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# Variable Sample Size Exponentially Weighted Moving Average Chart With Measurement Errors

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## Abstract

The effect of measurement errors on adaptive Shewhart charts have been investigated by several researchers. However, the effect of measurement errors on the performance of variable sample size EWMA control charts has not so far been investigated. In this paper, the performance of the VSS EWMA chart in the presence of measurement errors is investigated using a linear covariate error model and a Markov chain method. It is shown that the performance of the VSS EWMA chart is significantly affected by the presence of measurement errors. The effect of taking multiple measurements for each item in a subgroup on the performance of the VSS EWMA chart is also investigated. Moreover, the performance of the VSS EWMA control chart is compared with several other control charts in the presence of measurement errors. At last, an illustrative example is presented to show the application of the VSS EWMA control chart with measurement errors.

**Keywords:** adaptive control charts; EWMA control charts; linear covariate error model; Markov chain approach; variable sample size

## 1. Introduction

Although the Shewhart  $\bar{X}$  control chart is effective in detecting large process mean shifts; however, the Exponentially Weighted Moving Average (EWMA) control chart has a better performance in detecting small and moderate shifts. Many developments have been made to improve the performance of control charts in detecting process mean shifts. One of the main approach to improve the performance of EWMA charts is using adaptive procedures and designing adaptive control charts. An adaptive control chart is a chart for which at least one of its parameters (sample size, sampling interval, and the control limit coefficient) is variable throughout the process.

A lot of researches have been conducted to develop adaptive control charts and it has been shown that they have better performance than their Fixed Sampling Rate (FSR) control charts counterparts to detect

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small and moderate shifts. In this paragraph, the literature concerning adaptive Shewhart  $\bar{X}$  control charts is reviewed. Prabhu et al. [1] and Costa [2] presented the Variable Sample Size (VSS)  $\bar{X}$  control chart. Reynolds [3] investigated the Variable Sampling Interval (VSI)  $\bar{X}$  control chart. Amiri et al. [4] investigated the economic-statistical design of the VSI and VSS  $\bar{X}$  control chart by Taguchi loss function approach. Prabhu et al. [5] and Costa [6] investigated the combined Variable Sample Size and Sampling Interval (VSSI)  $\bar{X}$  control scheme. Costa [7] introduced the Variable Parameters (VP)  $\bar{X}$  chart. In the next paragraph, papers in the area of adaptive EWMA charts are introduced.

In addition to studies on adaptive Shewhart  $\bar{X}$  control chart, many researches have also been undertaken on adaptive EWMA control chart as well, for example, see Saccucci et al. [8], who investigated the VSI EWMA control chart. Reynolds and Arnold [9] considered the VSS and VSI EWMA control charts and they developed integral equation and Markov chain methods to evaluate the statistical performance of these charts. Castagliola et al. [10,11] presented the VSI R-EWMA and VSI S2-EWMA control charts to monitor the process variance. Kazemzadeh et al. [12] designed a VSI EWMA control chart based on the  $t$  distribution. Also, the efficiency of the VSI EWMA- $\bar{X}$  (median) control chart has been investigated by Tran et al. [13]. Ugaz et al. [14] proposed an Adaptive EWMA (AEWMA) control chart with time-varying smoothing parameter. Tang et al. [15] introduced an optimal design of the AEWMA chart for the mean based on the median run length and on the expected median run length. Amiri et al. [16] designed a new adaptive variable sample size EWMA control chart. Capizzi and Masarotto [17] proposed an AEWMA- $\bar{X}$  control chart. Ong et al. [18] investigated the VSI EWMA chart when the process parameters are unknown. Tran et al. [19] presented an EWMA type control chart to monitor the ratio of two normal quality characteristics. Chong et al. [20] investigated the VSI EWMA  $\bar{X}$  chart when the process parameters are unknown. Haq and Khoo [21] introduced an adaptive multivariate EWMA (AMEWMA) chart and they computed the run length characteristics of this chart by using Monte Carlo simulation runs. In the above mentioned papers, the measurement errors are not considered. However, in the real world, there are measurement errors in many processes. Some researchers studied the effect of measurement errors on the performance of Shewhart control charts. Benet [22] introduced the effect of measurement errors by using the model  $X = Y + \varepsilon$ , where  $X$  is the observed value,  $Y$  is the true value of the quality characteristic and  $\varepsilon$  is a random measurement error. Linna and Woodall [23] suggested to use the linearly covariate model  $X=A+BY+\varepsilon$ , where  $A$  and  $B$  are two constants.

Here, the papers which consider the effect of measurement errors on the adaptive Shewhart charts are summarized. The effect of measurement errors on the performance of  $\bar{X}$ -R control charts, has been investigated by Linna and Woodall [23] and Kanazuka [24]. Costa and Castagliola [25] investigated the effect of measurement errors on the  $\bar{X}$  control chart with autocorrelated data and Hu et al. [26] evaluated this effect on the synthetic  $\bar{X}$  control chart. Linna et al. [27] analyzed the performance of the FSR

Hotelling's  $T^2$  control chart in the presence of measurement errors. Hu et al. [28,29] considered the effect of measurement errors on the performance of the VSS  $\bar{X}$  and VSI  $\bar{X}$  control charts, respectively. Sabahno and Amiri [30] investigated the effect of measurement errors on the performance of the VSSI  $\bar{X}$  control chart. Sabahno et al. [31,32,33] investigated the effect of measurement errors on the performance of the VP  $\bar{X}$ , VSS- $T^2$  and VSI- $T^2$  control charts, respectively. Ghashghaei et al. [34] presented the effect of measurement errors on the joint monitoring of the process mean and variability under Ranked Set Sampling (RSS) scheme. Amiri et al. [35] investigated the effect of measurement errors on the performance of Exponentially weighted Likelihood Ratio (ELR) control chart for the simultaneous monitoring of the mean vector and covariance matrix in multivariate normal processes. Maleki et al. [36] also considered the ELR control chart for the simultaneous monitoring of the mean vector and covariance matrix and they investigated the effect of measurement errors with linearly increasing error variance on the performance of this control chart. Nguyen et al. [37] evaluated the performance of a VSI Shewhart control chart monitoring the coefficient of variation in the presence of measurement errors. Finally, Tran et al. [38] evaluated the performance of coefficient of variation charts in the presence of measurement errors.

Concerning adaptive EWMA type charts in presence of measurement errors, Stemmann and Weihs [39] introduced the EWMA- $\bar{X}$ -S control chart and evaluated its performance in the presence of measurement errors. Maravelakis et al. [40] considered the EWMA control chart with measurement errors. Yang et al. [41] investigated the effect of measurement errors on the performance of an EWMA control chart for monitoring two dependent process steps. Haq et al. [42] considered the effect of measurement errors on EWMA control chart under RSS scheme. Abbasi [43,44] studied the effect of measurement errors on a EWMA control chart with two-component measurement errors (i.e. additive and also multiplicative ones). Saghaei et al. [45] investigated the economic design of an EWMA control chart with measurement errors by using a genetic algorithm. Cheng et al. [46] showed the effect of measurement errors on the EWMA- $\bar{X}$  and CUSUM- $\bar{X}$  control charts. Most of studies about adaptive EWMA control charts have been done without considering the measurement errors. Tang et al. [47] presented the effect of measurement errors on the AEWMA- $\bar{X}$  chart.

Maleki et al. [48] performed a review on the effect of measurement errors on the control charts. Based on this review paper and, to the best of authors' knowledge, the effect of measurement errors on the VSS EWMA- $\bar{X}$  control chart has not been investigated yet. Hence, in this paper, we consider the effect of measurement errors on the performance of the VSS EWMA chart. For this aim, we use Linna and Woodall [23]'s linear covariate error model and we consider both fixed and linearly increasing error variances. Then, we investigate the effect of the measurement errors, multiple measurements and constant  $B$  on the performance of the adaptive VSS EWMA control chart. A modified Markov chain method

(inspired by the one proposed by Lucas and Saccucci [49]) is used in order to compute the Run Length properties of the VSS EWMA control chart. We also compare the performance of the proposed VSS EWMA with FSR EWMA as well as with the Shewhart VSS  $\bar{X}$  and VSI  $\bar{X}$  control charts by Hu et al. [28,29], respectively in detecting step shifts in the process mean in terms of the Average Run Length (ARL).

The structure of this paper is as follows: we introduce the linearly covariate error model in the next section. The VSS EWMA control chart with measurement errors is detailed in section 3. Then, in section 4, the effect of measurement errors and multiple measurements on the performance of the VSS EWMA chart is investigated in terms of the ARL and Average Number of Observations to Signal (ANOS) by using a Markov chain. The performance of the VSS EWMA and FSR EWMA control charts are compared in section 5. An illustrative example based on a real case is presented in section 6 and, finally, conclusions and suggestions for future researches are given in section 7.

