



Statistically data-driven modelling Location Uncertainty in mesoscale dynamics

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Long Li¹

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Joint works with Bertrand Chapron², Bruno Deremble³, Noé Lahaye¹ and Etienne Mémin¹

¹ FLUMINANCE - Inria Rennes France

² LOPS - Ifremer Brest France

³ MEOM - IGE Grenoble France

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Quick review of Models under Location Uncertainty (MLU)

Location Uncertainty

Let $\mathcal{A} \subset \mathbb{R}^d$ ($d = 2, 3$) be a domain and $\mathbf{X}_0 = \mathbf{x} \in \mathcal{A}$.

Deterministic flow:

$$\frac{d\mathbf{X}_t}{dt} = \mathbf{u}(\mathbf{X}_t, t) \quad (1)$$

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Stochastic flow [Mémin, 2014, Bauer et al., 2020]:

$$d\mathbf{X}_t = \mathbf{u}(\mathbf{X}_t, t) dt + \sum_n \boldsymbol{\xi}^n(\mathbf{X}_t, t) dW_t^n \quad (\text{Itô})$$

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$$= \underbrace{\left(\mathbf{u} - \frac{1}{2} \sum_n (\boldsymbol{\xi}^n \cdot \nabla) \boldsymbol{\xi}^n \right)}_{\mathbf{u}^*} dt + \sum_n \boldsymbol{\xi}^n \circ dW_t^n \quad (\text{Stratonovich})$$

- $\{W_t^n\}_n$ is a set of independent standard Brownian motions defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$ with $\mathbb{E}_{\mathbb{P}}[dW_t^n | \mathcal{F}_t] = 0$.
- $\{\boldsymbol{\xi}^n(\bullet, t)\}_n$ is a set of orthogonal eigenfunctions of the spatial covariance at each time t .

Transport under LU

Transport of tracer:

$$\Theta(\mathbf{X}_{t+\delta t}, t + \delta t) = \Theta(\mathbf{X}_t, t) \quad (2)$$

Deterministic transport:

$$\frac{D\Theta}{Dt} = \frac{\partial\Theta}{\partial t} + \mathbf{u} \cdot \nabla \Theta = 0 \quad (3)$$

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Stochastic transport [Bauer et al., 2020, Resseguier et al., 2020]:

$$\mathbb{D}_t \Theta = d_t \Theta + \left(\mathbf{u}^* dt + \sum_n \xi^n \circ dW_t^n \right) \cdot \nabla \Theta = 0, \quad (4)$$

where $d_t \Theta(\mathbf{x}, t) = \Theta(\mathbf{x}, t + \delta t) - \Theta(\mathbf{x}, t)$.

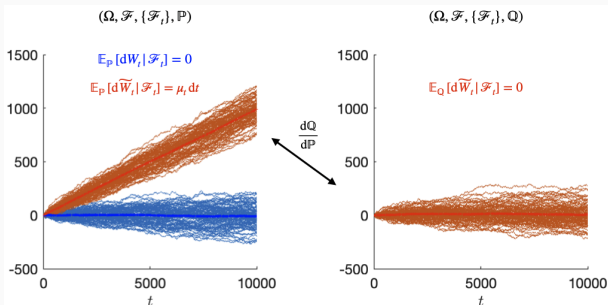
Re-weighting of Brownian motion

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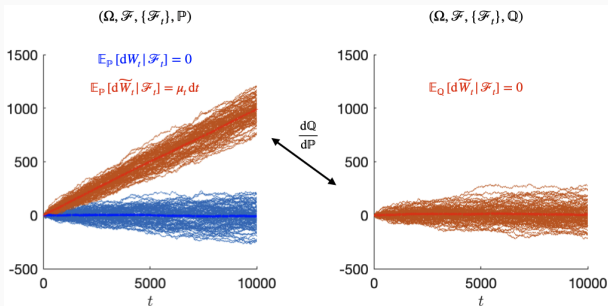
Change of probability measure:



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Cameron-Martin-Girsanov theorem:

$$\mathbb{E}_{\mathbb{P}} \left[\exp \left(\frac{1}{2} \int_0^T (\mu_t)^2 dt \right) \right] < \infty \implies \frac{d\mathbb{Q}}{d\mathbb{P}} = \exp \left[\int_0^T \mu_t dW_t - \frac{1}{2} \int_0^T \mu_t^2 dt \right]$$

Adjust drift in MLU

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$$d\mathbf{X}_t = \left(\mathbf{u} - \sum_n \xi^n \mu_t^n \right) dt + \sum_n \xi^n d\widetilde{W}_t^n \quad (\mathbb{Q})$$

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Stochastic transport under \mathbb{Q} :

$$\mathbb{D}_t \Theta = d_t \Theta + \left(\widetilde{\mathbf{u}}^* dt + \sum_n \xi^n \circ d\widetilde{W}_t^n \right) \cdot \nabla \Theta = 0 \quad (7)$$

Application to ocean quasi-geostrophic (QG) dynamics

Deterministic QG model

Governing equations:

$$\frac{DT}{Dt} = \mathcal{D}_T + \mathcal{F}_T$$

$$\frac{Dq}{Dt} = \mathcal{D}_q + \mathcal{F}_{q,\delta T}$$

$$\nabla^2 \psi + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2} \frac{\partial \psi}{\partial z} \right) = q - \beta y$$

$$\mathbf{u} = \nabla^\perp \psi$$

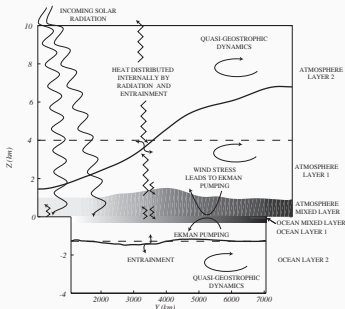


Figure 1: Illustration of the Quasi-Geostrophic Coupled Model (Q-GCM) from [Hogg et al., 2003, Hogg et al., 2004] (see details in <http://www.q-gcm.org>).

T = mixed layer temperature; q = potential vorticity (PV); \mathcal{D} = dumping terms; \mathcal{F} = forcing terms; ψ = streamfunction; $f_0 + \beta y$ = Coriolis approximation; N = buoyancy frequency; $\nabla^2 = \partial_x^2 + \partial_y^2$; $\nabla^\perp = (-\partial_y, \partial_x)$.

Deterministic simulations

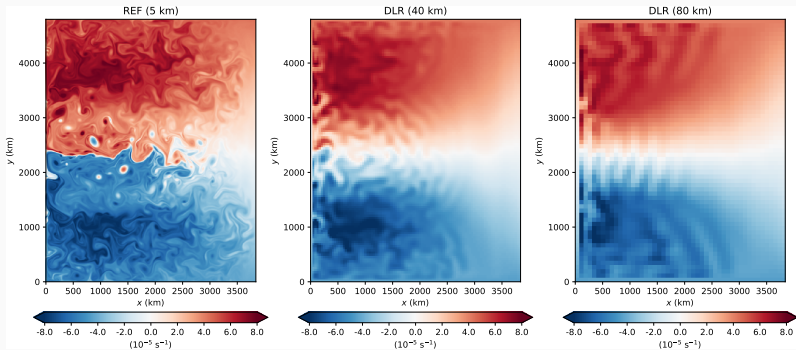


Figure 2: Snapshot (Year 95) of upper layer PV from deterministic simulations at different resolutions. (REF: eddy resolving reference; DLR: deterministic low-resolution.) Baroclinic Rossby radii are (39, 22) km.

Stochastic QG model

Governing equations:

$$\mathbb{D}_t T = (\mathcal{D}_T + \mathcal{F}_T) dt \quad (9a)$$

$$\mathbb{D}_t \mathbf{q} = (\mathcal{D}_q + \mathcal{F}_q) dt - \sum_{i=1,2} \mathbf{J}(u_i, (dX_t)_i) \quad (9b)$$

$$\nabla^2 \psi + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2} \frac{\partial \psi}{\partial z} \right) = \mathbf{q} - \beta y, \quad (9c)$$

where $\mathbf{J}(u, v) = \partial_x u \partial_y v - \partial_y u \partial_x v$, $(dX_t)_i = \widetilde{u}_i^* dt + \sum_n \xi_i^n \circ d\widetilde{W}_t^n$ and $\nabla \times ((\mathbf{v} \cdot \nabla) \mathbf{u}) = \mathbf{v} \cdot \nabla (\nabla \times \mathbf{u}) + \sum_{i=1,2} \mathbf{J}(u_i, v_i)$.

