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# Self-emitted surface corrugations in dynamic fracture of silicon single crystal

Meng Wang<sup>a</sup>, Marion Fourmeau<sup>a</sup>, Lv Zhao<sup>b,c,1</sup>, Franck Legrand<sup>a</sup>, and Daniel Nelias<sup>a,1</sup>

<sup>a</sup>Univ Lyon, INSA-Lyon, LaMCoS UMR CNRS 5259, F69621 Villeurbanne, France; <sup>b</sup>Department of Mechanics, Huazhong University of Science and Technology, Wuhan, 430074, China; <sup>c</sup>Hubei Key Laboratory of Engineering Structural Analysis and Safety Assessment, Wuhan, 430074, China

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1 **When dynamic crack front travels through material heterogeneities,**  
2 **elastic waves are emitted, which perturb the crack and change the**  
3 **morphology of fracture surface. For asperity-free crystalline ma-**  
4 **terials, crack propagation along preferential cleavage planes is ex-**  
5 **pected to present a smooth crack front and form a mirror-like frac-**  
6 **ture surface. Surprisingly, we show here that in single crystalline**  
7 **silicon without material asperities, the crack front presents a local**  
8 **kink during high-speed crack propagation. Meanwhile, local oscil-**  
9 **lations of the crack front, which can move along the crack front,**  
10 **emerge at the front kink position and generate periodic fracture sur-**  
11 **face corrugations. They grow from angstrom amplitude to few hun-**  
12 **dreds of nanometers and propagate with long lifetime at a frequency-**  
13 **dependent speed, while keeping a scale-independent shape. In par-**  
14 **ticular, the local front oscillations collide in a particle-like manner**  
15 **rather than proceeding with a linear superposition upon interaction,**  
16 **which presents the characteristic of solitary waves. We propose that**  
17 **such propagating mode of the crack front, which results from the**  
18 **fracture energy fluctuation at a critical crack speed in the silicon crys-**  
19 **tal, can be considered as novel nonlinear elastic waves that we call**  
20 **'corrugation waves'.**

silicon single crystal | dynamic fracture | crack front kink | corrugation waves | solitary waves

1 Crack propagation is the main cause of catastrophic material failure. Although dynamic fracture has been intensively  
2 studied for decades, crack dynamics in crystalline materials still challenge our understandings (1, 2), as they are more com-  
3 plicated than that can be predicted by the classical theories on brittle crack propagation due to the inelastic phenomena at  
4 the atomic scale (3, 4). Linear elastic fracture mechanics (5) describe a crack tip in a two-dimensional medium as an energy  
5 sink, around which all dissipation processes occur during crack propagation. Crack speed  $v$  is determined through the balance  
6 between the energy flux into the crack tip  $G$  and the fracture energy  $\Gamma(v)$  (energy needed to advance the crack tip of unit  
7 length at  $v$ ). Regarding three-dimensional systems, crack front (the leading edge of the crack) should be considered. Local  
8 crack speed  $v_l$  at each point of the crack front is governed by local energy balance  $\mathcal{G}_l = \Gamma_l(v_l)$ , and the ensemble of local  
9 energy balances in turn controls the whole crack front shape (6, 7).

10 The crack front is generally stable and produces 'mirror-like' fracture surface under pure tension (8, 9). However, it  
11 may lose stability at high speed and generate 3D markings on fracture surface (10–14). A recent study (7) showed that the  
12 crack front can develop localized topological defects drifting along the front that lead to faceted fracture surfaces. Elas-  
13 tic waves also perturb the crack front and generate fracture surface undulations. Shear waves that propagate in the bulk  
14 solids have been shown to emit from material surface flaws

and generate the well-known Wallner lines (15–19). Structural Lamb waves, generated upon debonding of implanted brittle layers and reflected from sample boundaries, can disturb the crack front and leave large scale periodic surface patterns (20). Moreover, a particular concept of one-dimensional elastic waves that exist only on the crack front, called crack front waves, emerged since the 1990s posterior to numerical predictions of persistent crack front oscillations caused by local heterogeneities of fracture energy in a linear elastic medium (6, 21–23). These waves propagate along the crack front at nearly the Rayleigh wave speed and locally change the in-plane and out-of-plane crack motion (24–26). Surface markings produced by the front waves were first reported in fracture tests on soda-lime glass incorporating artificial material asperities (27–29). Front wave traces emanated from both material asperities and crack micro-branching instabilities were observed (28, 29). Based on the post-mortem analysis of the fracture surface markings, the front waves were found to be solitary waves that are localized and long-lived. They present a unique scale-independent shape when scaling both the wavelength and the wave amplitude by the width of the initial perturbation, and exhibit a phase shift upon interaction (27). In this work, we report specific fracture surface corrugations in (110) cleavage of silicon single crystal, which only emerge in high-speed cracks. The corrugations initiate from local crack front kink and have a high correlation with the crack speed. Their morphology cannot be explained by any deflection scenario known for the (110) cleavage of silicon crystals. We propose that

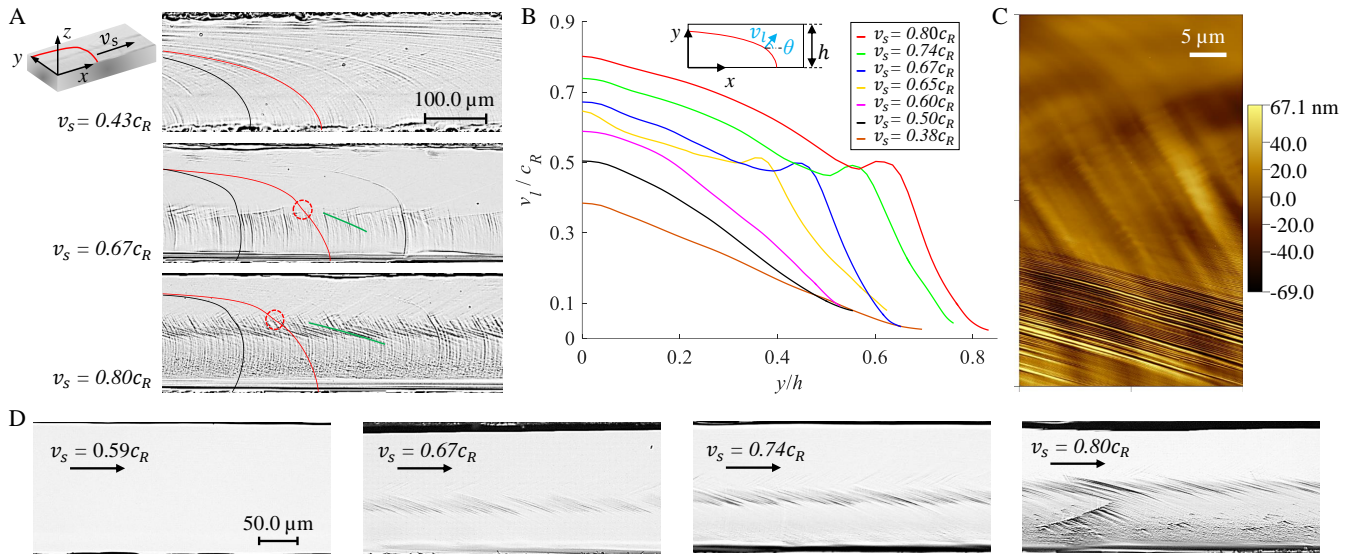
## Significance Statement

When moving crack front meets local material asperities, persistent nonlinear propagating mode of the crack front can be triggered, which results in non-trivial fracture surface patterns. Unexpectedly, we report continuous emergence of corrugation waves during high-speed crack propagation in asperity-free silicon single crystal. We reveal that these waves are driven by fracture energy fluctuation at critical crack speed. The corrugation waves are highly localized along the crack front. They twist the crack front and create fracture surface corrugations. Through meticulous quantitative characterization of the fracture surface markings, we evidence the nonlinear properties of these corrugation waves.

