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► To cite this version:

Leila Amgoud, Youssef Ouannani, Henri Prade. Arguing by analogy – Towards a formal view: A preliminary discussion. 1st Workshop on Similarity and Analogy-based Methods in AI (SAMAI 2012) co-located with ECAI 2012, Aug 2012, Montpellier, France. pp.65-68. <hal-03344977>

HAL Id: hal-03344977

<https://hal.science/hal-03344977v1>

Submitted on 15 Sep 2021

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Arguing by analogy – Towards a formal view

A preliminary discussion

Leila Amgoud and Youssef Ouannani and Henri Prade¹

Abstract. Although arguing by analogy is a current practice, little attention has been devoted to the study of this form of argumentation, especially at a formal level. This research note provides a preliminary study of what could be done in that direction. The discussion relies in particular on a logical modeling of analogical proportions (i.e., statements of the form “ A is to B as C is to D ”), in terms of similarity and dissimilarity, which has been recently proposed.

1 Introduction

The use of analogies plays an important role in many reasoning tasks, and analogical reasoning is usually recognized as a powerful, although heuristic, way to look for solutions by adaptation of existing ones, to jump to plausible conclusions, or to boost creativity in various areas [5] (including ancient [29] as well as modern mathematics [21]).

Analogical reasoning has been extensively studied in cognitive psychology [10, 8] and has been implemented in computational models [7, 17, 28]. The use of analogies in argumentation is often encountered, since arguments based on analogies are easy to grasp, are intuitively appealing, and may be especially convincing in public uses. However, analogical argumentation has been little studied if we except some works by philosophers [13, 2, 31] or linguists [20], or studies in legal reasoning [12, 14, 19]. Although argumentation has been extensively studied in artificial intelligence in the last two decades (see, e.g., [26]), analogical arguments have almost not been considered (an exception is [3, 4]).

A reason for this state of fact might be related to the difference of nature between deductive reasoning and analogical reasoning. Deductive reasoning relies on a well known formal apparatus developed for a long time, and provides conclusions that are as much reliable as the premises are. While deductive reasoning handles generic knowledge as well as pieces of factual evidence, analogical reasoning rather considers particular cases or situations, and is much more brittle since it only provides tentative conclusions. Moreover, the formal studies of analogical reasoning, even if there has been a number of proposals, remain less developed and somewhat scattered, and roughly speaking, analogical reasoning is often thought as something which is beyond logic. This probably contributes to make more difficult a formal theory of argumentation able to handle analogical arguments.

In this short note, we take advantage of the existence of a propositional logic modeling of analogical proportions, (i.e., statements of the form “ A is to B as C is to D ”) that has been recently developed, for offering some analysis of analogical arguments and for suggesting a formal view of their treatment. The rest of the paper is organized in two main parts. We first present an introductory overview of analogical reasoning based on analogical proportions, and then propose a preliminary study of analogical arguments.

2 A brief introduction to formal analogical reasoning

Analogy is currently understood as a weak form of similarity. For many authors, when comparing two objects S and T , one has to distinguish between identity, resemblance, and analogy. Resemblance is strictly weaker than identity. The fact that S resembles T if they belong to the same domain and have common features (which are easily observable), while S is analogous to T rather means that S and T may belong to different domains, and that S has the same relation with an object U as T has with another object V [11]. For instance, taking a famous example from Aristotle, “*Fish* (S) breathe through their *gills* (U), *mammal* (T) breathe through their *lungs* (V)”. This idea of viewing analogy as making a parallel between two system of objects, each related by similar relations, or even equations, has been investigated for a long time (see, e.g., [33]), and is at the core of the structure-mapping model [9, 7].

Case-based reasoning [1] also relies on the comparison between two pairs, which may be denoted $(Prob_1, Sol_1)$ and $(Prob_2, Sol_2)$, where $Prob_1$ and $Prob_2$ are the multiple-features descriptions of two problems, whose solutions Sol_1 and Sol_2 are respectively known and unknown. Case-based reasoning then amounts to suggest that Sol_2 may be obtained by adapting Sol_1 on the basis of the similarities and differences between $Prob_1$ and $Prob_2$. Indeed analogy is as much a matter of dissimilarity as a matter of similarity. This what has been also put in evidence in the logical definition of an analogical proportion, which is now recalled.

