Put Option Contracts in Newsvendor Model with Bankruptcy Risk

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Abstract. This paper studies a newsvendor problem in which the retailer can mix two contracts, a wholesale price and a put option contract. We consider that the newsvendor is financially constrained and may need to contract a loan to cover her ordering costs, with a probability that she becomes bankrupted. We show that when a put option contract is available, the retailer's order quantity increases, while the bankruptcy risk and therefore the loan's interest rate decrease. We illustrate these results with numerical experiments on a simple example for different demand sizes and variability.

Keywords: Newsvendor · Finance · Bankruptcy · Put option contracts.

1 Introduction

We consider a newsvendor problem in which a retailer has to decide its optimal inventory level for a future selling season. The classical objective is to find the best tradeoff between ordering too many units, inducing a holding cost for every unit of remaining product, or end up with insufficient inventory, leading to lost sales for unmet demands. This problem is rising because of the uncertainty in future demand and it becomes even more significant for the retailer when she needs to use a credit to cover her ordering costs and has to repay borrowed credit after realizing her sale. In that case, she usually refers to a bank to receive a loan to cover her expenses, possibly using a collateral to secure the loan and reduce her interest rate [17]. If the retailer cannot pay her credit obligation, she becomes bankrupted and loses all her wealth. Thus the demand's uncertainty is leading to newsvendor problem and may lead to a bankruptcy risk.

We focus on the above problem, that involves a financially constrained retailer, when she can orders her product through a mixture of wholesale price and put option contract. Units ordered with the latter are equipped with an option that allows the retailer to sell them back to the supplier at a predefined price.

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Traditionally, the put option contracts are a financial derivative tool used in the capital market, but it is a recent trend to use a similar concept in operations management. In essence, these contracts are similar to the well-known buyback contract, except that in this case the retailer needs to pay a premium for each unit she may sell back. In other words, the retailer can secure a part of her ordering quantity through a put option contract.

Different industries have used the put option contracts to deal with inventory risk management. Hewlett Packard (HP) are implying these kinds of contracts from 2000 to manage the inventory risk [15]. Put options have been used by the agricultural industries to hedge against unexpected weather situation in Chicago Mercantile Exchange [22]. Implying option contracts in the operation is an increasing trend to deal with supply chain management [19].

In this paper, we analyze the ordering decisions of the retailer with wholesale and put option contract, then we investigate how the bank selects a proper interest rate for a requested loan. Finally, we compare the performance, bankruptcy risk and received loan's interest rate of the retailer with and without a put option contract besides the wholesale price contract.

2 Literature review

Decentralizing the decision making in a supply chain generally leads to a global performance reduction [16]. Since demand uncertainty directly affects the retailer, she suffers from the lack of flexibility in her ordering decision and the induced risk on her profit[18], [3]. Different mechanisms have emerged to mitigate these effects, such as buyback, emergency purchase or option contracts. It is shown that option contracts can improve the flexibility of the supply chain [10] and can be used as a risk hedging tool [11] and [2]. In the operations management literature, three main kinds of option contracts have been studied: (i) call option contracts that the retailer can use to receive the product immediately at the selling season [1] (similar to capacity reservation contracts), (ii) put option contracts that the retailer can use to sell back unsold products [7], [2] (similar to buyback contracts) , or (iii) bidirectional option contracts that the retailer can choose to use it as a call or put option contract at the time of exercise [8], [21]. It is shown that under certain conditions, the supplier and retailer can both improve their performances with the existence of an option contract [4]. Other works illustrate that the option contracts can reduce the over/under inventory risk, see e.g. [23] for an application to the agricultural industry. In this study, we focus on the put option contract. In a recent article [19], the authors show that the supplier can design the put option contract in order to achieve the supply chain coordination, which means that the whole supply chain behaves like a centralized system. The put option contract is analyzed for a price setting newsvendor which can hedge her risk by using this contract [20]. [2] studied the put option contract under information asymmetry and risk-averse retailer.