M.W., M.F., and F.L. performed the experiment and analyzed the data. M.W. and L.Z. carried out the finite element calculations. D.N. and L.Z. were involved in all aspects of the analysis of the results. All authors contributed to the writing of the manuscript.

The authors declare no conflict of interest.

<sup>1</sup>To whom correspondence should be addressed. E-mail: daniel.nelias@insa-lyon.fr, lvzhao@hust.edu.cn



**Fig. 1. Dynamic crack front behavior in steady-state (110) cleavage of the single crystalline silicon samples under pure bending.** (A), Morphologies of the (110) cleavage plane ( $xy$  plane) of the as-sawn samples at different steady-state crack propagation speeds  $v_s$ . The crack propagates along the  $[1\bar{1}0]$  direction ( $x$  axis). The Wallner lines and crack fronts are highlighted by the black and red curves, respectively. The local kinks of the crack front are highlighted by the red dashed circles and the surface corrugations are highlighted by the green lines. (B), Normalized local crack front speed  $v_l/c_R$  as a function of normalized vertical position  $y/h$  along the crack front, for seven experiments with different  $v_s$ . The inset illustrates the sample thickness  $h$  and the angle  $\theta$  that represents the local normal direction of the crack front (red curve). (C), AFM measurement of the local front kink zone shows the emergence of the surface corrugations. (D), Morphologies of the (110) cleavage plane of the surface-polished samples at different  $v_s$ . Fracture surface morphologies exhibit the mirror-like feature at  $v_s = 0.59c_R$ , while the presence of surface corrugations at  $v_s = 0.67c_R$  (below  $0.45h \pm 0.01h$ ),  $0.74c_R$  (below  $0.51h \pm 0.02h$ ) and  $0.80c_R$  (below  $0.59h \pm 0.02h$ ).

56 these self-emitted corrugations are traces of a new kind of non-  
 57 linear elastic waves, namely corrugation waves, that nucleate  
 58 from the fracture energy fluctuation at a critical crack speed.  
 59 The corrugation waves share certain properties with the crack  
 60 front waves. We show, in addition to the scale-independent  
 61 characteristic shape (27, 28), two intriguing attributes of the  
 62 corrugation waves, i.e., nonlinear dispersion and particle-like  
 63 interaction dynamics.

## 64 Result

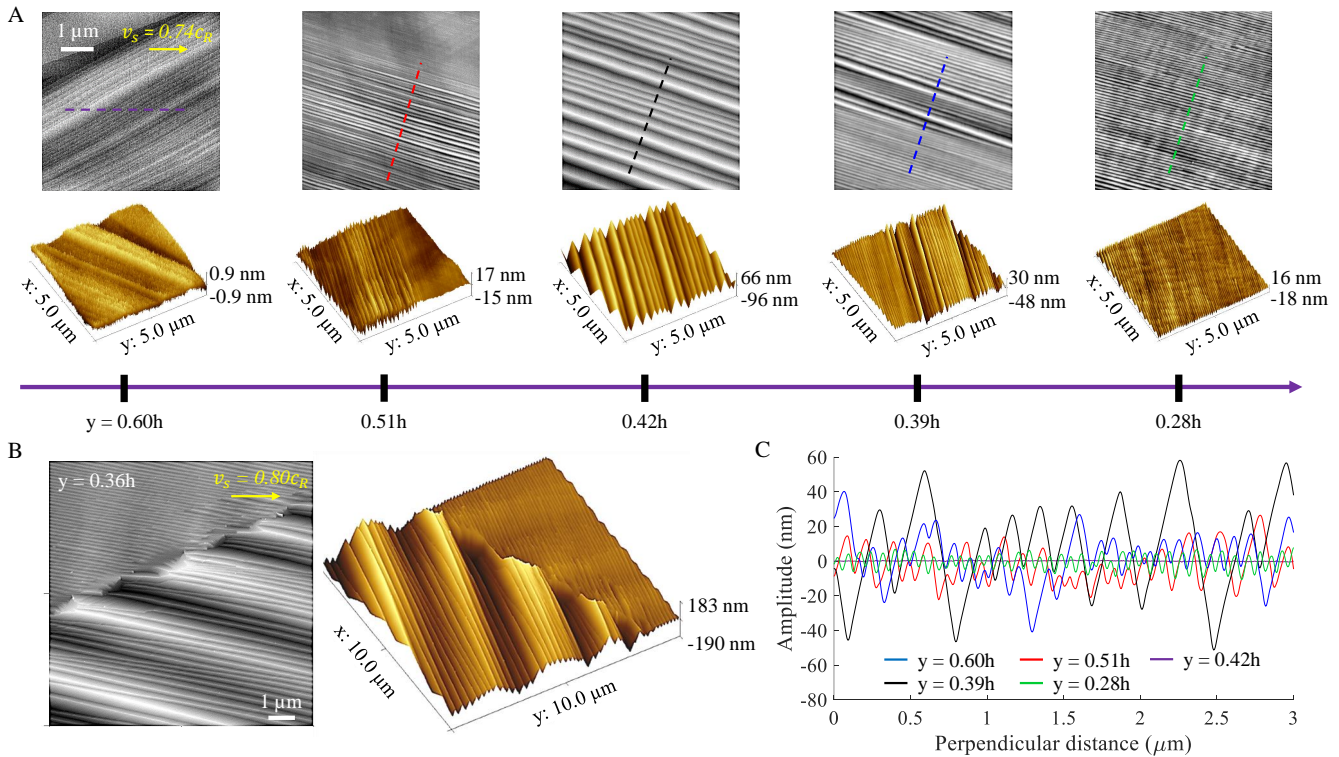
65 Our fracture experiments were first performed on as-sawn  
 66 single crystalline silicon plates under pure bending, which  
 67 leads to the (110) cleavage (see Materials and Methods). Due  
 68 to the bending stress gradient along the sample thickness, the  
 69 local crack speed decreases from the bottom side to the top  
 70 side and the leading front of the crack is curved. Note that  
 71 the bending stress is normal to the fracture surface, the crack  
 72 front thus propagates under pure opening mode (30). The  
 73 crack front is found to rapidly accelerate to the steady-state  
 74 regime (see SI Appendix, Texts S1-S2 and Figs. S1-S3), as  
 75 also reported in refs.(19, 31–34). Thanks to various seed  
 76 crack sizes, a wide range of steady-state crack speeds  $v_s$ , from  
 77  $0.23c_R$  to  $0.82c_R$  was obtained, where  $c_R$  represents the  
 78 Rayleigh wave speed and is equal to 4460 m/s for the (110)[ $1\bar{1}0$ ]  
 79 direction (35), allowing extensive examinations of crack dynamics.  
 80 Fig. 1A shows the (110) cleavage surfaces at different  $v_s$ .  
 81 Typical Wallner lines (black curves) generated from as-sawn  
 82 surface flaws can be observed (see also SI Appendix, Text S3  
 83 and Fig. S4). The invariant shape of the Wallner lines reveals  
 84 the steady-state propagation of the whole crack front. The  
 85 crack front shape (red curves in Fig. 1A) is reconstructed via  
 86 the retroactive decomposition of the Wallner lines (see SI  
 87 Appendix, Text S4 and Fig. S5). It is found that, at  $v_s < 0.62c_R$ , the