2.1 A propositional logic view of an analogical proportion

An analogical proportion is a statement of the form “ A is to B as C is to D ”, often denoted as $A : B :: C : D$, where A, B, C, D stand for objects, or situations. They may be described by means of sets of features. We assume here for simplicity that these features are binary. Thus, each of A, B, C , and D may be viewed as sets of properties (possessed by the corresponding items). Then, one may say that the

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analogical proportion $A : B :: C : D$ holds if we have both

$$A \cap \overline{B} = C \cap \overline{D} \text{ and } \overline{A} \cap B = \overline{C} \cap D,$$

where the overbar denotes set complementation. This means that “ A differs from B as C differs from D and B differs from A as D differs from C ”. This definition [18] refines previous proposals [15, 30]. A logical counterpart of this idea [18] amounts to state that for each binary feature viewed as a Boolean variable, the corresponding analogical proportion, denoted $a : b :: c : d$, is specified by the following pair of constraints:

$$((a \rightarrow b) \equiv (c \rightarrow d)) \wedge ((b \rightarrow a) \equiv (d \rightarrow c)) \text{ is true}$$

Thus, the proportion $a : b :: c : d$ can now be viewed as a Boolean formula. It can be checked that it takes the truth value 1 only for the 6 following 4-tuples (among 16 possible patterns) that are shown in Table 1. For all other valuations of (a, b, c, d) , the formula $a : b :: c : d$

Table 1. Truth table for analogical proportion

a	b	c	d
0	0	0	0
1	1	1	1
0	0	1	1
1	1	0	0
0	1	0	1
1	0	1	0

is false. It is easy to check that such a formal definition satisfies the properties that are usually assumed for an analogical proportion, namely:

- $a : b :: a : b$ and $a : a :: b : b$ hold, but $a : b :: b : a$ does not hold in general;
- if $a : b :: c : d$ holds then $a : c :: b : d$ should hold (central permutation);
- if $a : b :: c : d$ holds then $c : d :: a : b$ should hold (symmetry).

The reader is referred to [18] for a study of other properties with respect to connectives (e.g., $a : b :: \neg b : \neg a$ holds), to [24] for an overview of related notions and potential applications, and to [25] for illustrations and the relation to case-based reasoning.

2.2 Analogical-proportion based inference

An analogical equation $a : b :: c : x$ where the value of x is unknown, is solvable iff $(a \equiv b) \vee (a \equiv c)$ holds (e.g., $0 : 1 :: 1 : x$ has no solution). When it exists, the unique solution is given by $x = c \equiv (a \equiv b)$. This can be stated in terms of propositional logic entailments such as $a : b :: c : d, a, b, c \vdash d$, together with 5 other similar entailments (e.g., $a : b :: c : d, \neg a, b, \neg c \vdash d$) [24]. It provides a basis for the following inference pattern

$$\frac{\forall i \in [1, m], a_i : b_i :: c_i : d_i}{\forall j \in [m+1, n], a_j : b_j :: c_j : d_j}$$

In simple terms, this means that if the known part of the vector encoding D , say (d_1, \dots, d_m) is componentwise in analogical proportion with the corresponding parts (a_1, \dots, a_m) , (b_1, \dots, b_m) , (c_1, \dots, c_m) of the vectors encoding A , B , and C , then it should be also true for the unknown part (d_{m+1}, \dots, d_n) of the vector encoding D with respect to the corresponding parts of A , B , and

C . Thus, if (a_{m+1}, \dots, a_n) , (b_{m+1}, \dots, b_n) , (c_{m+1}, \dots, c_n) are known, then (d_{m+1}, \dots, d_n) can be obtained by equation solving. This extrapolation is exactly what analogical reasoning is about: we transfer the knowledge we have on the pair (A, B) to the pair (C, D) to predict the missing information about D , assuming a kind of regularity property. This is has been applied to classification problems, see [24] for references. This is obviously a form of reasoning that is not sound, but which may be useful for trying to guess unknown values.

