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A damage model for fretting contact between a sphere and a half space using semi-analytical method

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Abstract

This paper presents a fast method of solving 3D contact problems when one of the mating bodies has an elastic-damageable behavior. The damage model is implemented in a semi-analytical model using Eshelby's equivalent inclusion method in the contact solver. The proposed technique can be seen as an enrichment technique for which the effect of heterogeneous inclusions is surimposed on the homogeneous solution in the contact algorithm. Contact pressure and subsurface stress field computation time is kept small due to a massive use of 3D and 2D Fast Fourier Transforms. Cuboidal inclusions with the same size as the discretization of the half-space and with the same elastic properties are surimposed. The damage model affects the elastic properties of the cuboidal inclusions. The emphasis is put on the effects of the fretting regimes on the contact pressure and damage evolution.

Keywords: Contact Mechanics, Semi-Analytical Methods (SAM), Damage Mechanics, Fretting, Eshelby's Equivalent Inclusion Method (EIM), Inhomogeneity, Eigenstrain

1. Introduction

Fretting occurs when two body in contact are submitted to oscillatory tangential loading.

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Based on the amount of stick and slip that occurs in the contact two fretting
5 sliding conditions can be differentiated : the partial slip condition and the gross
slip condition [1]. For a circular contact in partial slip condition, the center of
the contact is stick and a slip annulus appears at the edge of the contact [2] but
in gross slip conditions all the contact surface is slipping.

Under fretting loading, cracks and surface degradation are appearing as a
10 result of fatigue phenomenon, void nucleation and sub-surface crack propagation
[3] [4]. Bryggman [5] explained the appearance of crack in partial slip conditions
by the cyclic shearing of material in the contact region and, in the same way,
wear would be caused by the propagation and intersections of these cracks.

Zhou and Vincent [6] have established three fretting regimes based on ex-
15 perimental results : the partial slip regime, the gross slip regime and the mixed
fretting regime which corresponds to sliding conditions evolving from gross slip
to partial slip due to a modification of the contact conditions. Wear is generally
associated with the gross slip regime while the partial slip regime to cracks [7].
Both damage phenomenon compete on the mixed regime and material response
20 fretting map associated with fretting regimes map has been proposed in [8].

Continuum Damage Mechanics (CDM) has been widely used to study fail-
ure due to fatigue [9]. Recently, CDM has been applied to study the damage
phenomenons leading to wear and cracks initiation in material during fretting
cycles [10, 11, 12, 13] and rolling/sliding contacts [14].

25 Raje et al. [15, 16] developed a damage model for rolling contact fatigue in
which the material microstructure is modeled using Voronoi tessellation. The
same model was used to study fretting fatigue [17] and fretting wear [18, 19].
Shen et al. [20] coupled an elastic-plastic damage model with wear in order
to simulate fretting fatigue life. Kumar et al. [21] proposed a finite element
30 simulation of fretting fatigue and studied the effects of the presence of voids
inside the material. Finite element (FE) models have been developed by many
authors for fretting [22] and wear computations [23, 24]. Recently, Yue et al.
[25] proposed a finite element model of fretting wear with a variable coefficient
of friction. But for three-dimensional problems and a moving load, a very fine

35 mesh is required for the contact interfaces and the computation costs become very high.

A numerical technique to simulate three-dimensional fretting has been investigated by Nelias and co-workers [26, 27, 28, 29] based on the semi-analytical method (SAM) initially proposed by Jacq et al. [30]. SAMs have been contin-
40 uously developed and applied to several problems such as thermo-elasto-plastic contact modeling [31], modeling plasticity and accumulation of plastic strains [31] [32], running-in [33], simulation of single impact [34], shot peening [35] and low plasticity burnishing [36] [33] [37], modeling of cuboidal inclusions [38] [39] [40] [41], ellipsoidal inclusions [42, 43] [44], heterogeneous viscoelastic behavior
45 [45] [46], heterogeneous elastoplastic behavior [47], as well as to account for material or coating anisotropy [48] [49].

The purpose of the present work is to use the damage mechanics approach to model the phenomenon leading to wear and cracks initiation in semi analytical methods. The proposed enrichment technique is using multiple damageable
50 cuboidal inclusions surimposed on the homogeneous solution in the contact algorithm. The damage algorithm only affects the Young's modulus of the inclusions and, by this way, modifies the contact solution. Fretting simulation have been performed in both gross slip and partial slip conditions The emphasis is put on the effects of the fretting regimes on the contact pressure and damage evolution.

55 **2. Theoretical Background and model description**

2.1. Contact problem formulation

Generally, the formulation of the contact between two finite bodies (Fig. 1) consists in a set of equations and inequalities that are recalled below:

- The load balance : the applied load W and the integration of the contact pressure $p(x, y)$ in the contact region Γ_c must be strictly equal.

$$W = \int_{\Gamma_c} p(x, y) d\Gamma \quad (1)$$

- The surface separation : the gap between the two contacting surfaces is:

$$h(x, y) = h_i(x, y) + \delta + u_z^{(B_1+B_2)}(x, y) \quad (2)$$

60 where $h_i(x, y)$ is the initial geometry, δ the rigid body displacement, and $u_z^{(B_1+B_2)}(x, y)$ the sum of normal displacements of surfaces 1 and 2, that can be due to elastic deflection (under loading only), plastic deformation or the presence of inhomogeneities.

- The contact conditions : the distance $h(x, y)$ is always positive, because the contacting bodies can not interpenetrate each other. The conditions are defined by the inequalities:

$$h(x, y) \geq 0$$

$$\text{contact : } h_i(x, y) = 0 \text{ and } p(x, y) > 0$$

$$\text{separation : } h_i(x, y) > 0 \text{ and } p(x, y) = 0 \quad (3)$$

- The tangential contact conditions can be written using the Coulomb's friction law to express the shear stresses q_τ in the contact. The tangential load Q and the contact zone Γ_c are known from the solution of the normal problem. The following system of equation needs to be solved to determine the sticking region Γ_{st} and the slipping region Γ_{sl} .

$$q_\tau(x, y) = -\mu \cdot p(x, y) \cdot \frac{\Delta s_\tau(x, y)}{\|\Delta s_\tau(x, y)\|} \quad \forall (x, y) \in \Gamma_{sl} \quad (4)$$

$$\Delta u_\tau(x, y) - \Delta \delta_\tau = \Delta s_\tau(x, y) \quad \forall (x, y) \in \Gamma_{sl} \quad (5)$$

$$\|q_\tau(x, y)\| < \mu \cdot p(x, y) \quad \forall (x, y) \in \Gamma_{st} \quad (6)$$

$$\Delta u_\tau(x, y) - \Delta \delta_\tau = 0 \quad \forall (x, y) \in \Gamma_{st} \quad (7)$$

$$\sum_{\Gamma_p} q(x, y) S = Q \quad (8)$$

$$\Gamma_{sl} \cup \Gamma_{st} = \Gamma_c \quad (9)$$

65 δ_τ is the tangential rigid body displacement and s_τ is the relative slip amplitude. The tangential surface displacement u_τ is the result of the displacements coming from both the shear stresses and the normal pressure field.

These equations are solved simultaneously using the Conjugate Gradient Method (CGM) as proposed by Polonsky and Keer [50].

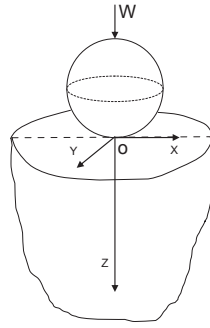


Figure 1: Contact of rigid indenter over a elastic half-space.

2.2. Eshelby's equivalent inclusion method in Contact Mechanics

The presence of a single or multiple inhomogeneities within one of the bodies in contact is taken into account by adding in Eq. (2) the eigendisplacement u_z^* induced by these inhomogeneities. Equation 2 is then modified as follows:

$$h(x, y) = h_i(x, y) + \delta + u_z(x, y) + u_z^*(x, y) \quad (10)$$

70 The bases of the method in the case of an elastic or elastic-plastic half-space are detailed extensively in [42, 43, 44, 47].

Sections 2.2.2 and 2.2.3 present the various steps for the calculation of u_z^* in the case of an elastic body containing elastic inhomogeneities. First, the calculation of the eigenstrain ε^* is presented followed by the calculation of the subsurface stresses and finally the calculation of the eigendisplacement u_z^* is detailed.
75

2.2.1. Eshelby's solution for an infinite space

An infinite matrix M with the elastic stiffness tensor C_{ijkl}^M containing an ellipsoidal domain Ω with the elastic stiffness tensor C_{ijkl}^I is submitted at infinity to a uniform strain ε^0 . The strain field is disturbed by the presence of the inhomogeneity.

The Eshelby's equivalent inclusion method (EIM) consists in representing the ellipsoidal inhomogeneity as an inclusion having the same elastic properties C_{ijkl}^M as the matrix but being subjected to an additional imaginary strain called eigenstrain ε^* giving:

$$C_{ijkl}^I(\varepsilon_{kl}^0 + \varepsilon_{kl}) = C_{ijkl}^M(\varepsilon_{kl}^0 + \varepsilon_{kl} - \varepsilon_{kl}^*) \quad \text{in } \Omega \quad (11)$$

The necessary and sufficient condition for the equivalence of the stresses and strains in the two above problems of inhomogeneity and inclusion is provided by Eq. (11). In particular, the eigenstrain ε_{ij}^* is related to compatibility strain ε_{ij} by:

$$\varepsilon_{ij} = S_{ijkl} \times \varepsilon_{kl}^*, \quad (12)$$

where S_{ijkl} is the Eshelby's tensor. Substitution of Eq. (12) into Eq. (11) leads to:

$$\Delta C_{ijkl} S_{klmnp} \varepsilon_{mn}^* + C_{ijkl}^M \varepsilon_{kl}^* = -\Delta C_{ijkl} \varepsilon_{kl}^0 \quad (13)$$

where

$$\Delta C_{ijkl} = C_{ijkl}^I - C_{ijkl}^M$$

Moschovidis and Mura [51] extended Eshelby's solution to two close ellipsoidal inhomogeneities. In recent work, multiple inclusions problems have been solved by using a conjugate gradient algorithm to determine each unknown eigenstrain [39, 40, 42].

2.2.2. Half-space solution

Three dimensional contact problems involve a half-space that is bounded by the surface plane $z = 0$ in the cartesian coordinate system (x, y, z) as shown in Fig. 1. Jacq et al. [30] and later Zhou et al. [38] proposed a method allowing

to extend previous solution, valid only for infinite spaces, to half spaces. The
 95 solution for an isotropic half space consists in decomposing the problem into
 three subproblems (Fig. 2), known as Chiu's decomposition [52].

(1) An inclusion with the prescribed eigenstrain $\varepsilon^* = (\varepsilon_{xx}^*; \varepsilon_{yy}^*; \varepsilon_{zz}^*; \varepsilon_{xy}^*;$
 $\varepsilon_{xz}^*; \varepsilon_{yz}^*)$ in an infinite space.

(2) A symmetric inclusion with a mirror eigenstrain $\varepsilon_s^* = (\varepsilon_{xx}^*; \varepsilon_{yy}^*;$
 100 $\varepsilon_{zz}^*; \varepsilon_{xy}^*; -\varepsilon_{xz}^*; -\varepsilon_{yz}^*)$ in the same space.

(3) A normal traction distribution $-\sigma^n$ at the surface of the half space
 ($z = 0$) which is a function of the eigenstrains ε^* and ε_s^* .