crack front exhibits a smooth quarter-ellipse shape, while it  
 incorporates a local kink at higher speeds, as highlighted by  
 the red circles in Fig. 1A. The front kink appears at an upper  
 position along the crack front when  $v_s$  increases. Based on  
 the steady-state crack front shape and speed, the local velocity  
 at each point of the crack front  $v_l$ , assumed to be normal to  
 the crack front, thus forming an angle  $\theta$  relative to the  $x$   
 direction (insert in Fig. 1B), can be calculated as  $v_s \cos(\theta)$   
 (30, 31). The local crack speed profiles, normalized by  
 anisotropic  $c_R$  for the (110) plane (35), are shown in Fig.  
 1B. In low-speed cases,  $v_l$  monotonically decreases from  
 the bottom to the top along the crack front, while in high-  
 speed cases, the local kink of the crack front is translated  
 into a fluctuation in  $v_l$ , which takes place at around  
 $0.5c_R$ , regardless of  $v_s$ .

When  $v_s > 0.62c_R$ , surface corrugations arise on the  
 fracture surface, jointly with the crack front kink. As  
 highlighted by the green lines in Fig. 1A, both the onset  
 height and the orientation of these corrugations relative to  
 the  $x$  direction have a strong dependency on the crack  
 front speed (see SI Appendix, Text S2 and Figs. S2-S3  
 for more details). They nucleate at a nearly constant  
 height during the steady-state crack propagation. Interestingly,  
 the nucleation height rises with the increase of  $v_s$ ,  
 adhering to the local front kink position (see Fig. 1C  
 and SI Appendix, Fig. S2-S3), where  $v_l$  fluctuates at  
 around  $0.5c_R$ . Therefore, the crack front kink, the  
 $v_l$  fluctuation and the appearance of surface corrugations  
 are coupled, crack speed-dependent events in high-speed  
 cracks.

To study accurately the morphological features of the  
 fracture surface corrugations, a second experimental  
 campaign was carried out on surface polished silicon  
 samples to exclude the Wallner lines (see Materials  
 and Methods). Fig. 1D shows the mirror-like fracture  
 surface at  $v_s = 0.59c_R$  and corrugated fracture surfaces  
 at  $v_s = 0.67c_R$ ,  $0.74c_R$  and  $0.80c_R$  for the pol-





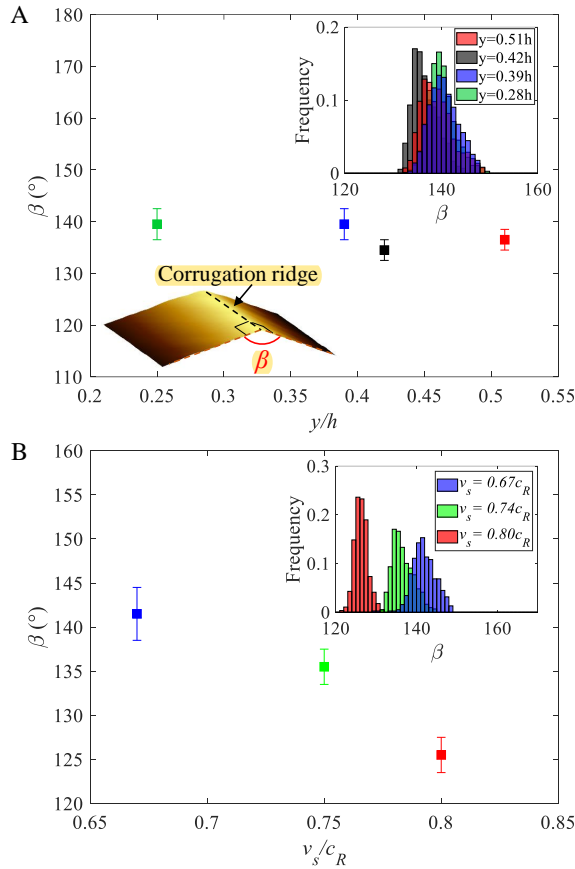
**Fig. 2. AFM measurement of the fracture surface corrugations in the polished single crystalline silicon samples.** (A), Topographies of the surface corrugations at  $v_s = 0.74c_R$  at different heights. (B), Collapse of previous corrugations and emergence of new ones at high-speed crack instabilities at  $v_{fo} = 0.80c_R$ . (C), Profiles of the surface corrugations (along the perpendicular direction of the corrugation ridge) at different life stages at  $v_s = 0.74c_R$ . The profiles were extracted along the dashed lines highlighted in (A) with the corresponding colors.

121 ished samples. The onset position of corrugations still adheres  
 122 to the crack front kink (see SI Appendix, Fig. S2). Note here  
 123 that the surface corrugations on the two opposing fracture sur-  
 124 faces involve a peak-to-valley match (see SI Appendix, Fig. S6).  
 125 This should be distinguished from the particular nanoscale  
 126 corrugations resulting from local plasticity ahead the crack  
 127 tip which presents a peak-to-peak match (36). Nanoscale to-  
 128 pographies of the surface corrugations are analyzed via the  
 129 atomic force microscope (AFM) measurements. Fig. 2A shows  
 130 five typical topographies at  $v_s = 0.74c_R$  for different locations  
 131 along  $y$  (the definition of  $y$  is given in the inset of Fig. 1A).  
 132 At  $y = 0.6h$ , the fracture surface is flat and contains only  
 133 sub-nano atom debonding lines pointing towards the upper  
 134 side of the sample. The surface corrugations nucleate at  $y$   
 135  $= 0.51h \pm 0.02h$ , which corresponds to the local front kink  
 136 position. Their amplitudes grow from an angstrom level ( $y =$   
 137  $0.51h$ ) to a maximum of roughly hundred nanometers ( $y =$   
 138  $0.42h$ ). Outside the front kink zone ( $y = 0.39h$ ), the corru-  
 139 gations rapidly decay in both amplitude and width. After the  
 140 decay (below  $y = 0.35h$ ), the surface corrugations extend at a  
 141 nearly constant amplitude of about 10 nm with long lifetime,  
 142 till the bottom of the fracture surface.