## 2. Linearly covariate error model

The linearly covariate measurement error model was first introduced by Linna and Woodall [23]. Let us assume that  $Y_{ij}$ , the  $j$ th observation of the quality characteristic at time  $i$  ( $i = 1, 2, \dots$ , and  $j = 1, 2, \dots, n_s$  with,  $n_s = \{n_1, n_2\}$  where  $n_1$  and  $n_2$  are the small and large sample sizes, respectively.) follows a normal  $N(\mu_0 + \delta\sigma_0, \sigma_0^2)$  distribution where  $\mu_0$  and  $\sigma_0$  are the in-control mean and standard deviation of the process, respectively, both assumed as known.  $\delta$  is the magnitude of the mean shift in unit of  $\sigma_0$  (note that  $\delta = 0$  when the process is in-control). In practice the true value of the quality characteristic  $Y_{ij}$  is not directly observed but it can be estimated from  $k = 1, \dots, m$  measurements  $X_{ijk}$  related to the true value  $Y_{ij}$  using the linearly covariate model

$$X_{ijk} = A + BY_{ij} + \varepsilon_{ijk}, \quad (1)$$

We assume that  $A$  and  $B$  are constants, and  $\varepsilon_{ijk}$  is the random error term which is assumed to be normally distributed with mean 0 and variance  $\sigma_M^2$ . Assume that, we have  $N$  standard parts with known but different quality characteristic values. Using some measurement system, the quality characteristic of these standard parts is measured. Due to the measurement errors, the observed values are different from the real ones. Using the true and observed values of the quality characteristic and fitting a simple linear regression, the parameters  $A$ ,  $B$  and  $\sigma_M^2$  can be estimated. Also,  $\mu_x$  and  $\sigma_x^2$  can be easily estimated by using the sample mean  $\mu_x$  and sample standard deviation  $\sigma_x$  of the measured values of the quality

characteristic and then the values  $\mu_Y$  and  $\sigma_Y^2$  are estimated using the following equations:

$$\mu_Y = \frac{\mu_X - A}{B} \text{ and } \sigma_Y^2 = \frac{\sigma_X^2 - \sigma_M^2}{B^2}.$$

From Montgomery and Runger [50] and Linna and Woodall [23], there are two possible scenarios for  $\sigma_M^2$ , where in the first one  $\sigma_M^2$  is considered as a constant and, in the second one,  $\sigma_M^2 = C + D\mu_Y$  where  $C$  and  $D$  are two extra constants. Similar to parameters  $A$  and  $B$ , parameters  $C$  and  $D$  can be estimated by using a dedicated simple linear regression model.

At time  $i = 1, 2, \dots$ , the sample mean  $\bar{X}_i$  of subgroup  $\{X_{i11}, \dots, X_{ilm}, \dots, X_{in1}, \dots, X_{inn}\}$  is equal to

$$\bar{X}_i = \frac{1}{mn_s} \sum_{j=1}^{n_s} \sum_{k=1}^m X_{ijk} \quad (2)$$

Substituting Equation 1 in Equation 2, we have

$$\bar{X}_i = A + \frac{1}{n_s} \left( B \sum_{j=1}^{n_s} Y_{ij} + \frac{1}{m} \sum_{j=1}^{n_s} \sum_{k=1}^m \varepsilon_{ijk} \right). \quad (3)$$

It can be easily shown that the expected value and the variance of  $\bar{X}_i$ , are respectively equal to

$$E(\bar{X}_i) = A + B(\mu_0 + \delta\sigma_0), \quad (4)$$

$$Var(\bar{X}_i) = \frac{1}{n_s} \left( B^2 \sigma_0^2 + \frac{\sigma_M^2}{m} \right). \quad (5)$$

### 3. VSS EWMA control chart with measurement errors

As mentioned before when the sample sizes are variable, the control chart for this scheme is called VSS control chart. As mentioned in the previous section, in this paper, we assume that there are two types of sample sizes ( $n_1 < n_2$ ). From Equation 1 relating  $Y$  and  $X$ , it is clear that  $X$  is normally distributed with mean  $A + B(\mu_0 + \delta\sigma_0)$  and variance  $B^2 \sigma_0^2 + \sigma_M^2$ . Assuming that at each sampling point we collect  $n_s = \{n_1, n_2\}$ , we calculate the mean of these observations as  $\bar{X}_i$  and we standardize it as bellow

$$U_i = \frac{\bar{X}_i - \mu_{0\bar{X}_i}}{\sigma_{\bar{X}_i}}, \quad (6)$$

where  $\sigma_{\bar{X}_i}$  is the standard deviation of  $\bar{X}_i$  obtained by taking the square root of Equation 5, also  $\mu_{0_{\bar{X}_i}}$  is the in-control expected value of  $\bar{X}_i$  obtained from Equation 4. After that,  $U_i \sim N(0,1)$  when the process is in-control.  $U_i$  is used in the EWMA statistic, as follows:

$$Z_i = \lambda U_i + (1 - \lambda) Z_{i-1}, \quad (7)$$

where  $Z_i$  follows a normal distribution with the following mean and asymptotic variance

$\mu_{Z_i} = 0, \sigma_{Z_i}^2 = \frac{\lambda}{2 - \lambda}$ , respectively. Hence, the steady state upper and lower control limits of the VSS

EWMA chart are obtained by using Equation 8 and Equation 9, respectively:

$$UCL = L \sqrt{\frac{\lambda}{2 - \lambda}}, \quad (8)$$

$$LCL = -L \sqrt{\frac{\lambda}{2 - \lambda}}. \quad (9)$$

Similarly, the steady state upper and lower warning limits, respectively, are:

$$UWL = W \sqrt{\frac{\lambda}{2 - \lambda}}, \quad (10)$$

$$LWL = -W \sqrt{\frac{\lambda}{2 - \lambda}}, \quad (11)$$

where  $0 < W < L$ . The VSS strategy is as follows:

- If  $Z_i \in \Omega_1 = [LWL, UWL]$ , the process is identified as “in-control”, and the small sample size  $n_1$  is taken for the next sampling period.
- If  $Z_i \in \Omega_2 = [LCL, LWL) \cup (UWL, UCL]$ , the process is also identified as “in-control”, but the large sample size  $n_2$  is chosen for the next sampling period.
- If  $Z_i \notin \Omega_3 = [LCL, UCL]$ , the process is declared as “out-of-control”, and the corrective actions must be undertaken.

For transient states, we have:

$$Pr(Z_i \in \Omega_1) = \Phi(W) - \Phi(-W) = 2\Phi(W) - 1, \quad (12)$$

$$Pr(Z_i \in \Omega_2) = \Phi(K) - \Phi(W) + \Phi(-W) - \Phi(-K) = 2(\Phi(K) - \Phi(W)), \quad (13)$$

$$Pr(Z_i \in \Omega_3) = \Phi(K) - \Phi(-K) = 2\Phi(K) - 1, \quad (14)$$

where  $\Phi(\cdot)$  denotes the cumulative distribution function (c.d.f) of the  $N(0,1)$  distribution. To obtain the warning limits coefficient, we use the balanced equation  $n_0 = n_1 P_1 + n_2 P_2$ , where  $n_0$  is average sample sizes (the weighted mean of  $n_1$  and  $n_2$ ), where  $P_1$  and  $P_2$  are the conditional probabilities of  $Z_i$  falling in  $\Omega_1$  and  $\Omega_2$ , respectively, provided that  $Z_i$  is “in-control”( $\Omega_3$ ). Hence, we have:

$$n_0 = n_1 \frac{Pr(Z_i \in \Omega_1)}{Pr(Z_i \in \Omega_3)} + n_2 \frac{Pr(Z_i \in \Omega_2)}{Pr(Z_i \in \Omega_3)} \quad (15)$$

By replacing Equations 12-14 in Equation 15 and solving for  $W$ , the following equation is obtained:

$$W = \Phi^{-1} \left( \frac{2\Phi(L)(n_0 - n_2) - n_0 + n_1}{2(n_1 - n_2)} \right), \quad (16)$$

where  $\Phi^{-1}(\cdot)$  denotes the inverse c.d.f. of the  $N(0,1)$  distribution.

#### 4. The effect of measurement errors on the VSS EWMA control chart

In this section, we focus on the performance of VSS EWMA control chart with a linearly covariate error model over a range of shift sizes. After obtaining  $W$  and considering the other parameters  $(n_1, n_2, n_0, m, A, B, L)$ , we compare the performance of the VSS EWMA control chart in terms of the ARL and ANOS. We use a Markov chain approach to compute the ARL and ANOS for EWMA control chart, according to the method presented by Lucas and Saccucci [49]. The distance between the control limits is divided into  $2p + 1$  states. Each state has a width of  $2d$ , where  $2d = \frac{UCL - LCL}{2p + 1}$ . The midpoint of each state is  $f_t$ ,  $t \in \{-p, \dots, p\}$ . The statistic  $Z_i$  is in the state  $t$ , if  $f_t - d \leq Z_i \leq f_t + d$ . When  $Z_i$  is within the control limits it is in a transient state, and when  $Z_i$  is outside the control limits, it enters the absorbing state. Let  $\mathbf{Q}$  be the  $(2p + 1, 2p + 1)$  matrix of transition probabilities  $Q_{gh}$  of going from state  $g$  to state  $h$ :

$$Q_{gh} = Pr(f_h - d < Z_i < f_h + d | Z_{i-1} = f_g), \quad (17)$$

$$Q_{gh} = Pr(Z_i < f_h + d | Z_{i-1} = f_g) - Pr(Z_i < f_h - d | Z_{i-1} = f_g). \quad (18)$$