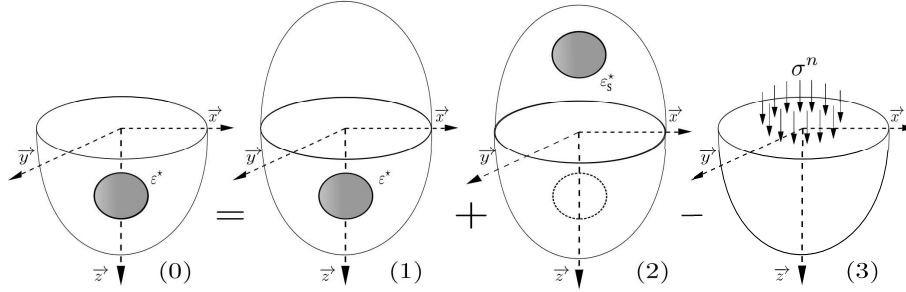


Figure 2: Decomposition of the half-space solution into three sub-problems.

The summation of the two solutions (1) and (2) leaves the plane of symmetry
 ($z = 0$) free of shear tractions. By adding an opposite normal stress σ^n , the
 105 condition of free surface traction is obtained. The stress at any point of the
 domain meshed with $n_x \times n_y \times n_z$ cuboids is given by:

$$\begin{aligned}
 \sigma_{ij}(x, y, z) = & \sum_{x^I=0}^{n_x-1} \sum_{y^I=0}^{n_y-1} \sum_{z^I=0}^{n_z-1} B_{ijkl}(x - x^I, y - y^I, z - z^I) \varepsilon_{kl}^*(x^I, y^I, z^I) \\
 & + \sum_{z^I=0}^{n_z-1} \sum_{y^I=0}^{n_y-1} \sum_{x^I=0}^{n_x-1} B_{ijkl}(x - x^I, y - y^I, z + z^I) \varepsilon_{skl}^*(x^I, y^I, -z^I) \\
 & - \sum_{y^I=0}^{n_y-1} \sum_{x^I=0}^{n_x-1} M_{ij}(x - x^I, y - y^I, z) \sigma^n(x^I, y^I, 0)
 \end{aligned}
 \tag{14}$$

where B_{ijkl} are the influence coefficients that relate the constant eigenstrain

at the point (x^I, y^I, z^I) which is the inclusion center in an infinite space to the stress σ_{ij} at the point (x, y, z) . M_{ij} represent the influence coefficients relating
110 the normal traction σ^n within a discretized area centered at $(x^I, y^I, 0)$ to the stress σ_{ij} at the point (x, y, z) .

$$B_{ijkl}(x) = C_{ijmn}^M D_{mnkl}(x) \quad \text{for } x \text{ in } D - \Omega \quad (15)$$

$$B_{ijkl}(x) = C_{ijmn}^M (D_{mnkl}(x) - I_{mnkl}) \quad \text{for } x \text{ in } \Omega \quad (16)$$

where $I_{ijkl} = \frac{1}{2} (\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl})$ is the fourth-order identity tensor.

The expression for D_{ijkl} is given in Mura [53].

$$D_{ijkl} = \frac{1}{8\pi(1-\nu)} [\Psi_{,ijkl} - 2\nu\delta_{kl}\phi_{,ij} - (1-\nu)(\delta_{kl}\phi_{,il} + \delta_{ki}\phi_{,jl} + \delta_{jl}\phi_{,ik} + \delta_{li}\phi_{,jk})] \quad (17)$$

$$\Psi(x) = \int_{\Omega} |x - x'| dx'$$

$$\phi(x) = \int_{\Omega} \frac{1}{|x - x'|} dx'$$

For a single inclusion centered at (x^I, y^I, z^I) in the half-space, the normal
115 traction σ^n at the surface point $(x', y', 0)$ is obtained as:

$$\begin{aligned} \sigma^n(x', y', 0) = & -B_{33kl}(x' - x^I, y' - y^I, -z^I,) \varepsilon_{kl}^*(x^I, y^I, z^I) \\ & - B_{33kl}(x' - x^I, y' - y^I, z^I,) \varepsilon_{s_{kl}}^*(x^I, y^I, -z^I) \end{aligned} \quad (18)$$

In Eq. (14), each component $M_{ij}()$ is obtained by a double integration of the function $F_{ij}()$ over a discretized surface area $2\Delta x \times 2\Delta y$ centered at $(x^I, y^I, 0)$, see appendices A and B.

$$M_{ij}(x - x^I, y - y^I, z) = \int_{x^I - \Delta x}^{x^I + \Delta x} \int_{y^I - \Delta y}^{y^I + \Delta y} F_{ij}(x - x', y - y', z) dx' y' \quad (19)$$

The 3D-FFT is used to accelerate the calculation of the first (1) and second
120 terms (2) and the 2D-FFT for the third term (3). Wrap around order and zero-padding techniques are used in order to remove the induced periodicity error [54].

2.2.3. Normal displacement of a surface point

The surface normal 'eigen-displacements' can be obtained when inserting the
125 eigenstrain into the total strain. They are generated by the pressure field σ^n
only. The normal displacements are calculated as:

$$u_z^*(x, y) = \sum_{y'=0}^{n_y-1} \sum_{x'=0}^{n_x-1} K^n(x-x', y-y') \sigma^n(x', y') \quad (20)$$

To solve the equation above numerically, the surface in contact is discretized
into $n_1 \times n_2$ rectangular elements of uniform size $2\Delta x \times 2\Delta y$. Then, pressure
and displacement within each discrete patch are treated as constant and their
130 values located at the center. The effect of a uniform pressure on a rectangular
area has been given by Love [55] and Johnson [56]. K^n denotes the influence
coefficients that relate the normal pressure at the surface point $(x', y', 0)$ to the
normal displacement at the surface point $(x, y, 0)$, recalled in Appendix C.

2.2.4. Integration of the inhomogeneity effects in the contact algorithm

135 In order to integrate the inhomogeneity effects in the contact algorithm, an
equivalent elastic algorithm is proposed. The effect of an inclusion on the contact
problem derives from the fact that the surface contact geometry is modified
by the eigen-displacement produces by the eigenstrain. The contact pressure
and shears are then updated, which modifies the eigenstrain value. The elastic
140 displacements are obtained from the updated contact pressure via the resolution
of the elastic contact problem. The algorithm is repeated until convergence of
the normal displacements is obtained. The equilibrium between the contact
problem and subsurface problem is checked at every time step of the loading
cycle.

145 2.3. Damage model

In the elastic heterogeneous contact solver presented in 2.2.4, a Continuum
Damage Mechanics (CDM) based model was implemented in order to describe
the degradation of material due to contact loading. During fretting cycles, the

high contact stresses are causing micro-cracks in the material responsible of
 150 wear or fatigue cracks [7, 3, 4, 5]. Sections 2.3.1 and 2.3.2 present the basics
 of continuum damage mechanic and his application when coupled with elastic-
 ity. Sections 2.3.3 highlights the implementation of the damage model in the
 heterogeneous contact algorithm.

2.3.1. Continuum Damage Mechanics

155 Continuum Damage Mechanics (CDM) background permits to describe the
 initiation and evolution of degradation in materials at the microscale such as
 micro cracks and voids. The damage model used in the current approach
 is isotropic and based on a single scalar damage variable D introduced by
 Kachanov [57]). Considering no healing of the material, D is monotonically
 160 increasing from $D = 0$, the undamaged state, to $D = 1$ the complete local
 rupture of the material.

The state of stress in the damaged material can be described by the effective
 stress introduced by Rabotnov [58]:

$$\tilde{\sigma} = \frac{\sigma}{(1 - D)} \quad (21)$$

Following Lemaitre [59] strain equivalence hypothesis, the strain behavior
 is modified by damage only through the effective stress. Hence, the strain
 associated with a damaged state under the applied stress is equivalent to the
 strain associated with its undamaged state under the effective stress. Applying
 the Hooke's law with E , the modulus of elasticity for the undamaged material,
 the elastic strain in the damaged material becomes:

$$\varepsilon = \frac{\tilde{\sigma}}{E} = \frac{\sigma}{(1 - D)E} \quad (22)$$

From this equation it can be deduced that an increase in the damage manifests as
 the reduction in the modulus of elasticity as shown in Fig. 3. D characterizes the
 effect of microscopic phenomenon on the macroscopic behavior of the material
 165 as a strength loss. These usually reproduce the presence of micro-cracks within
 an elastic material.

2.3.2. Elasticity Coupled with Damage

In order to model the damage of concretes, predominant in tension, Mazars [60] used the scalar damage parameter D coupled with elasticity in his 3D model. Mazars [60] choosed D as a function of the positive (tensile) strains and to evaluate these strains, he defined the following scalar called equivalent strain:

$$\tilde{\varepsilon} = \sqrt{\sum_{i=1,3} \langle \varepsilon_i \rangle_+^2} \quad (23)$$

with

$$\langle \varepsilon_i \rangle_+ = \left(\frac{\varepsilon_i + |\varepsilon_i|}{2} \right) \quad (24)$$

and ε_i the principal strains. However, to study the damage of contact under fretting, D needs to take into account tensile, compressive and shear stresses.

An adaptation of the expression of Mazars's equivalent strain is proposed here :

$$\tilde{\varepsilon} = \sqrt{\sum_{i=1,3} \langle \varepsilon_i \rangle_+^2 + \sum_{i=1,3} \langle \varepsilon_i \rangle_-^2} = \sqrt{\sum_{i=1,3} \varepsilon_i^2} \quad (25)$$

with

$$\langle \varepsilon_i \rangle_- = \begin{cases} |\varepsilon_i| & \text{if } \varepsilon_i \leq 0 \\ 0 & \text{if } \varepsilon_i > 0 \end{cases} \quad (26)$$

The equivalent strain is now traducing the local three-dimensional state of deformation of the solid via a uniaxial scalar variable.

The equivalent strain controls the growth of the damage variable according to an evolving threshold. At the end of every time step and for every point of the discretization of the half-space, the loading function can be defined as :

$$f(\varepsilon, D) = \tilde{\varepsilon} - K(D) \quad (27)$$

$K(D)$ takes the largest value of the equivalent strain $\tilde{\varepsilon}$ ever reached by the material during the loading history at the considered point. $K(D = 0)$ is initialized at ε_{d0} , the damage threshold strain, corresponding to the strain at

the elastic limit.

$$\begin{cases} K(D) = \varepsilon_{d0} & \text{if } D = 0 \\ K(D) = \max_t \tilde{\varepsilon} & \text{if } D \geq 0 \end{cases} \quad (28)$$

If the threshold is reached, a new increment of damage called δD is added to the damage variable at the considered point. The evolution law for the damage variable is defined as:

$$\begin{cases} \delta D = \frac{\tilde{\varepsilon} - K(D)}{\varepsilon_R - \varepsilon_{d0}} & \text{if } \tilde{\varepsilon} \geq K(D) \\ \delta D = 0 & \text{if } \tilde{\varepsilon} < K(D) \end{cases} \quad \rightarrow D = D + \delta D \quad (29)$$

with ε_R the strain leading to a macroscopic cracks.

The behavior of the material is linear elastic on the first part of Fig. 3. When the equivalent strain reaches the damage threshold, the local elastic properties of the matrix are modified through the presence of micro-cracks which decreases the strength of the material. This local decrease of the modulus of elasticity is irreversible. From ε_{d0} , the damage threshold strain, the damage variable will modify the elastic behavior of the material by decreasing the Young's modulus as illustrated in Fig. 3 and until the material strain reaches the macroscopic fracture strain ε_R (see Fig. 4b). The damage evolution is always increasing and locally linear by pieces (Fig. 4a).