The other aspect of our study focuses on the financial constraints of a retailer which can affect the performance of the global supply chain. Indeed, financial

considerations on the operational decisions of the retailer can help improve the supply chain efficiency [5], [9], [6]. In this case that retailer has a limitation on her capital and may refer to a creditor to cover her ordering cost, inducing a bankruptcy risk in the process. This scenario is analyzed comprehensively by Kouvelis and Zhao [12], [13]. In this article, we consider a financially constrained retailer who can use a put option contract to secure part of her ordering from the wholesale price contract. We first investigate the retailer and the bank decisions analytically. Then we analyze numerically the effect of a put option contract on the bankruptcy risk and performance of the retailer for different demand sizes.

3 Model Description

We consider a supply chain that consists of a financially constrained newsvendor (retailer) who faces a stochastic demand D at the beginning of the selling season (time 0). We assume that D follows an IFR (*Increasing Failure Rate*) distribution with probability density function (pdf) ϕ and cumulative distribution function (cdf) Φ and has finite mean μ. In order to satisfy her customers' demand, the retailer has the possibility to buy units of product using two types of contracts offered by her supplier. The first one is a classical wholesale price contract, in which the retailer purchases units at a given wholesale cost $w > 0$. With the second one, the retailer has the possibility to add a put option contract on some of the units purchased for an additional upfront payment $o > 0$ per unit. In return if the retailer decides to exercise her option once demand is realized, the supplier has to buy unsold units from her at a predefined per-unit exercise price e. Note that this option is only available for units to which the put option applies. At time 0, the retailer chooses a total quantity q to purchase from the supplier and an option quantity $q_p \leq q$. We note her decision vector at time 0 $\mathbf{q} = (q, q_p)$. At the end of the selling season (time 1), uncertainty is resolved and the retailer sells as much of her available inventory at a per-unit price p to satisfy the realized demand d. If she has leftover units that were purchased with a put option, she exercises her option and thus collects an additional revenue equal to $(e \min\{q-d, q_p\})^+$. The objective of the retailer is to decide how many units to purchase in total, as well as the quantity subject to the put option in order to maximize her profit at time T. Let r_f be the risk-free interest rate over the planning horizon. To avoid trivial cases, we consider in the remainder of this paper that $p > (w+o)(1+r_f) > e > o(1+r_f)$: The first inequality ensures that using the put option is profitable for the retailer, while the second and third one guarantee that she recovers a strictly positive part of the wholesale price when she exercises her option. We consider that the retailer is financially constrained, i.e. she has finite working capital y and collateral x. When the working capital is not enough to pay the quantity ordered by the retailer, she has the possibility to request a loan from a risk-neutral bank that offers a fairly priced loan. In that case, the interest rate offered by the bank depends on the risk incurred by the retailer and the collateral is used to secure the loan in order to reduce the bank's interest rate. In particular if the future value of the loan amount L is

lower that the retailer's collateral ($x \geq L(1+r_f)$), the loan is fully secured and the bank offers and interest rate equal to the risk-free rate r_f. On the other hand when the loan is not secured ($x < L(1+r_f)$) there exists a probability that the retailer does not have enough capital at the end of the selling season to fully repay the loan obligation. Therefore in that case, the bank chooses an interest rate $r > r_f$ to compensate the retailer's bankruptcy risk. Note that when the retailer becomes bankrupted, the bank seizes all her sales and collateral.

3.1 The Retailer's decision

We start by considering the case of a retailer with no financial constraint. The revenue generate by sales and buybacks at time 1 are equal to:

$$R(\mathbf{q}) = p\min\{q, \mathrm{D}\} + e\min\{q - \mathrm{D}, q_p\}$$

and we can express its expected cash flow as:

$$\Pi(\mathbf{q}) = \mathbb{E}\left[R(\mathbf{q}) + (1+r_f)(y - wq - oq_p)^+ + x\right]^+ \tag{1}$$

The following proposition states that the retailer can always find an unique optimal decision to maximize her expected cash flow.

