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# State Reconstructor for Real-Time Freeway Ramp Metering

Hirsh Majid, Hassane Abouaïssa, Daniel Jolly, Gildas Morvan

**Abstract**—The work presented in this paper aims to solve the problem of real-time ramp metering in the case of unavailable (not measured) traffic flow at an on-ramp or measurements provided by a faulty detector which lead to creation of unpredictable traffic states. A state reconstructor is then proposed and a systematic approach is developed for real time freeway ramp metering. The state reconstructor is developed using a second order sliding mode observer, while the control algorithm is based on the concept of differential flatness. Numerical simulations demonstrate the relevance of the provided algorithm for both state reconstructor and control of freeway networks.

**Index Terms**—Reconstructor, sliding mode, ramp metering.

## I. INTRODUCTION

The main objective of real-time traffic control is to deal with a daily problem of congestion. Such problem is well identified as a source of continuous growth of the direct and indirect costs (drivers health and stress problems, pollution, increase in energy consumption, ... etc.) [1], [2]. The principle of traffic control is based on the use of a variety of adequate actions such as: dynamic speed limits using variable message signs, driving information and alerts, tidal control, dynamic route guidance and ramp metering.

Ramp metering or admissible control, represents the most efficient and direct way to solve congestion problems and to upgrade freeway traffic [3], [4]. This control action consists in regulating the ramp flow at the entrance of the freeway [1]. As stated in [5], ramp metering strategies are a valuable tool for an efficient traffic management that can be classified into: *reactive strategies* that aim to maintain the traffic conditions in freeway close to pre-specified set values using real-time measurements, and *proactive strategies*, aiming at specifying optimal traffic conditions for a whole freeway network based on the demand and the model prediction over a time horizon [2].

Ramp metering strategies can be implemented locally (*isolated ramp metering*) in the vicinity of each ramp to calculate the corresponding ramp metering values. It can be implemented simultaneously (*coordinated ramp metering*) when the objective is to use the available traffic measurements from larger freeway sections.

However, the performance of any control strategy is closely related to the ability to have all needed information about the

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traffic state<sup>1</sup>. Indeed, traffic control problems usually require the knowledge of both the present and the future traffic behaviours. Such information is provided via a set of sensors already installed along the freeway infrastructure. The future traffic behaviour can be computed using a suitable traffic model.

The main objective of this paper is to deal with isolated ramp metering problems when no information about the traffic state variable at the merge segment is available. In other words, we assume that the measurements just downstream of the controlled ramp are provided by a faulty detector. Having the state reconstructor, a systematic approach for real time isolated ramp metering is provided in order to maintain the traffic conditions at a desired optimal operating level.

The paper is organized as follow:

- Section II, recalls the used macroscopic model and provides the basis of the state reconstructor scheme.
- Section III, presents the freeway ramp metering algorithm based on the concept of differential flatness .
- Section IV, provides some numerical simulations and demonstrates the relevance of the proposed approach.
- The conclusion, section V, summarizes the main obtained results and lists some promising perspectives and further researches.

## II. TRAFFIC FLOW MODELING AND STATE RECONSTRUCTOR

Several studies have demonstrated the relevance of macroscopic models for freeway traffic modeling, supervision and control [7], [8] and [9].

### A. Macroscopic models

The first macroscopic model was developed by Lighthill and Whitham [10] and Richard [11] which is based on the conservation law:

$$\frac{\partial}{\partial t}\rho(x,t) + \frac{\partial}{\partial x}q(x,t) = g(x,t) \quad (1)$$

where,  $\rho(x,t)$ ,  $q(x,t)$  and  $g(x,t)$  are, the traffic density in vehicles/km/lanes, the traffic volume in vehicles/h, the ramp generation term in vehicles/h/km, respectively.

Equation (1) is supplied by  $q(x,t) = \rho(x,t)v(x,t)\beta$ .

$v(x,t)$  is the mean speed and  $\beta$ , the number of lanes. The mean speed  $v(x,t)$  and the traffic density  $\rho(x,t)$  are related by

<sup>1</sup>Traffic density (or occupancy) constitutes the basis state variable of the most available traffic flow models and plays an important role for both traffic flow modeling and traffic control [6].