Replacing  $Z_i$  with  $U_i$  based on (7),

$$Q_{gh} = Pr(U_i < \frac{f_h + d - (1 - \lambda)f_g}{\lambda}) - Pr(U_i < \frac{f_h - d - (1 - \lambda)f_g}{\lambda}). \quad (19)$$

Substituting  $U_i$  with  $\bar{X}_i$  based on (6),  $Q_{gh}$  is obtained as follows:



$$Q_{gh} = Pr(\bar{X}_i < \left( \frac{f_h + d - (1-\lambda)f_g}{\lambda} \right) \sigma_{\bar{X}_i} + \mu_{0_{\bar{X}_i}}) - Pr(\bar{X}_i < \left( \frac{f_h - d - (1-\lambda)f_g}{\lambda} \right) \sigma_{\bar{X}_i} + \mu_{0_{\bar{X}_i}}). \quad (20)$$

For computing this probability, the Equation 20 should be standardized, hence, by using Equation 4 and Equation 5, we have:

$$Q_{gh} = Pr\left(\frac{\bar{X}_i - \mu_{\bar{X}_i}}{\sigma_{\bar{X}_i}} < \frac{\left[\left(\frac{f_h + d - (1-\lambda)f_g}{\lambda}\right) \sigma_{\bar{X}_i} + \mu_{0_{\bar{X}_i}} - \mu_{\bar{X}_i}\right]}{\sigma_{\bar{X}_i}}\right) - Pr\left(\frac{\bar{X}_i - \mu_{\bar{X}_i}}{\sigma_{\bar{X}_i}} < \frac{\left[\left(\frac{f_h - d - (1-\lambda)f_g}{\lambda}\right) \sigma_{\bar{X}_i} + \mu_{0_{\bar{X}_i}} - \mu_{\bar{X}_i}\right]}{\sigma_{\bar{X}_i}}\right). \quad (21)$$

Considering  $\mu_{0_{\bar{X}_i}} = A + B \mu_0$  (when  $\delta = 0$ ),  $\mu_{\bar{X}_i} = A + B (\mu_0 + \delta \sigma_0)$  and  $\sigma_{\bar{X}_i} = \sqrt{\frac{1}{n} \left( B^2 \sigma_0^2 + \frac{\sigma_M^2}{m} \right)}$  and

some simplifications in Equation 21, we have:

$$Q_{gh} = \Phi\left(\frac{f_h + d - (1-\lambda)f_g}{\lambda} - \frac{B \delta \sigma_0}{\sigma_{\bar{X}_i}}\right) - \Phi\left(\frac{f_h - d - (1-\lambda)f_g}{\lambda} - \frac{B \delta \sigma_0}{\sigma_{\bar{X}_i}}\right). \quad (22)$$

Let  $\mathbf{q}$  and  $\mathbf{n}$  be the  $(2p+1, 1)$  vector of initial probabilities and the  $(2p+1, 1)$  vector of sample sizes, respectively, associated to each transient states, i.e.

$$\mathbf{q}_t = \begin{cases} 0 & \text{if } Z_t \notin (f_t - d, f_t + d] \\ 1 & \text{if } Z_t \in (f_t - d, f_t + d] \end{cases}, \quad (23)$$

$$\mathbf{n}_t = \begin{cases} n_s & \text{if } LWL \leq f_t \leq UWL \\ n_L & \text{otherwise} \end{cases}. \quad (23)$$

Finally, the ARL and ANOS of the VSS EWMA control chart can be obtained using:

$$ARL_1 = \mathbf{q}^T (\mathbf{I} - \mathbf{Q})^{-1} \mathbf{1}, \quad (24)$$

$$ANOS_1 = \mathbf{q}^T (\mathbf{I} - \mathbf{Q})^{-1} \mathbf{n}, \quad (25)$$

where  $\mathbf{I}$  is the  $(2p+1, 2p+1)$  identity matrix and  $\mathbf{1}$  is a  $(2p+1, 1)$  unit column vector.

To evaluate the effect of measurement errors on the performance of the VSS EWMA chart, we used  $\lambda = 0.2$  and  $L = 2.962$  to achieve an in-control ARL of 500. Also, we used  $2p+1$  states where  $p = 105$ , based on Maravelakis [40], for the implementation of the Markov chain approach. Also, in our study, we considered both constant measurement errors variance as well as linearly increasing measurement errors variance.

#### 4.1. A constant measurement errors variance

The  $ARL_1$  and  $ANOS_1$  for the VSS EWMA chart with linearly covariate error model are obtained in Table 1 for different combinations of measurement errors ( $\sigma_M^2 = 0, 0.3, 0.7, 1$ ) and shifts magnitudes in

process mean ( $\delta = 0.1, 0.5, 1, 2$ ),  $n_1 = 1, 3, 5$  and  $n_2 = 6, 7, 10$  when  $m=1$  and  $B=1$ . From the results of this Table, we can easily see that when the measurement errors variance ( $\sigma_M^2$ ) increases, the  $ARL_1$  and  $ANOS_1$  also increase, i.e. the negative effect of measurement errors increases. For example in the case  $\delta = 0.1, n_1 = 1, n_2 = 6$  and  $\sigma_M^2 = 0$  we have  $ARL_1 = 184.8$  and  $ANOS_1 = 691.6$  while when  $\sigma_M^2 = 1$ , we have  $ARL_1 = 276.43$  and  $ANOS_1 = 1004.11$ . Table 2 is proposed to show the effect of the number of measurements ( $m$ ) when  $\sigma_M^2 = 1$  and  $B = 1$  for different shifts in the process mean ( $\delta = 0.1, 0.5, 1, 2$ ). Based on Table 2, using multiple measurements leads to decrease the negative effect of measurement errors. For example, considering the combination ( $m = 1, \delta = 0.1, n_1 = 1, n_2 = 6$ ) leads to  $ARL_1 = 276.43$  and  $ANOS_1 = 1004.11$  while, when  $m = 2$ , it results in  $ARL_1 = 238.2$  and  $ANOS_1 = 874.5$ .

In Table 3, we have performed a sensitivity analysis on the parameter  $B$ . From this Table, we can see the positive effect of parameter  $B$  on decreasing the effect of measurement errors, when  $\sigma_M^2 = 1$  and  $m=1$  for different shifts in the process mean. For example, considering the combination ( $\delta = 0.1, n_1 = 1, n_2 = 6$ ) for  $B=1$  leads to  $ARL_1 = 276.43$  and  $ANOS_1 = 1004.11$  while, for the same combination and  $B=2$ , we have  $ARL_1 = 213.9$  and  $ANOS_1 = 791.6$ . Table 4 shows the  $ARL$  and  $ANOS$  of the VSS EWMA chart for different values of  $\sigma_M^2$  when  $m = 5$  and  $B=1$ . Comparing the results of Tables 1 and 4 shows that increasing  $m$  for different values of  $\sigma_M^2$  decreases the negative effect of the measurement errors. For example considering the combination ( $\delta = 0.1, n_1 = 5, n_2 = 10, \sigma_M^2 = 0.3$ ) and  $m=1$  and  $m=5$  leads to  $ARL_1 = 139.2$  and  $ARL_1 = 117.4$ , respectively. Table 5 reports the  $ARL_1$  and  $ANOS_1$  for different values of  $B$ ,  $\sigma_M^2 = 1$  and  $m=5$ . Comparison between Tables 5 and 3 shows that when  $m=5$  is used, the effect of measurement errors is more than in the case of  $m=1$ . For example, considering the combination ( $\delta = 0.1, n_1 = 5, n_2 = 10$ ) when  $B=3$  and  $m=1$ ,  $ARL_1$  is equal to 122.3 while, when  $m=5$ , the  $ARL_1$  decreases to 113.8.

“Insert Table 1 about here”

“Insert Table 2 about here”

“Insert Table 3 about here”

“Insert Table 4 about here”

“Insert Table 5 about here”

#### 4.2. Linearly increasing measurement errors variance

In the case of linearly increasing measurement errors variance, we have  $\sigma_M^2 = C + D\mu_Y$ . We investigate the effect of parameter  $D$  on the performance of the VSS EWMA chart. Table 6 presents the  $ARL_1$  and  $ANOS_1$  performance of the VSS EWMA chart with linearly increasing variance for different values of  $D$ ,  $B=1$ ,  $C=0$  and  $m=1$ . From Table 6, increasing  $D$  significantly increases the  $ARL_1$  and  $ANOS_1$ , which indicates a negative effect of the linearly increasing variance on the performance of the VSS EWMA chart. Table 7 shows the  $ARL_1$  and  $ANOS_1$  based on different values of  $D$  for  $m=5$ . Comparison between Table 6 and 7 shows that when  $m=5$ , the effect of parameter  $D$  in increasing  $ARL_1$  and  $ANOS_1$  is less than in the case of  $m=1$ .