143 In particular, Fig. 2B highlights the occurrence of high-  
 144 speed crack instability, which induces an abrupt crack path  
 145 deviation and extends along the local front normal direction  
 146 (see SI Appendix, Text S5 and Fig. S7 for more details). In this  
 147 region, previous corrugation markings collapse and new single  
 148 or multiple corrugation markings arise along the instability  
 149 line. Their amplitudes are maximum upon nucleation (the  
 150 amplitude evolution is detailed in SI Appendix, Fig. S8). The

151 surface corrugations all exhibit wave-like feature along the  
 152 perpendicular direction of the corrugation ridge, as shown in  
 153 Fig. 2C. The angle between the corrugation planes, denoted by  
 154  $\beta$ , is measured to further reveal the geometrical characteristics  
 155 of the corrugations. Fig. 3A highlights the  $\beta$  distribution at  
 156 different life stages for  $v_s = 0.74c_R$ . The mean value of  $\beta$  is  
 157 about  $137^\circ$  at the nucleation stage of the corrugations. It then  
 158 decreases to approximately  $135^\circ$  during the growth and finally  
 159 increases to  $140^\circ$  after the decay of the corrugations. Fig. 3B  
 160 shows the dependency of  $\beta$ , measured in the zone presenting  
 161 the maximum amplitude of the surface corrugations, on the  
 162 crack speed  $v_s$ . It can be noticed that  $\beta$  decreases with the  
 163 increase of  $v_s$  (the distributions of  $\beta$  for  $v_s = 0.67c_R$  and  
 164  $0.80c_R$  at different locations are shown in SI Appendix, Fig.  
 165 S9). Moreover, the surface corrugations extend along various  
 166 crystallographic directions depending on the crack speed (see  
 167 SI Appendix, Figs. S2-S3). Therefore, one can conclude that  
 168 the surface corrugations do not coincide with any specific  
 169 crystal plane.

170 It is noteworthy that specific surface patterns, called facets,  
 171 were also reported in the (110) cleavage of silicon crystal  
 172 under tension when the crack speed exceeds  $0.67c_R$  (9). These  
 173 facets appear in the mirror-like zone of the fracture surface  
 174 and clearly lie on {111} planes towards the end of the mirror  
 175 zone when the crack speed is high. They display hills and  
 176 valleys with a length-scale of 100-1000 nm, which are similar  
 177 to the surface corrugations in the present study. However, the  
 178 facets reported in ref.(9) extend only in the  $[1\bar{1}0]$  direction  
 179 throughout the fracture surface, regardless of the crack speed.  
 180 This is well different from the surface corrugations shown



**Fig. 3. Distribution of angle  $\beta$  between corrugation planes.** (A), Variations of  $\beta$  as a function of the normalized vertical position  $y/h$  at  $v_s = 0.74c_R$ . The mean values of  $\beta$  and the standard deviations were determined from the histograms of  $\beta$  shown in the inset on the top right, which were computed from the profiles of the corrugations measured perpendicular to the corrugation ridge at different life stages presented in Fig. 2A. Schematic drawing of  $\beta$  is shown in the inset on the bottom left. (B), Dependency of  $\beta$  on  $v_s$ . The mean values of  $\beta$  and the standard deviations were determined from the histograms of  $\beta$  shown in the inset, computed from the profiles of the corrugations measured perpendicular to the corrugation ridge in the zone where the corrugations extend with the maximum amplitude at different  $v_s$ .

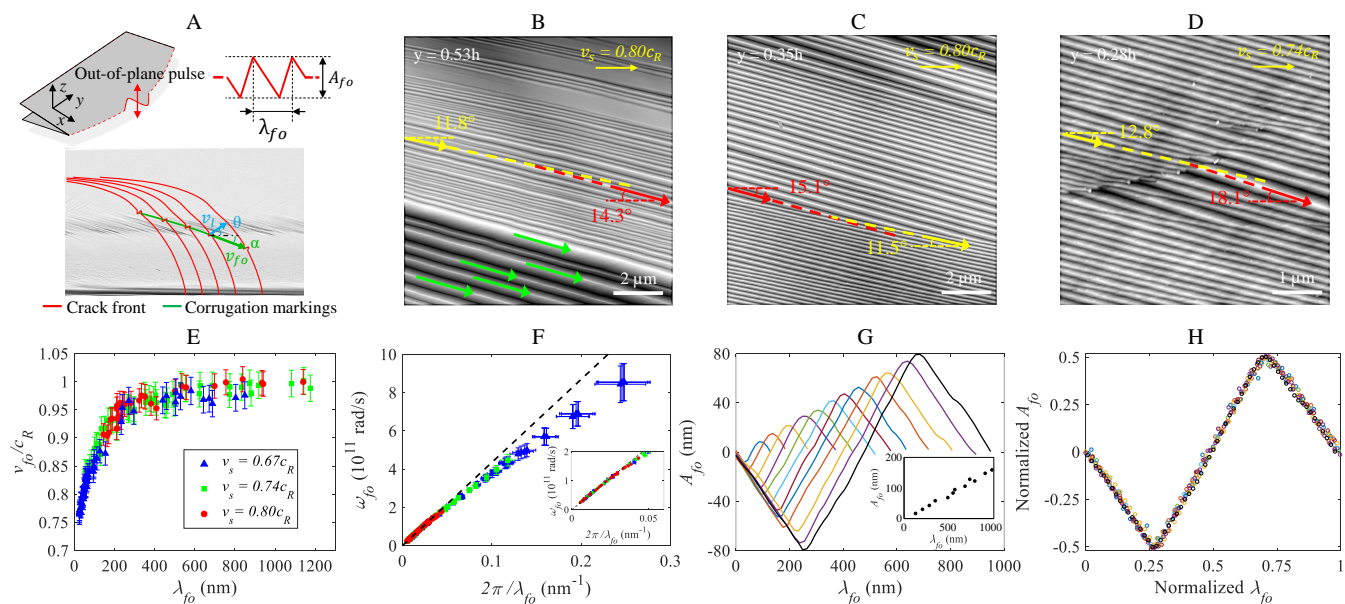
181 here, of which both the nucleation position and the extending  
 182 direction exhibit a strong dependency on the crack front speed.  
 183 Moreover, the surface corrugations do not coincide with the  
 184  $\{111\}$  planes even when  $v_s = 0.80c_R$  (the comparison of the  
 185 geometry between the surface corrugations and the  $\{111\}$   
 186 facets is presented in SI Appendix, Fig. S10). Hence, the  
 187 surface corrugations are different from the facets reported in  
 188 ref.(9). The long-lived propagation with a well-defined shape  
 189 of the surface corrugations (which extend over four decades  
 190 and keep a nearly invariant shape with an amplitude of one  
 191 decade) reveals that they are generated by highly localized  
 192 out-of-plane perturbations along the crack front, i.e., the local  
 193 crack front oscillations.