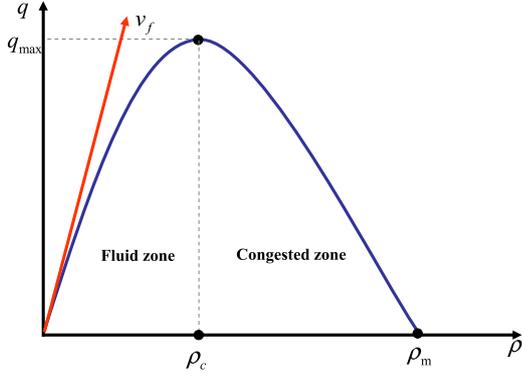


Fig. 1. Fundamental diagram

a so called "fundamental diagram"  $v(x,t) = V_e(\rho(x,t))$ . One of the most used form of such function is due to May [8]

$$V_e(\rho(x,t)) = v_f \exp\left(-\frac{1}{a} \left(\frac{\rho}{\rho_c}\right)^a\right) \quad (2)$$

where,  $a$  is a parameter,  $v_f$  is the free-flow speed and,  $\rho_c$  represents the critical density. Such fundamental diagram allows to identify the free flow zone and congested one (See Fig. 1).

As the first order macroscopic model is based on a static relationship between the main traffic variables, it presents several drawback such as the inability to describe the dynamic of the traffic behavior. In this context, the second order models seem to be more adapted. One of these models is the Payne's model [12]. In this work, we use the model proposed by [13], (See also [14]) which is discretized in time and space. Consider then the following freeway segment divided into  $n$  links (Fig. 2).

The studied freeway segment is then described by the following ordinary differential equations for each link  $i$  [15]

$$\dot{\rho}_i = \frac{1}{L_i} (q_{i-1} - \alpha \rho_i v_{i+1} - (1 - \alpha) \rho_{i+1} v_{i+2} + r_i(t) - s_i(t)) \quad (3)$$

$$\dot{v}_i = \frac{1}{\tau} [V_e(\rho_i) - v_i(t)] + \frac{\eta}{L_i} v_i(t) [v_{i-1} - v_{i+1}(t)] - \frac{v}{L_i} (\rho_{i+1} - \rho_i) - \frac{\delta}{L_i} (r_i(t) - s_i(t)) v_i \quad (4)$$

where,  $r_i(t)$  and  $s_i(t)$  are the on-ramp and off-ramp flows, respectively.  $\tau$ ,  $\eta$ ,  $v$  and  $\delta$  are model parameters.  $L_i$  is the length of the link  $i$ .

### B. State reconstructor

All current freeway traffic control and monitoring algorithms require an accurate knowledge of traffic states along the studied segments [6]. From the macroscopic point of view, the main variables of interest are the traffic density in vehicules/km/lane and the average speed in km/h [16]. These important traffic data are usually obtained via a set of sensors, for example, such as inductive loop detector, camera video.

Nevertheless, The high cost of installation and maintenance of a large number of sensors or the occurrence of sensor's failures, led many researchers to develop traffic control methods

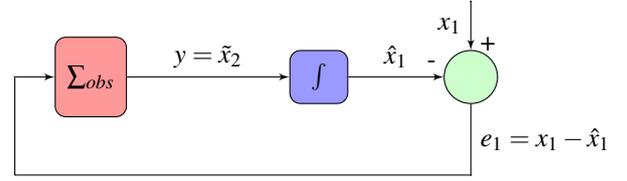


Fig. 3. Structure of the super twisting algorithm

and algorithms based on state estimation techniques in order to reduce the number of sensors or to reconstruct the missing state information.

Several methods were proposed, in [6], a state estimator was developed using Extended Kalman filters [17], other works use the well known Luenberger observer [18].

In [15], a first order sliding mode observer has been proposed in order to design a state observer. Such robust *variable structure observer* (see [19] and [20], for more explanation about Sliding mode control and observation) suffers from the high frequency chattering phenomenon generally present near the sliding surface. In this paper, a second order sliding mode observer is proposed in order to overcome this problem.