“Insert Table 6 about here”

“Insert Table 7 about here”

#### 5. Performance validation of the VSS EWMA

In this section, we provide a comparison between the VSS EWMA, FSR EWMA, adaptive Shewhart control charts (VSS  $\bar{X}$  and VSI  $\bar{X}$ ) for a wide range of mean shifts ( $\delta = [0, 0.9]$ ) with and without measurement errors in terms of the ARL criterion. For this aim, we considered the following parameters ( $\lambda = 0.2, n_1 = 3, n_2 = 7, n_0 = 5, B = 1$ ) and, an in-control ARL equal to 500 for all control charts. As a first case it is assumed that there is no measurement errors ( $\sigma_M^2 = 0$ ) and the results are shown in Figure 1. As illustrated in this figure, the VSS EWMA control chart outperforms the FSR EWMA, VSS and VSI  $\bar{X}$  charts for detecting shifts in the process mean. For example, when  $\delta = 0.2$ ,  $ARL_1$  for the VSS EWMA is equal to 41.28 while this value is equal to 52.48, 224.8 and 231.8 for the FSR EWMA, VSS  $\bar{X}$  and VSI  $\bar{X}$  control charts, respectively.

Figure 2 shows the ARL curves for the considered charts in presence of measurement errors ( $\sigma_M^2 = 1$ ). Actually, we compared the performance of proposed control chart with the FSR EWMA chart as well as adaptive Shewhart control charts (VSS  $\bar{X}$  and VSI  $\bar{X}$ ) by Hu et al. [28,29], respectively. The curves are plotted by considering  $m = 1$  and the parameters mentioned in the previous comparison. Figure 2 shows that the proposed VSS EWMA chart has a better performance than its FSR EWMA, VSS  $\bar{X}$  as well as VSI  $\bar{X}$  counterparts. For example, when  $\delta = 0.2$   $ARL_1$  for the VSS EWMA and FSR EWMA, VSS  $\bar{X}$  and VSI  $\bar{X}$  charts are equal to 83.49 and 101.9, 318.9 and 322.5, respectively. In Figure 3, we again considered the parameters used for Figure 2. However, the number of measurements has been increased

to  $m = 5$ . The results show an increasing sensitivity for all control charts since the out-of-control ARL values obtained for  $m = 5$  are smaller than the ones obtained for  $m = 1$ . Moreover, VSS EWMA control chart still outperforms the FSR EWMA, VSS  $\bar{X}$  and VSI  $\bar{X}$  control charts in this case as well.

“Insert Figure 1 about here”

“Insert Figure 2 about here”

“Insert Figure 3 about here”

## 6. An illustrative example

In this section we present an application of the VSS EWMA control chart with measurement errors. We use Costa and Castagliola [25] case corresponding to a production line of yogurts. The quality characteristic which is monitored is the weight of each yogurt cups. From a historical data, we know  $\mu_0 = 124.9$ ,  $\sigma_0^2 = 0.578$  and  $\sigma_M^2 = 0.058$ . We also assume  $\lambda = 0.2$ ,  $L = 2.962$ ,  $A = 0$ ,  $B = 1$ ,  $m = 2$ ,  $n_1 = 2$  and  $n_2 = 5$  for monitoring purpose based on Equations 8-11, we obtained  $UCL = 0.987$ ,  $LCL = -0.987$ ,  $UWL = 0.224$  and  $LWL = -0.224$ , respectively. Then, based on the mentioned parameters, we generated new data, computed the statistics  $\bar{X}_i$ ,  $U_i$  and  $Z_i$  until the VSS EWMA control chart signals the out-of-control status. The results are reported in Table 8 and graphically in Figure 4. As it is obvious from the results, the statistic  $Z_i$  is located below LCL, in sample 20, signaling an ‘out-of-control’ situation. Hence, the process needs corrective actions.

“Insert Table 8 about here”

“Insert Figure 4 about here”

## 7. Conclusion and suggestions for future research

In this paper, we investigated the effect of measurement errors on the VSS EWMA control chart using covariate error model. The performance of the proposed VSS EWMA chart is evaluated in terms of the ARL and ANOS performance measures using a Markov chain approach. The results show the negative effect of the measurement errors on the performance of the VSS EWMA control chart. Also, we showed that increasing multiple measurements as well as increasing the parameter  $B$  reduces the negative effect of the measurement errors on the performance of the VSS EWMA control chart. Also, we compared the performance of the proposed VSS EWMA control chart with FSR EWMA, VSS  $\bar{X}$  and VSI  $\bar{X}$  control charts in terms of ARL criterion in the presence of measurement errors, and the results showed that the

VSS EWMA chart outperforms the counterparts in detecting shifts in the process mean in presence of measurement errors.

Statistical and economic-statistical design of other adaptive procedures such as VSI, VSSI and VP for the EWMA control chart are suggested as future research. In this paper, the use of multiple measurements, which is the most common solution for reducing the effect of measurement errors was used. Evaluating other methods for reducing the effect of measurement errors on the performance the VSS EWMA control chart as well as for other adaptive EWMA control charts can be considered for future studies. For example, see Ghashghaei et al. [34] and Amiri et al. [35].

## References

- 1 Prabhu SS, Runger GC and Keats JB. “ $\bar{X}$  chart with adaptive sample sizes”, *International Journal of Production Research*, **31**(12), pp. 2895-2909 (1993).
- 2 Costa AFB. “ $\bar{X}$  charts with variable sampling size”, *Journal of Quality Technology*, **26**(3), 155–163 (1994).
- 3 Reynolds MR. “Variable sampling interval control charts with sampling at fixed times”. *IIE Transactions*, **28**(6), pp. 497–510 (1996).
- 4 Amiri F, Noghondarian K and Noorossana R. “Economic-statistical design of adaptive  $\bar{X}$  control chart: a Taguchi loss function approach”, *Scientia Iranica. Transaction E: Industrial Engineering*, **21**(3), p. 1096-1104 (2014).
- 5 Prabhu SS, Montgomery DC and Runger GC. “A combined adaptive sample size and sampling interval  $\bar{X}$  control scheme”, *Journal of Quality Technology*, **26**(3), pp. 164–176 (1994).
- 6 Costa A F B. “ $\bar{X}$  chart with variable sample size and sampling intervals”, *Journal of Quality Technology*, **29**(2), pp. 197–204 (1997).
- 7 Costa AFB. “ $\bar{X}$  charts with variable parameters”. *Journal of Quality Technology*, **31**(4), pp. 408-416 (1999).
- 8 Saccucci MS, Amin RW and Lucas JM. “Exponentially weighted moving average control schemes with variable sampling intervals”, *Communications in Statistics-Simulation and Computation*, **21**(3), pp. 627-657 (1992).
- 9 Reynolds MR and Arnold JC. “EWMA control charts with variable sample sizes and variable sampling intervals”, *IIE transactions*, **33**(6), pp. 511-530 (2001).
- 10 Castagliola P, Celano G and Fichera S. “Evaluation of the statistical performance of a variable sampling interval R EWMA control chart” *Quality Technology & Quantitative Management*, **3**(3), pp. 307-323 (2006).
- 11 Castagliola P, Celano G, Fichera S and Giuffrida F. “A variable sampling interval  $S^2$ -EWMA control chart for monitoring the process variance”, *International Journal of Technology Management*, **37**(1-2), pp. 125-146 (2006).
- 12 Kazemzadeh RB, Karbasian M and Babakhani MA. “An EWMA  $t$  chart with variable sampling intervals for monitoring the process mean”, *The International Journal of Advanced Manufacturing Technology*, **66**(1-4), pp. 125-139 (2013).
- 13 Tran K.P, Castagliola P, Nguyen TH and Cuzol A. “The efficiency of the VSI exponentially weighted moving average median control chart”, *Proceeding of 24<sup>th</sup> ISSAT International Conference on Reliability and Quality in Design*, Toronto, Canada (2018).
- 14 Ugaz W, Sánchez I and Alonso AM. “Adaptive EWMA control charts with time-varying smoothing parameter”, *The International Journal of Advanced Manufacturing Technology*, **93**(9-12), pp. 3847-3858 (2017).
- 15 Tang A, Castagliola P, Sun J, and Hu X. “Optimal design of the adaptive EWMA chart for the mean based on

