Monitoring the coefficient of variation: A literature review

Zahra Jalilibal, Amirhossein Amiri, Philippe Castagliola, Michael B.C. Khoo

Monitoring the Coefficient of Variation: A Literature Review

Zahra Jalilibal[a], Amirhossein Amiri[a*], Philippe Castagliola[b], Michael B. C. Khoo[c]

[a]Department of Industrial Engineering, Shahed University, Tehran, Iran

[b]Université de Nantes & LS2N UMR CNRS 6004, Nantes, France

[c]School of Mathematical Sciences, Universiti Sains Malaysia, Penang, Malaysia

[*]Corresponding author: amiri@shahed.ac.ir

Abstract

There are some situations when the process mean fluctuates from time to time but is still considered as in-control and the process standard deviation is a linear function of the process mean. In addition, in some cases, the mean and the variance of a process are actually dependent on each other. In these situations, many researchers have suggested to monitor the CV (Coefficient of Variation) as a single statistic. In this paper, a rigorous content analysis method (based on 71 related studies in this area from 2007 to 2021) is applied to categorize the articles that use the coefficient of variation in the SPM (statistical process monitoring) field, to identify the research gaps and to provide guidance to stimulate further researches in this direction.

Keywords: Control chart, Coefficient of variation (CV), Phase I, Phase II, Statistical process monitoring (SPM).

1.Introduction

Control charts are one of the main tools used in statistical process monitoring. If control charts are successfully implemented in many manufacturing and non-manufacturing environments, they can help experts to enhance the quality of a production by reducing process variability through the elimination of assignable causes.

Many studies have been conducted on the monitoring techniques (using control charts) in the past few decades and a large portion of them are focused on monitoring either the mean μ or the standard deviation σ of a process. However, in certain situations, the mean and the standard deviation are not independent of one another and the use of the classical $\bar{X}$, S or R control chart may lead to erroneous decisions. In such situations, the use of the CV (Coefficient of Variation), defined as the ratio $\frac{\sigma}{\mu}$, appears as a valuable alternative in process monitoring for which both the mean and the standard deviation are allowed to vary but their ratio should be kept constant when the process is in-control.

In this paper, we present an overview of control charts based on the coefficient of variation and present a perceptual categorization outline for papers in this field. In line with this aim, 71 papers have been reviewed and summarized.

The structure of the paper is as follows: Section 2 discusses some concerns in CV monitoring. Section 3 describes the survey methodology in searching for related publications and their classifications. Moreover, the perceptual classification scheme for the analysis of the papers is also discussed. Section 4 categorizes the papers by taking into account of the four criteria provided in Section 3 and discusses the analytical results. Finally, in Section 5, the research gaps are discussed and some suggestions are given for future research.

2. Concerns in CV monitoring

2.1.Initial papers and some important extensions of CV control charts

The first paper in this direction was written by Kang et al. (2007), which dealt with a Shewhart chart for monitoring the coefficient of variation and discussed the clinical chemistry control problems. Due to the weakness of the Shewhart control chart in detecting small and moderate shift sizes, Hong et al. (2008) were inspired to develop the EWMA CV control chart and assessed the chart's performance based on the average run length (ARL) criterion. Also, a cumulative sum (CUSUM) control chart for monitoring the

CV was presented by Lee et al. (2010). A two-sided EWMA control chart for monitoring the CV and a two-sided EWMA control chart with a squared CV were proposed by Castagliola et al. (2011), for unknown shifts and applications of these charts were illustrated using the metal sintering process. A double EWMA statistic was proposed for monitoring the CV by Hong et al. (2011) and the results were compared with the EWMA control chart for the CV. The results showed that the DEWMA CV chart outperforms the EWMA CV chart when the sample size is greater than five. A synthetic CV control chart was proposed by Calzada and Scariano (2013) who discussed only the performance of the synthetic CV chart in the case of an increasing CV shift. Castagliola et al. (2013a,b) presented adaptive variable sampling interval (VSI) Shewhart control charts to monitor the process CV and considered supplementary run-rules (RR). A modified EWMA CV control chart was developed by Zhang et al. (2014) which increases the sensitivity of the control chart by Castagliola et al. (2011). Castagliola et al. (2015) implemented a three-parameter logarithmic transformation for the variable sample size CV control chart. Tran and Tran (2016) applied a different CUSUM chart for monitoring the CV and reported that the proposed CV chart outperforms existing charts for different deterministic shift sizes. A variable sample size and sampling interval CV control chart was presented by Khaw et al. (2017). Based on the literature, we presented the control charts proposed by researchers in recent years (2018-2020), as well as the competing control charts and then showed the best control chart for different shift sizes. The results are summarized in Table 1.

Insert Table 1 about here

According to the results in Table 1, generally, it can be concluded that adaptive features, such as VSS, VSI, VSSI and VP improve the performance of the existing control charts. Moreover, memory-type control charts, such as EWMA and CUSUM outperform the Shewhart-type control charts in detecting small shifts. Finally, adding some features, such as auxiliary information and run rules have led to better performance of the proposed control charts, especially in detecting large shifts.

2.2.Categorizing literature based on type of control chart

2.2.1. The distributional properties of the univariate CV

In this section, a brief review of recent researches on univariate CV control charts, for monitoring shifts in the CV with a process from the normal distribution, is provided.

Let X be a positive random variable and suppose that $\mu = E(X) > 0$ and $\sigma = \sigma(X)$ are the mean and standard deviation of X, respectively. The population CV of the random variable X is computed as

$$\gamma = \frac{\sigma}{\mu}. \tag{1}$$

Now, suppose that $\{X_1, X_2, ..., X_n\}$ is a sample of n independent and identically distributed $N(\mu, \sigma)$ random variables. Assume that $\bar{X}$ and S are the sample mean and the sample standard deviation of $X_1, X_2, ..., X_n$, respectively. The sample CV, $\hat{\gamma}$, is computed using Eq (1) by replacing μ and σ with $\bar{X}$ and S, respectively.