1) *Short overview on the observers:* The sliding mode technique has been used for the observer's design in many applications. Consider the following nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(x, \dots, x_{n-1}) \\ y = x_1 \end{cases} \quad (5)$$

The sliding mode differentiator of order 2 (super twisting algorithm) is given in its general form by [21] (figure 3).

$$\sum_{obs} = \left\{ \begin{array}{l} y(e_1) = y + \lambda_1 |e_1|^{1/2} \text{sign}(e_1) \\ \dot{y} = \pi_1 \text{sign}(e_1) \end{array} \right\}, \lambda_1, \pi_1 > 0$$

Where,  $\lambda_1$  and  $\pi_1$  are positive tuning parameters of the differentiator whose the output is  $y$ . Applying the exact differentiator to system (5) when  $n=2$ , one obtains:

$$\dot{\hat{x}}_1 = \hat{x}_2 + \lambda_1 |x_1 - \hat{x}_1|^{1/2} \text{sign}(x_1 - \hat{x}_1) \quad (6)$$

$$\dot{\hat{x}}_2 = f_q(x_1, \hat{x}_2) + \pi_1 \text{sign}(x_1 - \hat{x}_1) \quad (7)$$

$f_q(x_1, \hat{x}_2) = \text{equation}(6)$ . Then, the estimated value of  $\hat{x}_2$  can be found from equation (7). For more details see ([22], [23], [24], [25], [26], [27], [28] and [29]).

2) *Traffic state estimation:* Consider the following freeway segments (Fig. 4): Using the relationship between the three aggregated variables (flow  $q$ , density  $\rho$ <sup>2</sup> and mean speed  $v$ ),

$$q_i(x,t) = \rho_i(x,t) v_i(x,t) \quad (8)$$

<sup>2</sup>loop-detector (sensor) measures directly the occupancy, which refers to the percentage of time a detector is occupied by vehicles. It can be transformed to density by  $TO = 100(L_b + L_v)\rho$  [30], where:  $L_b$  = length of the loop-detector,  $L_v$  = length of the vehicle,  $\rho$  = density and  $TO$  = occupation rate.

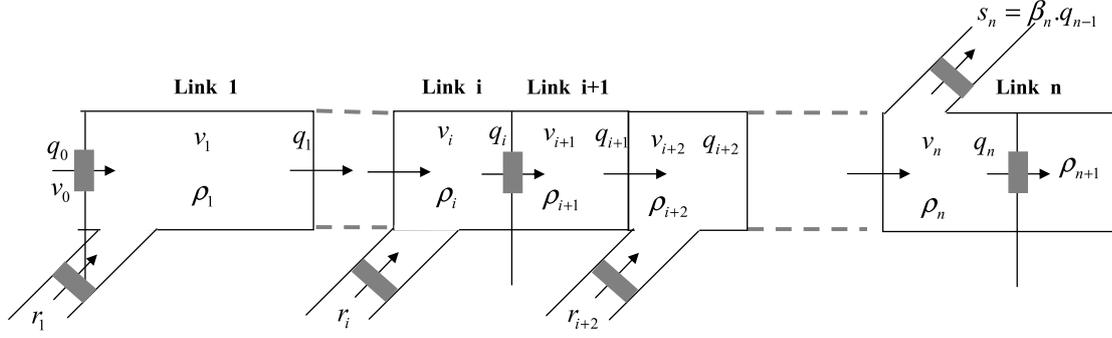


Fig. 2. Freeway example

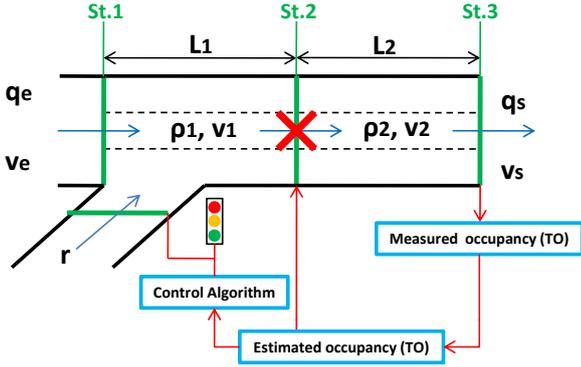


Fig. 4. Freeway segments example

The output vector is:  $y = [y_1 \ y_2]^T = [\rho_2 \ v_2]^T$

As long as the outgoing traffic speed satisfies the positivity condition:  $v_1, v_2 = v_s \geq V_{min} > 0$ , the studied freeway section (see figure 4) can be described by the following equations:

$$\dot{\rho}_1 = \frac{1}{L_1}(q_e + r - \alpha \rho_1 v_2 - (1 - \alpha) \rho_2 v_s) \quad (9)$$

$$\dot{\rho}_2 = \frac{1}{L_2}(\alpha \rho_1 v_2 - (1 - \alpha) \rho_2 v_s - q_s) = \dot{\rho} \quad (10)$$

$$\dot{v}_1 = \frac{1}{\tau}(V_e(\rho_1) - v_1) + \frac{\eta}{L_1} v_1 (v_e - v_2) - \frac{v}{L_1}(\rho_2 - \rho_1) - \frac{\delta}{L_1} r v_1 \quad (11)$$

$$\dot{v}_2 = \frac{1}{\tau}(V_e(\rho_2) - v_2) + \frac{\eta}{L_2} v_2 (v_1 - v_s) = \dot{v} \quad (12)$$

In expressions (9)-(12),  $\rho_2$  and  $v_2$  are assumed to be known.  $V_e(\rho_i)$  are given thanks to the May fundamental diagram (2).

It should be mentioned that  $q_s$  and  $v_s$  form the output measurements variables which are equal to  $q_2$  and  $v_2$  respectively, measured on sensor St.3.  $q_e$  and  $v_e$  (measured on sensor St.1) are external measured inputs (see figure 4). Assuming that sensor St.2 is broken down. We try to estimate the state variables  $\rho_1$  and  $v_1$  which are essential for the control system from the measured states on sensor St.3 :

Consider  $\hat{\rho}$  and  $\hat{v}$  are the estimated value of  $\rho$  and  $v$  respectively. Then, the observer errors are defined as:  $e_\rho = \rho - \hat{\rho}$  and  $e_v = v - \hat{v}$ .

The proposed observer (called super-twisting) has the form

$$\dot{\hat{\rho}}_1 = \frac{1}{L_1}(q_e + r - \alpha \hat{\rho}_1 v_2 - (1 - \alpha) \hat{\rho}_2 v_s) + z_{\rho 1} \quad (13)$$

Where  $z_{\rho 1}$  and  $z_{\rho 2}$  are the output injection of the form:

$$\dot{\hat{\rho}}_2 = \hat{\rho}_1 + z_{\rho 2} \quad (14)$$

$$z_{\rho 1} = \pi \text{sign}(\rho_2 - \hat{\rho}_2) \quad (15)$$

$$z_{\rho 2} = \lambda |\rho_2 - \hat{\rho}_2|^{\frac{1}{2}} \text{sign}(\rho_2 - \hat{\rho}_2) \quad (16)$$

For the speed estimation  $\hat{v}_1$ , the observer reads:

$$\dot{\hat{v}}_1 = \frac{1}{\tau}(V(\hat{\rho}_1) - \hat{v}_1) + \frac{\eta}{L_1} \hat{v}_1 (v_e - v_2) - \frac{v}{L_1}(\rho_2 - \hat{\rho}_1) - \frac{\delta}{L_1} r \hat{v}_1 + z_{v 1} \quad (17)$$

The output injection are of the form:

$$\dot{\hat{v}}_2 = \hat{v}_1 + z_{v 2} \quad (18)$$

$$z_{v 1} = \pi \text{sign}(v_2 - \hat{v}_2) \quad (19)$$

$$z_{v 2} = \lambda |v_2 - \hat{v}_2|^{\frac{1}{2}} \text{sign}(v_2 - \hat{v}_2) \quad (20)$$

### III. REAL-TIME RAMP METERING

The ramp metering algorithm is based on the differentially flat systems concept, (see [31], [32], [33] and [34] for more explanations about flat systems).

