Approaches for green roof dynamic model analysis using GSA

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Abstract:
Green roofs are a sustainable solution to manage water runoff from rain events in urban areas. Modeling hydrological phenomenon of green roofs is challenging because of the difficulties to characterize the soil parameters, inducing uncertainties on these parameters. Soil parameter uncertainties spread in the model and need to be analysed to better understand the dynamics of the water retention. For this purpose, the water retention capacity is represented by the Van Genuchten - Mualem model and sensitivity analysis is performed. Different methods of sensitivity analysis are considered to quantify effects of parameter uncertainties on the water retention capacity. The results of this study highlight the most influential soil parameters of the substrate on the water retention capacity.

Keywords: Uncertainty analysis, sensitivity analysis, green roof, Van Genuchten-Mualem model

1. INTRODUCTION

In the last decade, soil imperviousness has been one of the main urban issues. For strong rain events, runoff can lead to the discharge of high volume of water. Green Roofs (GR) are considered as a sustainable solution to contribute to rain water management [Mentens et al. (2006)]. Among all urban-water regulation systems, GR can be used to store and delay the release of rainwater to sewers. The GR performance can reduce storm-water runoff volume intercepted by the roof from 30% to 86%. The peak flow rate in water system can decrease from 22% to 93% and the peak flow can be delayed by 0 to 30 min [Li and Badcock Jr (2014)].
Typically, GR are structured in five layers: (i) the top layer is the vegetation plants, (ii) the layer underneath is the substrate, a growing medium, (iii) a geotextile is inserted to prevent the leaching of the substrate in the drainage layer, (iv) the drainage layer is employed to carry the infiltrated rainwater to drain and (v) a waterproof membrane protects the building concrete.
The outflow (Q_{out}) of GR is mainly related to the Volumetric Water Content (VWC) inside the substrate which depends on the physical properties of this type of soil. Few models exist to describe the hydrological infiltration throughout the substrate and are based on the Richards's equation [Richards (1931)]. This highly non linear partial differential equation describes the water infiltration in unsaturated porous media and makes it possible to determine the water retention capacity of GR. The characterization of the model parameters is difficult to handle because soil models cannot reproduce the living process of a GR. The soil parameters are considered uncertain and these uncertainties affect simulation of both VWC and Q_{out}.
Some studies have analysed parameter effects on water retention capacity. Usually, they focus on one parameter at a time and test different scenarios to evaluate the corresponding water retention capacity, for example, one type of plants at a time or one type of growing layer [Li and Badcock Jr (2014)]. This approach is limited since only a few scenarios among a wide range can be evaluated. The use of Sensitivity Analysis (SA) methods make it possible to analyse the model uncertainties. SA allows to quantify the contribution of parameter uncertainties on the output variation in order to better understand the model dynamics and improve design process [Saltelli et al. (2008)]. For instance, the studies can be focused on influential parameters and non-influential parameters can be fixed to a nominal value. Two types of SA can be distinguished: local SA which studies low variations of one parameter and is based on derivative and global SA which studies global simultaneous variation of all the parameters and makes it possible to study a larger interval including interactions between parameters. Global Sensitivity Analysis (GSA) are based on variance decomposition of the model output and are known as ANOVA (ANalysis Of VAriance) methods [Saltelli et al. (2008)].
For GR models, SA has been mostly used to analyse the thermal behavior in [Morau et al. (2014)] or the economic sustainability index of GR in [Ulubeyli and Arslan (2017)]. For hydric aspects, a local SA has been used to explain the qualitative impact of soil parameters on the water content and the output flow rate of GR in [Bouzouidja (2014)]. The data and the model studied represent a specific test bench. Moreover, only low variations of the parameters are analysed to prioritize parameter effects. The use of local SA limits the analysis on uncertainty interval and on effects of interaction between parameters which cannot be taken into account.
This paper focuses on GSA applied to a GR hydrological

model to assess the parameter effects on water retention capacity. The analysed data come from a GR in real conditions, in France, Europe. The objective of this study is twofold. First, exploratory analyses of the model uncertainties are carried out in order to better understand the GR retention capacity dynamics. To achieve it, GSA is performed on the VWC. Second, two methodologies to carry out GSA are presented: a sequential approach which provides parameter influence at each time instant and a multivariate approach which allows to sum up the influence dynamics to save computation time.
The paper is organized as follow. The GR hydrological model is presented in Section 2. Section 3 provides the key insights on GSA principles and methods used in this work. The application results are presented and discussed in Section 4. Finally, some conclusions are drawn in Section 5.