- median run length and expected median run length.” *Quality Technology & Quantitative Management*, **16**(4), pp. 439-45, (2018).
- 16 Amiri A, Nedaie A and Alikhani M. “A new adaptive variable sample size approach in EWMA control chart”, *Communications in Statistics-Simulation and Computation*, **43**(4), pp. 804-812 (2014).
  - 17 Capizzi G and Masarotto G. “An adaptive exponentially weighted moving average control chart”, *Technometrics*, **45**(3), pp. 199-207 (2003).
  - 18 Ong LV, Teoh WL, Khoo MBC, Chong ZL and Yeong WC. “A study on the variable sampling interval EWMA chart when the process parameters are unknown”, *In Journal of Physics: Conference Series*, **1132**(1), pp.012084 IOP Publishing (2018).
  - 19 Tran KP, Castagliola P and Celano G. “Monitoring the ratio of two normal variables using EWMA type control charts”, *Quality and Reliability Engineering International*, **32**(2), pp. 1853–1869 (2015).
  - 20 Chong, ZL, Castagliola P, Khoo MB, Ong LV and Teoh WL. “The Variable Sampling Interval EWMA  $\bar{X}$  Chart with Estimated Process Parameters”. *Journal of Testing and Evaluation*, published online: <https://doi.org/10.1520/JTE20180058>
  - 21 Haq A, Khoo MBC. “An adaptive multivariate EWMA chart”, *Computers and Industrial Engineering*, **131**, pp. 459, (2019)
  - 22 Bennett CA. “Effect of measurement error on chemical process control”, *Industrial Quality Control*, **10**(4), pp. 17–20 (1954).
  - 23 Linna KW and Woodall WH. “Effect of measurement error on Shewhart control charts”, *Journal of Quality Technology*, **33**(2), pp. 213-222 (2001).
  - 24 Kanazuka T. “The effect of measurement error on the power of  $\bar{X}$ -R charts”, *Journal of Quality Technology*, **18**(2), pp. 91-95 (1986).
  - 25 Costa AF and Castagliola P. “Effect of measurement error and autocorrelation on the  $\bar{X}$  chart”, *Journal of Applied Statistics*, **38**(4), pp. 661-673 (2011).
  - 26 Hu X, Castagliola P, Sun J and Khoo M.B. “The effect of measurement errors on the synthetic  $\bar{X}$  chart”, *Quality and Reliability Engineering International*, **31**(8), pp. 1769-1778 (2014)
  - 27 Linna KW, Woodall WH and Busby KL. “The performance of multivariate control charts in the presence of measurement error”, *Journal of Quality Technology*, **33**(3), pp. 349-355 (2018).
  - 28 Hu X, Castagliola P, Sun J and Khoo M.B. “The performance of variable sample size  $\bar{X}$  chart with measurement errors”, *Quality and Reliability Engineering International*; **32**(3), pp. 969-983 (2015).
  - 29 Hu X, Castagliola P, Sun J and Khoo M.B. “Effect of measurement errors on the VSI  $\bar{X}$  chart”, *European J. Industrial Engineering*, **10**(2), pp. 224–242 (2016).
  - 30 Sabahno H and Amiri A. “The effect measurement errors on the performance of variable sample size and sampling interval  $\bar{X}$  control chart”, *International Journal of Engineering-Transactions A: Basics*, **30**(7), pp. 995-1004 (2017).
  - 31 Sabahno H, Amiri A and Castagliola P. “Performance of the variable parameters  $\bar{X}$  control chart in presence of measurement errors”, *Journal of Testing and Evaluation*, **47**(1), pp. 480-497 (2019).
  - 32 Sabahno H, Amiri A and Castagliola P. “Optimal performance of the variable sample sizes Hotelling’s  $T^2$  control chart in the presence of measurement errors”, *Quality Technology & Quantitative Management*, **16**(5), pp. 588-612 (2019).
  - 33 Sabahno H, Amiri A and Castagliola P. “Evaluating the effect of measurement errors on the performance of the variable sampling intervals Hotelling’s  $T^2$  control charts”. *Quality and Reliability Engineering International*, **34**(8), pp. 1785-1799 (2018).
  - 34 Ghashghaei R, Bashiri M, Amiri A and Maleki MR. “Effect of measurement error on joint monitoring of process mean and variability under ranked set sampling”, *Quality and Reliability Engineering International*, **32**(8), pp. 3035-3050 (2016).
  - 35 Amiri A, Ghashghaei R and Maleki MR. “On the effect of measurement errors in simultaneous monitoring of

- mean vector and covariance matrix of multivariate processes”. *Transactions of the Institute of Measurement and Control*, **40**(1), pp. 318-330 (2018).
- 36 Maleki MR, Amiri A and Ghashghaei R. “Simultaneous monitoring of multivariate process mean and variability in the presence of measurement error with linearly increasing variance under additive covariate model”, *International Journal of Engineering-Transactions A: Basics*, **29**(4), pp. 471-480 (2016).
- 37 Nguyen, HD, Nguyen QT, Tran KP, and Ho DP.” On the performance of VSI Shewhart control chart for monitoring the coefficient of variation in the presence of measurement errors”. Published online in *The International Journal of Advanced Manufacturing Technology*, (2019). doi: 10.1007/s00170-019-03352-7.
- 38 Tran K P, Heuchenne C and Balakrishnan, N. “On the performance of coefficient of variation charts in the presence of measurement errors”. *Quality and Reliability Engineering International*, **35**(1), 329-350, (2019).
- 39 Stemann D and Weihs C. “The EWMA-X-S-control chart and its performance in the case of precise and imprecise data”, *Statistical Papers*, **42**(2), pp. 207–223 (2001).
- 40 Maravelakis P, Panaretos J and Psarakis S. “EWMA chart and measurement error”, *Journal of Applied Statistics* ; **31**(4):445-455 (2004).
- 41 Yang S F, Ho HW and Rahim MA. “Effects of measurement error on controlling two dependent process steps”, *Economic Quality Control*, **22**(1), pp. 127–139 (2007).
- 42 Haq A, Brown J, Moltchanova E and Al-Omari AI. “Effect of measurement error on exponentially weighted moving average control charts under ranked set sampling schemes”, *Journal of Statistical Computation and Simulation*, **85**(6), pp. 1224-1246 (2014).
- 43 Abbasi SA. “On the performance of EWMA chart in the presence of two-component measurement error”, *Quality Engineering*, **22**(3), pp. 199-213 (2010).
- 44 Abbasi SA. “Exponentially weighted moving average chart and two-component measurement error”, *Quality and Reliability Engineering International*, **32**(2), pp. 499-504 (2015).
- 45 Saghaei A, Fatemi Ghomi SMT and Jaber S. “Economic design of exponentially weighted moving average control chart based on measurement error using genetic algorithm”, *Quality and Reliability Engineering International*, **30**(8), pp. 1153-1163(2014).
- 46 Cheng X.B and Wang FK. “The performance of EWMA median and CUSUM median control charts for a normal process with measurement errors”, *Quality and Reliability Engineering International*, **34**(2), pp. 203-213 (2017).
- 47 Tang A, Castagliola P, Sun J and Hu X. “The effect of measurement errors on the adaptive EWMA chart”, *Quality and Reliability Engineering International*, **34**(4), pp. 609-630 (2018).
- 48 Maleki M.R, Amiri A and Castagliola P. “Measurement errors in statistical process monitoring: a literature review”, *Computers & Industrial Engineering*, **103**, pp. 316-329 (2016).
- 49 Lucas JM and Saccucci MS. “Exponentially weighted moving average control schemes: properties and enhancements”, *Technometrics*, **32**, pp. 1–12 (1990).
- 50 Montgomery DC and Runger GC. “Gauge capability and designed experiments. Part I: basic methods”. *Quality Engineering*, **6**(1), pp. 115–135 (1994).

## Appendix A: The proof of $Z_i$ distribution

Based on Equation 4 and Equation 5 we have,  $\bar{X}_i \sim N\left(A + B(\mu_0 + \delta\sigma_0), \frac{1}{n}(B^2\sigma_0^2 + \sigma_M^2)\right)$ . Let  $U_i$  be the standardized version of the statistic  $\bar{X}_i$  :

$$U_i = \frac{\bar{X}_i - \mu_{0_{\bar{X}_i}}}{\sigma_{\bar{X}_i}}, \quad (\text{A.1})$$

Since  $\bar{X}_i$  follows normal distribution with mean  $\mu_{0_{\bar{X}_i}}$  and variance  $\sigma_{\bar{X}_i}^2$ , Hence, the  $U_i \sim N(0,1)$ .  $U_i$  is used in the EWMA statistic, as follows:

$$Z_i = \lambda U_i + (1-\lambda)Z_{i-1}, \quad (\text{A.2})$$

The expansion of  $Z_i$  is provided as Equations A.3-A.5:

$$Z_i = \lambda U_i + (1-\lambda)(\lambda U_{i-1} + (1-\lambda)Z_{i-2}), \quad (\text{A.3})$$

where  $Z_{i-2} = \lambda U_{i-2} + (1-\lambda)Z_{i-3}$ , and in the same way

$$Z_i = \lambda U_i + (1-\lambda)(\lambda U_{i-1} + (1-\lambda)(\lambda U_{i-2} + (1-\lambda)Z_{i-3})) \quad (\text{A.4})$$

Finally, the  $Z_i$  expanded as below:

$$Z_i = \sum_{e=0}^{i-1} \lambda (1-\lambda)^e U_{i-e} + (1-\lambda)^i Z_0 \quad (\text{A.5})$$

where  $Z_0 = \mu_{U_i}$ .