Notice that the studied system (9)-(12) is flat with  $F = \rho_2$  its flat output (see [35], for the first application of flatness concept in the traffic flow control). It means that all the system variables can be parametrized in term of this flat output and a number of its time-derivative. The state variable  $\rho_1$  reads then

$$\rho_1 = L_2 \frac{\dot{F}}{\alpha v_2} + \frac{1 - \alpha}{\alpha} F + \frac{1}{\alpha v_2} q_s \quad (21)$$

From the expressions above, the control variable  $r(t)$  can be parametrized in term of the flat output and its time derivatives up to two

$$r(t) = \frac{L_1 L_2}{\alpha v_2} \ddot{F} + \left[ L_2 - \frac{L_1 L_2 \dot{v}_2}{\alpha v_2^2} + L_1 \frac{1 - \alpha}{\alpha} \right] \dot{F}$$

TABLE I  
PARAMETERS OF THE MODEL

Parameter	Value	Parameter	Value
Free-flow speed	105km/h	$\eta$	0.42
Jam density	180veh/km	$\nu$	$5.8E-2 * \frac{1000^3}{3600^2} / \frac{m^3}{sec^2 \cdot veh}$
Critical density	35.86 veh/km	$a$	1.66
$\alpha$	0.77	$\tau$	$7.8E-3 * 3600 \text{ sec}$
$L_1, L_2$	500m	$\delta$	0.17 m/veh

$$+ \frac{L_1}{\alpha v_2} \dot{q}_s + \left[ 1 - \frac{L_1 \dot{v}_2}{\alpha v_2^2} \right] q_s + 2(1 - \alpha) v_2 F - q_e \quad (22)$$

Given a desired reference trajectory for the traffic density, represented by  $F^*$ , a linearized control law achieving an exponential asymptotic tracking of the trajectory is given by the following expression:

$$r(t) = \frac{L_1 L_2}{\alpha v_2} \Gamma + \left[ L_2 - \frac{L_1 L_2 \dot{v}_2}{\alpha v_2^2} + L_1 \frac{1 - \alpha}{\alpha} \right] \dot{F} + \frac{L_1}{\alpha v_2} \dot{q}_s + \left[ 1 - \frac{L_1 \dot{v}_2}{\alpha v_2^2} \right] q_s + 2(1 - \alpha) v_2 F - q_e \quad (23)$$

$$\Gamma = \ddot{F}^* - \lambda_2 (\dot{F} - \dot{F}^*) - \lambda_1 (F - F^*) \quad (24)$$

$\ddot{F} = \Gamma$  is an auxiliary control input. The set of parameters  $\{K_1, K_2\}$ , is chosen so that the roots of the closed loop characteristic polynomial, in the complex variable  $s$ :  $p(s) = s^2 + K_2 s + K_1$  be a Hurwitz polynomial. Clearly, the tracking error,  $e = F - F^*$ , ( $F = \rho_2$ ) satisfies

$$\ddot{e} + K_2 \dot{e} + K_1 e = 0 \quad (25)$$

A suitable choice of the design parameters is obtained by matching the characteristic polynomial,  $p(s)$ , to a desired characteristic polynomial  $p_d(s)$ , with pre-specified root locations.

#### IV. NUMERICAL SIMULATIONS

For numerical simulations, consider the same example of the stretch depicted in Fig. 4. We suppose that the sensor on St.2 is broken down. Thus, in order to the control system do not disrupt, we try to estimate the state variables ( $\rho_1$  and  $v_1$ ), which are essential for the control algorithm, from the measured state variables ( $\rho_2$  and  $v_2$ ) on St.3. The traffic demand (veh/h/lane) is represented in Fig. 5. For simulation, the model parameters are given in table (I).

In the no-control case, Fig.6 and Fig. 7, show the densities and speed evolutions. Notice that in this case, the congestion appears for about 40 min. Without additive noise, we obtain a good estimation for both the traffic density and the mean speed of the first link.

Fig. 8 and Fig. 9 show the effectiveness of the proposed controller. Indeed, the proposed algorithm ensures convergence to the desired value. Notice also the good estimation results for the main state variables.

Obviously, the implementation of the ramp metering leads to a queue formation as depicted in Fig. 10.