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Energy conservation ($\mathcal{D} = \mathcal{F} = 0$) [Bauer et al., 2020]:

$$d_t \int \frac{1}{2} \left((\nabla \psi)^2 + \frac{f_0^2}{N^2} \left(\frac{\partial \psi}{\partial z} \right)^2 \right) d\mathcal{A} = 0 \quad (10)$$

Calibration of EOFs and correction drift

Given a set of eddy-resolving streamfunction $\{\psi_I(\mathbf{x}, t_i)\}_{n=1, \dots, N}$

- Spatially high-pass filtering data for coarse grid points, $\psi'_L = (\mathbf{I} - \mathcal{F}_L)\psi_I|_L$, and deriving the geostrophic velocity fluctuations, $\mathbf{u}'_L = \nabla_L^\perp \psi'_L$.

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- Detrending $\mathbf{u}''_L = \mathbf{u}'_L - \overline{\mathbf{u}'_L}^t$ and performing POD:
 $\mathbf{u}''_L(\mathbf{x}, t) = \sum_{n=1}^N \alpha^n(t) \phi^n(\mathbf{x})$ with $\langle \phi^n, \phi^m \rangle_{\mathcal{A}} = \delta_{nm}$ and $\overline{\alpha^n \alpha^{m^t}} = \lambda^n \delta_{nm}$.

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- Rescaling the noise EOFs by $\xi^n(\mathbf{x}) = \sqrt{\lambda^n \tau} \phi^n(\mathbf{x})$ with τ the small-scale correlation time (usually chosen as Δt_L).

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- Finding the coefficients of correction drift μ^n by solving $\min \left\| \sum_n \xi^n \mu^n - \overline{\mathbf{u}'_L}^t \right\|_{\mathcal{A}}^2$, i.e. $\mu^n = \langle \overline{\mathbf{u}'_L}^t, \xi^n \rangle_{\mathcal{A}}$.

Stochastic simulations

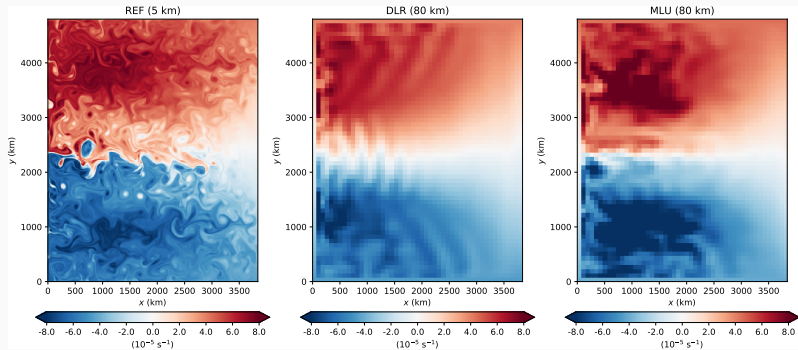


Figure 3: Snapshot (Year 120) of upper layer PV from different models. Baroclinic Rossby radii are (39, 22) km.

Long-term mean

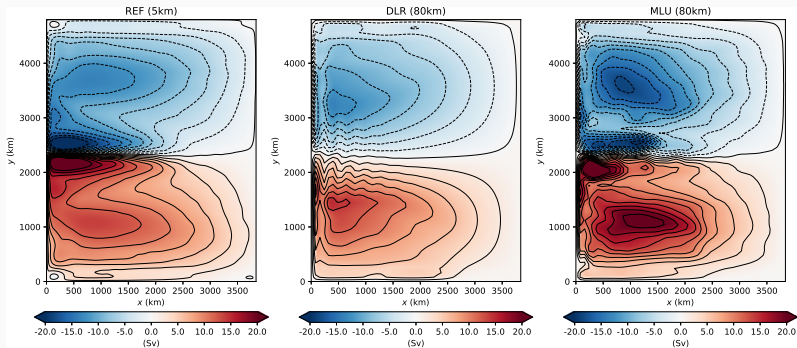


Figure 4: Temporal mean (from 110 yrs to 210 yrs) of upper layer streamfunction for different models.