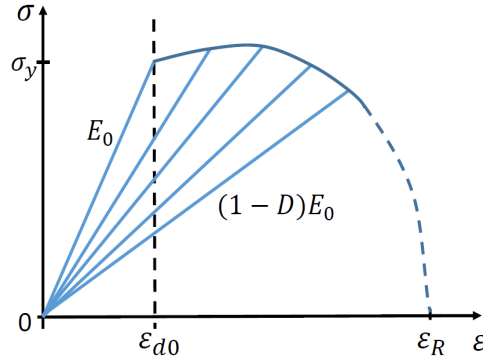


Figure 3: Stress-Strain plot with damage evolution.

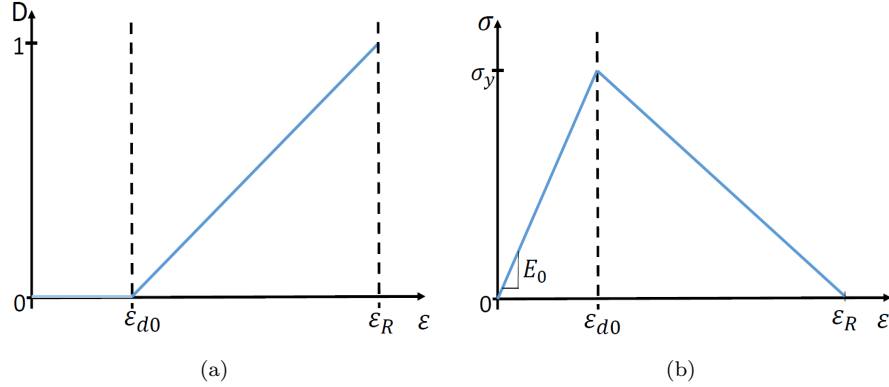


Figure 4: (a) Damage evolution as a function of strain showing the purely elastic domain and the damageable domain. (b) Purely elastic and damageable domain on a stress strain plot.

2.3.3. Integration of the damage model in the contact algorithm

The damage model defined in 2.3.2 is coupled with the semi-analytical contact solver developed by Nelias and co-workers [28, 43]. This method is based on the solution developed by Love [55] for a rectangular patch of pressure over a homogeneous half space. The Eshelby's equivalent inclusion method is used as an enrichment technique in order to introduce a local modification of the Young's modulus of the material. Multiple cuboidal inclusions with the same size as the discretization of the half-space and initially with the same elastic properties are surimposed on the half-space as presented in Fig. 5. The effect of damage on the material is traduced through the modification of the Young's modulus of the inclusions. Leroux et al.[42] and Koumi et al.[44] have shown that the contact pressure distribution may be significantly modified by the presence of inhomogeneities close to the surface, which subsequently affect the subsurface stress distribution. The Young's modulus of the inclusions are modified by the damage variable D and hence affect the contact pressure distribution through the contribution of eigenstrains. One of the main advantage of the method is to compute only the stress field in the area around the contact (at the surface but also in depth). It has been observed that damage usually happened just

under the contact during fretting [5]. For sake of computational efficiency, the
 205 half-space has been enriched only in this area. Note that no damage occurs far
 from the contact. When the first element is reaching a damage value equal to 1,
 the simulation is stopped. It should be noted that in CDM, $D = 1$ means that
 the element is too damaged to ensure continuity which can lead to the initiation
 of crack. In order to continue the simulation once an element damage value
 210 reaches 1, the contact surface should be modified by removing the element. De-
 pending of the area concerned and of the localization of the damage, it can be
 interpreted as wear or crack initiation. It should be noted that because simu-
 lations are stopped when a first element damage value reaches 1, the present
 model is only simulating the phenomena leading to the first appearance of wear
 215 or crack.

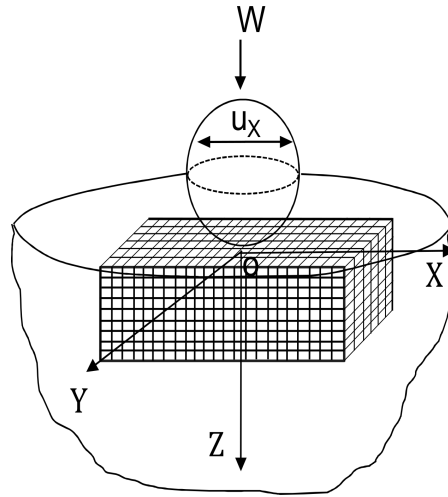


Figure 5: 3D view of a sphere on a elastic half-space with multiple cuboidal inclusions surimposition.

The main step of the algorithm, sumerized on the flowchart in Fig. 6 are
 described here :

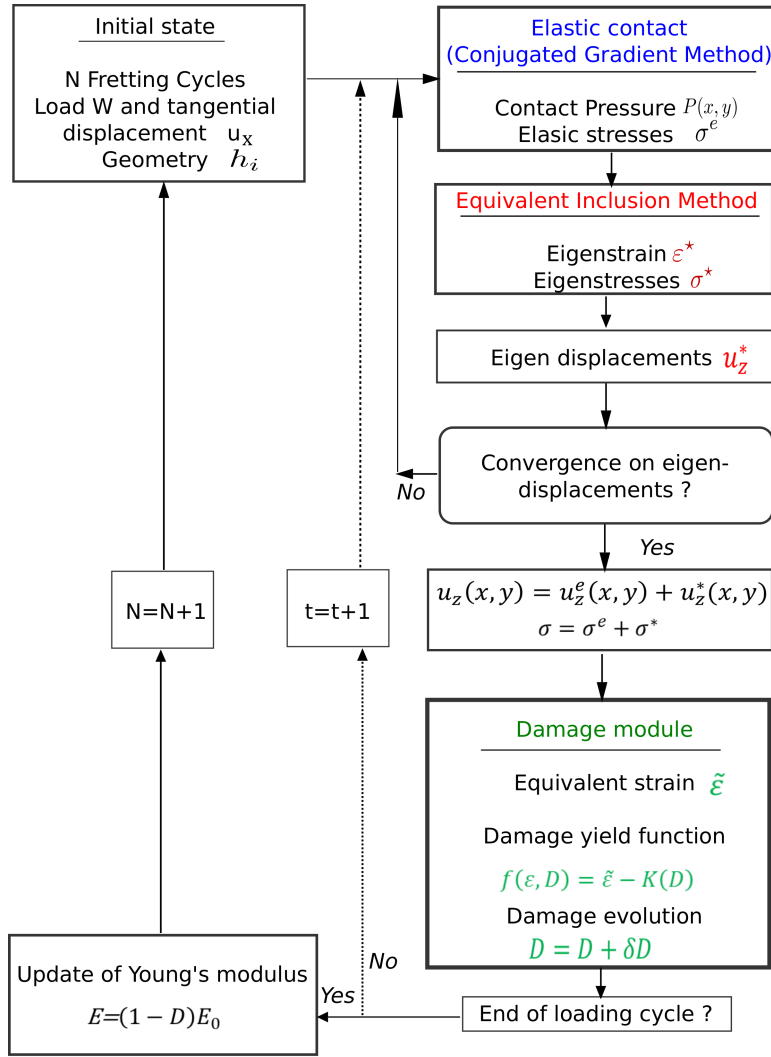


Figure 6: Algorithm of Heterogeneous Elastic-Damageable Contact Problem

1st Fretting Cycle :

- **Step1:**

220

Solve the elastic contact problem for the initial loading increment, and determine the elastic stresses and strains in the solid.

- **Step2:** Damage module

Compute the equivalent strains for the current loading increment.

225 Test the damage criterion for each inclusion and determine the inclusion's
damage increments δD for the current time step.
Increment the loading and solve the elastic problem again.
Compute the equivalent strains and, after testing the damage criterion,
determine the inclusion's damage increment δD .
Repeat until the last loading increment of the loading cycle.

230

- **Step3:** At the end of the cycle:
Update the corresponding Young's modulus with the damage variable
computed during the loading cycle.

235 *I*th Fretting Cycle :

- Assign to each inclusion the new Young's modulus.

- Repeat the previous steps for the considered loading cycle. Compute elastic stresses, equivalent strains and the damage increments δD for every
240 time step of the cycle.

- At the end of the fretting cycle, if one-or more-of inclusions are totally
damaged : stop the calculation.

3. Validation

245 For validation purpose of the enrichment technique, a comparison with the
analytical Hertzian solution is performed. Note that the heterogeneous semi-
analytical method has been compared and validated in previous work with both
analytical solutions [42] and finite element simulation [44]. A 3D rigid sphere
in contact with a heterogeneous half-space has been simulated with the semi-
250 analytical method. The half-space Young's modulus and Poisson's ration are

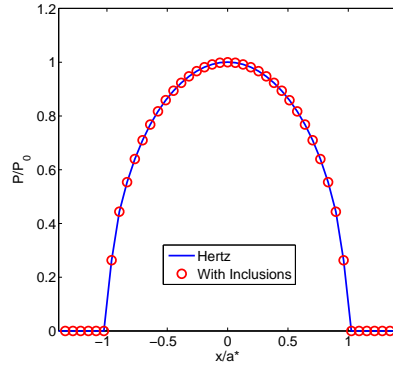
choosen as $E_0 = 210\text{GPa}$ and $\mu_0 = 0.3$, respectively. The normal applied load is $W = 410\text{N}$. For the homogeneous half-space, this load leads to a contact radius $a^* = 0.32\text{mm}$ and a maximum contact pressure $P_0 = 1890.8\text{MPa}$. The half-space is discretized in $67 \times 67 \times 51$ computation points such as the space between
 255 the constituted is $2\Delta x = 2\Delta y = 2\Delta z = 0.062a^*$. Then the half-space is filled from the free surface to a defined thickness with cuboidal inclusions centered on computation points and having the same dimensions as the discretization. The enrichment is constituted of $N_x \times N_y \times N_z = 62 \times 62 \times 36$ cuboids as illustrated in Fig. 5.

260 A comparison between the analytical Hertzian contact solution and the contact solution founded with the enriched half-space is given for the pressure distribution, for the half space stresses along axis z and axis y at surface ($z = 0$) as shown in Fig. 7. A very good agreement is found which in turn validates the numerical enrichment technique.

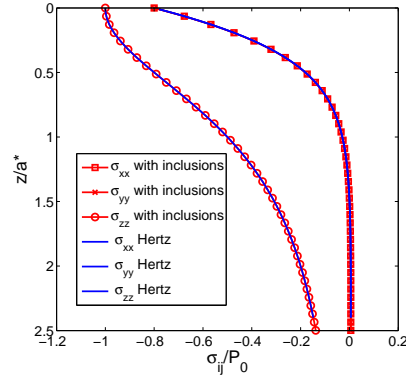
265 4. Results

4.1. Description of the problem

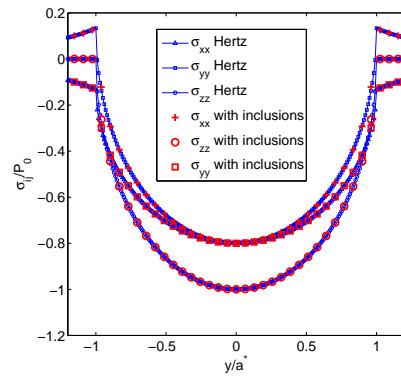
In this section the contact simulation between a rigid sphere of radius $R=25\text{mm}$ and a homogeneous half-space is presented. The contacting bodies are subjected to an oscillatory tangential motion. The two bodies are first brought into contact with a normal load $W = 410\text{N}$. A tangential displacement along the x
 270 direction is then applied. The half-space Young's modulus and Poisson's ration are choosen as $E_0 = 210\text{GPa}$ and $\mu_0 = 0.3$, respectively. For the homogeneous half-space, this load leads to a Hertzian contact radius $a^* = 0.32\text{mm}$ and a maximum contact pressure $P_0 = 1890.8\text{MPa}$. The imposed rigid body displacement is cycling between $u_x = 0.025\text{mm}$ and $u_x = -0.025\text{mm}$ ($u_x/a^* = 0.078$) to
 275 reproduce an entire fretting loop. The fretting cycle is decomposed into multiple time steps as shown in Fig. 8.