194 The characteristics of the local crack front oscillations are  
 195 assessed with the periodic surface corrugations. As illustrated  
 196 in Fig. 4A, when the front oscillations propagate at speed  $v_{fo}$ ,  
 197 they generate surface corrugations oriented with an angle  $\alpha$   
 198 relative to the  $x$  direction. As the surface corrugations are  
 199 the traces of the front oscillations, their profiles along the  
 200 crack front therefore provide the shape of the front oscillations  
 201 (wavelength  $\lambda_{fo}$  and amplitude  $A_{fo}$ ). When extracting

202  $\lambda_{fo}$  and  $A_{fo}$ , we assume that the crack front is smooth at  
 203 the length scale of the surface corrugations, hence the front  
 204 tangent coincides with that of the crack front reconstructed  
 205 at the length scale of the Wallner lines. It is found that the  
 206 orientation of the corrugations  $\alpha$  changes with the wavelength  
 207  $\lambda_{fo}$ . Figs. 4B-D show that  $\alpha$  increases (decreases) when the  
 208 width of the corrugations increases (decreases), as highlighted  
 209 by the yellow and red arrows. This dependence can be observed  
 210 during both the growth (see Fig. 4B) and the decay  
 211 (see Fig. 4C) of the corrugations, as well as upon nucleation  
 212 of new corrugations triggered by high-speed crack instabilities  
 213 (see Fig. 4D). Moreover,  $\alpha$  is constant for corrugations of large  
 214 width, as highlighted by the green arrows in Fig. 4B. Given  
 215 that the speed of the local front oscillations  $v_{fo}$  is related to  
 216  $\alpha$  through  $v_{fo} = v_l/\cos(\alpha+\theta)$ , the correlation between  $\alpha$  and  
 217  $\lambda_{fo}$  can be translated into the relation between  $v_{fo}$  and  $\lambda_{fo}$ .  
 218 As shown in Fig. 4E, for  $\lambda_{fo} > 220$  nm,  $v_{fo}$  is approximately  
 219 equal to the Rayleigh wave speed. However, for  $\lambda_{fo} < 220$   
 220 nm,  $v_{fo}$  significantly drops when  $\lambda_{fo}$  decreases, indicating  
 221 the dependence of  $v_{fo}$  of the front oscillation on its frequency.  
 222 This dependence can be characterized by the frequency of the  
 223 front oscillations  $\omega_{fo}$  (given by  $2\pi v_{fo}/\lambda_{fo}$ ) as a function of  
 224 the wavenumber (given by  $2\pi/\lambda_{fo}$ ). The relation becomes  
 225 nonlinear above  $\omega_{fo} = 1.23 \times 10^{11}$  rad/s (inset of Fig. 4F).

226 Furthermore, during the growth (decay) of the crack front  
 227 oscillations, we reveal that  $\lambda_{fo}$  increases (decreases) when  $A_{fo}$   
 228 increases (decreases). As a result, sparser corrugations exhibit  
 229 larger amplitude (see the AFM measurements in Fig. 2C). To  
 230 further quantify the relationship between  $\lambda_{fo}$  and  $A_{fo}$ , the  
 231 shapes of the front oscillations are extracted from the surface  
 232 corrugations at  $v_s = 0.80c_R$ , in the region where  $y$  varies between  
 233  $0.46h$  and  $0.36h$  (i.e.,  $\theta$  varies between  $42^\circ$  and  $33^\circ$ ).  
 234 The corresponding shapes of the front oscillations are plotted  
 235 in Fig. 4G. It can be noticed that  $\lambda_{fo}$  exhibits approximately  
 236 a linear correlation with  $A_{fo}$  (see inset, Fig. 4G). Interestingly,  
 237 the local front oscillations present a scale-independent  
 238 characteristic shape upon normalization over  $A_{fo}$  and  $\lambda_{fo}$  (see  
 239 Fig. 4H). It should be noted that the characteristic shape  
 240 is not affected by the lattice structure of the silicon crystal,  
 241 given that the front oscillations travel on the anisotropic (110)  
 242 crystal plane.

243 In addition to the frequency-dependent speed and the scale-  
 244 independent shape, we report another property of the local  
 245 front oscillations, which is reflected by their particle-like  
 246 interaction behavior. As can be appreciated in Figs. 5A and  
 247 B, when a fast moving oscillation (long  $\lambda_{fo}$ ) meets a slow  
 248 one (short  $\lambda_{fo}$ ), they exhibit a particle-like collision. The  
 249 interaction scenario highlighted here is that the fast moving  
 250 oscillation decays and decelerates while the slow one grows  
 251 and accelerates (see Fig. 5B). It is noteworthy that the linear  
 252 correlation between  $\lambda_{fo}$  and  $A_{fo}$  is retained, hence the shape  
 253 of the front oscillation remains scale-independent during the  
 254 interaction. After the collision, both the shape and the speed  
 255 of the front oscillations exchange and an orientation shift occurs  
 256 for the surface corrugations involved in the interaction.  
 257 The collision events are more frequent when successive inter-  
 258 actions of the crack front oscillations take place. As shown  
 259 in Figs. 5C-D, alternative growth (acceleration) and decay  
 260 (deceleration) of the front oscillations occur during the consecutive  
 261 collisions, which generate the surface corrugations with  
 262 repeated variations in orientation and amplitude.



**Fig. 4. Characterization of local crack front oscillations.** (A), Local crack front oscillations with the out-of-plane amplitude  $A_{fo}$  and wavelength  $\lambda_{fo}$  are localized along the crack front (red curve), propagate at  $v_{fo}$ , and leave corrugation markings forming an angle  $\alpha$  relative to the  $x$  direction.  $\lambda_{fo}$  is defined as the distance between two adjacent crests of the periodic oscillations. (B)-(D), Dependency of the tilt angle  $\alpha$  of the corrugation markings at different life stages when  $\lambda_{fo} < 220$  nm. The variations of  $\alpha$  are highlighted by the yellow and red lines. (B), The growth of the corrugation markings shows the increase of  $\alpha$  with the increase of  $\lambda_{fo}$ . The green arrows highlight that corrugation markings with  $\lambda_{fo} > 220$  nm present constant  $\alpha$ . (C), The decay of the corrugation markings shows the decrease of  $\alpha$  with the decrease of  $\lambda_{fo}$ . (D), The new corrugation markings with longer  $\lambda_{fo}$  triggered by the high-speed crack instabilities show larger  $\alpha$  in comparison to previous markings. (E), The normalized speed of front oscillations  $v_{fo}/c_R$  as a function of the wavelength  $\lambda_{fo}$ . (F), The frequency of local front oscillations  $\omega_{fo}$  as a function of the wavenumber  $2\pi/\lambda_{fo}$ . The linear relation is highlighted by the black dashed line. A zoom of the linear part with  $\omega_{fo} < 1.23 \times 10^{11}$  rad/s is highlighted in the inserted graph. The uncertainties of  $v_{fo}$  and  $\lambda_{fo}$  result from the  $v_s$  measurement by the high-speed camera and the surface marking measurement by the AFM. (G), Shape of the front oscillations. The shapes with  $A_{fo}$  varying between 16.6 nm and 159.6 nm were extracted from the AFM measurements along the crack front at  $v_s = 0.80c_R$ . The inset shows the linear correlation between the amplitude  $A_{fo}$  and the wavelength  $\lambda_{fo}$  of front oscillations. (H), Scale-independent characteristic shape of the local front oscillations. The shapes of front oscillations are normalized over their amplitudes and wavelengths, i.e., the abscissa and the ordinate are normalized by  $\lambda_{fo}$  and  $A_{fo}$ , respectively.