Many studies have been focussing on the distributional properties of the sample CV, such as Iglewicz et al. (1968), Iglewicz and Myers (1970), Warren (1982), Vangel (1996), and Reh and Scheffler (1996). Among the aforementioned papers, Iglewicz et al. (1968) pointed out that $\frac{\sqrt{n}}{\hat{\gamma}}$ follows a non-central t distribution with $n-1$ degrees of freedom and a non-centrality parameter $\frac{\sqrt{n}}{\hat{\gamma}}$. Based on this property, the cumulative distribution function (c.d.f.), $F_{\hat{\gamma}}(x \mid n, \gamma)$, of $\hat{\gamma}$ is derived as

$$F_{\hat{\gamma}}(x \mid n, \gamma) = 1 - F_t\left(\frac{\sqrt{n}}{x} \mid n-1, \frac{\sqrt{n}}{\gamma}\right), \tag{2}$$

where $F_t(.)$ is the c.d.f. of the non-central t distribution with $n-1$ degrees of freedom and non-centrality parameter $\frac{\sqrt{n}}{\gamma}$. The inverse c.d.f., $F_{\hat{\gamma}}^{-1}(\alpha \mid n, \gamma)$, of $\hat{\gamma}$ is computed as

$$F_{\hat{\gamma}}^{-1}(\alpha \mid n, \gamma) = \frac{\sqrt{n}}{F_{\hat{\gamma}}^{-1}\left(1-\alpha \mid n-1, \frac{\sqrt{n}}{\gamma}\right)}, \tag{3}$$

where $F_t^{-1}(.)$ is the inverse c.d.f. of the non-central t distribution. Please refer to Tian (2005), Verrill and Johnson (2007), and Mahmoudvand and Hassani (2009) for discussions on confidence intervals and hypothesis tests for the univariate CV.

- Shewhart CV chart

  Kang et al. (2007) proposed a monitoring scheme for the coefficient of variation with a Shewhart control chart, which is denoted as the $SH-\gamma$ control chart. The lower and upper control limits $LCL_{SH-\gamma}$ and $UCL_{SH-\gamma}$, adopted by Kang et al. (2007) are probability type control limits with an assumed Type-I error rate, i.e., to obtain a desired in-control ARL_0. The $LCL_{SH-\gamma}$ and $UCL_{SH-\gamma}$ are computed as:

$$LCL_{SH-\gamma} = F_{\hat{\gamma}}^{-1}(\frac{\alpha_0}{2} \mid n, \gamma_0), \tag{4}$$

$$UCL_{SH-\gamma} = F_{\hat{\gamma}}^{-1}(1-\frac{\alpha_0}{2} \mid n, \gamma_0), \tag{5}$$

  Note that the inverse cumulative distribution function of $\hat{\gamma}$ is denoted as $F_{\hat{\gamma}}^{-1}(\alpha \mid n, \gamma)$. When an assignable cause occurs, the sample CV, $\hat{\gamma}$, will fall outside the control limits. Otherwise, the process is in-control and without taking any corrective action, sampling is continued.

- Synthetic CV chart

A synthetic control chart with a standard CRL sub chart combin wit each other to monitor the process by any given procedure of control chart with a run length variable which is denoted by T, probability function Pr[T = t], and finite first and second expected value, E[T] and E[T^2]. The control chart procedure is applied on the smaples which obtained from the production process of samples at sequential inspection points. While a non-conforming sample happens, a count is imported to investigate the number of samples between the current non-conforming sample and the previous one.While the count number is less than a desirable value, namely *L*, the process is considered out-of-control. Otherwise, the number of count variable reset and the procedure of control chart restart.

The run length variable *T* for the CV follows a geometric distribution,

$$P[T=t]=(1-p)^{t-1}p, t=1,2,3,... \tag{6}$$

$$E[T]=\frac{1}{p}, \tag{7}$$

$$E[T^2]=\frac{2-p}{p^2}, \tag{8}$$

where $p=p(a,LCL,UCL\,|\,n,\gamma_0)$. The upper and lower control limits for the CV control chart and the threshold value *L* for the CRL chart are specified in a way that the desirable in-control ARL is achieved. The $\text{ARL}_{\text{SynCV}}$ and the $\text{SDRL}_{\text{SynCV}}$ of the Synthetic CV chart are

$$ARL_{SynCV}=ARL_{SynCV}(a,LCL,UCL\,|\,n,\gamma_0)=(\frac{1}{1-(1-p)^L})\times(\frac{1}{p}), \tag{9}$$

and

$$SDRL_{SynCV}=SDRL_{SynCV}(a,LCL,UCL\,|\,n,\gamma_0)=\left(\frac{2-p}{(1-(1-p)^L)p^2}+\left(\frac{1}{(1-(1-p)^L)^2}\right)[\frac{1}{p^2}-2\sum_{t=1}^{L}t(1-p)^{t-1}]\right)^{\frac{1}{2}}, \tag{10}$$

where $\alpha=1-F_{\hat{\gamma}}(UCL\,|\,n,p,\delta_1)$ and $\gamma=a\gamma_0$.

- CV-CUSUM control chart

Two one-sided CUSUM control chart for monitroring the squared CV were proposed by Castagliola et al. (2011), Zhang et al. (2014), and Tran and Tran (2016), for a process with normal distribution. This control chart is denoted as CUSUM-γ^2.

A CUSUM control chart for upward shifts which aims to detect an increase in the CV is defined as

$$C_i^+ = \max(0, C_{i-1}^+ + (\hat{\gamma}_i^2 - \mu_0(\hat{\gamma}_i^2)) - K^+), \quad (11)$$

where $C_0^+ = 0$ and $H^+ > 0$ are respectively the initial value and the upper control limit.

A CUSUM control chart for downward shifts which aims to detect a decrease in the CV is defined as

$$C_i^- = \max(0, C_{i-1}^- - (\hat{\gamma}_i^2 - \mu_0(\hat{\gamma}_i^2)) - K^-), \quad (12)$$

where $C_0^- = 0$ and $H^- > 0$ are respectively the initial value and the lower control limit. The mean of the sample CV and the reference parameter of CV squared CUSUM control chart are denoted as $\mu_0(\hat{\gamma}^2)$ and $K^+(K^-)$, respectively.