(a)



(b)



(c)

Figure 7: Validation of the enrichment technique with Hertzian analytical solution (a) Pressure distribution (b) Stresses along z direction (c) Stresses along y direction

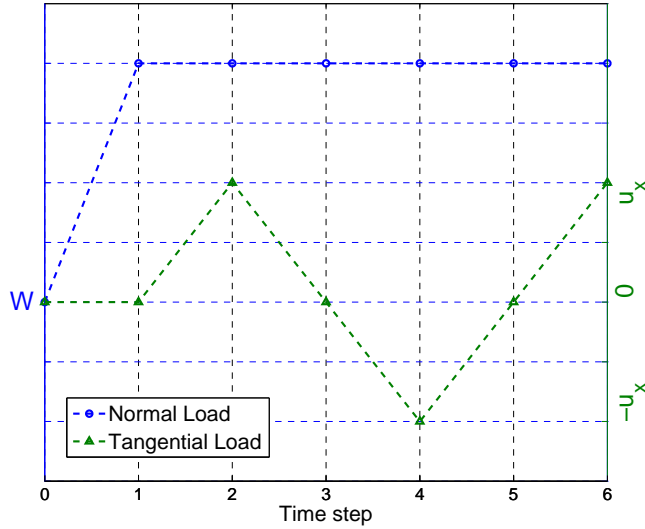


Figure 8: Normal and tangential load during one fretting cycle.

The half-space has been enriched with damageable elastic cuboidal inclusions with the same properties than the half-space. The damage model, as presented
 280 in section 2.3.2 is used with two parameters : $\varepsilon_R = 0.036$ and $\varepsilon_{d0} = 0.003225$. Firstly, results in the case of the gross slip regime are presented, afterwards the partial slip regime is investigated and finally, simulation in the case of coated materials are presented.

4.2. Simulation in gross slip regime

285 Here are presented the results of contact simulations in gross slip regime. According to Coulomb's law, the shear distribution is equal to the coefficient of friction times the pressure along the contact surface : $Q = \mu \times P$.

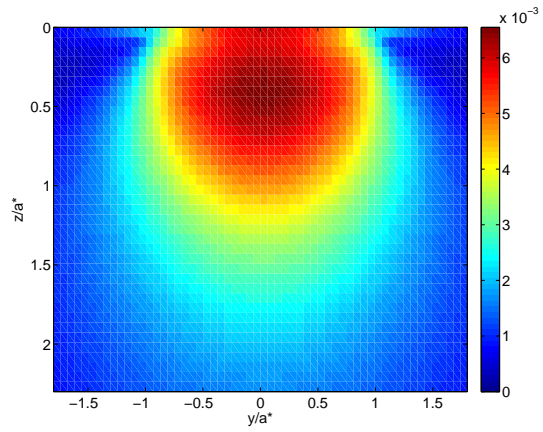
For unlubricated fretting, the coefficient of friction is generally high [61] and main phenomenon are wear and cracks. That's why all simulations have been
 290 performed with a friction coefficient $\mu \geq 0.3$.

A first fretting simulation is presented with a coefficient of friction $\mu = 0.5$. In Fig. 9, the distribution of the equivalent strain $\tilde{\varepsilon}$ is shown for normal and

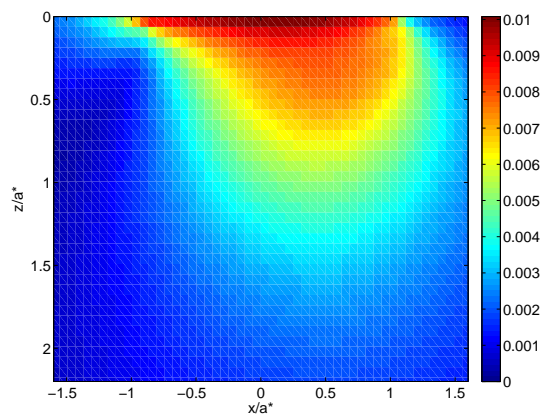
tangential loading conditions for the undamaged material. It can be observed that the maximum value of $\tilde{\epsilon}$ is reached at depth around $z/a^* = 0.5$ under the surface for a normal loading. When a tangential displacement is added, the maximum value of $\tilde{\epsilon}$ is reached at the surface and is more than 50% higher than with normal load only. All damage simulations have been performed until at least one point reached the critical damage value $D = 1$. The distribution of the scalar damage variable D at the end of fretting simulation is plotted at the contact surface $z = 0$ and in the plane $x = 0$ in Fig. 10a and Fig. 10b. It can be observed that the maximum damage is located at the center of the contact surface where the contact pressure is the higher and where the surface has seen the biggest sliding amplitude.

Figure 11 represents the evolution of the damage variable and the associated Young's modulus for the most damaged point of the material during fretting cycles. As the Young's modulus is decreasing, the equivalent strain is increasing and the material is becoming more damaged until the damage variable reaches its critical value and the Young's modulus has dropped to 0.

As the damage variable increases, contact pressure decreases (Fig. 12) and consequently, the contact area increases to respect the load equilibrium (Eq. 1). Moreover, the contact pressure drops locally by almost 40% at the center of the contact, where the surface is the more damaged. These results are in good agreements with the numerical results obtained by Shen et al. [20].

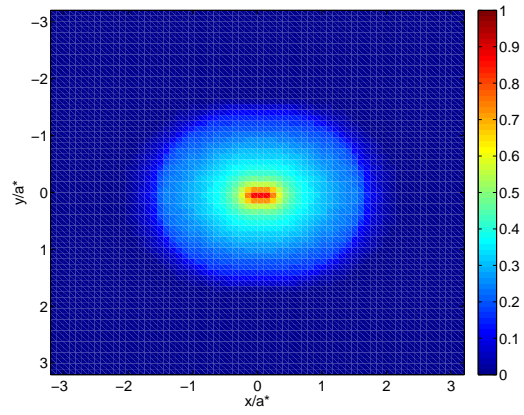


(a)

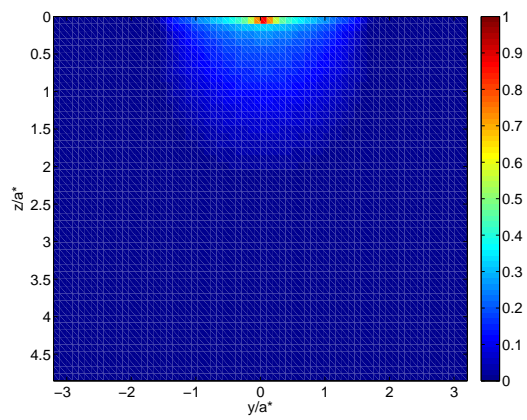


(b)

Figure 9: Equivalent strain $\bar{\varepsilon}$ (a) under normal loading in the plane $x = 0$. (b) under normal and tangential loading in the plane $y = 0$ with $\mu = 0.5$ during the first fretting cycle.



(a)



(b)

Figure 10: Damage variable D with $\mu = 0.5$ (a) in the plane $z = 0$. (b) in the plane $x = 0$ after 17 cycles.

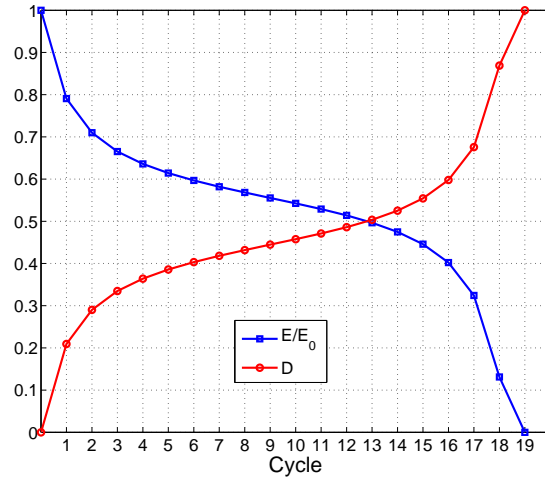


Figure 11: Evolution of damage and Young modulus with cycles for $\mu = 0.5$

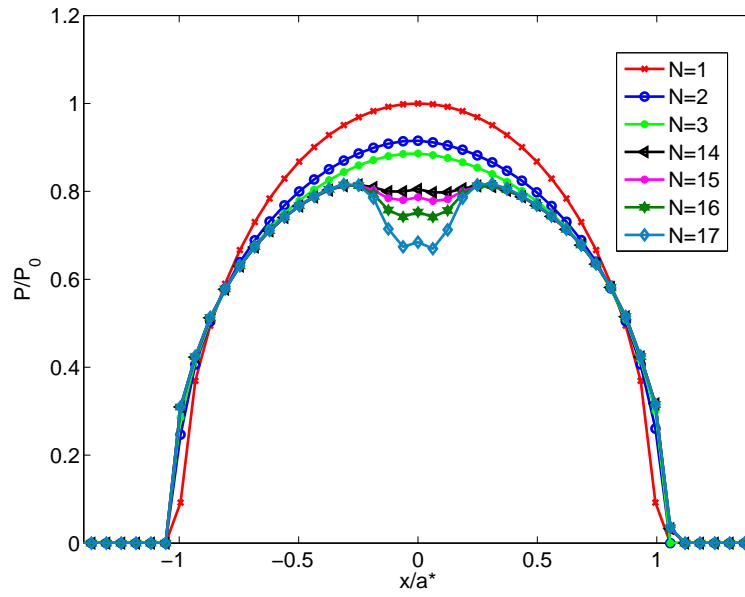


Figure 12: Effect of damage on contact pressure for $\mu = 0.5$

4.2.1. Effect of the coefficient of friction

315 Simulations were then performed in order to identify the influence of the friction coefficient on the model response. Firstly, values of the equivalent strain $\tilde{\epsilon}$ along direction z for different coefficients of friction are compared in Fig. 13. It can be observed that for $\mu < 0.3$, the highest value of $\tilde{\epsilon}$ is localized in the subsurface under the center of the contact while for $\mu \geq 0.3$, it is localized at the surface $z = 0$. Note that during unlubricated fretting conditions, the friction coefficient is usually high [61]. The following studies will focus on friction coefficient $\mu \geq 0.3$.

320

It can be observed in Fig. 16 that a higher coefficient of friction leads to an increased surface damage rate. It comes from the fact that, according to Coulomb's law, the higher is the coefficient of friction, the higher are the surface shear stresses. That is why the damage surface is larger with $\mu = 0.7$ (Fig. 14a and Fig. 14b) than for $\mu = 0.5$ and the pressure distribution is dropping on a larger surface (see Fig. 15). It should be noted that the number of cycles leading to failure is significantly lower than what can be found in literature.