2. HYDROLOGICAL MODELING

2.1 The governing equations

The governing equation of the infiltration is the one dimensional form of the Richards's equation [Richards (1931)]:

$$\frac{\partial \theta(h)}{\partial t} = \frac{\partial}{\partial x}\left(K(\theta)\frac{\partial h}{\partial x}\right) - \frac{\partial K(\theta)}{\partial x} \tag{1}$$

where $\theta(h)$ is the volumetric water content in the soil [$m^3 \cdot m^{-3}$] as a function of h which represents the pressure head [m], $K(\theta)$ is the unsaturated hydraulic conductivity [$m \cdot s^{-1}$], t is the time [s] and x is the depth [m].

The Van Genuchten-Mualem model (VGM) is considered to describe the hydraulic conductivity function [Van-Genuchten (1980)]. The fundamental equations are:

$$\theta(h) = \begin{cases} \theta_r + \dfrac{(\theta_s - \theta_r)}{(1 + |\alpha h|^n)^m} & h < 0 \\ \theta_s & h \geq 0 \end{cases} \tag{2}$$

$$K(S_e) = K_s S_e^l \left[1 - \left(1 - S_e^{1/m}\right)^m\right]^2 \tag{3}$$

where θ_r and θ_s are the residual and the saturated water content [$m^3 \cdot m^{-3}$], α is a fitting parameter for pressure head [m^{-1}], K_s is the saturated hydraulic conductivity [$m \cdot s^{-1}$], n is a dimensionless parameter defining pore size distribution in the substrate and related to m with $m = 1 - \frac{1}{n}$, l is the tortuosity and S_e the effective saturation [$m^3 \cdot m^{-3}$] given by:

$$S_e = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r} \tag{4}$$

The upper boundary condition at the soil-atmosphere interface is set as "atmospheric boundary conditions with run-off" and the lower boundary conditions is set as "seepage face".

2.2 Calibration

The VGM model is applied to each of the following three layers of a GR represented in Figure 1: substrate, geotextile and drainage layers.

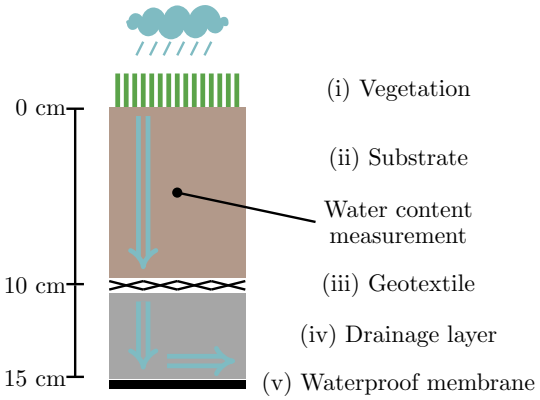

Fig. 1. Profile view of the experimental site including dimensions, materials typically used in extensive green roof and the volumetric water content sensor location within the substrate.

Table 1. Nominal value and uncertainty interval of ±10% for the substrate parameters

Uncertain parameter	Unit	Nominal value	Uncertainty interval	
			Inferior	Superior
θ_r	$m^3 \cdot m^{-3}$	0.08	0.072	0.088
θ_s	$m^3 \cdot m^{-3}$	0.43	0.387	0.473
α	m^{-1}	6.5	5.85	7.15
n	-	1.58	1.422	1.738
K_s	10^{-3} $m \cdot s^{-1}$	2.167	1.95	2.38
l	-	2.87	2.583	3.157

The model has one input – the rainfall – one output – the water content VWC in the substrate – and six parameters θ_s, θ_r, n, K_s, α and l. Their nominal value has been characterized in a previous study [Bouzouidja (2014)]. However, these parameters are affected by aging, soil compaction, roots and plants development and their characterization is challenging as discussed in [Solone et al. (2012)]. For these reasons, the parameters are considered uncertain and can be described as random variables defined by their probability distribution. Since no value inside the uncertainty interval is more probable than another, a uniform distribution is defined around the nominal value of each parameter (see Table 1).