- CV-EWMA control chart

Hong et al. (2008) proposed an EWMA control chart for monitoring the CV that responds sensitively to small shifts in the CV. The CV-EWMA statistic is represented by Z_t, which is defined as:

$$Z_t = \lambda W_t + (1-\lambda) Z_{t-1}, t \geq 1, 0 \leq \lambda \leq 1, \quad (13)$$

where λ is the smoothing parameter and W_t is the sample CV at the t^{th} time. Let $\mu(W)$ be the average and $\sigma^2(Z_t)$ be the variance of Z_t in Equation (14). Then

$$\sigma^2(Z_t) = \sigma^2(W_t)\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2t}]. \tag{14}$$

The estimation method suggested by Reh et al. (1996) is used to compute the average and variance of W, namely, $\mu(W)$ and $\sigma(W)$. The control limits and center line of the CV-EWMA control chart are given in the following equations.

$$\begin{aligned} UCL_t &= \mu_0(\hat{\gamma}) + L\sigma(W)\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2t}]}, \\ UCL_t &= \mu_0(\hat{\gamma}) - L\sigma(W)\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2t}]}. \end{aligned} \tag{15}$$

- CV-Run sum control chart

The *K* regions above the center line(*CL*) is represented in Fig.1 for the run sum CV (RS-γ) control chart. Moreover, the control limits, scores and probabilities are also illustrated in Fig. 1.

As it can be understood from Fig. 1, $(CL < UCL_1 < UCL_2 < ... < UCL_{k-1} < \infty)$ and $(LCL_k < LCL_{k-1} < ... < LCL_2 < LCL_1 < LCL_0 < \infty)$ are the upper and lower control limits (*UCL*, *LCL*), respectively, where $(0 \le S_1 \le S_2 \le ... \le S_k)$ and $(-S_k \le -S_{k-1} \le ... \le -S_2 \le -S_1 \le 0)$ are the integer scores in the regions above and below the *CL*, respectively. The regions $(R_{+0}, R_{+1}, ..., R_{+(k-1)})$ and $(R_{-0}, R_{-1}, ..., R_{-(k-1)})$, above and below the *CL*, are used to detect shifts (increasing or decreasing) in the CV, respectively. Also, S_t and $-S_t$, for $t = 1, 2, ..., k$, are the scores corresponding to the regions $R_{+(t-1)}$ and $R_{-(t-1)}$, respectively. The upper and lower control limits (*UCL*, *LCL*) and center line (*CL*) are calculated as

$$LCL_t = \max\left[0, \mu_0(\hat{\gamma}) - \left(\frac{3t}{k-1}\right)K\sigma_0(\hat{\gamma})\right], \tag{16}$$

and

$$UCL_t = \mu_0(\hat{\gamma}) + \left(\frac{3t}{k-1}\right) K\sigma_0(\hat{\gamma}), \tag{17}$$

for $t = 1, 2, ..., k-1$, where K is a parameter which is selected in a way that the desired ARL_0 is obtained. The in-control mean and standard deviation of $\hat{\gamma}$ (sample CV) are denoted as $\mu_0(\hat{\gamma})$ and $\sigma_0(\hat{\gamma})$, respectively.

Insert Figure 1 about here

2.2.2. The distributional properties of the Multivariate CV

In this section, a brief review of the literature on Multivariate CV control chart is provided, for detecting shifts in the multivariate CV of a process from the multivariate normal distribution.

It is also possible to define a multivariate analog of the univariate CV defined above. Assume that $\{\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_n\}$ is a random sample of size *n* from the *p*-variate normal distribution with mean vector, $\boldsymbol{\mu}$ and variance-covariance matrix, $\boldsymbol{\Sigma}$. The multivariate population CV is defined as

$$\gamma = (\boldsymbol{\mu}^{\mathbf{T}}\boldsymbol{\Sigma}^{\mathbf{-1}}\boldsymbol{\mu})^{-\frac{1}{2}}, \tag{18}$$

where $\bar{\boldsymbol{X}}$ and $\mathbf{S}$ are the sample mean vector and sample variance covariance matrix, computed from $\{\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_n\}$. Note that in the case of the *p*-variate normal distribution, $\bar{\mathbf{X}}$ and **S** are independent. The multivariate sample CV is obtained from Eq (18) by substituting $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ for $\bar{\mathbf{X}}$ and **S**, respectively.

The c.d.f. of $\hat{\gamma}$ was derived by Yeong et al. (2016b) using the Wijsman's theorem (Wijsman, 1957) as

$$F_{\hat{\gamma}}(x \mid n, p, \delta) = 1 - F_F\left(\frac{n(n-p)}{(n-1)px^2} \mid p, n-p, \delta\right), \tag{19}$$

where $F_F(.\,|\,p,n-p,\delta)$ is the c.d.f. of a non-central F distribution with p and

$UCL_1 = G^{-1}_{\hat{\gamma}_t}(1-\alpha_0; n, p, \theta),$

$SDRL = \frac{\sqrt{1-\pi_1}}{\pi_1}, \alpha_0 \in (0,1)$ degrees of freedom and non-centrality parameter $\delta = \frac{n}{(\tau\gamma_0)^2}$. The inverse c.d.f. of $\hat{\gamma}$ is obtained as

$$F^{-1}_{\hat{\gamma}}(\alpha\,|\,n,p,\delta) = \sqrt{\frac{n(n-p)}{(n-1)p}\left[\frac{1}{F^{-1}_F(1-\alpha\,|\,p,n-p,\delta)}\right]}, \tag{20}$$

where $F^{-1}_{\hat{\gamma}}(.\,|\,n,p,\delta)$ denotes the inverse c.d.f. of a non-central F distribution with p and $n-p$ degrees of freedom and non-centrality parameter δ (Giner-Bosch et al. 2019).