A basic pattern considered when trying to formalize analogical reasoning in the setting of first order logic (see, e.g., [27]) is the following: We have two objects represented by terms s and t , we observe that they share a property P , and knowing that another property Q also holds for s , we are tempted to infer that it holds for t as well (a conclusion that may turn to be plausible especially if some dependency is suspected between P and Q). This “analogical jump” corresponds to the following simple inference pattern:

$$\frac{P(s), P(t), Q(s)}{Q(t)}$$

The above pattern may be directly related to the idea of analogical proportion: One may consider that “ $P(s)$ is to $P(t)$ as $Q(s)$ is to $Q(t)$ ” (indeed they are similar changing s into t), or by central permutation that “ $P(s)$ is to $Q(s)$ as $P(t)$ is to $Q(t)$ ” (changing P into Q), the above pattern may be restated as

$$\frac{P(s) : P(t) :: Q(s) : Q(t)}{P(s), P(t), Q(s)} \quad Q(t)$$

which is a valid pattern of inference, from the propositional logic view of the analogical proportion. Similarly, one may consider richer patterns involving n-ary predicates, such as from $P(s), R(s, s'), Q(s'), P(t), R(t, t')$ infer $Q(t')$, which may correspond to the analogical proportion $P(s) \wedge R(s, s') : P(t) \wedge R(t, t') :: Q(s) : Q(t)$, itself possibly extrapolated from $P(s) : R(s, s') :: P(t) : R(t, t')$ and $R(s, s') : Q(s') :: R(t, t') : Q(t')$.

All the above patterns are quite different at first glance from a pattern of analogical reasoning proposed by Polya [22], which is now recalled.

2.3 Polya’s pattern of analogical reasoning

Polya [21] advocates the idea that analogical reasoning plays an important role when trying to solve problems in mathematics. Later, in [22] he proposed patterns of plausible reasoning in order to provide a more accurate view of reasoning mechanisms at work in problem solving. One of these patterns reads:

a and b are analogous
 a is true

—————
 b true is more credible

In [23], a modeling of “ a and b are analogous”, denoted $a \sim b$, has been proposed using a preferential nonmonotonic consequence relation \vdash , as $a \sim b$ iff $\vdash a \equiv b$. Clearly, $a \sim b$ iff $\neg a \sim \neg b$ holds. Semantically speaking, it amounts to state that $\Pi(a \equiv b) > \Pi(\neg(a \equiv b))$, where Π is a possibility measure based on a possibility distribution that rank-orders the interpretations. Viewing a and b

as compound descriptions of situations, and using a possibility distribution on the features for assessing their importance, another more intuitive view would amount to say that a and b are analogous as soon as they only differ on *non important* features. The following patterns have been established (among others) [23]

$$\frac{\sim a : b :: c : d \quad a \sim b}{c \sim d}$$

$$\frac{a \sim b \quad c \sim d}{\sim a : b :: c : d}$$

This shows a good agreement between the analogical proportion view and the relation \sim . The first inference pattern may be illustrated by an example mentioned by Aristotle [6]: Iphicrates, an Athenian general, provided the following argument about his son for whom one wanted that he serves in a public position, “if one deals with adults as tall children, are we going to deal with short adult as children?”. Indeed, it can be checked that $tall\ child : adult :: child : short\ adult$ holds (considering that child and adult are normally short and tall respectively). Then considering that $tall\ child \sim adult$ leads to admit that $child \sim short\ adult$.

3 Analogical argumentation

Let us start by quoting [2]: “An *analogy* is a comparison between two objects, or systems of objects, that highlights respects in which they are thought to be similar. *Analogical reasoning* is any type of thinking that relies upon an analogy. An *analogical argument* is an explicit representation of analogical reasoning that cites accepted similarities between two systems in support of the conclusion that some further similarity exists.” This well summarizes the basic issues.

In the previous section, we have recalled different patterns of analogical inference, and pointed out how they are underlain by the notion of analogical proportion, which itself puts in balance the ideas of dissimilarity and similarity. These different patterns provide a formal basis for discussing different issues regarding analogical arguments:

- how analogical argumentation differs from argumentation based on deductive reasoning,
- what kinds of attack exist against such arguments,
- how to evaluate analogical arguments.

