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Dynamic Identification of Physical Parameters with Subspace State Models

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1 Introduction

The ability to track the dynamic and physical behaviour of a class of large and flexible structures – typically discrete spatial structures or adaptable structures for which inertia or stiffness may vary significantly - is of great importance. Likewise, the development of new structural concepts and original applications such as tensegrity systems [1] made up of both cables and bars, shell's folding/unfolding and deployable covers involves creating skillfull tools to accurately identify the structure even after marked alterations.

In contrast to traditional approaches, we decided to build the model directly on the basis of the experimental results – the selected control and measurement points – without any preliminary theoretical model. The recent and relatively unknown subspace-state methods, which identify a discrete space state model (being composed of the following quadruple of matrices: A , B , C and D) of the studied system, has proven [2] very efficient for solving such problems.

Besides also identification of classical modal parameters, the state space representation allows relatively easy recognition of physical matrices provided that the number of outputs l tested is the same as the number of identified modes n . This ability to solve the inverse problem is of great interest in the control domain, particularly for discrete structures with concentrated masses and separate elements. Few preliminary numerical results have confirmed the principle. Unfortunately, the main theoretical condition, recalled just above ($l=n$), is difficult to apply practically.

In order to overcome this initial limitation, several methods have thus been developed which combine two at a time “sub-state space” models (e.g. partial models including only a few inputs/outputs and a few modes) of the same structure. Finally, the result is an enlarged space-state model including all inputs, outputs and modes coming from these different models. Several data records ¹ can thus be associated in order to yield an extended dynamic representation of the structure.

The theoretical background is then briefly summarized, and the paper presents numerical and experimental results acquired with this identification scheme.

¹ ...collected sucessively at different points.

First, to initially obtain good identifications, the algorithm - named $N4sid_r$ - derived from subspace methods is used. Secondly, the modal and physical parameters are computed when a sufficiently enlarged space-state model is reached. Physical matrices identified from corrupted data are compared with true matrices. A real variable mass system is also studied to assess the dynamic tracking of variable factors.

2 Theoretical background

In the identification field, frequency domain techniques are often the best known and the most frequently used. Yet many time domain identification methods have been described over the past few years. They are able to identify dynamic parameters and often produce models more accurately.

More recently, a class of subspace-state methods have been developed and shown their superiority in relation to the classical time domain approaches [2]. Instead of classically pre-estimating a large-sized block Hankel matrix made up of Markov parameters, subspace-state algorithms estimate the model directly from the input/output data without requiring such accuracy-consuming steps. Moreover, they make use of numerically stable methods such as RQ decomposition and singular value decomposition (SVD) [3].

The subspace identification algorithm taken into account in this paper is the robust version of $N4sid$ ($N4sid_r$), which stands for “Numerical algorithm for Subspace State Space System Identification” of P. Van Overschee and B. De Moor [4], [5].

2.1 State space representation

It provides a discrete time, linear, time-invariant state space model which is mathematically described by the following set of equations²:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + w_k \\ y_k = Cx_k + Du_k + v_k \end{cases} \quad \text{and} \quad E \left[\begin{Bmatrix} w_p \\ v_p \end{Bmatrix} \cdot \begin{Bmatrix} w_q^T & v_q^T \end{Bmatrix} \right] = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \cdot \delta_{pq} \quad (1)$$

where:

- $x_k \in \mathfrak{R}^{2n}$ is the state vector of the process which contains the numerical values of $2n$ states ³, $u_k \in \mathfrak{R}^m$ and $v_k \in \mathfrak{R}^l$ are respectively the measurements of the m inputs (forces) and l outputs (usually accelerations).
- $A \in \mathfrak{R}^{2n \times 2n}$, $B \in \mathfrak{R}^{2n \times m}$ and $C \in \mathfrak{R}^{l \times 2n}$ are respectively the state matrix, the input and the output matrix. $D \in \mathfrak{R}^{l \times m}$ is the direct feedthrough

² E is the expected value operator and δ_{pq} the Kronecker delta.