Also, the expected value and variance of  $Z_i$  are obtained as follows:

$$\begin{aligned} E(Z_i) &= E\left(\sum_{e=0}^{i-1} \lambda (1-\lambda)^e U_{i-e} + (1-\lambda)^i \mu_{U_i}\right) \\ &= \mu_{U_i} \left(1 - (1-\lambda)^i\right) + \mu_{U_i} (1-\lambda)^i = \mu_{U_i}. \end{aligned} \quad (\text{A.6})$$

$$\begin{aligned} \text{Var}(Z_i) &= \text{Var}\left(\sum_{e=0}^{i-1} \lambda (1-\lambda)^e U_{i-e} + (1-\lambda)^i \mu_{U_i}\right) \\ &= \sigma_{U_i}^2 \sum_{e=0}^{i-1} \lambda^2 (1-\lambda)^{2e} + 0 = \sigma_{U_i}^2 \frac{\lambda^2 (1 - (1-\lambda)^{2i})}{1 - (1-\lambda)^2} \\ &= \sigma_{U_i}^2 \frac{\lambda (1 - (1-\lambda)^{2i})}{2 - \lambda}. \end{aligned} \quad (\text{A.7})$$

Thus  $Z_i \sim N\left(\mu_{U_i}, \sigma_{U_i}^2 \frac{\lambda (1 - (1-\lambda)^{2i})}{2 - \lambda}\right)$  and for asymptotic variance, we have  $\sigma_{U_i}^2 \frac{\lambda}{2 - \lambda}$ . Since

$$U_i \sim N(0,1), \text{ then } Z_i \sim N\left(0, \frac{\lambda}{2 - \lambda}\right).$$



### Tables List:

- Table 1: (ARL<sub>1</sub>,ANOS<sub>1</sub>) when ARL<sub>0</sub>=500,  $k=2.962$ ,  $\lambda=0.2$ ,  $B=1$ ,  $m=1$   
Table 2: (ARL<sub>1</sub>,ANOS<sub>1</sub>) when ARL<sub>0</sub>=500,  $k=2.962$ ,  $\lambda=0.2$ ,  $B=1$ ,  $\sigma_M^2 = 1$   
Table 3: (ARL<sub>1</sub>,ANOS<sub>1</sub>) when ARL<sub>0</sub>=500,  $k=2.962$ ,  $\lambda=0.2$ ,  $m=1$ ,  $\sigma_M^2 = 1$   
Table 4: (ARL<sub>1</sub>,ANOS<sub>1</sub>) when ARL<sub>0</sub>=500,  $k=2.962$ ,  $\lambda=0.2$ ,  $B=1$ ,  $m=5$   
Table 5: (ARL<sub>1</sub>,ANOS<sub>1</sub>) when ARL<sub>0</sub>=500,  $k=2.962$ ,  $\lambda=0.2$ ,  $m=5$ ,  $\sigma_M^2 = 1$   
Table 6: (ARL<sub>1</sub>,ANOS<sub>1</sub>) when ARL<sub>0</sub>=500,  $k=2.962$ ,  $\lambda=0.2$ ,  $m=1$ ,  $B=1$   
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Table 8: The sample size  $n$  and statistics  $\bar{X}_i$ ,  $U_i$ ,  $Z_i$  and process status for simulated data generated based on the parameters reported by Costa and Castagliola [25]

### Figures List:

- Figure 1: The performance comparison of VSS EWMA control chart with FSR EWMA, VSS  $\bar{X}$  and VSI  $\bar{X}$  control charts with no measurement errors  
Figure 2: The performance comparison of VSS EWMA control chart with FSR EWMA, VSS  $\bar{X}$  and VSI  $\bar{X}$  control charts when  $\sigma_M^2 = 1$  and  $m = 1$   
Figure 3: The performance comparison of VSS EWMA control chart with FSR EWMA, VSS  $\bar{X}$  and VSI  $\bar{X}$  control charts when  $\sigma_M^2 = 1$  and  $m = 5$   
Figure 4: VSS EWMA control chart for the real case example

<b>Table 1</b>				
$n_I, n_2$	$n_I=1, n_2=6$	$n_I=5, n_2=10$	$n_I=3, n_2=7$	$n_I=3, n_2=10$
$W$	0.672	0.672	0.672	0.672
$\text{ANOS}_0$	1753.6	3751.7	2501.8	3254.2
$\sigma_M^2 = 0$				
$\delta = 0.1$	(184.8, 691.6)	(111.6, 892.6)	(152.4, 805.27)	(118.07, 838.01)
$\delta = 0.5$	(9.54, 40.07)	(5.68, 47.55)	(7.47, 43.00)	(6.16, 45.96)
$\delta = 1$	(4.13, 15.16)	(2.59, 20.31)	(3.18, 17.05)	(2.86, 19.47)
$\delta = 2$	(2.25, 7.51)	(1.67, 11.78)	(1.94, 9.56)	(1.93, 12.30)
$\sigma_M^2 = 0.3$				
$\delta = 0.1$	(219.1, 809.4)	(139.2, 1099.7)	(184.5, 964.4)	(146.7, 1024.5)
$\delta = 0.5$	(11.65, 50.06)	(6.82, 57.73)	(9.14, 53.19)	(7.36, 55.91)
$\delta = 1$	(4.74, 17.79)	(2.95, 23.42)	(3.66, 19.88)	(3.25, 22.51)
$\delta = 2$	(7.51, 8.26)	(1.84, 13.46)	(2.03, 10.12)	(2.00, 12.82)
$\sigma_M^2 = 0.7$				
$\delta = 0.1$	(254.99, 931.39)	(170.7, 1335.2)	(219.3, 1136.01)	(179.2, 1234.02)
$\delta = 0.5$	(14.57, 63.67)	(8.35, 71.37)	(11.42, 67.03)	(8.95, 69.24)
$\delta = 1$	(5.49, 21.18)	(3.39, 27.31)	(4.26, 23.47)	(3.73, 26.27)
$\delta = 2$	(2.75, 9.42)	(1.95, 14.53)	(2.18, 11.02)	(2.08, 13.41)
$\sigma_M^2 = 1$				
$\delta = 0.1$	(276.43, 1004.11)	(191.16, 1486.8)	(240.8, 1241.7)	(200.0, 1367.5)
$\delta = 0.5$	(16.85, 74.06)	(9.51, 81.72)	(13.17, 77.63)	(10.16, 79.35)
$\delta = 1$	(6.04, 23.67)	(3.71, 30.10)	(4.69, 26.08)	(4.07, 28.97)
$\delta = 2$	(2.96, 10.28)	(2.02, 15.14)	( 2.31, 11.81)	(2.16, 14.02)

<b>Table 2</b>				
$n_1, n_2$	$n_1=1, n_2=6$	$n_1=5, n_2=10$	$n_1=3, n_2=7$	$n_1=3, n_2=10$
$W$	0.672	0.672	0.672	0.672
$\text{ANOS}_0$	1753.6	3751.7	2501.8	3254.2
$m = 1$				
$\delta = 0.1$	(276.43, 1004.11)	(191.16, 1486.8)	(240.8, 1241.7)	(200.0, 1367.5)
$\delta = 0.5$	(16.85, 74.06)	(9.51, 81.72)	(13.17, 77.63)	(10.16, 79.35)
$\delta = 1$	(6.04, 23.67)	(3.71, 30.10)	(4.69, 26.08)	(4.07, 28.97)
$\delta = 2$	(2.96, 10.28)	(2.02, 15.14)	( 2.31, 11.81)	(2.16, 14.02)
$m = 2$				
$\delta = 0.1$	(238.2, 874.5)	(155.6, 1222.7)	(202.9, 1055.19)	(163.7, 1134.2)
$\delta = 0.5$	(13.10, 56.82)	(7.58, 64.53)	(10.27, 60.07)	(8.15, 62.56)
$\delta = 1$	(5.12, 19.50)	(3.17, 25.40)	(3.96, 21.69)	(3.498, 24.423)
$\delta = 2$	(2.60, 8.84)	(1.91, 14.07)	(2.10, 10.54)	(2.03, 13.09)
$m = 3$				
$\delta = 0.1$	(222.5, 820.9)	(142.06, 1121.06)	(187.7, 980.3)	(149.6, 1043.5)
$\delta = 0.5$	(11.89, 51.18)	(6.95, 58.86)	(9.33, 54.33)	(7.49, 57.02)
$\delta = 1$	(4.80, 18.08)	(2.99, 23.76)	(3.71, 20.19)	(3.29, 22.83)
$\delta = 2$	(2.48, 8.35)	(1.85, 13.58)	(2.043, 10.19)	(2.00, 12.86)
$m = 4$				
$\delta = 0.1$	(213.9, 791.6)	(134.8, 1067.2)	(179.6, 939.9)	(142.2, 995.37)
$\delta = 0.5$	(11.30, 48.38)	(6.63, 56.04)	(8.86, 51.48)	(7.16, 54.25)
$\delta = 1$	(4.64, 17.36)	(2.89, 22.9)	(3.58, 19.42)	(3.19, 22.02)
$\delta = 2$	(2.42, 8.12)	(1.82, 13.26)	(2.01, 10.03)	(1.99, 12.75)
$m = 5$				
$\delta = 0.1$	(208.5, 773.1)	(130.4, 1033.9)	(174.50, 914.6)	(137.6, 965.4)
$\delta = 0.5$	(10.94, 46.71)	(6.44, 54.3)	(8.58, 49.78)	(6.96, 52.60)

$\delta = 1$	(4.54, 16.93)	(2.83, 22.41)	(3.5, 18.95)	(3.12, 21.52)
$\delta = 2$	(2.38, 7.99)	(1.80, 13.04)	(2.00, 9.93)	(1.98, 12.68)