325

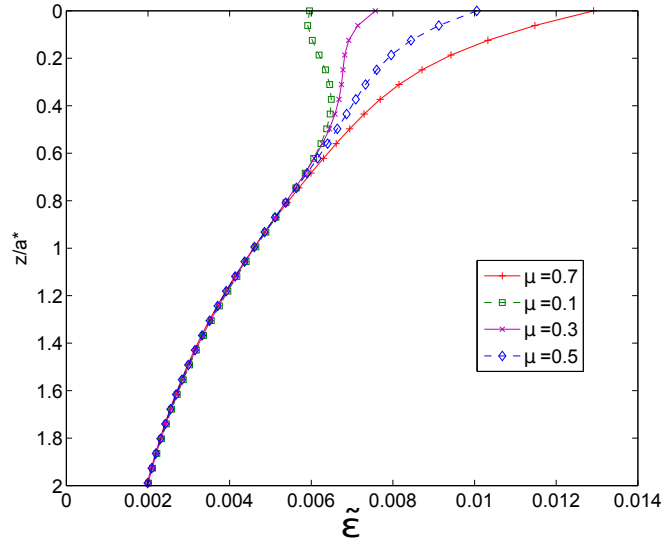
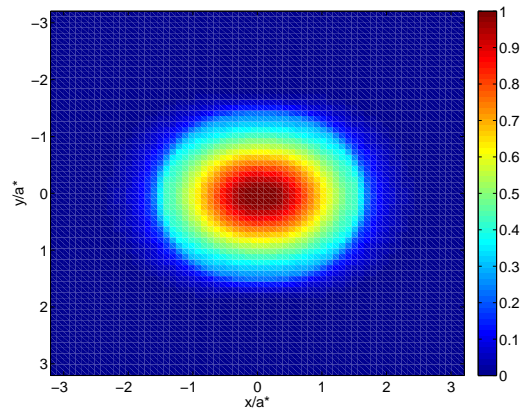
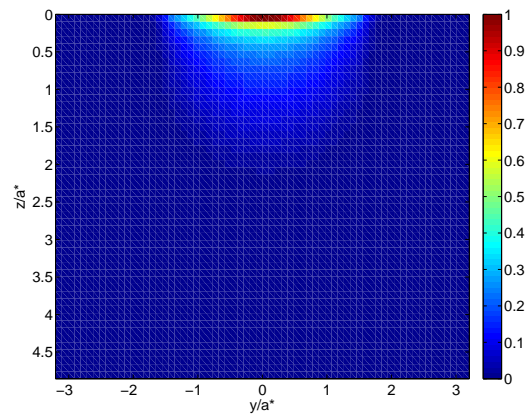


Figure 13: Equivalent strain $\tilde{\epsilon}$ along direction z for different friction coefficients.



(a)



(b)

Figure 14: Damage variable D with $\mu = 0.7$ (a) in the plane $z = 0$. (b) in the plane $x = 0$ after 4 cycles.

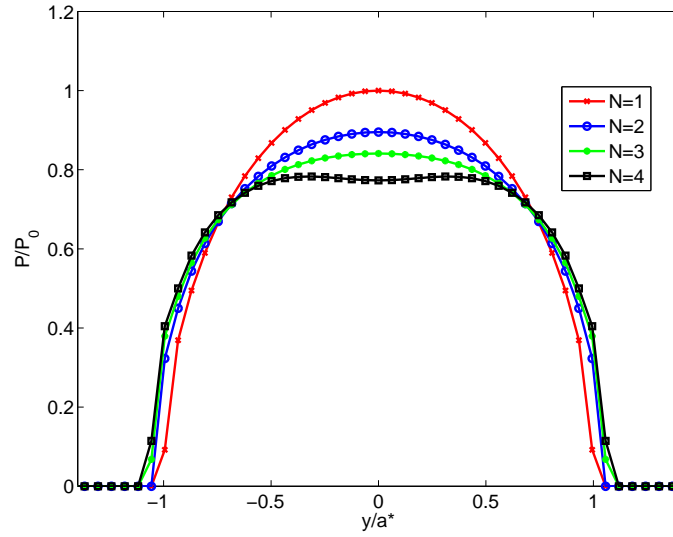


Figure 15: Effect of damage on contact pressure for $\mu = 0.7$

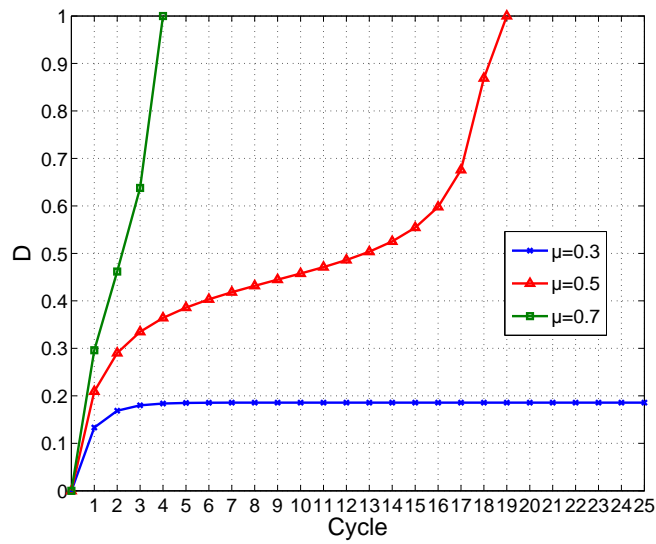


Figure 16: Evolution of damage function of friction coefficient

330 *4.2.2. Effect of computation discretization on damage evolution*

The reference space between the computation point is defined as $2\Delta x = 0.062a^*$ in section 3. Simulations are performed with different discretization sizes and the damage evolution of the most damage point is plotted in Fig. 17. There is no effect of the discretization size on the damage computation at the beginning of the simulation but after a few cycles, damage values are slowly diverging. For an identical damage value, different discretizations are modifying the Young's modulus in a different material volume. In the following cycles, the stress field will not be the same for the different discretizations.

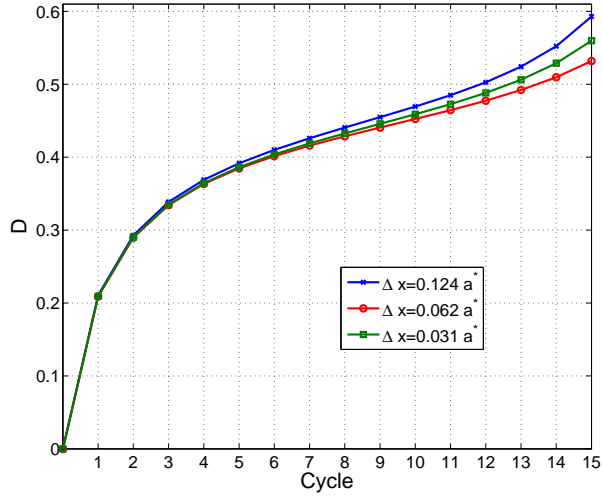


Figure 17: Evolution of damage function of enrichment width.

4.2.3. Effect of enrichment size on damage evolution

340 The half-space is constituted of $N_x \times N_y \times N_z$ cuboids as illustrated in Fig. 5. Fretting simulations are performed with $\mu = 0.5$ and with different enrichment sizes and plotted in Fig. 18. The enrichment depth $N_z = 2.23a^*$ is kept the same for every simulation. N_x and N_y are equals and vary from $2a^*$ to $4a^*$. One can observe that no effect of the size of the enrichment is observed on the damage evolution nor on contact pressure (see Fig. 19).

345

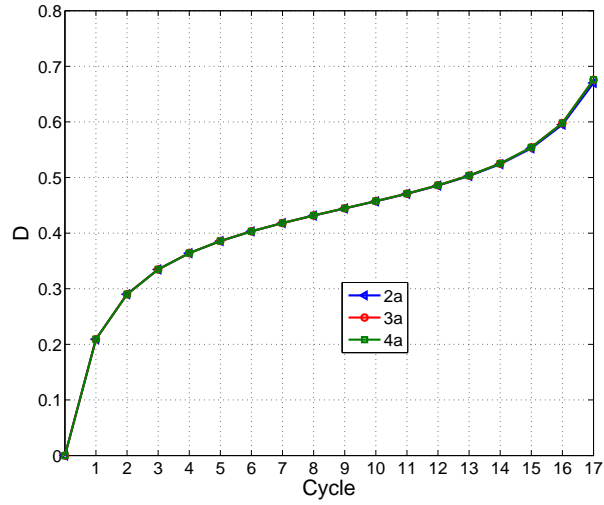


Figure 18: Evolution of damage function of enrichment width.

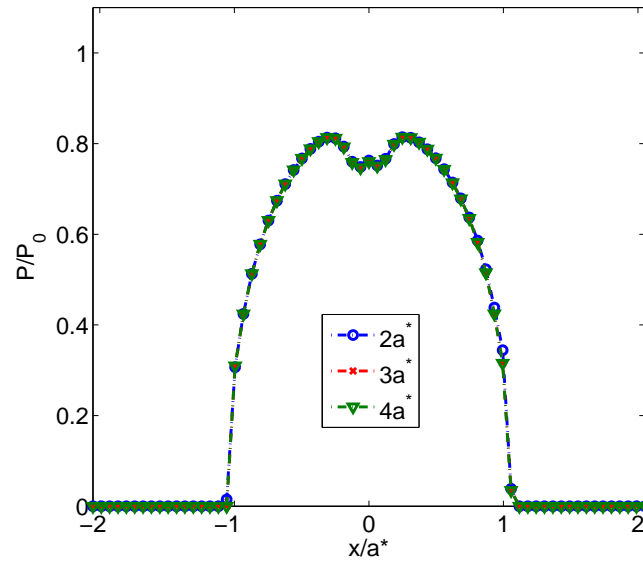


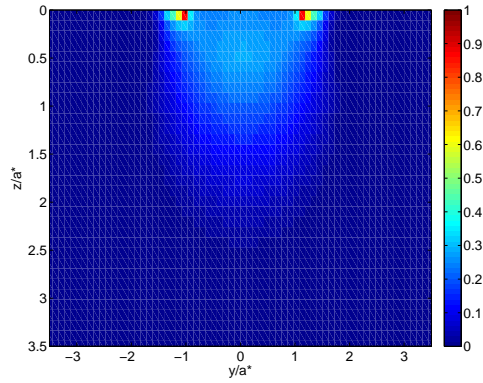
Figure 19: Comparison of contact pressure at the end of the damage simulation function of enrichment width.

4.3. Simulation in partial slip regime

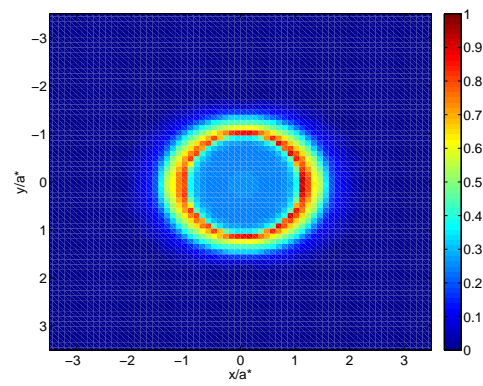
In this section, the same damage model is used in the partial slip regime. A gross slip fretting loop is associated to wear whereas the partial slip regime is associated to cracking appearance. In partial slip regime the center of the contact is sticking and an annular slip zone is appearing at the edge of the contact area as shown in [2]. Accordingly to fretting material response fretting map, cracking appearance is generally associated with smaller displacement amplitude and higher normal loading than in gross slip conditions [7].

A simulation is performed using a coefficient of friction of $\mu = 0.7$ and a normal load $W = 900\text{N}$. Accordingly with Hertzian theory, this load leads to a contact radius $a^* = 0.41817\text{mm}$ and a maximum contact pressure $P_0 = 2457.386\text{MPa}$. The tangential displacement is imposed with a value of $u_x = 0.001\text{mm}$ or $u_x/a^* = 0.0023$. The damage model parameters are kept the same as in section 4.1.

The state of the scalar damage variable D at the end of fretting simulation can be observed at the contact surface $z = 0$ and in the plane $x = 0$ in Fig. 20a and Fig. 20b after five fretting cycles. The higher damage values are localized in the slip circular zone at the edge of the contact. It is where the material sees the maximum strain during fretting cycles. In agreement to that, the pressure distribution is locally dropping at the contact edge as the damage is increasing (Fig. 21). During the damage progression, the contact area is increasing to respect the load equilibrium (Eq. 1) and the shear maximum value is moving out of the initial contact area (Fig. 22). These results exhibits the same effects of damage on contact pressure that the ones found in Ghosh et al. [18].