³ Consequently, n is the number of modes.

term⁴. A characterizes the complete dynamics of the system: the n frequencies ω_i and n damping ratios ε_i are easily determined from its complex eigenvalue pairs λ_i

$$(i = 1, \dots, 2n) \text{ with } \lambda_i = \pm (\varepsilon_i \times \omega_i) + i \times \omega_i \sqrt{(1 - \varepsilon_i^2)}.$$

w_k and v_k are unmeasurable vector signals; they are supposedly zero mean, stationary, white noise vector sequences. Q , S and R are their associated covariance matrices.

2.2 State space identification problem

Given N measurement of the input and output of the unknown real system, the multi input/output identification problem as described by (1) involves estimating:

- the order n of the unknown system.
- the quadruple state space matrices within a similarity transformation T , i.e. $(TAT^{-1}, TB, CT^{-1}, D)$.

The key elements of subspace methods are using geometric tools and manipulation of subspaces spanned by rows and columns. Irrespective of an specific subspace algorithm, the identification process is twofold [3]. The first step performs both state sequence X_s and extended observability matrix Γ_s directly from the given input/output data, where:

$$X_s = [x_s \quad x_{s+1} \quad \dots \quad x_{s+r-1}]_{(2n,r)} \quad \Gamma_s = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{s-1} \end{bmatrix}_{(ls,2n)} \quad (2)$$

In a second step, the state space representation up to a similarity transformation is obtained. Distinctions between the different subspace algorithms are ultimately due to different numerical implementations.

2.3 Physical parameters identification

In order to subsequently identify m , c and k (mass, damping and stiffness matrix respectively), the main problem involves retrieving the “physical” form of the estimated state space model usually expressed in continuous time ($[]_c$) as:

$$A_c = \begin{bmatrix} 0_{n \times n} & I_n \\ -m^{-1}k & -m^{-1}c \end{bmatrix} \quad B_c = \begin{bmatrix} 0_{n \times n} \\ m^{-1} \end{bmatrix} \quad C_c = [I_n \quad 0_{n \times n}] \quad D_c = [0] \quad (3)$$

⁴ D is most often 0 in continuous time.

To solve this problem, it has been shown in [6] that there is a unique similarity transformation P , defined as $P = \begin{bmatrix} C_c & C_c \\ C_c & A_c \end{bmatrix}$, that reaches this “physical” space.

Note that m , c and k can be determined only if the number of output l equals the number of modes n . P must be invertible and then square; a condition that is seldom satisfied under real tests.

2.4 Re-association methods

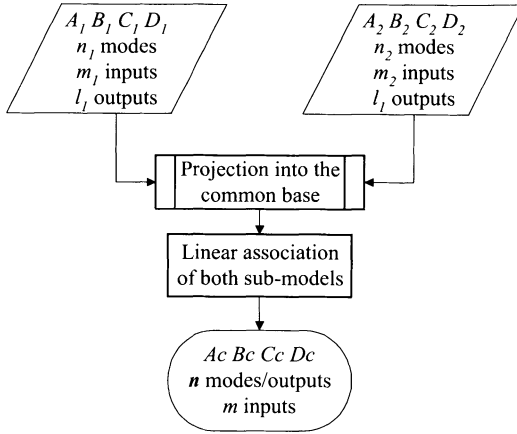


Fig. 1. Re-association principle

Methods that combine two at a time sub-state space representations (Figure 1) have been developed in order to increase the number of outputs but also the number of inputs and the number of modes of a reduced model. A detailed description of three related methods can be found in [7]. The key step lies in the projection properties into a common base [8] of each sub-state space representation allowing subsequently linear association of input, output and modes.