Vertical effect

“Barotropic noise”: EOFs and drift learned from $\int \mathbf{u}'_L dz$

“Baroclinic noise”: EOFs and drift learned from $\mathbf{u}'_L - \int \mathbf{u}'_L dz$

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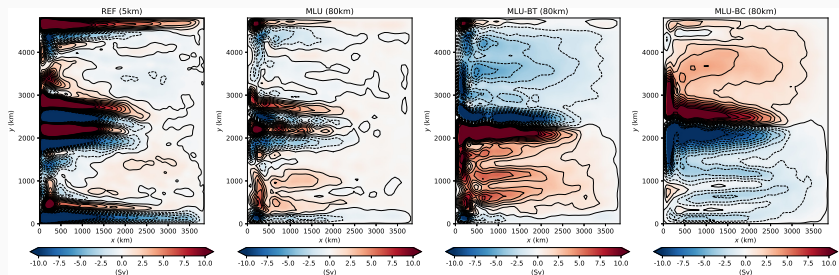


Figure 5: Temporal mean (from 110 yrs to 210 yrs) of bottom layer streamfunction for different models. (MLU-BT: MLU with “barotropic noise”; MLU-BC: MLU with “baroclinic noise”.)

Dynamical parametrization

Updating noise in time by projection:

$$\mathbf{P} = \mathbf{I}_2 - \frac{\nabla\eta(\nabla\eta)^T}{\|\nabla\eta\|^2}, \quad \eta = \frac{\partial}{\partial\mathbf{z}} \left(\frac{f_0^2}{N^2} \frac{\partial\psi}{\partial\mathbf{z}} \right) \quad (11)$$

“New noise” = $\sum_n \mathbf{P}\xi^n \circ dW_t^n$ (recalling that $\mathbf{q} = \nabla^2\psi + \eta + \beta\mathbf{y}$)

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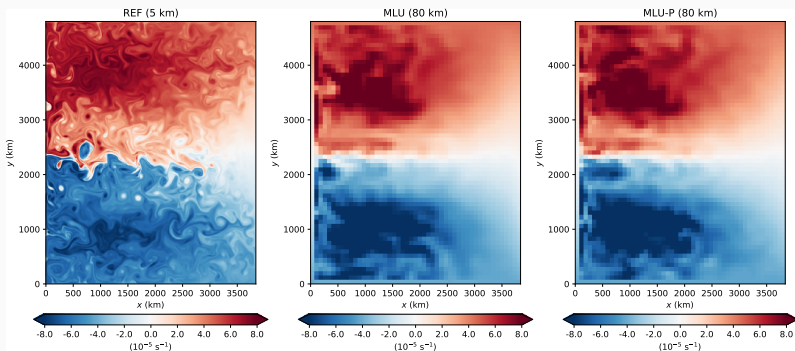
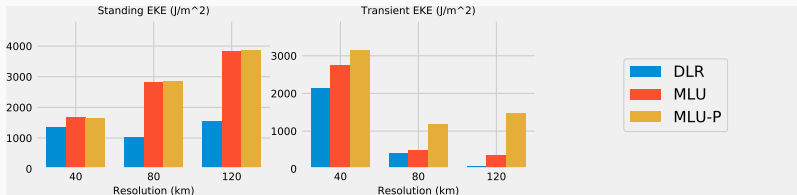


Figure 6: Snapshot (Year 120) of upper layer PV from different models. (MLU-P: MLU with projection noise.) Baroclinic Rossby radii are (39, 22) km.