(a)



(b)

Figure 20: Damage variable D with $\mu = 0.7$ (a) in the plane $z = 0$. (b) in the plane $x = 0$ after 5 cycles.

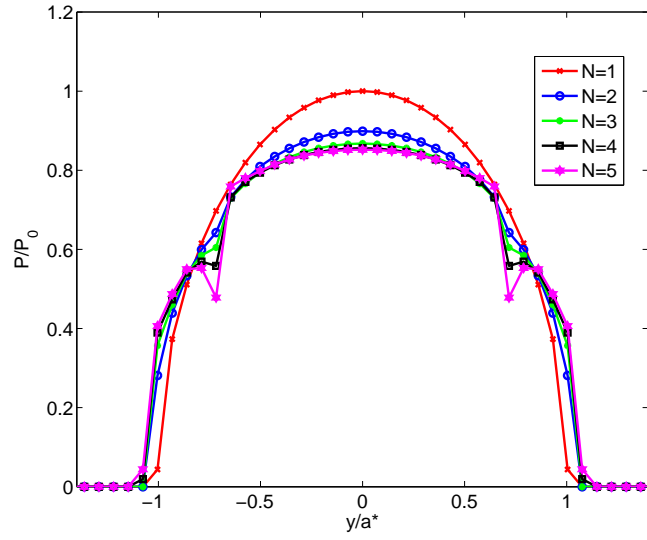


Figure 21: Evolution of contact pressure with damage in the plane $x=0$ for $\mu=0.7$

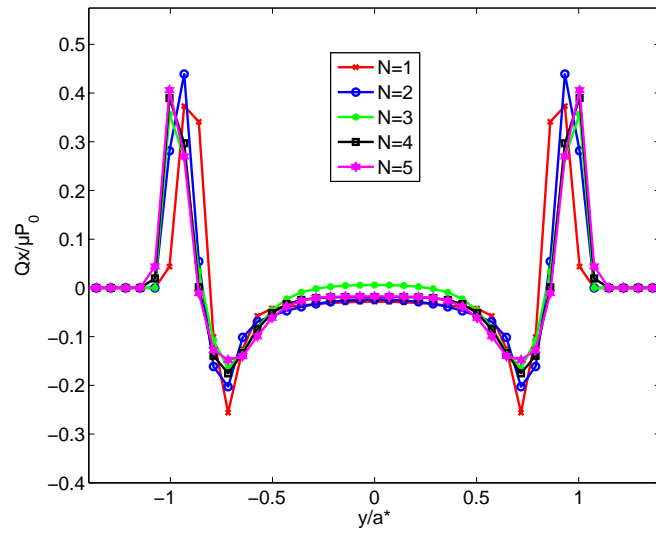


Figure 22: Evolution of contact shear distribution with damage in the plane $x=0$ for $\mu=0.7$

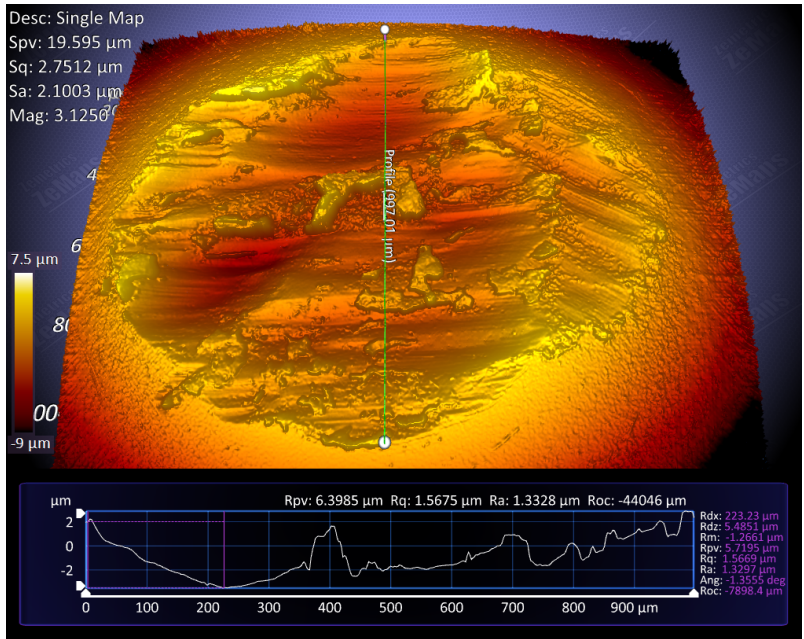


Figure 23: Wear scar of a steel ball under gross sliding

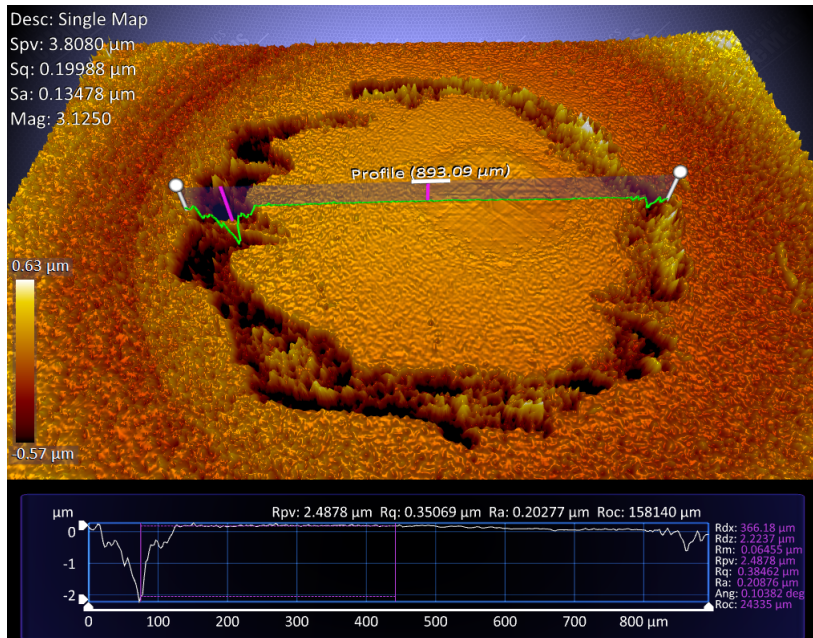


Figure 24: Wear scar of a steel ball under stick-slip

370 The results obtained in sections 4.2 and 4.3 are qualitatively in good agree-
 ment with the surface damage phenomenon usually observed in fretting condi-
 tions, see for example our experimental results in Figs. 23 and 24 – that can
 be compared to Figs. 14 and 20 – or to literature data [7].

4.4. Comparison with another damage model

375 In this section, the damage law used in Ghosh et al. [18] is presented and
 implemented into the semi-analytical contact solver. Fretting simulations are
 performed in gross slip regime and results are compared between the two damage
 models. A damage evolution law with an isotropic damage variable D based on
 the work of Chaboche and Lesne [62] is implemented and recalled here:

$$\frac{dD}{dN} = \frac{0.8S_{ut}}{E} \frac{\Delta\tau}{H(1-D)} \quad (30)$$

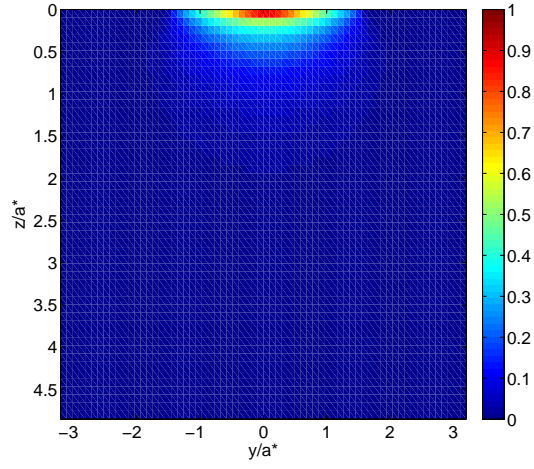
380 with N the number of stress cycles, S_{ut} the ultimate tensile strength of the
 material, E the Young’s modulus and H the hardness of the material. $\Delta\tau$ is
 the shear stress reversal at the considered point during a fretting cycle. Details
 leading to this equation can be found in Ghosh et al. [18].

The shear stress amplitude during one fretting cycle is calculated using the
 385 semi-analytical contact solver and damage evolution is calculated at every point
 of the material domain using Equation 30. For sake of computational efficiency,
 the number of cycles leading to the first fully damage element is computed using
 the jump-in-cycles algorithm proposed by Lemaitre [9] and already used in finite
 element simulation by Slack et al. [63]. This method assumes a linear damage
 390 evolution over a block of cycles.

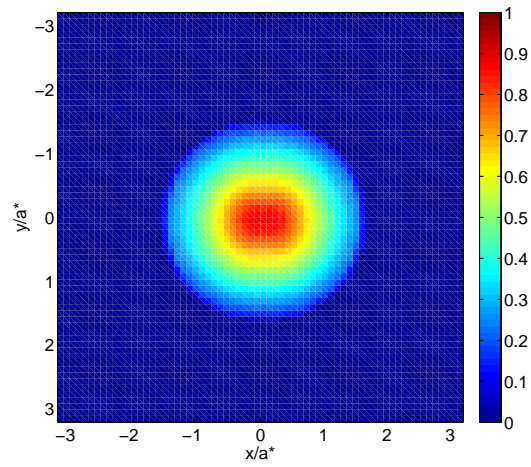
4.4.1. Gross slip regime

Contact between a rigid sphere of radius $R = 25\text{mm}$ and a homogeneous
 half-space is simulated using the semi-analytical solver. The same material and
 loading characteristics than in section 4.1 are used with a coefficient of friction
 395 $\mu = 0.7$.

The half-space has been enriched with damageable elastic cuboidal inclu-
 sions with initially the same properties than the half-space. The damage model
 proposed in Ghosh et al. [18] is used with two parameters: $S_{ut} = 2500$ MPa
 and $H = 1$ GPa. The damage increment used in the jump-in-cycles algorithm
 400 is chosen to be $\Delta D = 0.01$. All damage simulations have been performed until
 at least one point reached the critical damage value $D = 1$. Results in the case
 of gross slip regime and partial slip regime are investigated. The distribution
 of the scalar damage variable D at the end of fretting simulation is plotted at
 the contact surface $z = 0$ and in the plane $x = 0$ in Fig. 25. It can be observed
 405 that the maximum damage point is located at the center of the contact surface
 as observed with the previous model in Fig. 14. As the damage variable in-
 creases, contact pressure decreases (Fig. 26) and consequently, the contact area
 increases to respect the load equilibrium (see Eq. 1). Moreover, the contact
 pressure found at the last cycle with the two models are similar. A small dif-
 410 ference is found between the two pressures due to the fact that simulation is
 stopped once the damage variable reaches the value $D = 1$ for the first time.
 The represented contact pressure is computed with the damage state from the
 previous cycle and depending on the damage evolution law, the damage level at
 this previous cycle is not exactly the same in the two models.



(a)



(b)

Figure 25: Damage variable D with $\mu = 0.7$ (a) in the plane $x = 0$. (a) in the plane $z = 0$ after 28 loading cycles using damage model from Ghosh et al. [18].

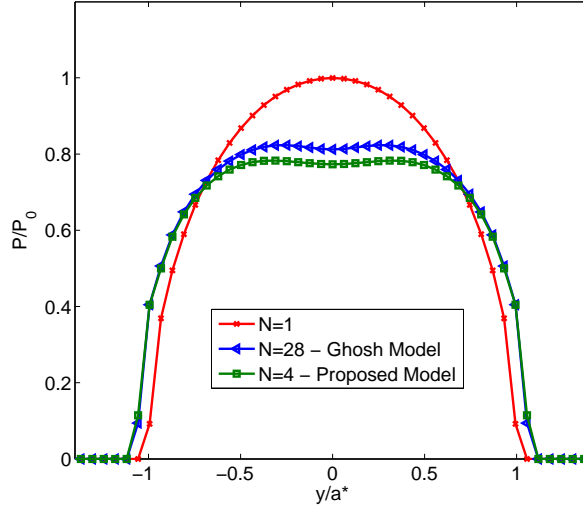


Figure 26: Contact pressure with damage in the plane $x=0$ for $\nu=0.7$ at the last iteration of the simulation for the two models.

