \rho_{21})^2$, $\sigma_{10} = (\kappa + \rho_{11})^2$, $\sigma_{11} = exp\left(\frac{1}{a}(\frac{\rho_{12}}{\rho_{c_{12}}})^a\right)$, $\sigma_{12} = exp\left(\frac{1}{a}(\frac{\rho_{22}}{\rho_{c_{22}}})^a\right)$, $\sigma_{13} = \frac{v_{12}\sigma_{17}}{L_{12}}$, $\sigma_{14} = \frac{v_{22}\sigma_{18}}{L_{22}}$, $\sigma_{15} = (\frac{\rho_{12}}{\rho_{c_{12}}})^{(a-1)}$, $\sigma_{16} = (\frac{\rho_{22}}{\rho_{c_{22}}})^{(a-1)}$, $\sigma_{17} = v_{11} - v_{12}$, $\sigma_{18} = v_{21} - v_{22}$, $\sigma_{19} = exp(\frac{1}{a}(\frac{\rho_{11}}{\rho_{c_{11}}})^a)$ and $\sigma_{20} = exp(\frac{1}{a}(\frac{\rho_{21}}{\rho_{c_{21}}})^a)$.

The studied system is characterized by one input (control) variable, we have then one flat output $F = y$, which represents the difference in travel time function between the two routes. Thus the equation (15) can be rewritten as:

$$\dot{F}(t) = A + B + C + D + \alpha E \tag{16}$$

from which

$$\alpha(t) = \frac{\dot{F}(t) - (A + B + C + D)}{E} \tag{17}$$

The expression of the state variable allows to choose a suitable trajectory of the travel time (the flat output). The equation of the control (input) variable allows to add additional constraints to this travel time trajectory. This means that all important properties of the system (see equation (15)) are contained in such a differential parametrization.

B. Trajectory Planning:

The equation (17), corresponds to an open loop control algorithm. In order to define the trajectory planning, a suitable desired trajectory F^* has to be defined. According to the expression of the control variable in equation (17), this trajectory must have smooth derivatives up to order two. In order to reduce computational effort in real time situation, one can build this reference trajectory for the travel time (flat output) using a polynomial interpolation [23] from the initial and final conditions of the travel time $(F(t_i) = F_i,\ \dot{F}(t_i) = 0)$ and $(F(t_f) = F_f,\ \dot{F}(t_f)) = 0)$. This is accomplished by prescribing the following desired trajectory for the flat output F:

$$F^*(t) = \begin{cases} F_{t_i} & \text{for } \ t < t_i \\ F_{t_i} + (F_{t_f} - F_{t_i})\varphi(t, t_i, t_f) & \text{for } \ t_i \leq t \leq t_f \\ F_{t_f} & \text{for } \ t > t_f \end{cases} \tag{18}$$

where $\varphi(t, t_i, t_f)$ is a polynomial function of time which exhibit a sufficient number of zero derivatives at times t_i and t_f. (For the polynomial calculation see e.g. [28] [23]):

$$\varphi(t) = \begin{cases} 0 & \text{if } \ t < t_i \\ 3\left(\frac{t-t_i}{t_f-t_i}\right)^2 - 2\left(\frac{t-t_i}{t_f-t_i}\right)^3 & \text{if } t_i \leq t \leq t_f \\ 1 & \text{if } t > t_f \end{cases} \tag{19}$$

Thus, by replacing the term $\dot{F}(t)$ in equation (17) by the term of desired trajectory $\dot{F}^*(t)$, we obtain the nominal open loop control:

$$\alpha^*(t) = \frac{\dot{F}^*(t) - (A + B + C + D)}{E} \tag{20}$$

C. Trajectory Tracking:

Due to the parameter variations and disturbances in traffic flow and frequent changes in the traffic conditions, the open loop control is not sufficient. To ensure a steady state and reduce the influence of parameter variations, the traffic flow has to be operated in closed loop. Using the flatness-based open loop presented in equation (17), an additional feedback can be determined in order to achieve a desired dynamic behaviour and to compensate the external disturbances. Here, a first order sliding mode controller is used to feedback the system which asymptotically regulates the output towards the desired equilibrium position (see Fig. 3). Thus, the closed-loop control schema including the flatness-based loop can be obtained as follows:

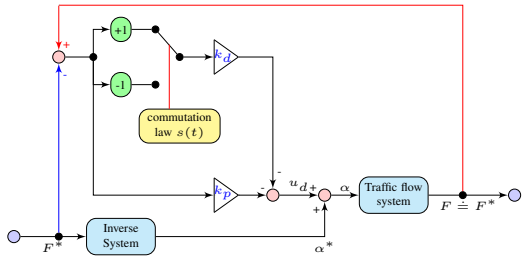

Fig. 3. Closed loop control structure

TABLE I

THE MODEL AND SIMULATION PARAMETERS

Parameter	value	Parameter	value
a	2.34	ρ_c	36 $veh/km.$
τ	18 s	ρ_{max}	180 $veh/km.$
ν	60 km^2/h	vf	90 $km/h.$
κ	40 veh/km	q_{max}	$\rho_c vf exp(\frac{-1}{a})$.
Time step	20 s	Cell length	400 m

$$\psi(t) = \dot{F}^*(t) - K_d sign(s(t)) - K_p s(t) \tag{21}$$

where k_d and k_p are parameters that must be selected so as to satisfy the desired performances of the closed loop system and to ensure asymptotically stabilization of the input variable. The control law α now reads:

$$\alpha(t) = \left[\frac{\dot{F}^*(t) - K_d sign(s(t)) - K_p s(t)}{E} - \frac{A + B + C + D}{E}\right] \tag{22}$$

V. NUMERICAL SIMULATIONS

For the numerical simulations, consider the freeway section depicted in Fig. 2. It shows two routes with the same length and geometric conditions. Each route is partitioned into 2 identical cells (each cell = 400m). At the end of each cell, one loop detector (sensor) is installed. The model parameters are depicted in table I. The control algorithm value is shown via variable message signs (VMS). For the sake of simplicity, we assume a full compliance of the drivers.

The used data is collected between 6 AM - 10 PM. The simulation time step is about 20 s. The simulations have been done in two different cases; when the two routes have the same geometric conditions as well as when the two routes have different geometric conditions (L11, L12 = 300 m). The traffic demand used in the simulations is shown in Fig. 4. Fig. 5, 6, 7 and 8 show the densities, mean speeds, control values and the travel times evolution in the case when the two routes have the same geometric conditions. From these

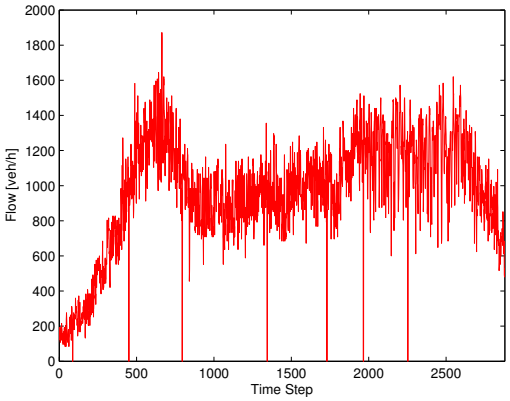

Fig. 4. Traffic demand used in the simulation

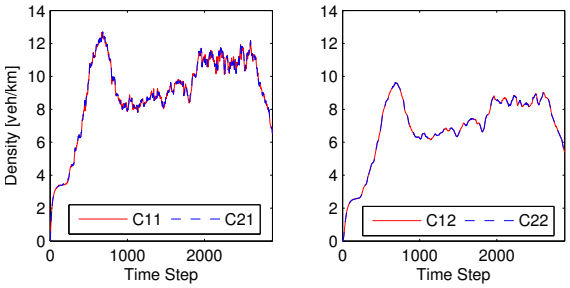

Fig. 5. Traffic densities in the case of the same geometric conditions

figures, it is clear that the densities and mean speeds of the system in each reciprocal cell are equal, it means that the control algorithm distribute the traffic demand between two routes somehow one route not be overloaded and the other underused. Fig. 9, 10, 11 and 12 show the densities, mean speeds, control values and the travel times evolution in the case when the two routes have some different in theirs geometric conditions (L11, L12 = 300 m, L21, L22 = 400 m). We can realize the control algorithm distribute always the traffic demand between the two routes equally. Contrary, because of different length of the two routes, the mean speed of the two routes are different.

