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NONFACTORIZABLE VISCOELASTIC BEHAVIOR: MODELING AND IDENTIFICATION

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ABSTRACT

A three dimensional viscoelastic model at finite strain representing nonfactorizable behaviour of rubber like materials is proposed. The model is based upon the internal state variables approach within the framework of rational thermodynamics such that the second principle of thermodynamics is satisfied. Motivated by experimental and rheological results, the nonfactorizable aspect of the behavior was introduced via strain dependent relaxation times which leads to a reduced time with a strain shift function. The identification of the models parameters and its capacity to predict the nonfactorizable behaviour of rubber like materials with respect to experimental data and the multi-integral viscoelastic model of Pipkin have been addressed in a previous work. The main concern of this work is the representation of the implementation of constitutive equations following from this model into Abaqus finite element software via a Fortran UMAT subroutine. To this end, the deformation gradient split into a volumetric and isochoric parts was employed, then the so-called tangent module was computed for a Mooney-Rivlin hyperelastic material. Finally, the computation of the integral in the model was computed using midpoint method. The validation of the UMAT was carried using homogeneous transformations of simple extension and pure shear.

1 INTRODUCTION

It is well known that rubber-like materials exhibit nonlinear viscoelastic behavior over a wide range of strain and strain rates confronted in several engineering applications such as civil engineering, automotive and aerospace industries. Further, the time dependent properties of these materials, such as shear relaxation module and creep compliance, are, in general, functions of the history of the strain or the stress [1]. Therefore, in a wide range of strain, a linear viscoelasticity theory is no longer applicable for such material and new constitutive equations are required to fully depict the behavior of rubber-like. In this work we shall develop a nonlinear model at finite strain for nonfactorizable viscoelastic materials within the framework of rational thermodynamics and the approach of internal state variables, see [2], [3] and [4] taking into account the dependence of the time dependent functions upon the state of the strain. The implementation of the model within Abaqus software is made thanks to a Umat subroutine Fortran. This paper is organized as follows: in section 2 the mechanical framework and the model are recalled. In section 3 the implementation in Matlab software is presented and validated by a comparison to the results from Abaqus software and a summarize of the Umat for the implementation in Abaqus is provided.

2 MECHANICAL FRAMEWORK AND CONSTITUTIVE EQUATIONS

Consider a viscoelastic material with reference placement Ω_0 in the reference configuration C_0 . It occupies at the time t the placement Ω in the deformed configuration C_t . Let φ denote a macroscopic motion between the two configurations, which maps any point X in the reference configuration C_0 to the point x in the deformed configuration. Let $F(X, t) = \partial x / \partial X$ be the deformation gradient tensor. Likewise, let $J = \det(F)$ be the jacobian of the deformation gradient tensor. From the deformation gradient $F(X, t)$, the right and left Cauchy-Green strain tensors $C = F^t F$ and $B = F F^t$ are obtained. The formulation of the constitutive equations in the nonlinear range of behavior is based upon the decomposition of the deformation gradient tensor $F(X, t)$ into volumetric and isochoric parts such that:

$$F = J^{-1/3} \bar{F} \quad \text{where } \det(\bar{F}) = 1 \quad (1)$$

in which \bar{F} is the isochoric part of the deformation gradient tensor, the right and left Cauchy-Green strain tensors associated with it reads:

$$\bar{C} = \bar{F}^t \bar{F} = J^{-2/3} C, \quad \bar{B} = \bar{F} \bar{F}^t = J^{-2/3} B \quad (2)$$

The free energy density according to [2] is expressed as follows:

$$\Psi(\bar{C}, Q) = U^0(J) + \bar{\Psi}^0(\bar{C}) - \frac{1}{2} Q : \bar{C} + \Psi_I(Q) \quad (3)$$

in which Q is a second order tensor internal variable akin to the second Piola-Kirchhoff stress tensor, its evolution law is expressed as follow:

$$\frac{\partial Q}{\partial \xi} + \frac{1}{\tau} Q = \frac{\gamma}{\tau} DEV \left[2 \frac{\partial \Psi^0(\bar{C})}{\partial \bar{C}} \right] \quad \text{with } \xi(t) = \int_0^t \frac{dt'}{a(\bar{C})} \quad (4)$$

γ and τ are the viscoelastic parameter and the relaxation time of the Prony series respectively, in relation 4 ξ denotes the reduced time which is an increasing function of real time t and $a(\bar{C})$ is a positive function of the left Cauchy-Green strain tensor called a strain-shift function.

Application of the Clausius-Duhem inequality and the resolution of the evolution equation 4 along with the form of the free energy density of equation 3 lead to the expression of the second Piola-Kirchhoff stress tensor.

$$S = J^{-2/3} \int_0^\xi G(\xi - \xi') \frac{\partial}{\partial \xi'} \text{DEV} \left(2 \frac{\partial \Psi^0(\bar{C})}{\partial \bar{C}} \right) d\xi' + JpC^{-1} \quad (5)$$

3 IDENTIFICATION OF THE PIPKIN MODEL

3.1 Pipkin isotropic model

Pipkin [5] proposed a third order development of the tensorial response function Q for an isotropic incompressible material. The principle of material indifference requires that the Cauchy stress tensor takes the following form:

$$\sigma = RQR^t + pI \quad (6)$$

R is the rotation tensor obtained from the polar decomposition of the transformation gradient tensor F and p is the indeterminate parameter due to incompressibility. The third functional development of Q reads

$$Q(t) = \int_0^t r_1(t-t') \dot{E}(t') dt' + \int_0^t \int_0^t r_2(t-t', t-t'') \dot{E}(t') \dot{E}(t'') dt' dt'' + \int_0^t \int_0^t \int_0^t r_3(t-t', t-t'', t-t''') \text{tr} \left[\dot{E}(t') \dot{E}(t'') \right] \dot{E}(t''') dt' dt'' dt''' + \int_0^t \int_0^t \int_0^t r_4(t-t', t-t'', t-t''') \dot{E}(t') \dot{E}(t'') \dot{E}(t''') dt' dt'' dt''' \quad (7)$$

r_i $i = 1..4$ are the relaxation kernels expressed by a decaying exponential functions and $\dot{E}(t)$ is the time derivative of the Green-Lagrange deformation tensor $E = 1/2(C - I)$.

3.2 Identification of the model's functions

The free energy density Ψ^0 , the viscoelastic kernel $G(\xi)$ and the reduced time $\xi(t)$ of relation 5 are identified separately. To this end data in pure shear and simple extension were generated following relations 6 and 7. Equilibrium tests of simple extension and pure shear are used in the identification of Ψ^0 , relaxation tests with small level of strain in pure shear are used in the identification of $G(\xi)$ and monotonic tests of simple extension are used in the identification of $\xi(t)$ and then the whole identification procedure is validated by predicting the response of the model to a monotonic test of pure shear. Each identification procedure turns out to a least square minimization problem.

4 CONCLUDING REMARKS

A nonlinear viscoelastic model at finite strain to describe nonfactorizable behavior of rubber like materials has been proposed. The model is formulated using the decomposition of the deformation gradient tensor which make it applicable to both compressible and incompressible materials. The implementation of the model is based on the application of the midpoint algorithm to the convolution integral in the constitutive equation of the stress. This algorithm was validated using homogeneous transformation of simple extension for several strain configurations. Henceforth, the algorithm could be used in the Umat Fortran to implement the fully three dimensional model within Abaqus software.

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