In the presence of additive noise, Fig.12 and Fig. 13 show a good robustness of the identification method as well as the

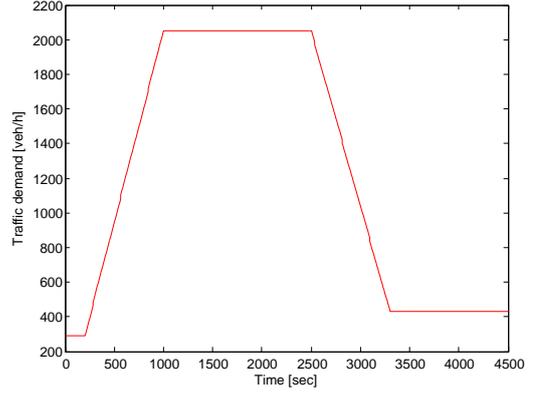


Fig. 5. Traffic demand

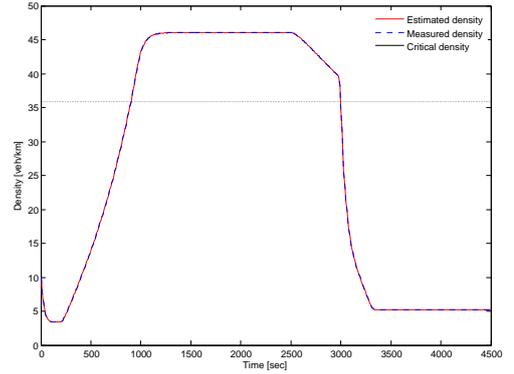


Fig. 6. Traffic densities evolution: no-control case

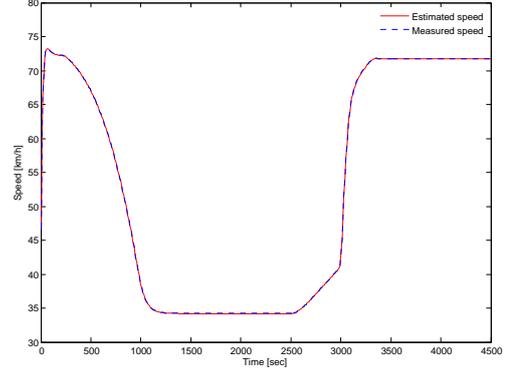


Fig. 7. Mean speed evolution: no-control case

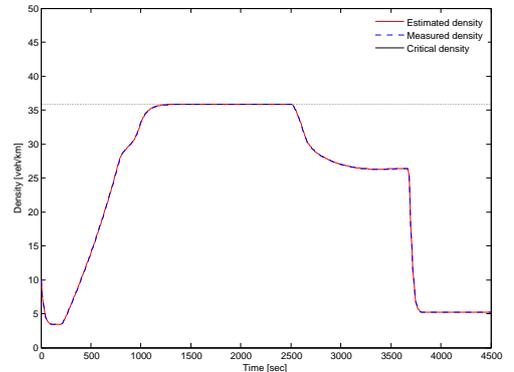


Fig. 8. Traffic densities evolution: control case

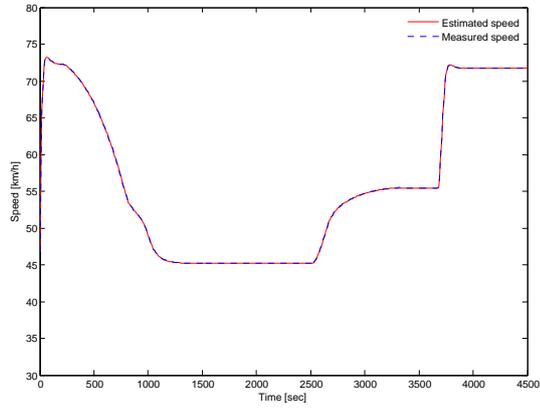


Fig. 9. Mean speed evolution: control case

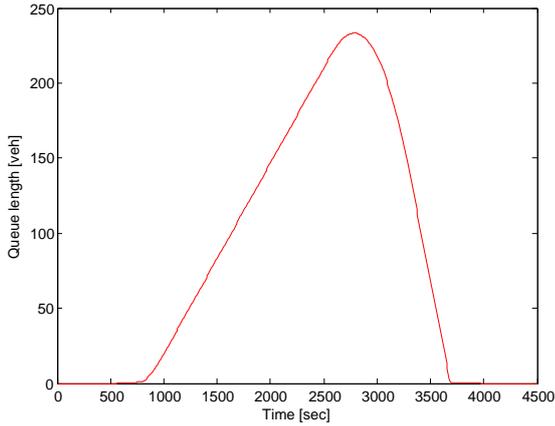


Fig. 10. Queue length evolution

control algorithm to keep the trajectory around the desired value. In table (II), the standard error occur in the estimated values of  $\rho$  and  $v$ , in the case of without additive noise and with the presence of additive noise, are given.

## V. CONCLUSION

The work presented in this paper exploits the relevance of high order sliding mode observer for state estimation. The proposed super twisting algorithm seems to be very efficient for the case where any information about the state of the controlled ramp is available. On the other hand, the application