2.3 Experimental GR and data strategy

The data have been collected on an in-situ experimental GR installed in Tomblaine (4840'N 613'E, North-East of France), under a Cfb climate according to the Köppen-Geiger classification. The GR plot is placed at the top of a 6 m in height flat-roof building and its surface is about 98 m^2. The vegetation is a mix of species of sedum. The substrate (10 cm height) is a man-made porous medium composed of 80% pozzolana and 20% organic part.
A five months period has been chosen from July to November 2011 which represents two periods (dry and wet) and different hydrological phenomena. To compute volumetric water content, the Richards's equation for the parameters given in Table 1 is solved using the Hydrus-1D© software [Simunek et al. (2008)]. The rainfall (blue), the measured VWC (black), the simulated VWC (red) and the simulated outflow Q_{out} (green) are displayed in Figure 2.

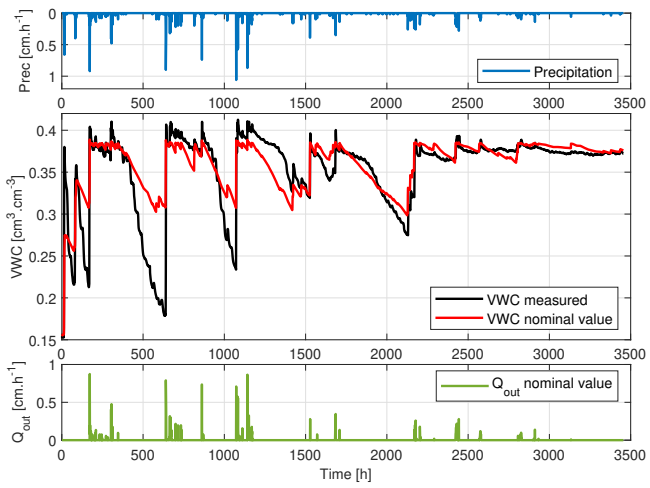

Fig. 2. Comparison between the numerical simulation (red) and the measurement (black) of the volumetric water content VWC. Rainfall is shown at the top (blue) and water outflow Q_{out} at the bottom (green)

Over the first 185 hours of simulation, large variations appear due to the initialization step of the software and are not taken into account for the result analysis. On the following hours of simulation, different hydrological phenomena can be observed. During a rain event, the VWC reaches its saturation value ($\theta_s = 0.43$ $m^3 \cdot m^{-3}$) and decreases when water comes down to the drainage layer (not shown). This hydrological process is quite well captured by the VGM model. When rainfall stops, the VWC decreases with a linear shape until the next rain event. Some events are not exactly reproduced by simulation. This is caused by the parameter uncertainty as discussed in Section 2.2 and also to more complex phenomena, difficult to model such as vegetation evolution [Emilsson (2008); Berretta et al. (2014)].

The aim of the study is to use GSA to determine the influence of the uncertain parameters on the VWC. In the next section, GSA theory is recalled and several methods are presented.

3. GLOBAL SENSITIVITY ANALYSIS METHODS

Global sensitivity analysis makes it possible to quantify uncertain parameter effect on the output model.

Consider a discrete model ($t = 1, 2, ...T$),

$$Y(\omega, t) = \mathcal{G}(X(\omega), t) \tag{5}$$

where $X = [X_1, ..., X_i, ..., X_k]^T$ gathers the k model uncertain parameters with $X_i \in \mathbb{R}$, $Y \in \mathbb{R}$ the output, $\mathcal{G}$ the model function which is known and $\omega \in \Omega$ where Ω is the space of random events of a probabilistic space. ω is used to indicate the randomness of the parameter and the output. In the following, in order to simplify the notations the stochastic variable ω is omitted. The random parameters X_i, $i = 1, ..., k$, are assumed independent and defined by their marginal distribution.