<b>Table 3</b>				
$n_1, n_2$	$n_1=1, n_2=6$	$n_1=5, n_2=10$	$n_1=3, n_2=7$	$n_1=3, n_2=10$
$W$	0.672	0.672	0.672	0.672
$\text{ANOS}_0$	1753.6	3751.7	2501.8	3254.2
$B = 1$				
$\delta = 0.1$	(276.43, 1004.11)	(191.16, 1486.8)	(240.8, 1241.7)	(200.0, 1367.5)
$\delta = 0.5$	(16.85, 74.06)	(9.51, 81.72)	(13.17, 77.63)	(10.16, 79.35)
$\delta = 1$	(6.04, 23.67)	(3.71, 30.10)	(4.69, 26.08)	(4.07, 28.97)
$\delta = 2$	(2.96, 10.28)	(2.02, 15.14)	(2.31, 11.81)	(2.16, 14.02)
$B = 2$				
$\delta = 0.1$	(213.9, 791.6)	(134.8, 1067.2)	(179.6, 939.9)	(142.2, 995.37)
$\delta = 0.5$	(11.30, 48.38)	(6.63, 56.04)	(8.86, 51.48)	(7.16, 54.25)
$\delta = 1$	(4.64, 17.36)	(2.89, 22.92)	(3.58, 19.42)	(3.19, 22.02)
$\delta = 2$	(2.42, 8.12)	(1.82, 13.26)	(2.01, 10.03)	(1.99, 12.75)
$B = 3$				
$\delta = 0.1$	(198.4, 738.5)	(122.3, 972.7)	(165.04, 867.7)	(129.1, 910.4)
$\delta = 0.5$	(10.31, 43.75)	(6.10, 51.32)	(8.09, 46.76)	(6.60, 49.65)
$\delta = 1$	(4.36, 16.15)	(2.73, 21.49)	(3.36, 18.12)	(3.01, 20.62)
$\delta = 2$	(2.32, 7.76)	(1.75, 12.56)	(1.97, 9.77)	(1.96, 12.53)
$B = 4$				
$\delta = 0.1$	(192.6, 718.5)	(117.7, 938.2)	(159.6, 841.00)	(124.3, 879.2)
$\delta = 0.5$	(9.97, 42.14)	(5.92, 49.67)	(7.82, 45.11)	(6.41, 48.04)
$\delta = 1$	(4.26, 15.72)	(2.67, 20.98)	(3.28, 17.66)	(2.95, 20.12)
$\delta = 2$	(2.29, 7.65)	(1.72, 12.25)	(1.96, 9.68)	(1.95, 12.44)

<b>Table 4</b>				
$n_1, n_2$	$n_1=1, n_2=6$	$n_1=5, n_2=10$	$n_1=3, n_2=7$	$n_1=3, n_2=10$
$W$	0.672	0.672	0.672	0.672
$\text{ANOS}_0$	1753.6	3751.7	2501.8	3254.2
$\sigma_M^2 = 0$				
$\delta = 0.1$	(184.8, 691.6)	(111.6, 892.6)	(152.4, 805.27)	(118.07, 838.01)
$\delta = 0.5$	(9.54, 40.07)	(5.68, 47.55)	(7.47, 43.00)	(6.16, 45.96)
$\delta = 1$	(4.13, 15.16)	(2.59, 20.31)	(3.18, 17.05)	(2.86, 19.47)
$\delta = 2$	(2.25, 7.51)	(1.67, 11.78)	(1.94, 9.56)	(1.93, 12.30)
$\sigma_M^2 = 0.3$				
$\delta = 0.1$	(192.3, 717.4)	(117.4, 936.4)	(159.3, 839.6)	(124.1, 877.6)
$\delta = 0.5$	( 9.96, 42.05)	(5.91, 49.59)	(7.81, 45.03)	(6.40, 47.95)
$\delta = 1$	(4.26, 15.69)	(2.67, 20.95)	(3.28, 17.63)	(2.94, 20.10)
$\delta = 2$	(2.29, 7.64)	(1.72, 12.23)	(1.96, 9.68)	(1.95, 12.44)
$\sigma_M^2 = 0.7$				
$\delta = 0.1$	(201.8, 750.02)	(124.9, 992.9)	(168.1, 883.3)	(131.9, 928.5)
$\delta = 0.5$	(10.52, 44.71)	(6.21, 52.30)	(8.25, 47.74)	(6.72, 50.61)
$\delta = 1$	(4.42, 16.40)	(2.76, 21.79)	(3.41, 18.39)	(3.05, 20.92)
$\delta = 2$	(2.34, 7.83)	(1.77, 12.73)	(1.98, 9.82)	(1.97, 12.58)
$\sigma_M^2 = 1$				
$\delta = 0.1$	(208.5, 773.1)	(130.4, 1033.9)	(174.50, 914.6)	(137.6, 965.4)
$\delta = 0.5$	(10.94, 46.71)	(6.44, 54.3)	(8.58, 49.78)	(6.96, 52.60)
$\delta = 1$	(4.54, 16.93)	(2.83, 22.41)	(3.5, 18.95)	(3.12, 21.52)
$\delta = 2$	(2.38, 7.99)	(1.80, 13.04)	(2.00, 9.93)	(1.98, 12.68)

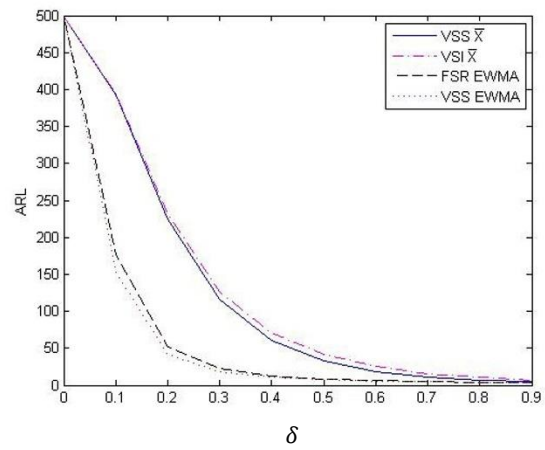
<b>Table 5</b>				
$n_1, n_2$	$n_1=1, n_2=6$	$n_1=5, n_2=10$	$n_1=3, n_2=7$	$n_1=3, n_2=10$
$W$	0.672	0.672	0.672	0.672
ANOS <sub>0</sub>	1753.6	3751.7	2501.8	3254.2
$B = 1$				
$\delta = 0.1$	(208.5, 773.1)	(130.4, 1033.9)	(174.5, 914.6)	(137.6, 965.4)
$\delta = 0.5$	(10.94, 46.71)	(6.44, 54.34)	(8.58, 49.78)	(6.96, 52.60)
$\delta = 1$	(4.54, 16.93)	(2.83, 22.41)	(3.50, 18.95)	(3.12, 21.52)
$\delta = 2$	(2.38, 7.99)	(1.8, 13.04)	(2.00, 9.93)	(1.98, 12.68)
$B = 2$				
$\delta = 0.1$	(191.09, 713.2)	(116.5, 929.2)	(158.2, 833.9)	(123.1, 871.1)
$\delta = 0.5$	(9.89, 41.72)	(5.87, 49.25)	(7.75, 44.69)	(6.36, 47.62)
$\delta = 1$	(4.23, 15.61)	(2.65, 20.84)	(3.26, 17.54)	(2.93, 20.00)
$\delta = 2$	(2.28, 7.62)	(1.71, 12.16)	(1.95, 9.66)	(1.950, 12.42)
$B = 3$				
$\delta = 0.1$	(187.6, 701.3)	(113.8, 909.03)	(155.04, 818.14)	(120.3, 852.8)
$\delta = 0.5$	(9.69, 40.80)	(5.76, 48.30)	(7.60, 43.75)	(6.25, 46.70)
$\delta = 1$	(4.181, 15.36)	(2.62, 20.55)	(3.22, 17.27)	(2.89, 19.70)
$\delta = 2$	(2.269, 7.56)	(1.69, 11.96)	(1.949, 9.60)	(1.943, 12.36)
$B = 4$				
$\delta = 0.1$	(186.4, 697.1)	(112.9, 901.9)	(153.9, 812.54)	(119.3, 846.3)
$\delta = 0.5$	(9.62, 40.48)	(5.73, 47.97)	(7.54, 43.42)	(6.21, 46.38)
$\delta = 1$	(4.161, 15.27)	(2.61, 20.45)	(3.20, 17.17)	(2.88, 19.60)
$\delta = 2$	(2.263, 7.542)	(1.68, 11.88)	(1.946, 9.58)	(1.940, 12.33)