3 Simulation results

In this section, a constant 4 degrees of freedom system is considered, defined as:

$$m = \text{diag}([300 \ 220 \ 300 \ 310])$$

$$c = 10^2 \begin{bmatrix} 23 & -15 & 0 & 0 \\ -15 & 26 & 11 & 0 \\ 0 & -11 & 13 & -2 \\ 0 & 0 & -2 & 12 \end{bmatrix} \quad k = 10^4 \begin{bmatrix} 32 & -16 & 0 & 0 \\ -16 & 36 & -20 & 0 \\ 0 & -20 & 35 & 15 \\ 0 & 0 & -15 & 29 \end{bmatrix} \quad (4)$$

In order to validate the efficiency of the physical parameters autonomous⁵ identification principle, we first excite the studied system with 1 white noise input, and 4 outputs are used. Given that the number of modes (4) equals the number of outputs, no combining methods are necessary here.

1200 data samples are used with the sampling frequency set at 30Hz. The number of data block rows s is set at 12. 50 runs are systematically performed and the true system order (namely 4) is specified for each one. The data are then averaged.

For zero noise level, the identification results agree perfectly with the true physical matrices. For 1% and 10% noise level, the Figure 2 shows the bands covered by the evaluated relative estimation errors⁶ when the four inputs are selected successively.

Examination of these results highlights the fact that the stiffness matrix is the easiest to identify with the mass matrix. On the other hand, increasing the noise level is detrimental to the overall consistency; the damping matrix remains the most sensitive.

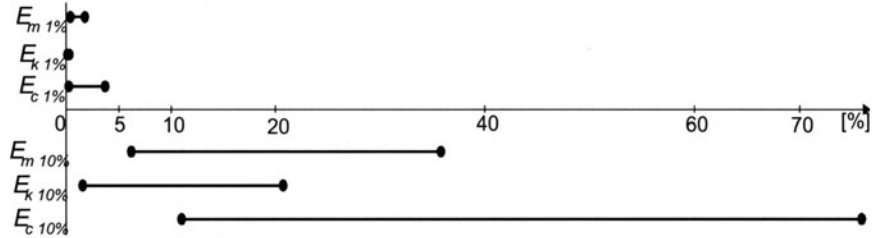


Fig. 2. Relative estimation errors of m, c & k versus selected input

Finally, Table 1 gives the estimated matrices for 10% noise and demonstrates a good identification process.

⁵ Nothing of m , c or k is known.

⁶ $E_m = 100 \frac{|m - \hat{m}|}{|m|}$ where \hat{m} is the true mass matrix. Alike for c and k .

$m = \text{diag}([302.56 \quad 222.97 \quad 317.42 \quad 290.85])$	
$c =$	$k =$
$10^2 \begin{bmatrix} 22.81 & -14.738 & -0.162 & 0.05 \\ -14.373 & 25.223 & -10.679 & 0.107 \\ -0.072 & -11.094 & 13.054 & -2.053 \\ 0.313 & -0.48 & -1.782 & 12.273 \end{bmatrix}$	$10^4 \begin{bmatrix} 32.026 & -15.94 & -0.106 & -0.071 \\ -15.94 & 35.784 & -19.988 & -0.109 \\ 0.036 & -19.988 & 34.911 & -14.947 \\ 0.255 & -0.444 & -14.947 & 29.443 \end{bmatrix}$

Table 1. Estimated matrices – 10% noise level

In a second step, with the same 4 d.o.f. system and the same numerical conditions, the capacities of the re-association methods are evaluated. Many inputs/outputs sequences are tested (with at least here one input and two outputs per identification) with the aim of producing an enlarge state-space representation made of 4 outputs. In order to identify the physical matrices, we assume the mass matrix m is entirely known.

For zero noise level, the results are perfect and are not shown. For 1% and 10% noise level, Figure 3 summarizes the evaluated level errors. Comparatively, they are almost equivalent to previous simulations performed without sub-models association highlighting the efficiency of such combining techniques. But lower results can be expected if m is autonomously identified rather than assumed to be known.