3.1 Sequential sensitivity analysis

Concepts of global sensitivity analysis recalled in this section are detailed in [Saltelli et al. (2008)]. To measure the effect of the parameter uncertainty on $Y(t)$ at each time instant, the first-order sensitivity index $S_i(t)$ is defined as:

$$S_i(t) = \frac{Var[E[Y(t)|X_i]]}{Var[Y(t)]} \tag{6}$$

where $E[Y(t)|X_i]$ is the expectation of the output $Y(t)$ given X_i, $Var[.]$ is the variance operator. $S_i(t)$ is between 0 and 1. Close to 0, $S_i(t)$ indicates non-influential parameter X_i and on the contrary, if $S_i(t)$ is close to 1, the parameter X_i is influential.
To measure the effect of the parameter X_i and all its interactions, the total sensitivity index $S_{T_i}(t)$ is defined as:

$$S_{T_i}(t) = \sum_{l \# i} S_l(t) \tag{7}$$

where $l\#i$ include all the index involving X_i.

The computation of $S_i(t)$ and $S_{T_i}(t)$ are complicated because of the variance integral. Some methods exist to estimate these indices. Very often, it is computed using Monte Carlo simulations [Saltelli et al. (2008)], but for computationally demanding models, this can become intractable. An alternative consists in expanding the model into orthogonal polynomials known as Polynomial Chaos (PC) [Crestaux et al. (2009)], which is less expensive. The sensitivity indices are then obtained in a straightforward way from the algebraic expression of the polynomial expansion coefficients. In this paper, sensitivity indices will be estimated using Polynomials Chaos Expansion (PCE), detailed in the next section.

3.2 Estimation based on polynomial chaos expansion

As explained in [Haro-Sandoval et al. (2012)], the homogeneous chaos expansion can be used to approximate any function in the Hilbert space of square-integrable functions. Therefore, the model output Y can be decomposed as follows:

$$Y(t) = \sum_{j=0}^{+\infty} a_j(t) \psi_j(X_1, ..., X_k) \tag{8}$$

where the multivariate polynomial ψ_j of degree j is defined by the tensor product of the corresponding one-dimensional polynomials $\phi_{\alpha_i^j}(X_i)$:

$$\psi_j(X_1, ..., X_k) = \prod_{i=1}^{k} \phi_{\alpha_i^j}(X_i) \tag{9}$$

where α_i^j is the PC degree following $\alpha^j = \sum_{i=1}^{k} \alpha_i^j \leq p$.

Different types of PC $\phi_{\alpha_i^j}(X_i)$ exist according to the parameter distribution. When the random variable distribution is uniform ($X_i \sim \mathcal{U}(a; b)$), Legendre PC are chosen.

In practice, the PC expansion is truncated up to a finite degree p and the number of coefficients in the expansion equals:

$$M + 1 = \frac{(k + p)!}{k!p!} \tag{10}$$

The optimal degree p could be selected by incrementing its value until a target accuracy, for instance the determination coefficient R^2, is reached. The deterministic PC coefficients (i.e. the $a_j(t)$'s in (8)) are the unknowns and several approaches have been proposed to compute them

[Berveiller et al. (2006); Crestaux et al. (2009); Rahman (2011)].

From the determination of the $a_j(t)$ coefficients, the first-order sensitivity index (6) which quantifies the effect of X_i on $Y(t)$ can be estimated by:

$$S_i(t) = \frac{\sum_{l \in \mathcal{I}_i} a_l^2(t) E[\psi_l^2(X_i)]}{\sum_{j=1}^{M} a_j^2(t) E[\psi_j^2(X_1, ..., X_k)]} \quad (11)$$

where $\mathcal{I}_i$ represents all polynomial index ψ_j which only depends on X_i.
The total sensitivity index of X_i (7) can be estimated by:

$$S_{T_i}(t) = \frac{\sum_{l \in \mathcal{I}_{\#i}} a_l^2(t) E[\psi_l^2(X_i)]}{\sum_{j=1}^{M} a_j^2(t) E[\psi_j^2(X_1, ..., X_k)]} \quad (12)$$

where $\mathcal{I}_{\#i}$ represents all polynomial index ψ_j which depends on X_i and all its interactions.