## Discussion and conclusion

Why the crack front locally kinks and the local crack speed fluctuates when  $v_s > 0.62c_R$ ? According to the linear elastic fracture mechanics, the local energy balance for each point  $y$  of a propagating mode I crack front reads (6):

$$\mathcal{G}_l[\{F(y', t' \leq t)\}, y, t] = \Gamma_l[x = F(y, t), y] \quad [1]$$

The region  $x < F(y, t)$  represents the crack wake and  $x = F(y, t)$  corresponds to the current crack front. The coordinates  $x$  and  $y$  are the same as in Fig. 1A. The local energy flux  $\mathcal{G}_l$  depends on the crack velocity as well as the history of the whole crack front  $\{F(y', t' \leq t)\}$ . The local energy flux  $\mathcal{G}_l$  is given by the general form:

$$\mathcal{G}_l = G_l[\{F(y', t' < t)\}, y, t]g(v_l) \quad [2]$$

with

$$g(v_l) \simeq 1 - v_l/c_R \quad [3]$$

where  $G_l$  is the local strain energy release rate at  $(y, t)$  for a crack at rest, which is independent of the crack velocity, and  $v_l$  is the local velocity normal to the crack front. In our case, since we only consider the crack propagation at a steady-state speed  $v_s$  with an invariant shape  $F^*(y)$ ,  $F \rightarrow F + \delta F = F^*(y) + v_s t$ . The dependency of  $G_l$  on the front history  $\{F\}$  vanishes. The local energy balance can be simplified as:

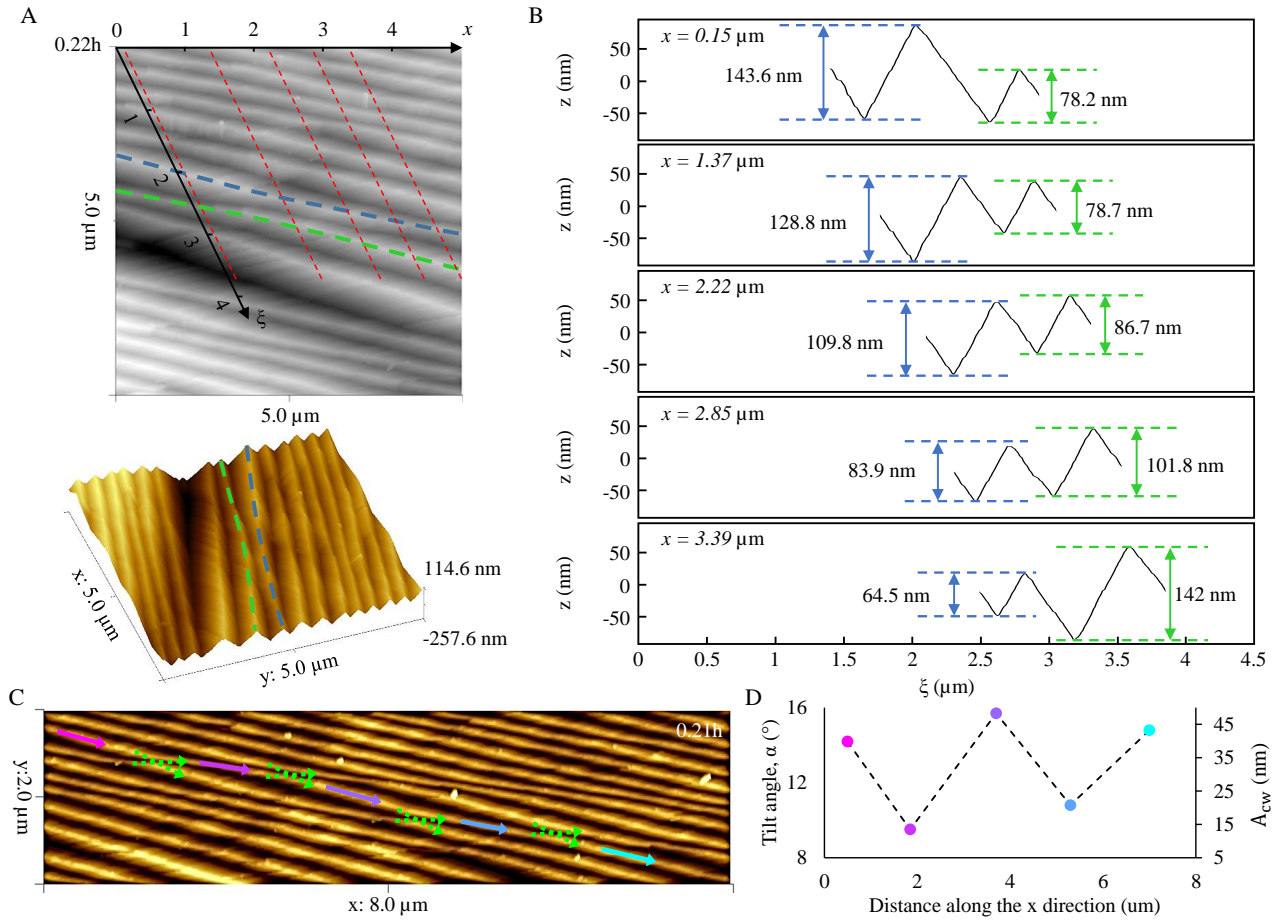
$$\Gamma_l[F^*(y) + v_s t, y] = G_l^s[F^*(y) + v_s t, y]g(v_l) \quad [4]$$

where  $G_l^s$  is the static local strain energy release rate that can be determined at a given moment  $t = t_0$  with the cracked sample geometry  $F^*(y) + v_s t_0$  and the applied load.

Equation. (4) implies that the  $v_l$  fluctuation related to the front kink at the high-speed crack front should be the response to an abnormal variation in  $\Gamma_l$ . To verify this conjecture, the instantaneous  $G_l^s$  was computed using 3D finite element simulations where the real steady-state crack front shapes and the experimental conditions were considered (see SI Appendix, Text S7 and Fig. S11). Only the crack front at the very beginning of the steady-state regime was taken into account, where  $t_0$  was very small since the crack accelerates very rapidly to the steady state, as reflected by the invariant Wallner lines (see SI Appendix, Fig. S3C). Hence, the stress field ahead of the crack front would not be affected by the stress waves reflected from the sample boundaries (see SI Appendix, Text S7), and Eq. (4) therefore remains valid, as also proposed in ref.(5). Moreover, as we just consider the steady-state crack propagation at very small  $t_0$ , the applied load at  $t = t_0$  could be assumed to be the same as that at the crack initiation. The local fracture energy  $\Gamma_l$  is then calculated using Eq. (4) with the known  $v_l$  (Fig. 1B),  $G_l^s$  (SI Appendix, Fig. S11E) and  $c_R$  (35).

Seven cracks with  $v_s$  varying from  $0.38c_R$  to  $0.80c_R$  are analyzed. Normalized  $\Gamma_l$  (over  $\Gamma_0$  that represents 2 times the surface energy) along the crack front is displayed in Fig. 6A. One can see that  $\Gamma_l$  is almost constant for  $v_s < 0.60c_R$ , while it significantly increases at certain height for  $v_s > 0.65c_R$ . In particular, the jump occurs at higher position for larger