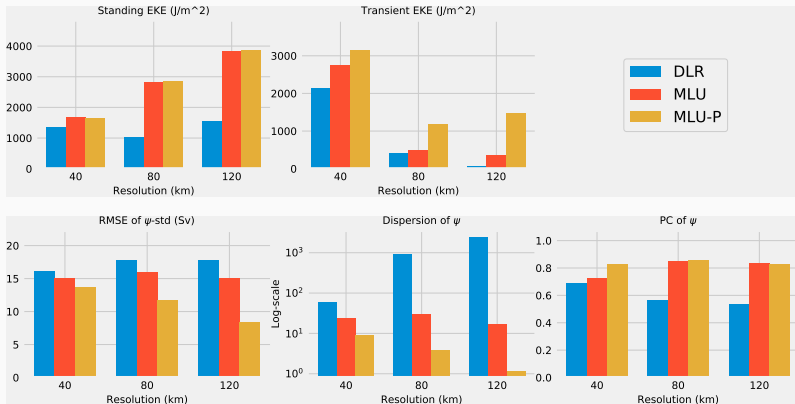
Diagnosis of variability

$$\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}', \quad \text{EKE} = \underbrace{\frac{\rho H}{2} \|\bar{\mathbf{u}}\|_{\mathcal{A}}^2}_{\text{Standing}} + \underbrace{\frac{\rho H}{2} \|\mathbf{u}'\|_{\mathcal{A}}^2}_{\text{Transient}} \quad (12)$$



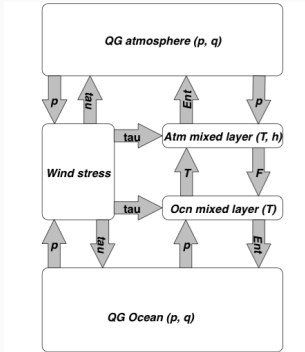
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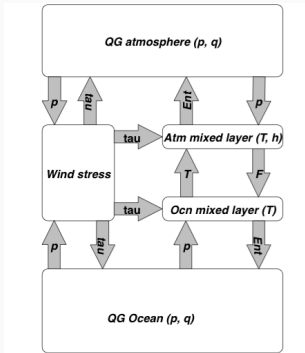
$$\text{Dispersion} = \left\langle \frac{\sigma_{\text{ref}}^2}{\sigma_{\text{mod}}^2} - 1 - \log \left(\frac{\sigma_{\text{ref}}^2}{\sigma_{\text{mod}}^2} \right) \right\rangle_{\mathcal{A}}, \quad \text{PC} = \frac{\|\sigma_{\text{ref}} \sigma_{\text{mod}}\|_{\mathcal{A}}^2}{\|\sigma_{\text{ref}}\|_{\mathcal{A}} \|\sigma_{\text{mod}}\|_{\mathcal{A}}} \quad (13)$$

Application to ocean-atmosphere coupling



Ekman layer momentum:

$$f_0 \times \mathbf{u} = -\nabla p + \partial_z \tau$$

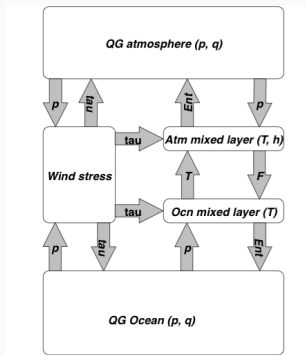


Ekman layer momentum:

$$f_0 \times \mathbf{u} = -\nabla p + \partial_z \boldsymbol{\tau} \quad (14a)$$

Wind stress [Duhaut and Straub, 2006]:

$$\boldsymbol{\tau}^a = \rho^a C_D |\mathbf{u}^a - \mathbf{u}^o| (\mathbf{u}^a - \mathbf{u}^o)$$



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Current stress:

$$\boldsymbol{\tau}^o = (\rho^a / \rho^o) \boldsymbol{\tau}^a \quad (14c)$$

ρ – pressure; $\boldsymbol{\tau}$ – stresses; a – atmosphere; o – ocean; C_D – drag coefficient (assumed to be constant here); ρ – density.

Mixed layer noise

Let $d\mathbf{U}_t = \sum_n \xi^n dW_t^n$, dP_t be a random pressure and $d\mathbf{T}_t$ be a (possible) random stress.