Summary of the sequential GSA approach

The different steps of the sequential GSA approach are:

(1) Determination of uncertain parameters and their distribution functions by domain experts
(2) Generation of N parameter samples following the prescribed distribution functions using specific sampling method
(3) Generation of the output $Y(\omega, t)$ (5) corresponding to the N parameter combinations (dim. $N \times T$)
(4) PC decomposition of $Y(\omega, t)$ for each instant (column) according to (8)
(5) Computation of sensitivity indices of the parameters for each instant of the PC decomposition of $Y(\omega, t)$ according to (11) and (12) (dim. $k \times T$)

The benefit of the sequential approach is to reveal the sensitivity index dynamics. However, the computation pf T indices is time consuming and leads to a huge amount of information difficult to interpret. Moreover, the sequential approach does not take into account the time correlation between two instants, thereby creating redundancy between indices.

3.3 Multivariate sensitivity analysis

To reduce the number of indices, different methods exist as multivariate GSA based on Principal Component Analysis (PCA) [Lamboni et al. (2011)]. Output time correlations are analysed by PCA and allow the definition of an adapted basis of projection. PCA is an optimal method of dimension reduction which minimizes data loss [Jolliffe and Cadima (2016)]. Output data are projected in the new basis and then each component.

Denote $Cov[Y]$ the $T \times T$ covariance matrix of $Y(\omega, t)$ over time. Its expansion based on eigenvalues is:

$$Cov[Y] = \frac{1}{N} Y_c' Y_c = \sum_{q=1}^{T} \lambda_q v_q v_q' \quad (13)$$

where Y_c is the matrix obtained by centering each column of $Y(t)$, $\lambda_q \in \mathbb{R}, q = 1, ..., T$ the eigenvalues of $Cov[Y]$ ($\lambda_1 \geq ... \geq \lambda_q \geq ... \geq \lambda_T$), $v_1, ..., v_q, ..., v_T$ a set of normalised and mutually orthogonal eigenvectors associated to the eigenvalue λ_q. The adapted basis corresponds to Q vectors v_q satisfying $Q \ll T$ with maximum of inertia $\mathbb{I}$.

$$\mathbb{I} = trace(Cov[Y]) = \sum_{q=1}^{T} \lambda_q \quad (14)$$

For a given random scenario ω_n, the model output $Y(\omega_n, t)$ can be approximated with the following expansion:

$$Y(\omega_n, t) \approx \overline{Y}(t) + \sum_{q=1}^{Q} h_q(\omega_n) v_q(t) \quad (15)$$

where $\overline{Y}(t)$ is the statistical mean of $Y(\omega, t)$ over the uncertainty domain Ω. The projection of $Y(\omega_n, t)$ in the new basis defined by the eigenvectors v_q corresponds to the principal components $h_q(\omega_n)$. It is defined by $h_q(\omega_n) = Y_c(\omega_n, t) v_q(t)$. According to (15), the output variability is contained in $h_q(\omega_n)$ with decreasing importance when q increases.
Output projected data correspond to a $N \times Q$ matrix and each column corresponds to a principal component h_q. Sensitivity indices are computed according to (11) and (12) for each principal component h_q.

Summary of the PCA-based GSA approach

The different steps of the multivariate GSA approach are:

(1) Steps (1), (2) and (3) are the same as for the sequential approach
(4) Decomposition of the covariance matrix of $Y(\omega, t)$ with an eigenvector v_q (dim. $T \times 1$) associated to each T eigenvalues λ_q, according to (13)
(5) Selection of the Q eigenvectors associated to the highest eigenvalues λ_q which represented the maximum of inertia $\mathbb{I}$ (new basis dim. $T \times Q$)
(6) Projection of $Y(\omega, t)$ in the new basis to obtain the principal components h (dim. $N \times Q$)
(7) PC decomposition for each principal component h according to (8)
(8) Computation of sensitivity indices (11) and (12) of the parameters for each principal component h (dim. $k \times Q$)

The advantage of PCA decomposition is to reduce the index number from T to Q and each index reveals a trend of sensitivity dynamics. Correlations between principal component and each instant of the output can be computed to interpret the meaning of each principal component.

4. RESULTS AND DISCUSSIONS

4.1 Sequential GSA applied to the GR model

Sequential SA is applied to the VGM model (1), (2) and (3) for the 5 months period, presented in Section 2.3. It represents a simulation time of $T = 3455$ hours for a computational time of 15 seconds.

