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Benefits of stochastic arithmetic in high performance simulations and arbitrary precision codes

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DSA (Discrete Stochastic Arithmetic) [Vig04] enables one to estimate the rounding error propagation which occurs with floating-point arithmetic. This probabilistic method uses a random rounding mode: at each elementary operation, the result is rounded up or down with the same probability. Therefore, the computer's deterministic arithmetic is replaced by a stochastic arithmetic, where each arithmetic operation is performed several times before the next one is executed. With DSA, temporary results that are actually numerical noise can be discarded and iterative algorithms can be stopped in an optimal way that does not rely on any parameter. DSA can be used to control the accuracy of programs in half, single, double and/or quadruple precision via the CADNA library [CAD, EBFJ15, GJP⁺18, JSHH21], and also in arbitrary precision via the SAM library [SAM, GJWZ11].

We present an algorithm that takes benefits of DSA to efficiently and accurately compute polynomial roots, in particular multiple roots. Thanks to a stochastic version of the polynomial GCD algorithm and the polynomial Euclidean division, the proposed algorithm provides a low-degree polynomial with single roots. Then Newton method can be applied to get fast and accurate approximations of the roots in arbitrary precision [GJQMS21].

Thanks to DSA, the accuracy estimation and the detection of numerical instabilities can be performed in parallel codes on CPU and on GPU [EBFJ15, EBFJ16, ELB⁺18]. However its performance overhead may be large compared with the standard floating-point operations. We show that with perturbed data it is possible to use standard floating-point arithmetic instead of DSA for the purpose of numerical validation. For instance, for codes including matrix multiplications, we can directly utilize the matrix multiplication routine (GEMM) of level-3 BLAS that is performed with standard

floating-point arithmetic. Consequently, we can achieve a significant performance improvement by avoiding the performance overhead of DSA operations as well as by exploiting the speed of highly-optimized BLAS implementations [JGM⁺20].

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