Proposition 1. *When the retailer is not financially constrained, her expected cash flow function is jointly concave. The corresponding optimal total quantity q and put option quantity q_p are:*

$$\begin{aligned} q^* &= \Phi^{-1}\left(1 - \frac{(w+o)(1+r_f) - e}{p-e}\right) \\ q_p^* &= q^* - \Phi^{-1}\left(\frac{o(1+r_f)}{e}\right) \end{aligned} \tag{2}$$

We now focus on the case where the retailer as a limited working capital y, with x the equities that can be used as collateral to secure the bank loan. If the retailer working capital is not sufficient to cover her cost at time 0, i.e. $wq + oq_p > y$, she needs to borrow a amount equal to $L = (wq + oq_p - y)^+$ from the bank. At time 1, the bank loan's repayment is then equal to $L(1+r)$, where r is the interest rate decided by the bank. Since the bank is risk neutral, its expected repayment Π_{bk} at time 1 must equal the present value of the amount borrowed by the retailer:

$$\mathbb{E}\left[\Pi_{bk}\right] = (1+r_f)(wq + oq_p - y)^+ \tag{3}$$

When the loan is secured by the collateral of the retailer ($L(1+r_f) \leq x$), the bank bears no risk of default from the retailer and receives a repayment $\Pi_{bk} = (wq + oq_p)(1+r_f)$ from the retailer with probability 1. As a consequence, it sets an interest rate equal to the risk-free rate r_f. Otherwise, the bank uses

equation (3) to define an interest rate $r > r_f$. The expected cash flow of the retailer then depends on the value of her collateral:

$$\Pi(\mathbf{q}) = \begin{cases} \mathbb{E}\left[R(\mathbf{q}) - L(1+r_f) + x\right]^+ & \text{if } L(1+r_f) \leq x \\ \mathbb{E}\left[R(\mathbf{q}) - L(1+r) + x\right]^+ & \text{otherwise} \end{cases} \tag{4}$$

At the end of the selling season, the retailer has to payback the loan with its interest rate. If the retailer's wealth (possibly including a portion of its collateral) is sufficient, the bank receives a full repayment of the loan. Otherwise, the retailer becomes bankrupted and the bank seizes all of her revenue and collateral. Formally, the expected repayment of the bank loan is:

$$\begin{aligned} \mathbb{E}[\Pi_{bk}] &= \mathbb{E}\left[\min\left\{R(\mathbf{q}) + x, L(1+r)\right\}\right] \\ &= \mathbb{E}\left[R(\mathbf{q}) + x\right] - \mathbb{E}\left[R(\mathbf{q}) + x - L(1+r)\right]^+ \end{aligned} \tag{5}$$

Since the bank is risk neutral and chooses an interest rate that satisfies equation (3), we can substitute the right-hand side of equation (5) to derive the following expression of the cash flow of the retailer:

$$\Pi(\mathbf{q}) = \mathbb{E}\left[R(\mathbf{q}) + x\right] - (1+r_f)(oq_p + wq - y)^+ \tag{6}$$

Equation (6) shows that the retailer decisions are not influenced by the bank loan and thus she always behaves as an unconstrained newsvendor. This result is compatible with M&M theory [14]. It says that under perfect market assumption (including no bankruptcy cost) and competitively priced loan (fairly), the retailer choose her oprational and financial decision separately.

3.2 Bank's problem

According to our assumptions, the bank always offers a fairly priced loan and is risk neutral. Thus when the loan is fully secured by the collateral of the retailer, the bank chooses risk-free rate r_f as the interest rate of the loan. When the loan is not secured, the bank bears the risk of default loan and uses the probability of bankruptcy of the retailer to compute an interest rate $r = r(\mathbf{q})$ in order to compensate the risk of default on average. As in [12], we can use on the bankruptcy threshold $b(\mathbf{q})$ instead of $r(\mathbf{q})$ from the following relationship:

$$pb(\mathbf{q}) + e\min\left\{q - b(\mathbf{q}), q_p\right\} = \left[(oq_p + wq - y)^+(1+r) - x\right]^+ \tag{7}$$