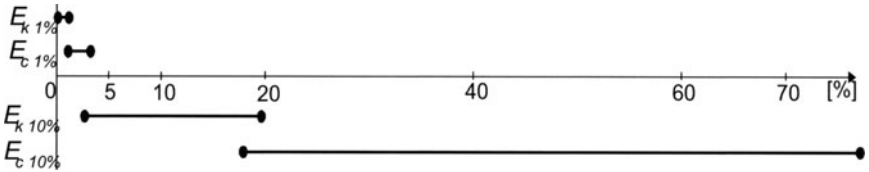


Fig. 3. Relative estimation errors of c & k with sub-models association

4 Experimental results

This section describes the results of tests which were conducted on a simple cantilevered beam. This structure was excited consecutively at different locations using only one random noise input. 2 or 4 accelerometers were used for measuring the dynamic response. Two analyses were carried out on this structure.

→ The first experimental sequence involved evaluating the capacity of the whole identification scheme to track noticeable alterations in the physique of the system. We chose to modify the mass distribution of an equivalent

2 degrees of freedom structure by successively overloading each node with a 1.2 kg mass. The analysis was performed on a 0-100 Hz frequency range. The sampling frequency was set at 1000 Hz and 2000 data points were collected during the test. The number of data block rows s was preset at 10 to effectively identify 10 modes (one input only). Nevertheless, after the reduction process (unstable and out of range modes are removed), only 2 modes were clearly selected.

Inspection of the identified mass matrices (Table 2) clearly highlighted the capacity to track weight variations despite a slight overestimation probably due to the comparatively high mass of the electromagnetic exciter.

no overloading	1.2 kg on node 1	1.2 kg on node 2
$\begin{bmatrix} 3.80 & . \\ . & 1.25 \end{bmatrix}$	$\begin{bmatrix} 5.94 & . \\ . & 1.51 \end{bmatrix}$	$\begin{bmatrix} 4.28 & . \\ . & 2.85 \end{bmatrix}$

Table 2. Identified mass matrices under test

As for the stiffness matrix, its different estimations were logically stable (less than 5.5% variation). On the other hand, the damping matrix was more affected; its high sensitivity has already been shown with numerical simulations.

	no overloading	1.2 kg on node 1	1.2 kg on node 2
k	$10^5 \begin{bmatrix} 6.91 & -2.62 \\ -2.62 & 1.13 \end{bmatrix}$	$10^5 \begin{bmatrix} 7.22 & -2.73 \\ -2.73 & 1.19 \end{bmatrix}$	$10^5 \begin{bmatrix} 6.59 & -2.49 \\ -2.49 & 1.08 \end{bmatrix}$
c	$\begin{bmatrix} 72.27 & 166.45 \\ -137.43 & -24.06 \end{bmatrix}$	$\begin{bmatrix} 113.79 & 183.19 \\ -153.82 & -26.17 \end{bmatrix}$	$\begin{bmatrix} 126.63 & 316.60 \\ -147.73 & -77.26 \end{bmatrix}$

Table 3. Identified stiffness and damping matrices under test

↪ The second field tests were aimed at experimentally validating the capacities of the re-association methods. In order to obtain an equivalent 4 d.o.f. model having 4 outputs, each sub-identification involved 3 measurement outputs and one input (either at the middle or end of the beam). 2000 data points were gathered on a 0-500 Hz frequency range with a 2000 Hz sampling frequency (just 1 second!). The same four modes were systematically selected among 10 identified ($s = 10$), thus avoiding the need to combine modes during these tests. Simultaneously, direct identification (without any

sub-models association) using whole outputs was performed to obtain a comparative base.

Regardless of the test, all physical matrices were entirely identified without any preliminary knowledge of them. As shown in Figure 4, the reconstructed outputs calculated with the combined state space model or with the direct identification were very close to the real data. These traces illustrate the high efficiency of the $N4sid_r$ algorithm and the low effects of such re-association processes on each other. Good subsequent autonomous identifications of the physical matrices could thus be expected. The Table 4 sums up the values of m , c and k obtained by the direct or the combining method respectively.