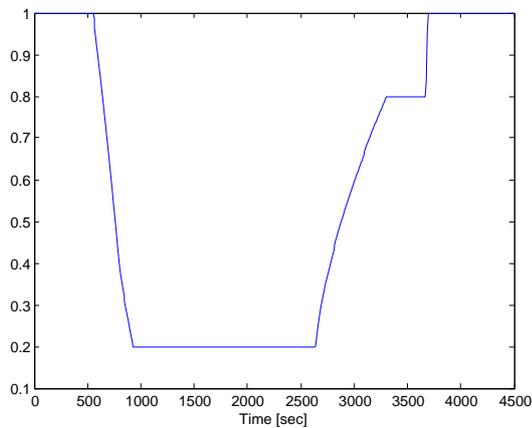


Fig. 11. Control variable

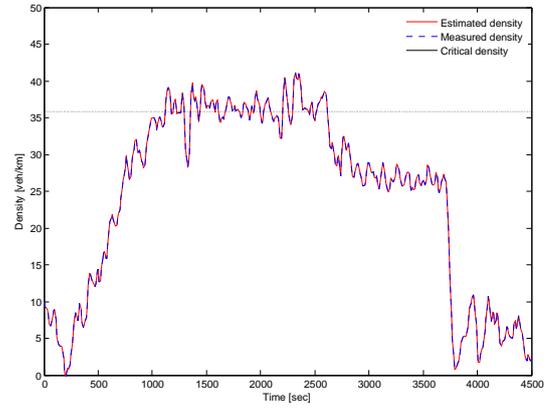


Fig. 12. Densities evolution with additive noise

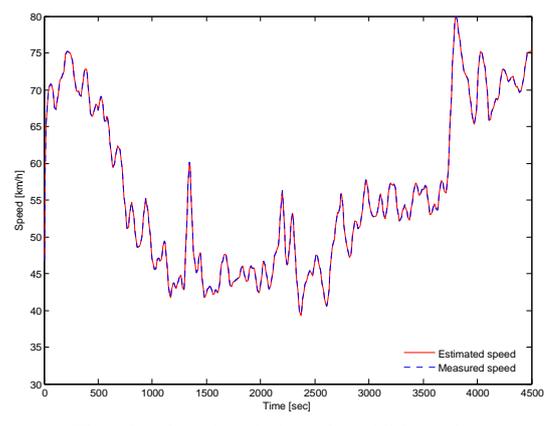


Fig. 13. Speed evolution with additive noise

of the concept of differential flatness for real time ramp metering is based on the system inversion concept, which do not requires integration of any differential equations. Notice that, the flat output is defined by the state of the downstream segment. This means that in addition of on-ramp metering of the merge section, we are able to control all the downstream ones. This fact leads to propose in future works a coordinated ramp metering algorithm which takes measurement of all studied freeway network.

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TABLE II  
ERROR NORM

		No control	Control
Without additive noise	$v_1$	0.131	0.147
	$\rho_1$	0.01	0.01
With additive noise	$v_1$	0.158	0.166
	$\rho_1$	0.01	0.01

(variance= 0.001, mean=0)

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