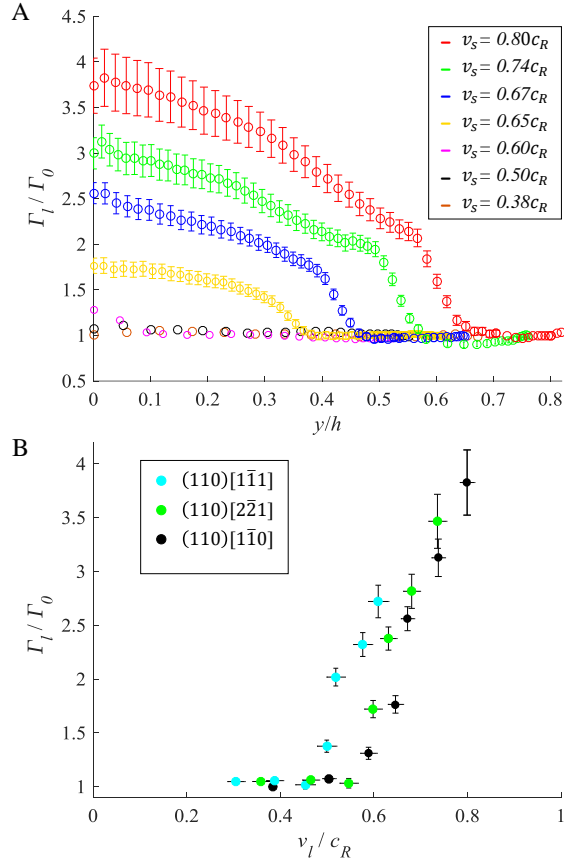


**Fig. 5. Collision behavior of crack front oscillations.** (A), Topographies of the surface corrugations at  $v_s = 0.80 c_R$ . The  $\xi$  axis is the tangent direction of the crack front. Corrugation markings generated by the collision of front oscillations, as highlighted by the blue and green dashed lines, respectively. The corrugation markings exchange their orientation  $\alpha$  after the collision. (B), Sequences of shape evolution of the front oscillations during the collision mentioned in (A). The five sequences along the  $x$  direction are highlighted in (A) by the red dashed lines. At  $x = 0.15 \mu\text{m}$ , different front oscillations propagate at  $v_{fo} = 0.965c_R$  (blue) and  $v_{fo} = 0.935c_R$  (green), respectively, then they exchange their speeds at  $x = 3.39 \mu\text{m}$ . (C), Topographies of the surface corrugations at  $v_s = 0.74 c_R$ ,  $y = 0.21 h$  incorporating successive collisions that result in successive phase shifts, as highlighted by the green dashed arrows. (D), Simultaneous variation of  $\alpha$  and  $A_{fo}$  of the corrugation marking generated by successive collisions of front oscillations, measured from the arrows with the corresponding colors in (C).

315  $v_s$ , which coincides with the crack front kink as well as the  
 316 onset of the corrugation markings. When plotting  $\Gamma_l$  as a  
 317 function of  $v_l$  along various crack fronts (see SI Appendix, Fig.  
 318 S12), we find that  $\Gamma_l$  involves a jump at  $v_l$  of about  $0.5c_R$ .  
 319 This corresponds well to the fact that  $v_l$  fluctuates at  $0.5c_R$   
 320 whatever the global crack speed (see Fig. 1B). Moreover, the  
 321 numerous crack fronts provide a possibility to assess  $\Gamma_l$  versus  
 322  $v_l$  for a given crystallographic direction, which can be analogue  
 323 to the straight tensile crack scenario. Fig. 6B shows  $\Gamma_l$  for  
 324 the  $[1\bar{1}1]$ ,  $[2\bar{2}1]$  and  $[1\bar{1}0]$  directions, represented by the blue,  
 325 green and black marks, respectively. One can see that  $\Gamma_l$   
 326 is nearly equal to the initiation fracture toughness  $2\gamma$  (3, 37),  
 327 when  $v_l$  is low. It then jumps at a critical speed  $v_c$ , which  
 328 depends on the crystallographic direction. For instance,  $v_c$  is  
 329 about  $0.50c_R$  for the  $[1\bar{1}1]$  direction and  $0.60c_R$  for the  $[1\bar{1}0]$   
 330 direction, which are in excellent agreement with refs.(4, 38)  
 331 (see SI Appendix, Fig. S12), and  $v_c = 0.58c_R$  for the  $[2\bar{2}1]$   
 332 direction is for the first time presented here.

333 With the characteristic shape of the front oscillations, we  
 334 assess that the increase of the fracture surface due to the  
 335 surface corrugations is about 6%, which is much lower than  
 336 the increase in  $\Gamma_l$ . No subsurface crack was found under the

337 surface corrugations (see SI Appendix, Fig. S13). Therefore, we  
 338 may adhere to the conjecture made in ref.(9), recently proved  
 339 by molecular dynamics simulations (4, 38), that the sharp  
 340 increase in dynamic fracture energy in silicon crystal results  
 341 from the additional dissipation by phonon emission, which is  
 342 also found to depend on the crystallographic structure of the  
 343 silicon crystal. For the crack front containing the local crack  
 344 speed gradient, the increase in  $\Gamma_l$  induced by phonon emission  
 345 occurs along the front portion where  $v_l \geq v_c$ , while  $\mathcal{G}_l$  is equal  
 346 to  $2\gamma$  for the rest of the front where  $v_l < v_c$ . The jump in  $\Gamma_l$   
 347 takes place at the location where  $v_l = v_c$ . The critical speed  
 348  $v_c$  is found to be about  $0.5c_R$  for the local crystallographic  
 349 direction  $\theta$  varying between  $30^\circ$  and  $50^\circ$ , and it changes for  
 350 other directions, which is in agreement with the anisotropic  
 351 dissipation of the phonon emission revealed in ref.(4, 38) (see  
 352 Fig. 6B and SI Appendix, Fig. S12). Indeed, a sharp increase  
 353 in  $\Gamma_l$  will locally slow down the crack front and the resulting  
 354 fluctuation in  $v_l$  consequently produces a kink of the crack  
 355 front. Note that all material-dependent, dissipative processes  
 356 are embraced in  $\Gamma_l$ , we point out that the local fracture energy  
 357 jump induced by the phonon emission and the consequent  
 358 crack front kink can be well interpreted through local energy



**Fig. 6.** (A), Variation of the normalized local dynamic fracture energy  $\Gamma_l/\Gamma_0$  along the crack front as a function of  $y/h$ , for the same seven fracture cases as in Fig. 1(B) with the corresponding colors. (B),  $\Gamma_l/\Gamma_0$  versus  $v_l/c_R$  along the  $[\bar{1}\bar{1}\bar{1}]$ ,  $[2\bar{2}\bar{1}]$  and  $[\bar{1}\bar{1}\bar{0}]$  directions, presented by the blue, green and black marks. The error bars are obtained from the uncertainties in the crack speed measurements.

balances  $\mathcal{G}_l = \Gamma_l(v_l)$  in the framework of LEFM.

The local fracture energy jump reported here is supposed to induce only an in-plane fluctuation of local crack speed and consequently alter the planar crack front dynamics. Nevertheless, it breaks down the translational invariance of the moving front and induces abnormal gradient of the energy flux  $d\mathcal{G}_l/dy$  at the front kink site. As suggested by theoretical predictions and experimental observations, the abnormal  $d\mathcal{G}_l/dy$  fulfills the nucleation condition of crack front waves (6, 21, 29). Therefore, the emission of the fracture surface corrugations at the  $\Gamma_l$  jump zones indicates that the front oscillations share the generation condition with the front waves, i.e., local fracture energy fluctuation ahead of the crack front. We conjecture that, like the out-of-plane corrugations (24) and the front waves (39), any slight out-of-plane perturbation (for instance the sub-nano atom debonding lines, see Fig. 2A) could trigger the out-of-plane component of the front oscillations which sprouts from the atomic scale.