In other words, $b(\mathbf{q})$ is the minimum realized demand such that the cash flow of the retailer is enough to repay the loan with its interest. Since the probability of bakruptcy for the retailer is equal to $\Phi(b(\mathbf{q}))$, the bank can calculate its expected loan repayment as a function of $b(\mathbf{q})$:

$$\mathbb{E}\left(\Pi_{bk}(b(\mathbf{q}))\right) = \begin{cases} p\int_0^{b(\mathbf{q})} \xi\phi(\xi)d\xi + eq_p + x + pb(\mathbf{q})\bar{\Phi}(b(\mathbf{q})) & \text{if } b \leq q - q_p \\ (p-e)\int_0^{b(\mathbf{q})} \xi\phi(\xi)d\xi + e(q_p - q)\Phi(q - q_p) & \\ \quad +eq + x + (p-e)b(\mathbf{q})\bar{\Phi}(b(\mathbf{q})) & \text{otherwise} \end{cases} \tag{8}$$

offered contracts	μ	σ	q	q_p	Π_r	%	r	$b(\mathbf{q})$
put & wholesale	100	30	96.99	25.61	19.29	4.80%	2.00%	0.00
only wholesale	100	30	83.05	-	18.04	-	2.00%	0.00
put & wholesale	200	60	193.97	51.22	38.56	4.82%	2.01%	12.19
only wholesale	200	60	166.09	-	36.79	-	2.01%	17.84
put & wholesale	300	90	290.96	76.84	57.72	4.93%	2.08%	68.82
only wholesale	300	90	249.14	-	55.01	-	2.14%	77.28
put & wholesale	400	120	387.95	102.45	76.70	5.03%	2.18%	125.64
only wholesale	400	120	332.19	-	73.03	-	2.30%	136.98
put & wholesale	500	150	484.93	128.06	95.51	5.17%	2.29%	182.59
only wholesale	500	150	415.23	-	90.81	-	2.48%	196.87

Table 1: Numerical results with different demand size.

Thus we can find $b(\mathbf{q})$ by substituting equation (8) in the risk neutral equation:

$$(oq_p + wq - y)(1 + r_f) = \mathbb{E}\left(\Pi_{bk}(b(\mathbf{q}))\right) \tag{9}$$

Finally, $r(\mathbf{q})$ is easily found from equation (7). Note that $\mathbb{E}\left(\Pi_{bk}(b(\mathbf{q}))\right)$ is a increasing function in $b(\mathbf{q})$ and $b(\mathbf{q})$ is a increasing function in $r(\mathbf{q})$. Therefore, there exists a unique interest rate for the requested loan from the bank's perspective.

4 Numerical experiments

In this section, we present the results of numerical experiments that illustrate the effect of put option on the bankruptcy risk, loan's interest rate and the performance of the retailer. First we consider that the retailer is facing two contracts, wholesale price $w = 0.7$, the put option with option price $o = 0.1$ and exercise price $e = 0.6$. The retailer has a working capial $y = 40$, a collateral $x = 60$ and sells her product to the final customers price $p = 1$. The demand is normally distributed (hence IFR). We assume that the risk-free rate is equal to $r_f = 2\%$.

We conduct these numerical experiments for different demand sizes. For the same capital structure, increasing the demand size is equivalent to consider a poorer retailer. Table 1 summarizes the results of our experiments and shows the performance of the retailer, the bankruptcy risk (indicated by $b(\mathbf{q})$) and the loan interest rate proposed by the bank. For each demand size we consider the results of two scenarios, namely the presence of absence of a put option contract. The numerical results show that the existence of a put option always increases the expected profit of the retailer and her order quantity while having financial constraint or not. When the retailer receives a loan and there exists a bankruptcy risk, the interest rate of the loan and the bankruptcy risk are reduced by using a put option contract in addition to the wholesale price contract. The effect of the put option contract on the retailer's performance is more significance when the retailer is facing financial constraints.

5 Conclusion and further research

We investigate the effect of complementing a wholesale price contract with a put option contract on the performance of a newsvendor with limited capital. The retailer uses her working capital to cover her costs but has the possibility to receive a loan from a bank if necessary. Our numerical results illustrate that existence of the put option contract increases her order quantity to reach a greater expected terminal cash flow while lowering both the bankruptcy risk and the loan's interest rate.

It is shown in the literature that the supplier can design such contracts to achieve the supply chain coordination [19]. An interesting question is thus whether a put option contract benefits every partner in the supply chain by reducing the probability of bankruptcy for the retailer. One may also consider the behavior of a supplier with the different contracts presented above, or even add the possibility of a trade credit instead of a bank loan. Finally as option contracts are risk hedging tools, the risk attitude of the retailer is likely to play a significant role in the performance of the whole system, hence incorporating this aspect into the model may bring valuable information for managers.

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