415 The number of damage cycles found in this damage model is equal to 28 compared to 4 in the proposed model. Therefore one damage cycle in our model is equivalent to approximately 7 cycles in the model of Ghosh et al. [18] in this specific example i.e. with the defined hardness and tensile strength.

4.5. Application to coated materials

420 A lot of materials used in the industry are coated to protect the surface from damage or to help keeping the integrity of the substrate. Hard metallic coatings can mitigate fretting by reducing friction and resisting crack initiation [64, 65]. Using the same enrichment technique as before gives the possibility to simulate coated materials. From the top surface until a defined thickness
 425 z_c cuboidal inclusions with different material properties than the substrate are used. Above this layer, the same material properties than the substrate are used for the inclusions (see Fig. 27).

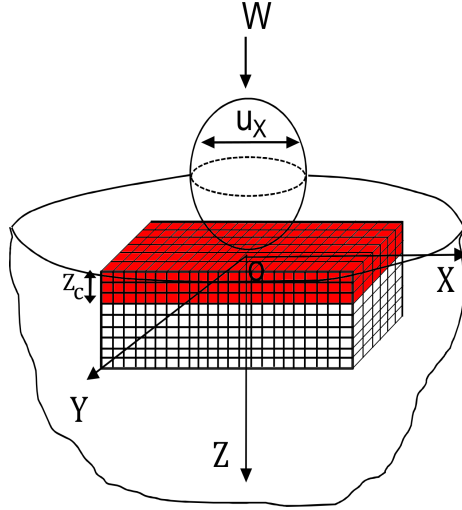
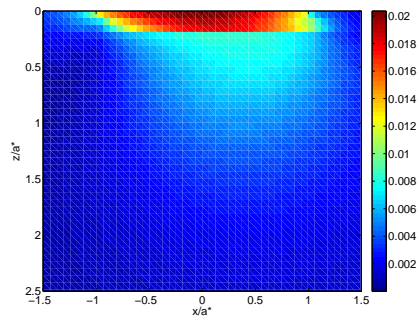


Figure 27: Enrichment of the half space with cuboidal inclusions to simulate the effect of coatings.

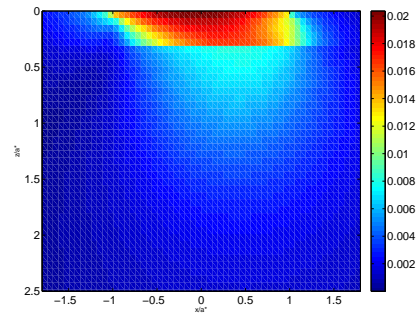
This section only aims to investigate the effects of elastic coatings on the localization of damage in the material. Firstly, only the distribution of equivalent strain $\tilde{\epsilon}$ is studied. Different coating's stiffnesses (with a Young modulus ratio defines as $\gamma = E/E_0$ with E and E_0 the modulus of the coating and substrate, respectively.) and thickness (z_c) are studied and the effect on damage evolution is analyzed. Two different coating stiffnesses are studied here, a harder one with $\gamma = 2$ and a softer one with $\gamma = 0.5$. The equivalent strain $\tilde{\epsilon}$ is plotted in order to represent the coating effect on the damage localization. The same parameters as in section 4.1 are used for the contact loading.

In Figs. 28 and 29, the equivalent strain $\tilde{\epsilon}$ is plotted for the undamaged material during the first cycle in the plane $y = 0$ for four different coating thicknesses z_c and for both coating stiffness ratios γ . These results are compared to the uncoated model along the axe z in Figs. 30 and 31.

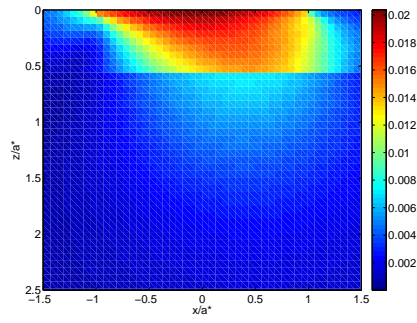
For $\gamma = 0.5$, the maximum equivalent strain $\tilde{\epsilon}$ is always located in the coated material (see Fig. 28) and $\tilde{\epsilon}$ is very attenuated in the substrate. Moreover, the equivalent strain $\tilde{\epsilon}$ in the coating is almost twice the value of $\tilde{\epsilon}$ in the uncoated half-space (Fig. 30).



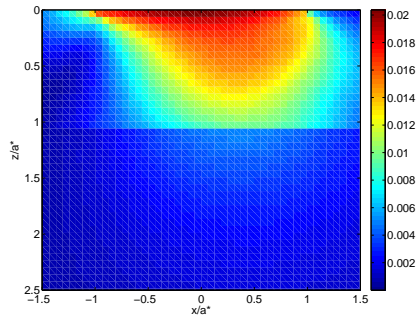
(a)



(b)



(c)



(d)

Figure 28: Equivalent strain $\bar{\epsilon}$ with a soft coating ($\gamma = 0.5$) of thickness (a) $z_c = 0.125a$ (b) $z_c = 0.25a$ (c) $z_c = 0.5a$ (d) $z_c = a$

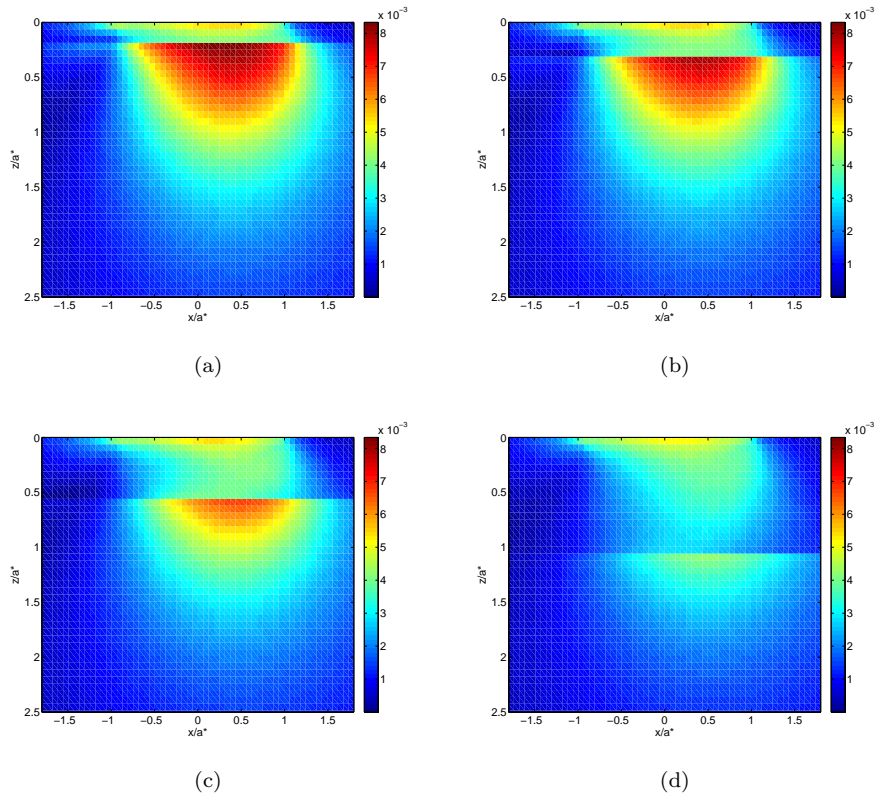


Figure 29: Equivalent strain $\tilde{\varepsilon}$ with a hard coating ($\gamma = 2$) of thickness (a) $z_c = 0.125a$ (b) $z_c = 0.25a$ (c) $z_c = 0.5a$ (d) $z_c = a$

445 In contrary to soft coatings, the maximum equivalent strain $\tilde{\varepsilon}$ is always
 located in the substrate material (see Fig. 29) for hard coatings ($\gamma = 2$). More-
 over, the maximum equivalent strain $\tilde{\varepsilon}$ in the hard coating is almost two times
 lower than with the uncoated half space (Fig. 31). It is important to notice
 that the maximum of the equivalent strain $\tilde{\varepsilon}$ is located at the interface between
 450 the coating and the substrate for a coating thickness $z_c/a^* \approx 0.5$ (close to the
 maximum shear stress localisation). In the case of $z_c = a^*$, there is no high
 equivalent strain $\tilde{\varepsilon}$ at the interface. The layered structure of hard coatings leads
 to damage propagation at the interface between the coating and the substrate
 parallel to the surface as shown in [66].

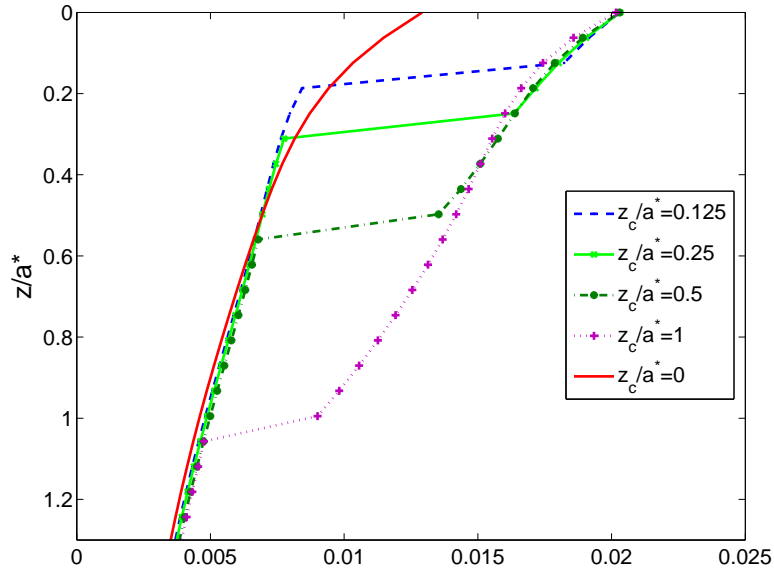


Figure 30: Equivalent strain along z for different thickness z_c and with a soft coating ($\gamma = 0.5$)

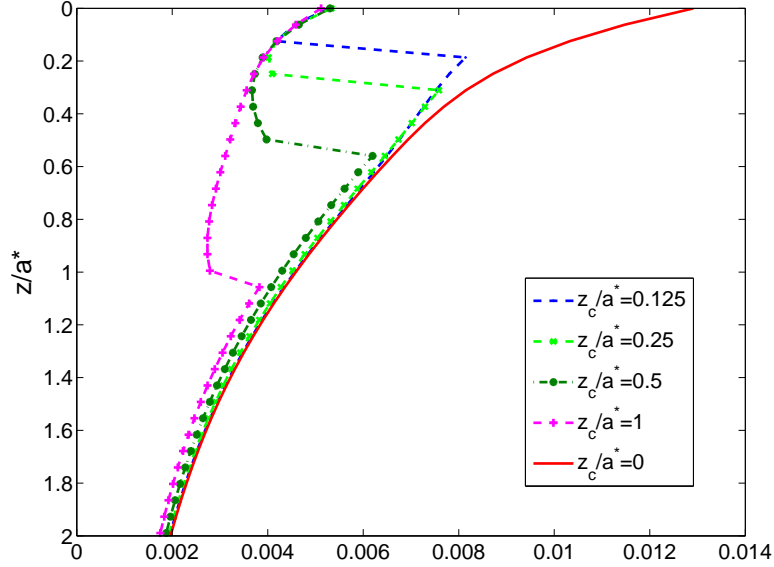


Figure 31: Equivalent strain along z for different thickness z_c and with a hard coating ($\gamma = 2$)

455 The damage variable D is plotted in Fig. 32, along the z -axis for the two different coating stiffnesses for a coating thickness $z_c = 0.25a$ after one fretting cycle. A strong hypothesis is made here that the damage parameters are the same for the coating and for the substrate even if there are not from the same material.