- Shewhart- MCV chart

  Yeong et al. (2016) proposed two one-sided Shewhart charts for monitoring the MCV. As we know $\tau = \frac{\gamma_1}{\gamma_0}$, where γ_0 and γ_1 are respectively the values of the MCV for the in-control and out-of-control processes. The process is in-control when $\tau = 1$; and when $\tau \neq 1$, the process $\{\mathbf{X}_t\}$ is out-of-control due to a shift (upward or downward) in γ_0. *LCL* and *UCL* of the one-sided Shewhart MCV charts for detecting upward and downward shifts in the process MCV are given as

$$LCL_1 = G^{-1}_{\hat{\gamma}_t}(\alpha_0; n, p, \theta), \tag{21}$$

$$UCL_1 = G^{-1}_{\hat{\gamma}_t}(1-\alpha_0; n, p, \theta), \tag{22}$$

  where the Type-I error size is denoted as $\alpha_0 \in (0,1)$, which leads to an in-control ARL equal to $\frac{1}{\alpha_0}$. The out-of-control ARL and SDRL of the proposed control chart are calculated as

$$ARL = \frac{1}{\pi_1} \text{ and } SDRL = \frac{\sqrt{1-\pi_1}}{\pi_1}, \tag{23}$$

where $\pi_1 = P(\hat{\gamma}_t < LCL_1) = G_{\hat{\gamma}_t}(LCL_1, n, p, \theta_1)$ or $\pi_1 = P(\hat{\gamma}_t > UCL_1) = 1 - G_{\hat{\gamma}_t}(UCL_1, n, p, \theta_1)$

with $\theta_1 = \frac{n}{\gamma_1^2}$. Interested readers may refer to Yeong et al. (2016) for more details on the Multivariate CV Shewhart control chart.

- Synthetic MCV control chart

By integrating two sub-charts, namely SH MCV and CRL sub-charts, the Syn MCV chart is formed. It is used for detecting shifts (τ > 1), upward multivariate CV. Detecting upward increasing shifts in CV (upward multivariate CV) is more essential than detecting decreasing shifts in CV (downward multivariate CV) in the multivariate process. So, Yeong et al. (2016b) proposed the upward Syn MCV chart .

The upper control limits of the MCV sub-chart (of the Syn-MCV chart) can be computed from the following equation:

$$UCL = F_{\hat{\gamma}}^{-1}(1-\alpha \mid n, p, \delta_0). \tag{24}$$

The ARL and SDRL of the Syn MCV control chart can be calculated as

$$ARL_{SynMCV} = \left(\frac{1}{1-(1-\alpha)^L} \right) \times (\frac{1}{\alpha}), \tag{25}$$

and

$$SDRL_{SynMCV} = \left(\frac{2-\alpha}{(1-(1-\alpha)^L)\alpha^2} + \left(\frac{1}{(1-(1-\alpha)^L)^2} \right) [\frac{1}{\alpha^2} - 2\sum_{t=1}^{L} t(1-\alpha)^{t-1}] \right)^{\frac{1}{2}}, \tag{26}$$

where $\alpha = 1 - F_{\hat{\gamma}}(UCL \mid n, p, \delta_1)$ and $\delta_1 = \frac{n}{\gamma_1^2}$.

- EWMA MCV control chart

According to an EWMA design, $Z_t = \lambda W_t + (1-\lambda)Z_{t-1}, t \geq 1, 0 \leq \lambda \leq 1,$ the proposed statistic can be calculated in the MCV- squared control chart for each sample $t \geq 1$, where the value of MCV-squared is denoted by $\hat{\gamma}_t^2$ and calculated at sample t, while λ is a fixed smoothing parameter ($0 < \lambda \leq 1$). The expected value of $\hat{\gamma}^2$ is denoted by Z_0 or $\mu_0(\hat{\gamma}^2)$, when the process is in-control (i.e, $\gamma = \gamma_0$).

$$Z_0 = \mu_0(\hat{\gamma}^2) = E[\hat{\gamma}^2 \mid n, p, \gamma = \gamma_0] \tag{27}$$

The center line (*CL*) of the EWMA MCV control chart is given by the value $\mu_0(\hat{\gamma}^2)$. The mean and standard devition of Z_t are defined ($\mu(Z_{+\infty})$ and $\sigma(Z_{+\infty})$) as asymptotic limits of an EWMA control chart.

$$\sigma(Z_{+\infty}) = \sqrt{\frac{\lambda}{2-\lambda}}\sigma_0(\hat{\gamma}^2), \tag{28}$$

when the process is in-control. Note that $\sigma_0(\hat{\gamma}^2)$ is the standard deviation of $\hat{\gamma}^2$ when the process is in-control.

$$\sigma_0(\hat{\gamma}^2) = \sqrt{Var[\hat{\gamma}^2 \mid n, p, \gamma = \gamma_0]}. \tag{29}$$

UCL is defined for the proposed control chart more precisely as

$$UCL = \mu_0(\hat{\gamma}^2) + K\sqrt{\frac{\lambda}{2-\lambda}}\sigma_0(\hat{\gamma}^2), \tag{30}$$

while the distance between the center line and the upper control limit is denoted by $(K > 0)$ which is expressed in terms of the number of standard deviations of Z_t. Two constant parameters, namely, λ and K, should be determined by the practitioner.

- Run sum MCV control chart

Two one-sided run sum MCV control charts containing an upward control chart for detecting increasing shifts and a downward control chart for detecting decreasing shifts in the multivariate CV are suggested. The following upper control limits are defined for the upward run sum MCV chart with k regions:

$0 < UCL_0 < UCL_1 < UCL_2 < ... < UCL_k$, while the median of sample CV is denoted by $UCL_0 = F_{\hat{\gamma}}^{-1}(0.5 | n, v, \delta_0)$ and $UCL_k = \infty$. The score S_j is assigned if $\hat{\gamma}_t$ falls in the interval $[UCL_{j-1}, UCL_j)$, i.e.

$$S(\hat{\gamma}_t) = S_j, \text{if} \quad \hat{\gamma}_t \in [UCL_{j-1}, UCL_j), \ j = 1, 2, ..., k, \tag{31}$$

where the sample number for Phase II is $(t = 1, 2, ...,)$. The values of the scores are $(0 \le S_1 \le S_2 \le ... \le S_k)$. The following lower control limits are defined for the downward run sum MCV chart with k regions:

$LCL_k < LCL_{k-1} < ... < LCL_2 < LCL_1 < LCL_0 < \infty$, while the median of sample MCV is denoted as $LCL_0 = F_{\hat{\gamma}}^{-1}(0.5 | n, v, \delta_0)$ and $LCL_k = 0$. The score $-S_j$ is assigned if $\hat{\gamma}_t$ falls in the interval $(LCL_j, LCL_{j-1}]$,

$$S(\hat{\gamma}_t) = -S_j, \text{if} \quad \hat{\gamma}_t \in (LCL_j, LCL_{j-1}], \ j = 1, 2, ..., k. \tag{32}$$

with t = 1, 2, … representing the sample number of the Phase-II process. Note that $(-S_k \le -S_{k-1} \le ... \le S_2 \le S_1 \le 0)$.