<b>Table 6</b>				
$n_I, n_2$	$n_I=1, n_2=6$	$n_I=5, n_2=10$	$n_I=3, n_2=7$	$n_I=3, n_2=10$
$W$	0.672	0.672	0.672	0.672
$\text{ANOS}_0$	1753.6	3751.7	2501.8	3254.2
$D = 1$				
$\delta = 0.1$	(438.8, 1550.4)	(393.5, 2976.1)	(422.2, 2126.8)	(399.8, 2629.4)
$\delta = 0.5$	(98.12, 388.2)	(52.78, 440.02)	(77.02, 424.6)	(55.98, 421.5)
$\delta = 1$	(24.91, 109.4)	(13.51, 117.1)	(19.33, 114.04)	(14.36, 113.8)
$\delta = 2$	(8.23, 33.91)	(4.96, 41.13)	(6.44, 36.68)	(5.40, 39.70)
$D = 2$				
$\delta = 0.1$	(465.9, 1641.3)	(438.2, 3303.6)	(456.1, 2291.1)	(442.2, 2895.8)
$\delta = 0.5$	(168.6, 636.05)	(99.63, 801.3)	(137.8, 732.3)	(105.4, 755.01)
$\delta = 1$	(49.06, 206.8)	(25.57, 220.2)	(37.81, 217.8)	(27.08, 213.4)
$\delta = 2$	(13.56, 58.95)	(7.82, 66.67)	(10.63, 62.24)	(8.40, 64.64)
$D = 3$				
$\delta = 0.1$	(476.3, 1676.05)	(456.4, 3436.7)	(469.3, 2355.3)	(459.3, 3003.02)
$\delta = 0.5$	(219.1, 809.4)	(139.2, 1099.7)	(184.5, 964.4)	(146.7, 1024.5)
$\delta = 1$	(73.29, 298.04)	(38.49, 326.1)	(56.87, 319.7)	(40.80, 314.3)
$\delta = 2$	(19.30, 85.01)	(10.74, 92.63)	(15.05, 88.84)	(11.45, 90.00)
$D = 4$				
$\delta = 0.1$	(481.8, 1694.3)	(466.3, 3508.8)	(476.3, 2389.5)	(468.6, 3060.8)
$\delta = 0.5$	(256.5, 936.6)	(172.2, 1345.9)	(220.9, 1143.6)	(180.7, 1243.4)
$\delta = 1$	(96.13, 381.0)	(51.60, 430.6)	(75.38, 416.2)	(54.72, 412.7)
$\delta = 2$	(25.43, 111.6)	(13.77, 119.4)	(19.73, 116.3)	(14.63, 116.09)

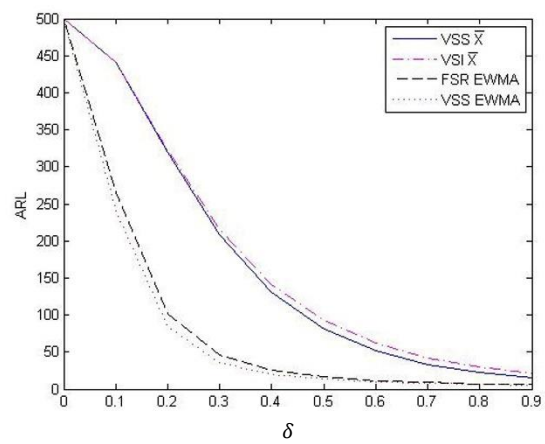
<b>Table 7</b>				
$n_I, n_2$	$n_I=1, n_2=6$	$n_I=5, n_2=10$	$n_I=3, n_2=7$	$n_I=3, n_2=10$
$W$	0.672	0.672	0.672	0.672
$\text{ANOS}_0$	1753.6	3751.7	2501.8	3254.2
$D = 1$				
$\delta = 0.1$	(327.7, 1177.5)	(245.1, 1886.3)	(294.6, 1504.8)	(254.6, 1714.6)
$\delta = 0.5$	(25.75, 113.01)	(13.93, 120.7)	(19.97, 117.75)	(14.79, 117.41)
$\delta = 1$	(8.15, 33.50)	(4.91, 40.70)	(6.37, 36.26)	(5.34, 39.28)
$\delta = 2$	(3.81, 13.79)	(2.41, 18.68)	(2.93, 15.57)	(2.65, 17.85)
$D = 2$				
$\delta = 0.1$	(381.6, 1358.7)	(310.9, 2370.7)	(354.4, 1796.9)	(319.8, 2126.8)
$\delta = 0.5$	(44.14, 187.6)	(23.07, 199.1)	(34.02, 197.00)	(24.42, 193.2)
$\delta = 1$	(12.01, 51.75)	(7.01, 59.43)	(9.42, 54.90)	(7.55, 57.57)
$\delta = 2$	(5.02, 19.07)	(3.12, 24.91)	(3.89, 21.24)	(3.43, 23.95)
$D = 3$				
$\delta = 0.1$	(409.7, 1453.2)	(349.8, 2655.7)	(387.3, 1956.7)	(357.7, 2365.1)
$\delta = 0.5$	(62.84, 259.2)	(32.80, 279.9)	(48.58, 275.8)	(34.75, 270.4)
$\delta = 1$	(16.08, 70.58)	(9.12, 78.26)	(12.58, 74.08)	(9.76, 75.97)
$\delta = 2$	(6.13, 24.08)	(3.76, 30.55)	(4.76, 26.51)	(4.12, 29.41)
$D = 4$				
$\delta = 0.1$	(427.09, 1511.2)	(375.3, 2843.3)	(408.02, 2057.4)	(382.4, 2520.3)
$\delta = 0.5$	(80.95, 326.1)	(42.79, 360.6)	(63.02, 352.02)	(45.36, 346.9)
$\delta = 1$	(20.39, 89.87)	(11.28, 97.48)	(15.89, 93.83)	(12.02, 94.73)
$\delta = 2$	(7.19, 29.00)	(4.37, 35.91)	(5.61, 31.63)	(4.77, 34.62)



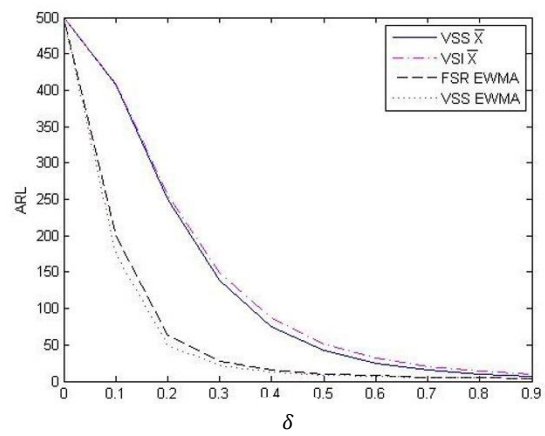
<b>Table 8</b>					
Time( $i$ )	$n_i$	$\bar{X}_i$	$U_i$	$Z_i$	Status
1	2	124.86	-1.12	-0.12	In-control
2	2	124.69	-1.43	-0.38	In-control
3	5	125.49	0.02	-0.30	In-control
4	5	125.11	-1.06	-0.45	In-control
5	5	125.58	0.28	-0.30	In-control
6	5	125.53	0.14	-0.21	In-control
7	2	125.59	0.19	-0.13	In-control
8	2	125.69	0.38	-0.03	In-control
9	2	124.88	-1.08	-0.24	In-control
10	5	125.45	-0.08	-0.21	In-control
11	2	125.54	0.10	-0.14	In-control
12	2	124.79	-1.24	-0.36	In-control
13	5	125.82	0.97	-0.09	In-control
14	2	126.68	2.16	0.35	In-control
15	5	125.64	0.45	0.37	In-control
16	5	125.93	1.28	0.55	In-control
17	5	125.99	1.44	0.73	In-control
18	5	126.15	1.91	0.97	In-control
19	5	125.61	0.37	0.85	In-control
20	5	126.06	1.64	1.01	Out-of-control



**Figure 1**



**Figure 2**



**Figure 3**

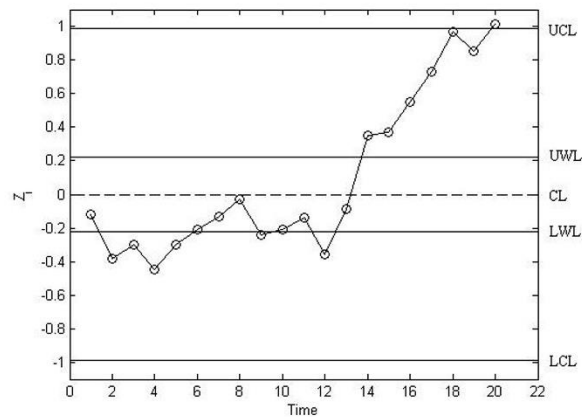


Figure 4

## Biographies

**Zeynab Hassani** is a M.S. student in Industrial Engineering at Shahed University in Iran. She received her BS degree in Torbat Heydarieh University. Her research interests include statistical process monitoring, measurement errors and adaptive control charts. In addition, she is interested in the location emergency equipment and stochastic problems.

**Amirhossein Amiri** is an Associate Professor at Shahed University in Iran. He holds a BS, MS, and PhD in Industrial Engineering from Khajeh Nasir University of Technology, Iran University of Science and Technology, and Tarbiat Modares University in Iran, respectively. He is now Vice Chancellor of Education in Faculty of Engineering at Shahed University in Iran and a member of the Iranian Statistical Association. His research interests are statistical process monitoring, profile monitoring, and change point estimation. He has published many papers in the area of statistical process control in high quality international journals such as *Quality and Reliability Engineering International*, *Communications in Statistics*, *Computers & Industrial Engineering*, *Journal of Statistical Computation and Simulation*, *Soft Computing* and so on. He has also published a book with John Wiley and Sons in 2011 entitled *Statistical Analysis of Profile Monitoring*.

**Philippe Castagliola** graduated (PhD 1991) from the Université de Technologie de Compiègne, France (UTC). He is currently a professor at the Université de Nantes, Nantes, France, and he is also a member of the Laboratoire des Sciences du Numérique de Nantes (LS2N), UMR CNRS 6004. He is an associate editor for the *Journal of Quality Technology*, *Communications in Statistics (LSTA, LSSP, UCAS)*, *Quality Technology and Quantitative Management* and for the *International Journal of Reliability, Quality and Safety Engineering*. His research activity includes developments of new statistical process monitoring techniques.