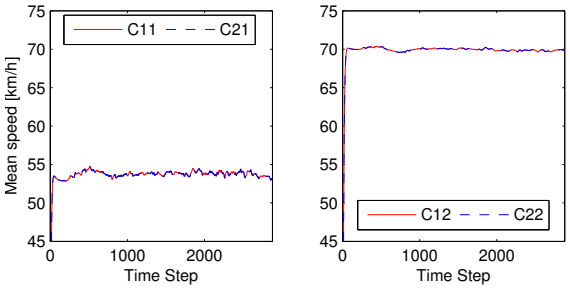

Fig. 6. Mean speeds in the case of the same geometric conditions

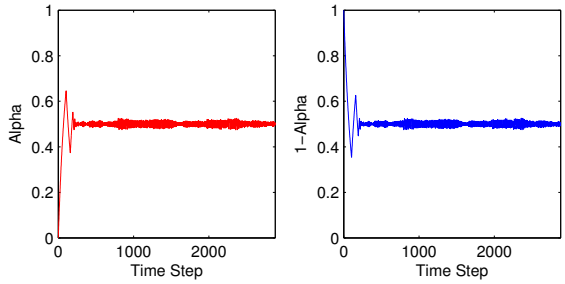

Fig. 7. Control value in the case of the same geometric conditions

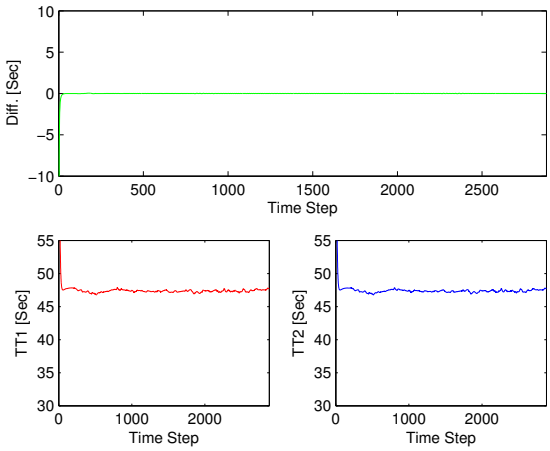

Fig. 8. Travel times of the two routes with the same geometric conditions

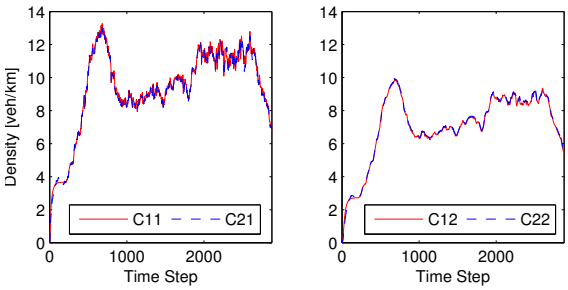

Fig. 9. Traffic densities in the case of different geometric conditions

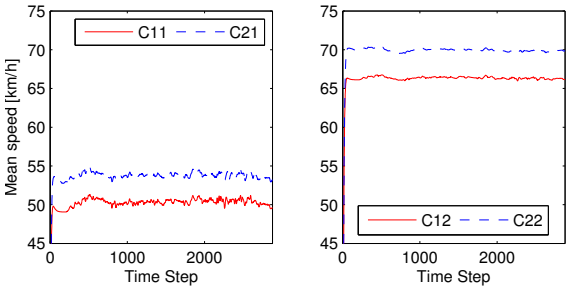

Fig. 10. Mean speeds in the case of different geometric conditions

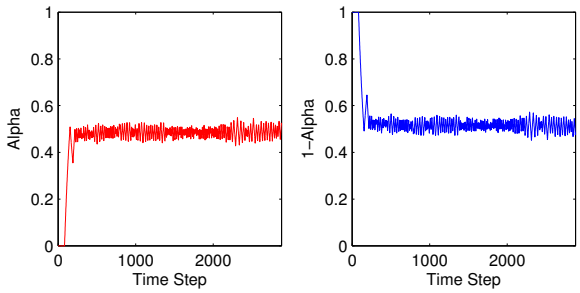

Fig. 11. Control value in the case of different geometric conditions

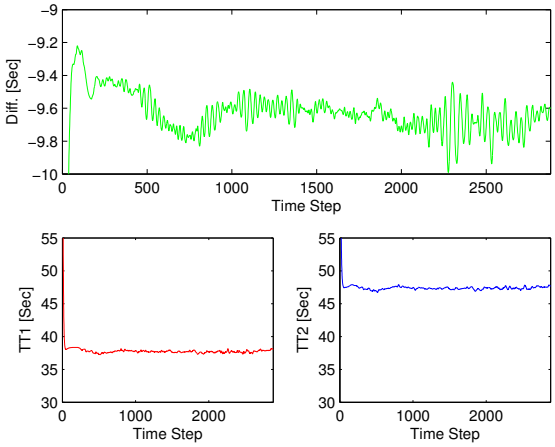

Fig. 12. Travel times of the two routes with different geometric conditions

VI. Conclusion

In this paper, a first order sliding mode flatness-based control algorithm was designed in order to control the incoming flow into two alternate routes which have the same origin and destination. The objective of the control algorithm is to minimize the difference between the travel times of two alternate routes, accordingly, to optimize the flow on the freeway portion. The control algorithm was applied using the METANET model which is a nonlinear traffic flow model. The results show the relevance of the control algorithm in the cases of the same and different geometric conditions of the two alternate routes. These results inspire us to extend this work to deal with more complex network and to design an integrated control in the future.

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