Step 1: As detailed in Section 2, there are $k = 6$ uncertain parameters – θ_s, θ_r, n, K_s, α and l – defined by a uniform distribution given in Table 1.
Step 2: Several sampling methods have been tested with different sample size. The comparison reveals Sobol sequence and Latin Hypercube efficiency compared to Monte Carlo. In the following, the output will be computed with $N = 500$ samples generated using Sobol sequence. Therefore, a 500×6 matrix is generated following a uniform distribution using substrate uncertainty interval (Table 1).

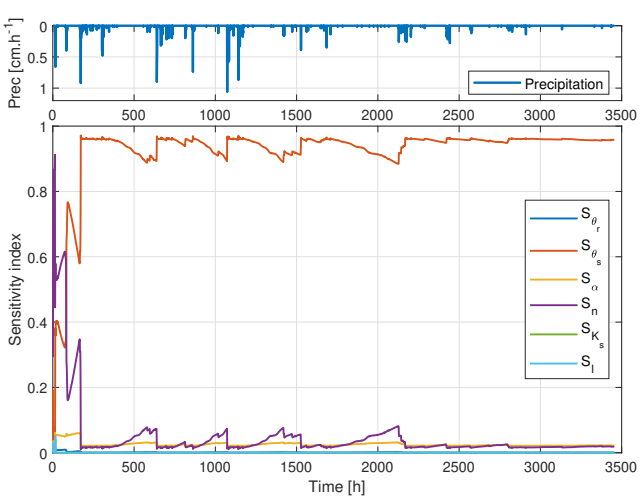

Fig. 3. First-order sensitivity indices for sequential SA applied to quantify substrate parameter effects on VWC of the VGM model (at the bottom). Rainfall is shown at the top (blue).

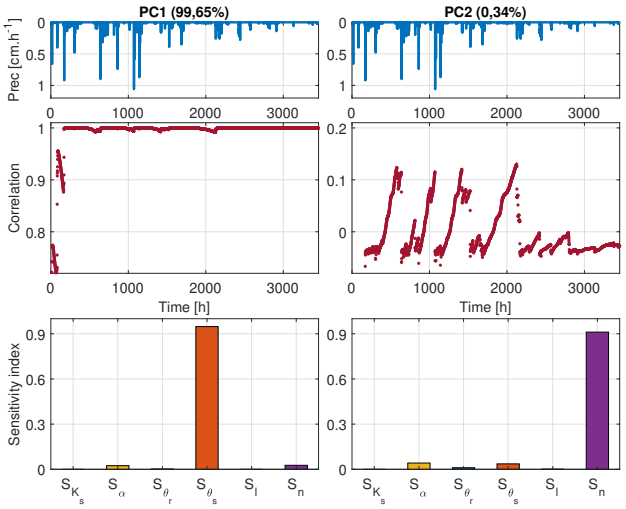

Fig. 4. First-order sensitivity indices for the two principal components of multivariate SA applied to quantify substrate parameter effects on VWC of the VGM model (bottom plots). Correlation between output data and principal components (red - middle plots). Rainfall events are shown on the top (blue).

Step 3: The volumetric water content is obtained by using Hydrus-1D© for each parameter combination. It requires 2h to compute $N = 500$ output evaluations. Next, the output are processed with Matlab© to obtain a 500×3455 output matrix.
Step 4: The output matrix is decomposed for each instant according (8). Sequential method requires the identification of $(M + 1) \times T$ coefficients. The choice of the development degree p also influences the computation time as explained in Section 3.2. Increasing p directly increases the computation time without improving the index quality. Tests on different values of p showed that $p = 3$ gives the best compromise.
Step 5: The sensitivity indices are computed according to (11) and (12) at each instant.