The *UCL* and *LCL* of the upward and the downward run sum MCV control charts, respectively, are calculated as

$$UCL_j = K \times F_{\hat{\gamma}}^{-1}(\alpha_j | n, v, \delta_0), \ j = 1, 2, ..., k-1. \tag{33}$$

$$LCL_j = K \times F_{\hat{\gamma}}^{-1}(1 - \alpha_j | n, v, \delta_0), \ j = 1, 2, ..., k-1. \tag{34}$$

Note that K is a parameter which is selected in a way that a desirable in-control ARL can be achieved.

2.3. Sample sizes

As we can see in the distributional properties of the CV in Subsection 2.2, the sample size is denoted as n. We investigated recent papers in the area of monitoring the CV and reported the considered sample sizes, as well as the performance criteria. Additionally, the best sample size based on the simulation studies is investigated and the findings are provided. It is obvious that by increasing the sample size, the performance of each control chart in detecting shifts in the CV gets better. The results are summarized in Table 2.

Insert Table 2 about here

2.4. Applications of CV and CV control charts

There are some situations when the process mean fluctuates over time (or from time to time) but the process is still considered as in-control, while the process standard deviation is a linear function of the process mean. In addition, in some cases, the mean and variance of a process are actually dependent on one another. In the above situations, many researchers have suggested to monitor the CV (Coefficient of Variation) as a single statistic such as Kang et al. (2007), Hong et al. (2008) and etc. CV control charts are applied by many researchers and several applications have been investigated in the literature.

In some situations, the in-control parameter shifts from sample to sample. For example, Castagliola et al. (2015a) have demonstrated that the in-control process does not have a stable parameter. Therefore, it is not suitable to adopt the $\bar{X}$ and S charts in process monitoring. In these situations and for similar processes, where the process output changes from time to time according to the planning decisions but the output variance is a linear function of the mean, the CV chart can be adopted in process monitoring.. As industrial engineers frequently encounter scenarios where the process output changes, CV charts will be very useful to monitor such processes. Kang et al. (2007) demonstrated the necessity of CV monitoring in a clinical chemistry-control problem. As the mean varies from patient to patient, it is not useful to monitor the sample mean, $\bar{X}$. Moreover, the variance is a function of the mean, thus, the R or S chart cannot be adopted. However, such a process usually illustrates a constant proportionality between the process mean and the process standard deviation. Therefore, the CV can be adopted as the monitoring statistic to measure the inherent variability in the process.

CV monitoring is different from joint monitoring. In a joint monitoring of the process mean and standard deviation, a process is deemed as in-control only when both the mean and standard deviation are in-control. If one of these parameters or both of them change, the process is out-of-control and the control chart in use will signal an out-of-control situation. However, in CV monitoring, both the process mean and standard deviation may change due to the nature of the process but the process may still be in-control. In other words, the ratio $\frac{\sigma}{\mu}$ should be monitored over time and if this ratio is in-control, the process will be in-control.

In these situations, CV control charts are applied by many researchers and several applications have been investigated in the literature. For example, the CV is generally applied in finance. In this field, the risk confronted by investors is measured with the CV, by specifying the volatility of the return on an asset to the expected value of the return (Sharpe, 1994). Moreover, the portfolio performance is measured by the CV and the risk of the market (Knight and Satchell, 2005). In the field of manufacturing and materials engineering, CV is applied to investigate the failure rate, reliability and fatigue limit of materials (He and Oyadiji, 2001). CV was adopted in mechanical engineering (Mucha and Witkowski, 2013), manufacturing (Gauri, 2005), geotechnical engineering (Jiang et al., 2014) and landfill engineering (Dixon and Jones, 2005), where some quality characteristics related to the physical properties of products constituted by metal alloys or composite materials often have a standard deviation which is proportional to their population mean. In the area of medical and biological sciences, Shechtman (2013) used the CV as an influential indicator of reliability and variability. CV is utilized in assessing some specified chemicals in the blood and urine (serum) of a patient, in monitoring the chemistry control of a process (Apple et al., 2002; Chadban et al., 2003). Another application of the CV in health sciences was investigated by Gulhar et al. (2012) for illative and delineative matters. Furthermore, evaluating the resistance of ceramics (Gong and Li, 1999) and also spectrophotometric repeatability (Centore, 2016) were made by using the CV.

As the application of the CV is remarkable in numerous fields, monitoring the CV with control charts becomes an important research topic among researchers. Kang et al. (2007) exploited a Shewhart CV control chart for monitoring the levels of cyclosporine in patients undergoing organ transplantation which illustrates the necessity of CV monitoring in a common clinical chemistry-control problem where repeated measurements of some characteristics (such as the amount of a chemical in a patient's blood) are taken and quality-control checks are required to be carried out on the laboratory's measurements. Castagliola et al. (2011) discussed the applications of SPC for monitoring the CV in materials engineering by an example. The example deals with monitoring the pressure test drop time from an Italian company that manufactures sintered mechanical parts. Castagliola et al. (2015a) implemented the VSS CV control chart on a real dataset from the sanitary sector for the die casting manufacturing parts and showed the necessity to monitor the CV in a die casting hot chamber process from a Tunisian company manufacturing zink alloy (ZAMAK). The weight of scrap zinc alloy material to be eliminated between the molding process and successive continuous plating surface treatment requires to be monitored. Teoh et al. (2017) showed the applications of run sum CV control chart on real industrial data of a die casting hot chamber process. These data are provided by a zinc-alloy (ZAMAK) manufacturing company in Tunisia. Ye et al. (2018) represented that monitoring the CV is useful to detect the presence of chatter, which is a severe form of self-excited vibration in a machining process which results in many machining problems. By adopting the CV, the process monitoring can be made on various machining materials and machining parameters. Abbasi (2020) demonstrated the application of auxiliary information based CV charts on a real data set on air quality. As polluted air seriously affects human health, monitoring and controlling the pollution levels in air can improve air quality.