The comparison gives similar results. However, note that parameters that are supposed to be zero in the stiffness and damping matrices were not correctly identified.

Re-building of the output y_4 with $N4sid_r$.

Identifications performed with filtered data (hanning) - Id n°1: u_2y_{123} - Id n°2: u_2y_{234}

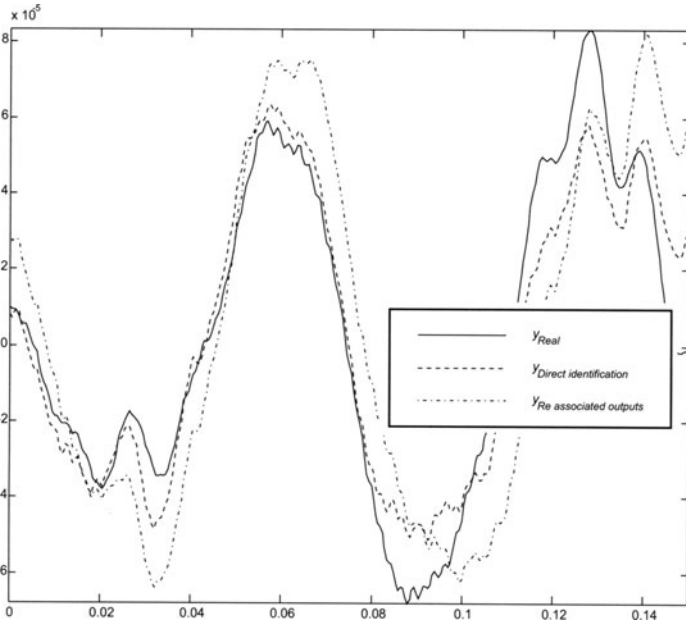


Fig. 4. Comparison between reconstructed output and real output

Direct identification

m	$k \times 10^7$	$c \times 10^3$
$\begin{bmatrix} 3.56 & & & \\ & 4.60 & & \\ & & 3.19 & \\ & & & 0.90 \end{bmatrix}$	$\begin{bmatrix} 1.12 & -0.92 & 0.35 & -0.08 \\ -0.92 & 1.31 & -0.79 & 0.24 \\ 0.35 & -0.79 & 0.71 & -0.27 \\ -0.11 & 0.27 & -0.27 & 0.11 \end{bmatrix}$	$\begin{bmatrix} 0.03 & 1.09 & -0.45 & 0.45 \\ -1.04 & -0.34 & 0.07 & -0.89 \\ 0.85 & -0.53 & 0.67 & 0.61 \\ -0.43 & 0.50 & -0.53 & -0.11 \end{bmatrix}$

Identification with combining method

m	$k \times 10^7$	$c \times 10^3$
$\begin{bmatrix} 3.69 & & & \\ & 4.77 & & \\ & & 3.30 & \\ & & & 0.93 \end{bmatrix}$	$\begin{bmatrix} 1.15 & -0.95 & 0.36 & -0.09 \\ -0.95 & 1.35 & -0.81 & 0.24 \\ 0.36 & -0.81 & 0.73 & -0.28 \\ -0.11 & 0.28 & -0.28 & 0.12 \end{bmatrix}$	$\begin{bmatrix} 0.10 & 1.34 & -0.01 & 0.71 \\ -1.52 & -0.57 & -0.42 & -1.18 \\ 0.93 & -0.42 & 0.98 & 0.78 \\ -0.46 & 0.46 & -0.65 & -0.17 \end{bmatrix}$

Table 4. Identified physical matrices with and without re-associated outputs

5 Conclusion

In order to perform high level dynamic and autonomous identification of physical parameters of structures, an algorithm from the class of subspace state methods was successfully evaluated. Subsequently, the principles of original methods which are able to combine several sub-state space models of the same system have been succinctly outlined and numerically validated.

Then these different identification processes were tested on a simple real structure and showed their high robustness, particularly concerning mass and stiffness matrices.

These encouraging results allow to extend this identification scheme to larger structures.

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