Despite its large computation time, the sequential GSA approach allows to obtain sensitivity index dynamics over time and to point out influence evolution. Sequential sensitivity indices for substrate SA are represented in Figure 3. It is worth noting that first-order indices are equal to total indices $S_i = S_{T_i}$. Such equality shows that higher-order indices are null which means there is no influence of interaction between parameters.
As a result, sensitivity indices over time have revealed three influential parameters: S_{θ_s} the saturated water content index (orange), S_n the porosity index (purple) and S_α the pressure head fitting parameter index (yellow).
S_{θ_s} index follows the water content dynamics i.e its influence rises when it rains and decreases when soil dries. Between 97 % and 88 % of water content variation are explained by θ_s variations. On the contrary, S_n index inversly follows VWC and between 1 % and 8.2 % of VWC variation are explained by n. Moreover, S_α is a quasi-constant value around 0.24. The other parameters represent less than 0.1 % of the influence.
For further work, the parameters θ_r, K_s and l can be fixed to their nominal value. Moreover, to optimise the GR water retention, the most important parameter on which to focus is the saturated water content θ_s.

Sequential sensitivity analysis gives sensitivity indices trends over time. To sum up this trends, the multivariate approach is applied.

4.2 Multivariate GSA applied to the GR hydrological model

Multivariate analysis is applied to the $N \times T$ output matrix (500×3455). The first steps are the same as in the sequential procedure described in Section 4.1.

Step 4: The covariance of the output matrix is decomposed using (13). From this decomposition, the most informative eigenvectors are selected to create a new basis.
Step 5: Two vectors ($Q = 2$) are selected by analysing inertia (14). The first one represents 99.65% of data information and the second one represents 0.35%. These vectors composed an 3455×2 adapted basis to project output data.
Step 6: The VWC output matrix (dim. 500×3455) is projected on the 3455×2 adapted basis. The projected data correspond to a new matrix of size 500×2. The columns of the projected data correspond to the principal components (PC1 and PC2).
Step 7: Each principal component is decomposed according (8).
Step 8: Two sensitivity indices are computed from each principal component using (11) and (12) and are sufficient to sum up the dynamics of the parameter influences.

The results of the multivariate GSA for the variation of substrate parameters are represented in Figure 4.
The PC1 correlation plot of Figure 4 shows an intense linear link between most of instants and the first component. During saturation periods (VWC close to 40%), correlations are equal to 1 and slightly decrease towards 0.99 during drying periods. The corresponding first-order sensitivity indices (bottom left) show that the most influential parameter is the saturated water content θ_s explaining 94.78% of VWC variations. The porosity parameter n

explains 2.66% of the VWC variations and the pressure head fitting parameter α explains 2.42%. All other parameters are considered non-influential because they affect less than 1% of the output variation. The PC2 correlation plot of Figure 4 shows an oscillation between 0.13 and -0.06 of the linear correlations and completes information provided by PC1 during drying periods. The corresponding first-order indices (bottom right) show a loss of θ_s influence which explains 3.61% of output variations and an increase of n sensitivity index influence explaining 91.11% of the output variations. The influence of the parameter α explains 2.42% of VWC variation. Other parameters are non-influential. To interpret these results, they must be taken into account in their entirety. Saturated water content θ_s is the most influential parameter in particular during rainfall event when VWC is at its maximum. However, θ_s influence decreases during drying periods and n influence increases. The influence of α is constant whatever the rainfall events and the other parameters are non-influential.

All these results are consistent with the sequential ones presented in Section 4.1. It is worth noting that the sequential analysis requires the computation of $T = 3455$ indices, which is done in 13 seconds. The PCA based approach requires the computation of only two indices, which takes 6 seconds of simulation, to represent the same influences, underlining the advantage of this approach.

5. CONCLUSION

In this paper, the effects of the soil parameter uncertainties on the water retention capacity have been investigated for a green roof in real conditions. Sequential and multivariate GSA have been carried out to study the behavior of the parameter influences with respect to time. Similar results are obtained with these two approaches but multivariate GSA significantly reduces the indices number to compute. It has been shown that the saturated water content is the most influential parameter, followed by the pore size distribution and the pressure head fitting parameter. No matter the season, the influence of the saturated water content follows the dynamics of the water content, and its influence rises when it rains and decreases when the soil dries. On the contrary, the influence of the porosity parameter decreases when it rains and rises when soil dries. These results highlight the importance of the saturated water content parameter to optimise the GR water retention. Sensitivity analysis can be seen as a helpful tool to better understand the green roof behavior. In the future, it would be interesting to study the influence of the soil parameter of the drainage on the outflow of the green roof. This could help to choose the most appropriate substrate and drainage to minimize the outflow to the sewer system during rainfall events.

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