3. Survey methodology, questions and possible answers for the perceptual categorization outline

3.1. Survey methodology

In this subsection, a classification of the papers in the area of process monitoring of the CV is conducted on the basis of a two-step content analysis (Kolbe & Burnett, 1991):

Step 1: Literature search and survey sources

To provide the relevant papers and sources, only journal papers are selected in the survey. The related studies are conducted by means of a digital search. For this purpose, phrases such as ‘‘coefficient of variation control chart”, ‘‘Multivariate coefficient of variation”, “coefficient of variation monitoring” and so on are used as keywords. Subsequently, a “snow ball” approach is carried out by investigating the references and citations of each paper for obtaining the previous researches. During the process, the completion of the paper is accompanied by adding new papers to our results.

Step 2: Categorizing the selected papers

Relevant papers are categorized by considering the following four criteria:

- ✓ *Analysis of the related publications in each year*
- ✓ *Number of published papers in each journal*
- ✓ *Name of authors/co-authors in the published papers*
- ✓ *Perceptual categorization scheme to appraise the papers*

Insert Table 3 about here

3.2. Questions and possible answers

Questions and possible answers for each content are considered and suggested. The perceptual categorization scheme, including three general criteria and the corresponding sub-criteria for each one, are considered and listed in Table 3. Each criterion is defined and their possible states for each one are described in detail. Then, each criterion and the corresponding sub-criteria specified in Table 1 are explained in this subsection.

3.2.1. Areas of SPM

Relying on the reviewed papers, the following three areas of SPM can be considered:

• ***Statistical design:*** Checking the stability of a process during the period of time using statistical approaches is the principal aim of statistical process monitoring (SPM). The most helpful tool for this purpose are control charts. According to Woodall (1985), control charts can be designed by considering some measures such as average run length (ARL) and standard deviation of the run length (SDRL) and any other metrics based on the run length (RL) criterion.

• ***Economic and/or economic-statistical design:*** Obtaining the optimal parameters of a control chart by minimizing the expected value of some cost function, is known as the "economic design" of a control chart (Lorenzen and Vance (1986)). In the "Economic-statistical design" of a control chart, the expected value of a cost function is minimized by also considering some statistical constraints based on the in-control and out-of-control ARLs or average time to signal (ATS) criterion (Saniga (1989)).

3.2.2. Types of CVs

Univariate coefficient of variation:

In the case that only a single quality characteristic is available and that it is necessary to monitor the coefficient of variation instead of the mean or the standard deviation, a univariate coefficient of variation should be considered.

Multivariate coefficient variation:

The multivariate CV is helpful in monitoring the performance of multivariate processes. For instance, the relative variability of the gene expression levels, which are attained by the microarray methods, can be monitored by the multivariate CV chart. Furthermore, to detect signs of evolution, several features or specifications of a given species are measured, where the multivariate CV chart is employed to monitor the multivariate CV.

3.2.3. Types of shifts

Known shift size: If the shift size is deterministic, in other words, when a particular shift size can be specified, then the shift size is declared as known.

Unknown shift size: In the case where the magnitude of a shift is unknown, which is true in most applications, the shift size is said to be unknown.

3.2.4. Types of control charts

The papers selected in this survey are categorized based on the types of control charts, which include the Shewhart-type and memory based control charts. The definitions of these types of control charts are given as follows:

Shewhart-type control chart: This type of control chart, first suggested by Dr.Walter Shewhart in 1924 to monitor quality characteristics, only uses the information of the last sample taken and it is sensitive in detecting large shifts in the process.

***Memory-based control chart*:** This kind of control chart relies on the previous and current observations of the process. The main goal of this type of control chart is to serve as an alternative to the Shewhart-type control charts for a quicker detection of small and medium shifts in the process. The Exponentially Weighted Moving Average (EWMA) (Roberts, 1959) and the Cumulative Sum (CUSUM) (Page, 1954) control charts are the most common charts in this category.

Adaptive control charts: In adaptive control charts, one or more parameters of the charts are allowed to change over time, based on the position of the previous control charting statistic plotted on the control charts. Adaptive control charts are generally categorized into four types:

1. variable sample size (VSS), 2. variable sampling interval (VSI), 3. variable sample size and sampling interval (VSSI), 4. variable parameters (VP) and 5. Fixed parameters (FP).

VSS: when the sample size changes from one sample to the other.

VSI: when the sampling interval changes from one sample to the other.

VSSI: when both sampling interval and sample size are variable.

VP: when all the parameters are allowed to be variable through time.

FP: when all the parameters are fixed through time, the chart is non-adaptive and it is known as a fixed parameters chart.

Other control charts: This category contains synthetic and run sum (Davis et al. 1990) control charts. The design of synthetic control charts in the monitoring of both univariate and multivariate quality characteristics were presented by many researchers. Wu and Spedding (2000) were the pioneers of the synthetic chart to detect shifts in the process mean. They concluded that their suggested control chart outperforms the Shewhart $\bar{X}$ chart in detecting shifts in the process mean.

The run sum control chart is a powerful method which is easy to implement. The run sum chart can be considered as a zone control chart. The run sum control charting procedure involves dividing the control chart into regions and assigning scores to these regions. The run sum control charts have received a noticeable attention among practitioners.

3.2.5. Analysis phase

As it can be seen in the literature, control charts for monitoring the CV are applied in Phase I or Phase II. The main concentration of Phase I analysis is to figure out the process variability, checking the process stability using historical data, examining the ideas of process-improvement, choosing a suitable in-control model, and estimating the parameters of the in-control model. As mentioned before, the practitioner selects a suitable in-control model. Then, the parameters of the process are estimated to determine a proper Phase II monitoring scheme. The purpose of a Phase II analysis is to detect out-of-control situations in the process as quickly as possible, based on the information of Phase I.

3.2.6. Evaluation criterion

As mentioned, control charts for monitoring the CV are classified into the Phase I and Phase II categories. The performance of a control chart for monitoring the CV in Phase I is assessed by the signal probability criterion. This criterion is defined as the probability of a charting statistic to fall beyond the control limits of the chart. To evaluate the performance of Phase II charts, it is prevalent to use some metrics based on the run-length characteristics, such as the Average Run Length (ARL), Standard Deviation of the Run Length (SDRL), Expected Average Run Length (EARL) [When the practitioners are unable to specify the exact value of τ (shift size), then EARL can be used as a measure of the chart's performance (Lim et al. (2017)], Average Time to Signal (ATS) and Standard Deviation of the Time to Signal (SDTS). Note that the run length is defined as the number of samples (or time) taken from the beginning of process monitoring until an out-of-control signal occurs. Moreover, Time to signal is defined as the time taken from the beginning of process monitoring until an out-of-control signal happens.