An argument by analogy involves at least one premise which refers to an analogy, and as such departs from deductive (as well as inductive, or abductive) arguments [13]. An analogy may be a simple statement relating two objects “ a is analogous to b ” (or “ a is like b ”), or the statement of an analogical proportion. Thus, Polya’s pattern of plausible reasoning provides the simplest form of argument by analogy, which departs from a deductive argument “ a is true” and “ a implies b ”, then “ b is true”.

Note also that rather stating “ a is like b ”, one may use premises of the form “Objects A and B are similar in having properties P_1, \dots, P_n ”, making explicit in what respects the objects are analogous. For instance, given that “Peter is like Paul, they like good life”, and that “Paul spoilt his fortune in a few years”, one may argue that “Peter (who is presently rich) will do the same”. An example of argument involving an analogical proportion is the following: “credit rating agencies are useful”, since “credit rating agency

is to crisis as thermometer is to fever” and “thermometers are useful”.

Analogical argument, as any argument may be attacked, or used in attacks against other arguments (which may have or not an analogy form). An example of this latter case, is provided by the Iphicrates example, where the analogical proportion is not challenged. On the contrary, it is used to show that given this analogical proportion, as soon as one accepts to consider $a = tall\ child$ and $b = adult$ as analogous, one is led to accept an absurd conclusion, namely considering $c = child$ and $s = short\ adult$ as analogous.

An analogical argument may be attacked by

- *disputing the relevance of the similarities* that are pointed out (in terms of features or relations) with respect to the conclusion. This amounts in the “analogical jump” pattern of the previous section to say that properties P and Q are in fact unrelated. This may be done by providing a kind of *counterexample* by pointing out an object for which property P is true, but for which property Q is false.
- *disputing the alleged similarity* between two objects, or *challenging an analogical proportion* by pointing out that the two objects are in fact dissimilar with respect to another (relevant) property, or by exhibiting another (relevant) feature where the analogical proportion fails to hold. Thus, if we take the “credit rating agency” example, the analogy can become debatable once we remark that “credit rating agencies have an effect on the crisis” while “thermometers have no effect on the fever”.
- *pointing out undesirable consequences*. A well-known example is given by the philosopher David Hume who attacked the teleological argument according to which since a complex object like a watch requires an intelligent designer, a (more) complex object like the universe should also have an intelligent designer. Apart from attacks of the two previous types, Hume argued for instance that since watches are often the result of the work of several people, the reasoning support polytheism as well.

Besides, it is also of interest to notice that a sequence of analogical arguments may be also lead to consider analogical proportions. Typically in a debate, a discussant d may state that situation S_2 is like situation S_1 and that what took place in S_1 will happen in S_2 as well. The opponent, discussant d' , will argue that in fact there is an (important) feature where they differ, and that what took place in S_1 may not happen in S_2 . Then d may produce another pair of situations S_3, S_4 , where the same difference can be observed without affecting the conclusion advocated by d for S_2 . Then d' may counter-argue if he knows another pair of situations $S'3, S'4$ where the same difference does lead to a different conclusion. Thus this kind of exchange can be analyzed in terms of analogical proportions. Indeed, depending if we consider $S_3 : S_4 :: S_1 : S_2$, where the same effects have been observed for S_1, S_3, S_4 , or if we consider $S'3 : S'4 :: S_1 : S_2$ where different effects have been reported, one may conclude in opposite ways about S_2 (using the transfer pattern of the previous section for inferring new analogical proportions). It suggests that analogical proportions should play a role in the analysis of analogical arguments.

4 Concluding remarks

Analogical argumentation, although it is currently used in practice, and has been discussed by philosophers, has received very little attention in artificial intelligence until now. The study of [3, 4] based

on the structure-mapping model appears to be an exception. This research note has tried to provide some formal basis for the analysis of analogical arguments, by emphasizing the role played by analogical proportions in providing a logical view of analogical reasoning. What has been presented is clearly preliminary and much remains to be done for developing a formal model for analogical argumentation.

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