460 It can be observed that, accordingly with what was previously observed with $\tilde{\varepsilon}$, the soft coating is a lot more damaged than the hard one. Moreover, for the hard coating, the maximum damage is located at the interface between the substrate and the coating. But for the soft coating, the maximum of damage is located at the surface and is much higher (around 5 times).

465 The results presented here show that hard coatings are effective to protect the material from fretting surface degradations while soft coatings are damaging faster than uncoated material assuming that the damage threshold in term of yield strength (ε_{d0}) and ductility (ε_R) are the same.

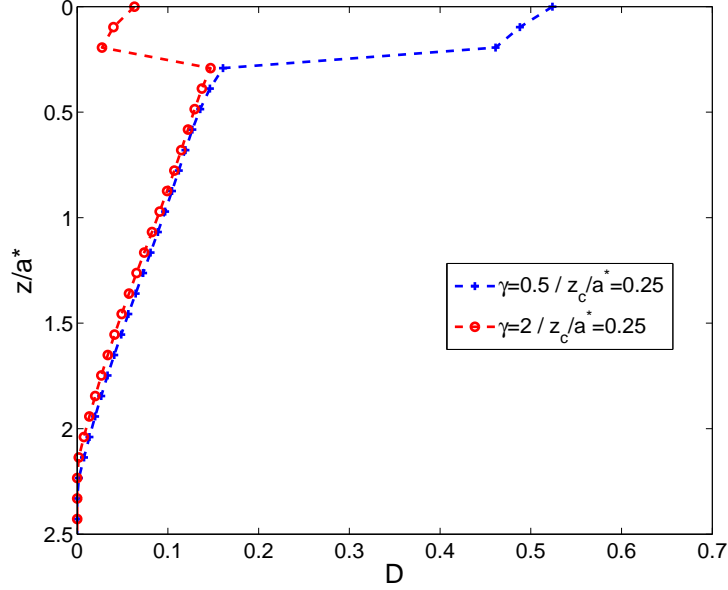


Figure 32: Damage after one fretting cycle along z direction for two different coating stiffnesses

5. Conclusion

470 In this paper, a numerical method has been proposed to model the effect of fretting on surface damage and contact solution. A three-dimensional contact solver with heterogeneous elastic damageable model is developed based on the Eshelby's equivalent inclusion method. Multiple cuboidal inclusions are surimposed on the half space solution as an enrichment technique. The model has
 475 been validated by performing a comparison with the the Hertzian contact solution. The model allow to simulate fretting cycles while taking into account the damage evolution of the surface and his influence on the contact solution. The following major conclusions have been reached :

- The proposed method permits to couple the contact problem, the presence of heterogeneous inclusions and a damage law. The algorithm is very
 480 robust and convergence can be easily reached even with high level of dam-

age. Influence of the computation discretization and enrichment size have been performed to proof the accuracy of the simulations.

- Contact pressure and shear distributions have been investigated along with the damage evolution for both gross slip and partial slip regimes. 485
- The results obtained with our model are in good agreement with the surface damage phenomenon caused by fretting. The proposed method reproduce accurately some classical results of the literature [7]. The model proposed in Ghosh et al. [18] have been implemented in the semi-analytical solver along with the jump in cycle algorithm and a good agreement have 490 been found between the two models.
- The enrichment technique allows to simulate fretting contact on coated material. The influence of these coatings on the damage localization in the material has been explicated. In agreement with the literature [64, 65], 495 it is found again that hard coatings are protecting the surface from fretting damage.

The present work is made on the assumption of linear elasticity to keep the model simple and demonstrate its capability. To be more realistic, the present model could be improved by taking into account plasticity effects during the 500 fretting cycles.

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Nomenclature

Letters

a^*	contact radius
B_{ijkl}^*	influence coefficients that relating the stress σ_{ij} at point (x_1^3, x_2, x_3) to the constant eigenstrain at the point (x_1^k, x_2^k, x_3^k)
C_{ijkl}^M, C_{ijkl}^I	elastic constants of the matrix and the inhomogeneity
E^I	Young's modulus of the inhomogeneity
H	Hardness of the material
S_{ut}	Ultimate tensile stress
h	distance between the two surfaces of the contacting bodies
I_{ijkl}	the fourth-order identity tensor
K^n	coefficients in the normal displacement at the contact surface due to the contact pressure
M_{ij}	influence coefficients relating the stress σ_{ij} at the point (x_1, x_2, x_3) to the normal traction σ^n within a discretized area centered at $(x_1^k, x_2^k, 0)$
n_1, n_2, n_3	grid sizes in the half-space along the Cartesian directions x_1, x_2, x_3 , respectively
W	normal applied load
P_0	maximum Hertzian pressure
P	contact pressure distribution
R	indenter radius
S_{ijkl}	components of the Eshelby's tensor
u_i^0	displacements corresponding to the infinite applied strain ε_{ij}^0
u_i	disturbed contribution of the displacements
$x^I = (x^I, y^I, z^I)$	Cartesian coordinates of the inclusion center
D	damage variable
δD	damage increment
E	undamaged Young's of the inhomogeneity
N	Number of cycles
\tilde{E}	effective Young's modulus
z_c	thickness of the coating.
N_i	Number of inclusions in the enrichment along direction i .

Greek letters

$\tilde{\sigma}$	effective stress
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Appendix A. Stress in a half-space due to a concentrated unit normal force at the surface origin(F_{ij})

$$F_{11}(x, y, z) = \frac{1}{2\pi} \left[\frac{1-2\nu}{r^2} \left(1 - \frac{z}{\rho}\right) \frac{x^2 - y^2}{r^2} + \frac{zy^2}{\rho^3} - \frac{3zx^2}{\rho^5} \right],$$

$$F_{22}(x, y, z) = F_{11}(y, x, z),$$

$$F_{33}((x, y, z)) = -\frac{3}{2\pi} \frac{z^3}{\rho^5},$$

$$F_{12}((x, y, z)) = \frac{1}{2\pi} \left[\frac{1-2\nu}{r^2} \left(1 - \frac{z}{\rho}\right) \frac{xy}{r^2} + \frac{zyx}{\rho^3} - \frac{3zyx}{\rho^5} \right],$$

$$F_{13}((x, y, z)) = -\frac{3}{2\pi} \frac{xz^2}{\rho^5},$$

$$F_{23}((x, y, z)) = F_{12}(y, x, z),$$

where

$$r^2 = x^2 + y^2, \quad \rho = \sqrt{x^2 + y^2 + z^2},$$

with ν , the Poisson's ratio of the isotropic half-space.

Appendix B. Stresses in a half-space subject to normal pressure (M_{ij})

685 An isotropic half-space is submitted a uniform normal pressure σ^n in a discretized surface area of $2\Delta x \times 2\Delta y$ at the center point $P(x', y', 0)$. The stress at an observation point $Q(x, y, z)$ is given in [38] and [56]:

$$\begin{aligned} \sigma_{ij}(x, y, z) &= M_{ij}(x - x', y - y', z)\sigma^n(x, y) \\ \sigma_{ij}(x, y, z) &= \frac{\sigma^n}{2\pi} [h_{ij}(\xi_1 + \Delta x, \xi_2 + \Delta y, \xi_3) - h_{ij}(\xi_1 + \Delta x, \xi_2 - \Delta y, \xi_3) \\ &\quad + h_{ij}(\xi_1 - \Delta x, \xi_2 - \Delta y, \xi_3) - h_{ij}(\xi_1 - \Delta x, \xi_2 + \Delta y, \xi_3)] \end{aligned}$$

where

$$\xi_1 = x - x', \xi_2 = y - y', \xi_3 = z - z'.$$

The functions $h_{ij}()$ in Eq.(B1) are defined by

$$h_{11}((x, y, z)) = 2\nu \tan^{-1} \frac{y^2 + z^2 - \rho y}{xz} + 2(1 - \nu) \tan^{-1} \frac{\rho - y + z}{x} + \frac{xyz}{\rho(x^2 + z^2)},$$

$$h_{22}((x, y, z)) = h_{11}(y, x, z),$$

$$h_{33}((x, y, z)) = \tan^{-1} \frac{y^2 + z^2 - \rho y}{xz} - \frac{xyz}{\rho} \left(\frac{1}{x^2 + z^2} + \frac{1}{y^2 + z^2} \right),$$

$$h_{12}((x, y, z)) = -\frac{z}{\rho} - (1 - 2\nu) \log(\rho + z),$$

$$h_{13}((x, y, z)) = -\frac{yz^2}{\rho(x^3 + z^2)},$$

$$h_{23}((x, y, z)) = h_{13}(y, x, z),$$

where

$$\rho = \sqrt{x^2 + y^2 + z^2}.$$

Appendix C. Normal displacement at the surface subject to normal pressure (K^n)

690 The contact between a sphere and an elastic half-space having respectively elastic constants (E_1, ν_1) and (E_2, ν_2) , where the surface $z = 0$ is discretized into rectangular surface area of $2\Delta_1 \times 2\Delta_2$, is now considered. The initial contact point coincides with the origin of the Cartesian coordinate system $((x, y, z))$. The relationship between the normal displacement at an observation point
695 $P(\xi_1, \xi_2, 0)$ and the pressure field at the center $Q(\xi'_1, \xi'_2, 0)$ is built using the function K^n .

$$K^n(c_1, c_2) = \left[\frac{1 - \nu_1^2}{\pi E_1} + \frac{1 - \nu_2^2}{\pi E_2} \right] \sum_{p=1}^4 K_p^n(c_1, c_2),$$

$$K_1^n(c_1, c_2) = (c_1 + \Delta_1) \log \left(\frac{(c_2 + \Delta_2) + \sqrt{(c_2 + \Delta_2)^2 + (c_1 + \Delta_1)^2}}{(c_2 - \Delta_2) + \sqrt{(c_2 - \Delta_2)^2 + (c_1 + \Delta_1)^2}} \right),$$

$$K_2^n(c_1, c_2) = (c_2 + \Delta_2) \log \left(\frac{(c_1 + \Delta_1) + \sqrt{(c_2 + \Delta_2)^2 + (c_1 + \Delta_1)^2}}{(c_1 - \Delta_1) + \sqrt{(c_2 + \Delta_2)^2 + (c_1 - \Delta_1)^2}} \right),$$

$$K_3^n(c_1, c_2) = (c_1 - \Delta_1) \log \left(\frac{(c_2 - \Delta_2) + \sqrt{(c_2 - \Delta_2)^2 + (c_1 - \Delta_1)^2}}{(c_2 + \Delta_2) + \sqrt{(c_2 + \Delta_2)^2 + (c_1 - \Delta_1)^2}} \right),$$

$$K_4^n(c_1, c_2) = (c_2 - \Delta_2) \log \left(\frac{(c_1 - \Delta_1) + \sqrt{(c_2 - \Delta_2)^2 + (c_1 - \Delta_1)^2}}{(c_1 + \Delta_1) + \sqrt{(c_2 - \Delta_2)^2 + (c_1 + \Delta_1)^2}} \right),$$

where

$$c_1 = \xi_1 - \xi'_1 \quad \text{and} \quad c_2 = \xi_2 - \xi'_2$$