3.2.7. Data generating techniques

Only two data generating techniques are used for monitoring the CV, which include the SRS and RSS methods defined as follows:

SRS: A statistical subset from the population where each member has an equal chance of being selected is called the Simple Random Sampling.

RSS: Combining SRS with the investigator's professional knowledge and judgment in selecting situations for sampling is called the Ranked Set Sampling.

3.2.8. Measurement errors

In most applications, the presence of measurement errors is inevitable and the actual values of the quality characteristics are unavailable. Hence, the following covariate error model is utilized to show the relationship between the observed (Y) and the actual (X) values of the quality characteristic:

$$Y = A + BX + \varepsilon. \tag{13}$$

The actual values of the quality characteristic (X) usually follow a normal distribution with mean μ_X and variance σ_X^2, where A and B are two fixed constants. In Equation (14), ε is the measurement error term which is supposed to follow a normal distribution with mean zero and variance σ_ε^2 (Maleki et al., 2017).

4. Analyzing the Results

The four criteria listed in Section 3 are used to investigate the literature. They provide a general view about the trend of publications in different years, the distribution of papers in each journal, the list of authors and co-authors who contributed in the area of monitoring the CV, and finally the classification of the papers based on the conceptual framework discussed in Section 3.

Trend of publications in each year

The number of publications in the area of monitoring the coefficient of variation is depicted in Fig. 1. As shown in this figure, at least three papers have been published annually from 2011 to 2021, except in 2012 and 2014 where only two papers have been published. Also, the number of publications in each year has been increasing in recent years, which shows the importance and interest researchers have in this topic.

Insert Figure 2 about here

Number of published papers in each journal

Table 4 lists the number of published papers among 36 journals. By considering the frequency of the published papers, ‘‘Quality and Reliability Engineering International” and ‘‘Computers & Industrial Engineering” are ranked first and second, with 10 and 7 papers in this area, respectively. After these journals, the “The International Journal of Advanced Manufacturing Technology” with 5 papers, ranked as third journal.

Insert Table 4 about here

Authors/co-authors contributing in this area

The names of authors who published more than 2 papers in this area accompanied by their affiliations and countries are given in Table 5. Based on the number of papers, “Khoo, M. B. C.”, “Yeong, W. C.” and “Castagliola, P.” are the most active authors in this area, with 25, 18 and 16 papers, respectively.

Insert Table 5 about here

Perceptual categorization scheme to appraise the related papers

All the 71 reviewed papers on monitoring the coefficient of variation in various areas of SPM are categorized in Table 4, on the basis of the perceptual categorization scheme. Section 4 articulates an investigation of the literature with reference to the perceptual categorization. For each content, a comprehensive investigation is conducted in Section 4. Accordingly, the research gaps and orientation for further studies are discussed in Section 5.

An extensive analysis about each content expressed in Section 3, including areas of SPM, type of shift size, type of control chart, type of quality characteristic, phase of analysis, evaluation criterion and type of adaptive control chart is conducted.

Areas of SPM

Fig. 3 summarizes the percentage of the published papers in different areas of SPM. Based on Fig. 2, the most endeavors, i.e. 94.3% (67 out of 71 articles) were carried out on "statistical design" in the area of monitoring the CV. The first paper in this direction was written by Kang et al. (2007), which deals with a Shewhart chart for monitoring the coefficient of variation. As their proposed chart is less sensitive to small and moderate shifts, other researchers were inspired to develop CV control charts. Thus, Hong et al. (2008) studied the monitoring of the univariate CV by using an EWMA control chart. Most researches in the area of statistical design of control charts have been proposed for monitoring univariate CV. Some papers have been recently focused on monitoring the multivariate CV, such as Khaw and Chew (2019), Nguyen et al. (2019), Chew et al. (2020) and Ayyoub et al. (2020 a, b). There are only 4 papers (5.6%) that are related to "economic/economic-statistical designs". Yeong et al. (2015), Yeong et al. (2016a), Yeong et al. (2017b) and Chew et al. (2020) are the only papers in the field of Economic design/Economic-statistical design for monitoring the CV.

Insert Figure 3 about here

Types of quality characteristics

Fig. 4 presents the percentage of papers with different types of quality characteristics. As shown in Fig. 3, 77% of the published papers (55 papers) are concentrated on the univariate CV, while only 23% (16 papers) on the multivariate CV. From 2007 to 2018, all the published papers have concentrated on the univariate CV, except Yeong et al. (2016b) which is the first paper on monitoring the multivariate CV. In recent years, papers that considered multivariate CV were made by Khaw and Chew (2019), Nguyen et al. (2019), Chew et al. (2020) and Ayyoub et al. (2020a, b).

Insert Figure 4 about here

Types of shift sizes

Based on the 4th column of Table 6, 50 papers (70%) have considered only the known shift size, while only 6 papers (9%) have focused on only the unknown shift size. Also, 15 studies (21%) investigated both known and unknown shift sizes. Fig. 5 presents these percentages in a pie chart.

Insert Figure 5 about here

Insert Table 6 about here

Types of control charts

The types of control charts used to monitor the CV are listed in the 5th column of Table 6. The results show that most of the papers have utilized Shewhart-type control charts (37 papers-52.1 %). 26 papers have used memory-based control charts (36.6%). Only 8 papers (11.3%) have used other control charts, such as synthetic and run sum charts.

The analysis phase

The analysis Phase of each reviewed paper is listed in the 5th column of Table 4. Fig.6 illustrates the results concerning the analysis phase. Note that 65 papers (92%) have been developed for a Phase II analysis and just 6 papers (8%) considered a Phase I analysis. It seems that most of the papers in this field focused on Phase II analysis and can be used as further studies.

Insert Figure 6 about here

Evaluation criterion

The 7th column of Table 6 shows the different criteria used in each paper analyzed in this survey, in assessing the performance of the proposed control charts. In "statistical design", the PTS, as well as the ARL and SDRL are the most prevalent criteria for Phase I and Phase II analyses, respectively. There is only one paper investigating the change point estimation, which uses criteria such as the average and the standard deviation of the estimated change point, represented by $\bar{\hat{\tau}}$ and $sd(\hat{\tau})$, respectively.