Mixed layer noise satisfying **geostrophic** equilibrium:

$$f_0 \times (\mathbf{u}_t dt + d\mathbf{U}_t) = -\nabla(p dt + dP_t) + \partial_z \boldsymbol{\tau} dt \quad (15a)$$

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Mixed layer noise satisfying **ageostrophic** regime:

$$f_0 \times (\mathbf{u}_t dt + d\mathbf{U}_t) = -\nabla(p dt + dP_t) + \partial_z(\boldsymbol{\tau} dt + d\mathbf{T}_t) \quad (16a)$$

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$$d\mathbf{T}_t^a = \rho^a C_D |d\mathbf{U}_t^a - d\mathbf{U}_t^o| (d\mathbf{U}_t^a - d\mathbf{U}_t^o) \quad (16c)$$

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$$d\mathbf{T}_t^o = (\rho^a / \rho^o) d\mathbf{T}_t^a \quad (16d)$$

First results

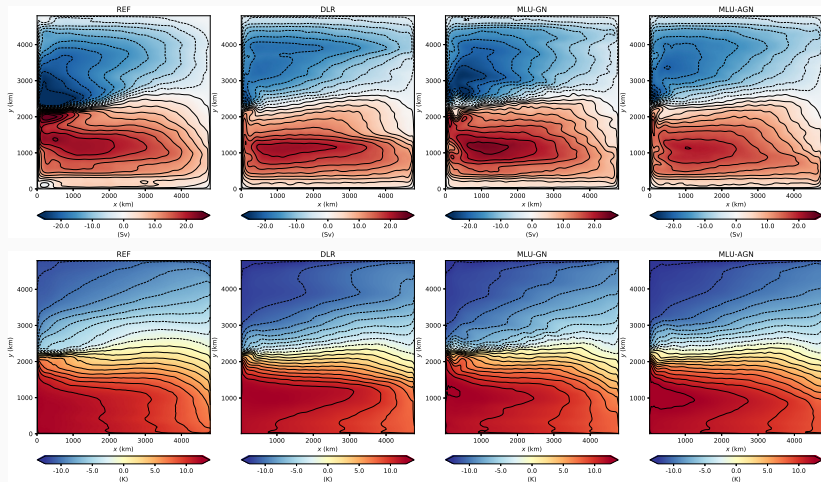


Figure 7: Comparison of 15-years-mean of the ocean upper layer stream function (top) and the sea surface temperature anomaly (bottom) for different coarse-grid (40 km for ocean and 160 km for atmosphere) runs. (MLU-GN: MLU with geostrophic mixed layer noise; AGN: with ageostrophic noise.)

Current feedback

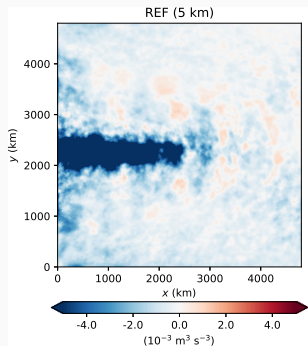


Figure 8: Eddy wind work for eddy-resolving simulation data (5km for ocean and 80km for atmos). Here a 2-years high-pass filter is used for 80-years data.

Eddy wind work [Renault et al., 2016, Renault et al., 2017]:

$$F_e K_e = \overline{(\tau^o)' \cdot (\mathbf{u}^o)'} \quad (17)$$

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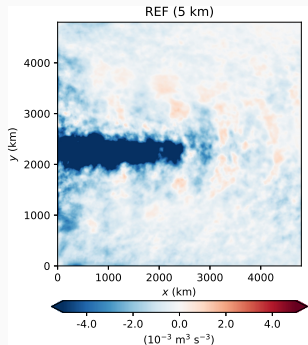


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Under **ageostrophic** mixed layer noise:

$$\begin{aligned} & \overline{((\boldsymbol{\tau}^o)' dt + d\boldsymbol{\Upsilon}_t^o) \cdot ((\mathbf{u}^o)' dt + d\mathbf{U}_t)} \\ &= \overline{(\boldsymbol{\tau}^o)' \cdot (\mathbf{u}^o)'} dt + \overline{d\boldsymbol{\Upsilon}_t^o \cdot d\mathbf{U}_t} \quad (18) \end{aligned}$$

(for a long term mean)

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- Combine the stochastic QG model and the stochastic SQG model [Lapeyre and Klein, 2006]

Thank you very much!

Questions?



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