Identification of nonlinear viscoelastic model

Adel Tayeb, Makrem Arfaoui, Abdelmalek Zine, Adel Hamdi, Jalel Benabdallah, Mohamed Ichchou

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Tayeb Adel[a,b], Arfaoui Makrem[a], Zine Abdelmalek[b], Hamdi Adel[a], Benabdallah Jalel[a] and Ichchou Mohamed[b]

[a] Laboratoire de Mécanique appliquée et Ingénierie
Université de Tunis ElManar, Tunis, TUNISIE
[b] Laboratoire de Tribologie et Dynamique des Systèmes
Université de Lyon, Lyon, FRANCE

Abstract – Elastomers are widely used in several engineering applications such as aerospace, automotive and civil engineering applications thanks to their ability to undergoing high strains and strain rates in large temperature range. Several models have been developed in the literature to investigate those nonlinearities.

In this work we expose a nonlinear viscoelastic model at finite strain for rubber-like materials based upon the theory of irreversible thermodynamic and the time strain superposition principle. The identification of several model's parameters is highlighted. A systematic identification procedure is used. First, we start by the identification of the shear relaxation module using relaxation tests at low levels of deformation and the identification tools in Abaqus software, then we identify the hyperelastic potential using equilibrium data of simple extension and pure shear experiment and a constrained linear least square minimization with Matlab software, finally we identify the reduced time function using monotonic tests of simple extension for different strain rates and the discretization of the behaviour law.

The capacity of the model to predict the behaviour of the material is illustrated via the Cauchy stress relative error.

Key words: modelling / identification/ linear least square/ reduced time.

Introduction

The behaviour of rubber-like materials is known to be hyperviscoelastic at different deformation levels. In this paper we carry out a systematic identification procedure of fitting an incompressible hyperviscoelastic constitutive law [1] to experimental data. To this end we use an experimental characterization performed with Bromobutyl rubber (BIIR) [2] and a linear and nonlinear least square optimization tool in Matlab and Abaqus. Thus, a numerical discretization of the stress-strain integral relation was performed and validated with homogeneous transformation tests. Shear relaxation tests were used to identify the relaxation shear module, tests of simple shear and simple extension at equilibrium were exploited to identify the hyperelastic part of the behaviour whereas monotonic tests were utilized to identify the strain shift function. Results of the identification are highlighted and capacity of the model to predict the behaviour of the material of interest is discussed.

1 Hyperviscoelastic model

In figure 1 is plotted the shear relaxation module for two different levels of deformation: 10% and 50%. The material of interest shows a significant dependence of the relaxation shear module upon deformation level. Motivated by this fact, we developed a nonlinear viscoelastic model taking into account the dependence upon the deformation via a deformation shift function and thermodynamic principles [1].

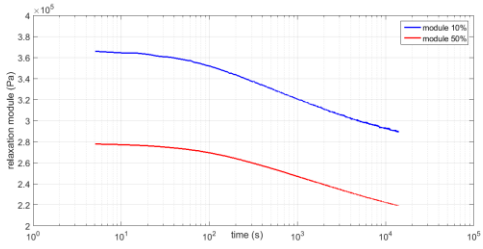

Figure 1: dependence of the relaxation module upon deformation level

$$\underline{\underline{\sigma}} = 2\underline{\underline{F}}DEV\left(2\frac{\partial\psi_0\left(\underline{\underline{C}}\right)}{\partial\underline{\underline{C}}}\right)\underline{\underline{F}}^t + dev\left[\underline{\underline{F}}\int_0^{\xi}\frac{\partial G\left(\xi-\xi'\right)}{\partial\xi'}DEV\left(2\frac{\partial\psi_0\left(\underline{\underline{C}}\right)}{\partial\underline{\underline{C}}}\right)d\xi'\underline{\underline{F}}^t\right] - p\underline{\underline{I}} \quad (1)$$

This model (equation (1)) involves three different functions which are the instantaneous hyperelastic potential ψ_0, the shear relaxation module G and the reduced time function ξ. In what follows we expose a systematic procedure to identify these three functions.