Since the local front oscillations can travel along the crack front, they present a propagating mode of the crack front. This propagating mode can not be considered as the front waves, as they propagate approximately at the Rayleigh wave speed only for the large length scale ( $\lambda_{fo} > 220$  nm). We suggest that the frequency-dependent speed for  $\omega_{fo} > 1.23 \times \text{rad/s}$   $10^{11}$  is due to the nonlinear dispersion of the front oscillations

(40, 41). Nevertheless, the unique characteristic shape of the front oscillations coincides with the property of the front waves (27). It reveals an intrinsic nonlinear attribute of the front oscillations. This is well distinguished from the undulation of the crack front induced by linear elastic waves, for which the wavelength is independent of the amplitude. Taking the Wallner lines (note that the Wallner lines are commonly believed to be generated by shear waves) observed in the as-sawn samples for comparison, it is found that the shear waves nucleated at the surface flaws exhibit a constant wavelength, as reflected by the constant width of the Wallner lines during their decay (see SI Appendix, Fig. S4). Besides, the shape of the shear wave packets is found to depend on the generation source and exhibits no well-defined nature (29) (see also SI Appendix, Fig. S4). Furthermore, the collision events reveal another nonlinear attribute of the front oscillations. Instead of a linear superposition as the linear elastic waves would proceed, the particle-like interaction of the front oscillations is similar to the collision of solitary waves, during which the two pulses exchange their speeds and shapes, resulting in a phase shift (27, 29, 42). Hence, we conclude that the front oscillations reported in this work, which share certain properties with crack front waves, can be considered as nonlinear elastic waves. We call them 'corrugation waves' since they produce periodic out-of-plane corrugations on the fracture surface.

The corrugation waves deform the crack front and lead to an increase of the fracture surface. They create extra energy dissipations and hence can propagate only in the region when  $\mathcal{G}_l > 2\gamma$ . On the upper side of the nucleation site of the corrugation waves,  $\mathcal{G}_l = 2\gamma$  (see Fig. 6A). Therefore, no additional energy can support the corrugation waves to extend to the upper side. This explains why the corrugation waves extend solely to the lower side where  $\mathcal{G}_l > 2\gamma$ . The most distinguishable behavior of the corrugation waves reported here is that they are continuously self-emitted from the moving front and propagate with the characteristics of solitary waves. As a result, periodic surface corrugations are created and cover the whole fracture ligament, as experimentally observed. The continuous nucleation of corrugation waves in response to the fracture energy jump at the critical crack speed could be analogous to the consequence of a crack traveling through substantial heterogeneities, which continually twist the crack front (43, 44). The initial state of the corrugation waves depends on the source that creates them. The gradual growth of corrugation waves at the front kink zone (Fig. 2A) could be explained by the continuous increase in the fracture energy (see Fig. 6A), while the corrugation waves incorporating the maximum amplitude upon emission is due to the abrupt change in fracture energy caused by the high-speed crack instabilities (see Fig. 2B).

In summary, the nonlinear attributes of the corrugation waves reported here, such as the scale-independent characteristic shape, the nonlinear dispersion and the particle-like collision construct a new propagating mode of crack front, which is different from what has been described for the front waves with the numerical modeling and experiments incorporating individual material asperities. Our results may give new insights on soliton-like crack front dynamics, as well as roughness formation on the fracture surface of asperity-free crystalline materials.



## Materials and Methods

**Silicon sample preparation.** Solar-grade single crystalline silicon wafer with a purity of 99.9999% was used. We prepared two types of silicon samples: as-sawn silicon samples and mirror-polished silicon samples. (001) silicon wafers were cut by diamond wire from boron-doped single crystalline silicon ingot with a dopant concentration of  $5.44 \times 10^{15}$  atoms/cm<sup>3</sup>. The error in sawing direction relative to the (001) plane is 3°. The damaged surface of wafers was cleaned up by chemical etching to remove a thin layer, but the wire sawing flaws were partially kept. As-sawn silicon samples of size  $50 \times 50 \times 0.19$  mm<sup>3</sup> were obtained by cleavage from the (001) silicon wafers. For preparing mirror-polished silicon samples, a layer of 10 μm was eliminated by mechanical polishing for both surfaces of the (001) silicon wafers. Mirror-polished silicon samples of  $50 \times 30 \times 0.17$  mm<sup>3</sup> were then cleaved from the polished silicon wafers. The single crystal for all the samples is oriented such that the sample surface is perpendicular to the [001] direction and the two edges are parallel to the <110> directions.

**Fracture experiments.** 100 experiments, including 70 on as-sawn silicon samples and 30 on mirror-polished silicon samples were carried out to ensure the reliability of the experimental results. The samples were loaded under four-line bending condition along the [110] direction (*y* direction) via quasi-static displacement at a strain rate of  $10^{-6}$  s<sup>-1</sup>. The spans of the punch roller and the support are 21 mm and 40 mm, respectively. A single crack that propagates on the (110) plane was driven from a seed crack introduced at the center of the sample edge using a Vickers indenter. The indentation force was well controlled to obtain a sharp seed crack of desired length located on the (110) plane. Fracture process of the bottom surface of the sample was captured with a high-speed camera (Phantom V710). For as-sawn silicon samples, we recorded images with a spatial resolution of 97.6 μm/pixel and an acquisition rate of 180,000 frames per second. For mirror-polished silicon samples, we recorded images at a spatial resolution of 89.3 μm/pixel and an acquisition rate of 340,000 frames per second (the acquisition rate can be increased when the image window is reduced to match the sample dimension). The crack position was identified by the gradient of light intensity resulting from the curvature variation around the crack tip, as the sample was illuminated by a continuous light source (MultiLED G300). The uncertainties on crack speed measurement are 70 m/s and 121 m/s for as-sawn and mirror-polished samples, respectively, since we took an uncertainty of 4 pixels on the crack tip location.

**Post-mortem analysis of fracture surface.** The fracture surfaces of all the tested samples were observed with a digital microscope (Keyence VHX-2000). The fracture surface morphologies are almost identical for similar steady-state speed. The Wallner lines were intensive for the as-sawn samples and few could be seen for the polished samples as surface defects were not completely removed. Their shapes can be strikingly observed and then be extracted after contrast enhancement of fractography. The 3D topographies of the fracture surfaces of mirror-polished samples were measured using AFM (Bruker Dimension 3100 Nanoscope V) with an out-of-plane (along *z* direction) resolution of 0.1 nm. The silicon sample was aligned in the AFM setup so that the edge of the fracture surface was

parallel to one line of the crosshair. The misalignment error is in the order of  $\pm 0.5^\circ$ . The measurements were carried out in a square region of  $5 \times 5 \mu\text{m}^2$ ,  $10 \times 10 \mu\text{m}^2$  or  $20 \times 20 \mu\text{m}^2$ , corresponding to an in-plane (*xy* plane) resolution of 5 nm, 10 nm or 20 nm, respectively. Each direction was scanned back and forth to ensure that there was no bias due to the scanning direction and the speed of the AFM tip. The analysis of AFM measurements was carried out with the Gwyddion software (45).

**Data Availability.** All materials, methods and data supporting the findings of this study are available within the paper and SI Appendix. The AFM measurement of the surface corrugation markings, custom Matlab code for the crack front reconstruction and the finite element model reported in this work have been deposited in Zenodo: <https://doi.org/10.5281/zenodo.3601407>

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