2 Identification procedure

The identification procedure contains three main steps:

– Identification of the shear relaxation module G using shear relaxation tests

– Identification of the hyperelastic potential ψ_0 using equilibrium curves for pure shear and simple extension experiments

– Identification of reduced time function ξ using monotonic tests of simple extension and the discretized stress-strain relation.

3.1 Identification of the shear relaxation module

In literature there are several decay functions to describe the shear relaxation module. In this work we adopted the decay exponential functions (equation (2)). For a nonlinear viscoelastic model it's more convenient to identify the relaxation function at low level of deformation [seddiq]. Thus, the shear relaxation at 10% and the identification module of Abaqus software are used to fit the relaxation module.

$$G(t) = G_0\left(1-\sum_i g_i\left(1-\exp\left(-\frac{t}{\tau_i}\right)\right)\right) \quad (2)$$

In expression (2) G_0 is the initial shear module whereas g_i and τ_i are the coefficients of the Prony series, these coefficients are reported in figure 1.

	Coefficients g_i	Time constants τ_i (s)
1	$4,46E-03$	$14,79$
2	$3,77E-02$	$125,71$
3	$5,69E-02$	$460,7$
4	$5,84E-02$	$1761,6$
5	$8,76E-02$	$9598,5$

Table 1: Prony series coefficients

Using the values in table 1 one could simply calculate the relative error between experimental and fitted shear relaxation module, this error is about 0.001% .

3.2 Identification of the hyperelastic potential

The identification of hyperelastic potential relies on two main steps which are the choice of the free energy density to use in the first place and the check of the stability of the potential in the second place. Whether the hyperelastic potential is linear or nonlinear in terms of its parameters a linear or nonlinear least square optimization is used respectively. In this work we adopted the Mooney-Rivlin [3] polynomial free energy density for an isotropic incompressible material (equation (3)). In

this expression c_{ij} are the parameters to be identified, I_1 and I_2 are the first and the second invariant of the right Cauchy-Green strain tensor $\underline{\underline{C}}$.

$$\psi = \sum_{i,j} c_{ij} \left(I_1 - 3\right)^i \left(I_2 - 3\right)^j \quad (3)$$

The nominal stress of such model is expressed as follow in term of the principle stretch λ :

$$\pi = \frac{\partial \psi}{\partial \lambda} = \sum_{i,j} c_{ij} \phi(i,j,\lambda) \quad (4)$$

In the above expression (equation (4)) $\phi(i,j,\lambda)$ is a nonlinear function of λ, for a simple extension experiment it takes the following form

$$\phi(i,j,\lambda) = 2i\left(\lambda^2 + \frac{2}{\lambda} - 3\right)^{i-1}\left(\lambda - \frac{1}{\lambda}\right)\left(2\lambda + \frac{1}{\lambda^2} - 3\right)^j + \\ +2j\left(\lambda^2 + \frac{2}{\lambda} - 3\right)^i\left(\lambda - \frac{1}{\lambda}\right)\left(2\lambda + \frac{1}{\lambda^2} - 3\right)^{j-1} \quad (5)$$

In order to obtain more useful form of equation (4) with respect to the identification procedure, it's possible to express the material's parameters c_{ij} with the vector [4] $c \in \mathbb{R}^{ij}$ i.e.

$$c^T = \left\{c_{01}, \ldots\ldots, c_{0j}, c_{10}, \ldots\ldots\ldots, c_{1j}, c_{i0} \ldots\ldots\ldots c_{ij}\right\} \quad (6)$$

And the function $\phi(i,j,\lambda)$ with the matrix $\Phi \in M_{k \times ij}(\mathbb{R})$, k is the number of experimental points, hence we have the following expression for the discrete nominal stress

$$\Pi = \Phi c \quad (7)$$

$$\min_{c \in \mathbb{R}^{ij}} \left\| \Phi c - \tilde{\Pi} \right\|_2^2 \quad (8)$$

$$\frac{\partial \psi}{\partial I_1} > 0, \frac{\partial \psi}{\partial I_2} \geq 0 \quad (9)$$

In light of expression (7), the identification process reduces to the minimization of expression (8), $\tilde{\Pi}$ is the vector of experimental nominal stresses. The condition of stability is expressed in terms of empirical inequalities (9). Hence, we use a constrained linear least square minimization via "lsqlin" tools in Matlab software. In figure 2 and 3 are plotted the relative error of the Cauchy stress for simple extension and pure shear respectively.

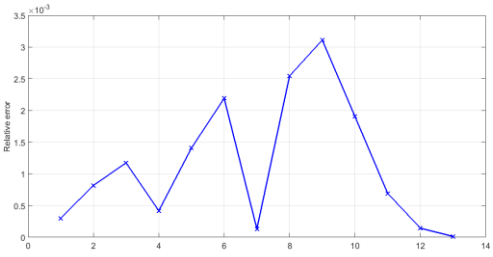

Figure 2: Cauchy stress relative error for simple extension

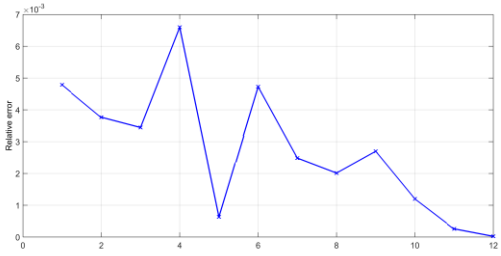

Figure 3: Cauchy stress relative error for pure shear

3.3 Identification of the reduced time function

The identification of the reduced time function is obtained thanks to the discretization of the behaviour law equation (1), thus, at each experimental point t_k the minimization of the Cauchy stress error between the model and the and the recoded stress gives the corresponding reduced time ξ_k . The experiment used here is the monotonic simple extension test at different strain rates combined with a dichotomy algorithm implemented in Matlab software. As a result we got the reduced time as a function of real time, the nominal stress error for the three strain rates 10%,100% and 200% are reported in figure 4, 5 and 6 respectively.

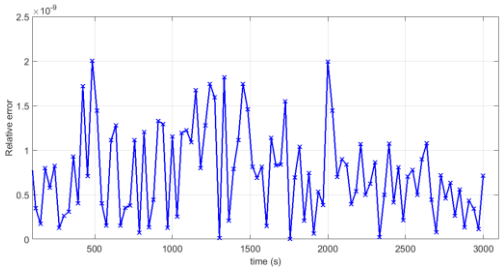

Figure 4: Cauchy stress relative error for simple extension at $10\%\, mn^{-1}$

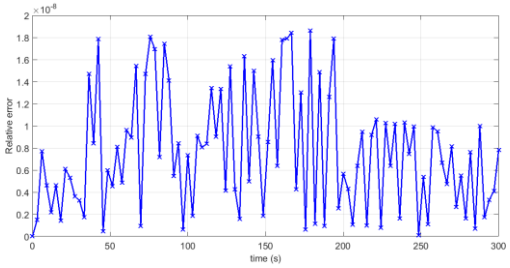

Figure 5: Cauchy stress relative error for simple extension at $100\%\, mn^{-1}$

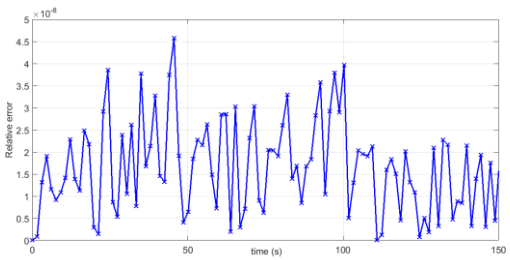

Figure 6: Cauchy stress relative error for simple extension at $200\%\, mn^{-1}$

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