Types of data generating techniques

The different *data generating techniques* utilized in the published papers in the area of monitoring the CV are summarized in the 8th column of Table 6. Among all the 71 papers, 69 of them used simple random sampling (SRS) and only 2 papers utilized RSS sampling methods.

Measurement errors

The 9th column of Table 6 deals with the existence/absence of measurement errors in the 71 published papers in the area of monitoring the CV. It can be seen that only 9 papers have considered the presence of measurement errors. A noticeable point in this area is that most of the papers which considered measurement errors have focused on monitoring the univariate CV and only one paper focused on monitoring the multivariate CV.

Types of adaptive control charts

The last column of Table 6 lists the types of adaptive control charts used in monitoring the CV. The outcomes in the last column of Table 6 are summarized in Fig. 7. As it can be seen, about 75% of the papers (53 papers) have utilized non-adaptive (FP) control charts and just 18 papers have considered adaptive charts (25 %). Among the adaptive charts, 6 papers (8%) deal with VSI charts and 5 papers (7%) are based on VSS charts. The VSSI and VP charts are considered in 5 papers (7%) and 2 papers (3%), respectively.

Insert Figure 7 about here

5. Concluding remarks and some suggestions for future studies

CV control charts are applied by many researchers and several applications have been investigated in the literature which are discussed in detail in Subsection 2.4.

Some important concluding remarks are as follows:

1. A noticeable growth in the different research areas of monitoring the CV has taken place since 2007, especially in the last decade and notably from 2012 to 2019, in which the number of publications has grown significantly.
2. All of the reviewed papers were published in 36 different journals, where 17 (out of 71) papers appeared in 2 journals, namely, Quality and Reliability Engineering International and Computers

& Industrial Engineering. These two journals are ranked first and second, with 10 and 7 papers in this area, respectively.

3. A large percentage of the published papers in monitoring the CV originated from researchers from three countries. As seen in the results, over the past decade, the researchers from Malaysia, France and Korea have noticeable contributions in monitoring the CV.
4. A great number of papers focused on the "statistical design" of CV control charts. Consequently, it can be concluded that only few researches were made on the "economic-statistical design" of CV control charts.
5. Even though remarkable extensions have been made in the last 10 years in the literature of monitoring the CV, based on the analytical results in this paper, several topics are identified and proposed for further studies.

A review of papers based on a perceptual categorization outline in the area of monitoring the CV has been carried out in this paper. Based on the content analysis provided, some gaps and potential areas for further studies in SPM are highlighted.

1. Most of the papers which considered measurement errors focused on the univariate CV charts. Therefore, investigating the effect of measurement errors on the performances of different types of multivariate CV charts can be explored in the future, as currently only the research by Ayyoub et al. (2020a) on multivariate CV chart with measurement errors is available in the literature. Moreover, designing both univariate and multivariate adaptive control charts in the presence of measurement errors are potential areas for future studies.
2. There is no paper in the literature on change point estimation. As a result, research in this direction is an important topic in a future study. In line with this aim, different change point estimators for the monitoring of both univariate and multivariate CV based on various types of shifts, including step, drift, monotonic, as well as sporadic can be developed.

3. Based on the content analysis, most of the papers used the SRS sampling method. Hence, developing CV control charts with other types of sampling methods is suggested for further research.
4. As only a few papers considered adaptive control charts for monitoring both univariate and multivariate CV, more studies are needed to address the applications of adaptive control charts for monitoring the CV.
5. In many manufacturing and non-manufacturing environments, it is important to monitor the CV in multi-stage processes. Therefore, it is beneficial for future studies to consider the monitoring of the CV in multi-stage processes.
6. It is impossible to collect adequate data to perform a Phase I analysis and estimate the process parameters, in some production environments. As sufficient reference data in Phase I are unattainable, self-starting control charts could be an alternative. As a further extension in monitoring the CV, designing self-starting control charts is suggested.
7. In some applications, the observations are auto-correlated. In these cases, developing control charts for monitoring the CV when the observations are auto-correlated can be a potential area for future research. Thus, investigating the effect of measurement errors on the monitoring of multivariate CV by considering auto-correlated multivariate observations for different time series models is suggested.
8. Since 2019, only 2 papers which incorporated auxiliary information into the monitoring of the CV were proposed. Therefore, this area can be investigated further in a future research. Moreover, it is also suggested that the incorporation of runs rules and auxiliary information into CV charts can be explored further.
9. Developing control charts for monitoring the CV in new areas of SPM, such as high dimensional process monitoring or big data are potential areas for further research.
10. Investigating the effect of parameter estimation in Phase I on the performance of control charts for monitoring the CV in Phase II should be considered in future studies.

11. Applying runs rules in memory-type control charts, such as EWMA and CUSUM charts for monitoring both univariate and multivariate CV can be an important topic for further studies.
12. Based on the literature of CV charts, the economic-statistical design of control charts for monitoring multivariate CV is identified as a research gap which ought to be addressed in the near future.
13. Most of the papers adopt numerical methods to obtain the optimal chart parameters in the field of CV monitoring. Future research can study the use of meta-heuristic methods to obtain the optimal chart parameters, and studies can be done on whether the optimal chart parameters obtained through meta-heuristic methods results in better performance.
14. An interesting area other than RSS is the dynamic sampling scheme proposed by Li and Qiu (2014) with a cumulative sum (CUSUM) control chart.

References

Abbasi, S. A., & Adegoke, N. A. (2018). Multivariate coefficient of variation control charts in phase I of SPC. *The International Journal of Advanced Manufacturing Technology*, *99*(5-8), 1903-1916.

Abbasi, S. A. (2020). Efficient control charts for monitoring process CV using auxiliary information. *IEEE Access*, *8*, 46176-46192.

Amdouni, A., Castagliola, P., Taleb, H., & Celano, G. (2015). Monitoring the coefficient of variation using a variable sample size control chart in short production runs. *The International Journal of Advanced Manufacturing Technology*, *81*(1-4), 1-14.

Amdouni, A., Castagliola, P., Taleb, H., & Celano, G. (2016). One-sided run rules control charts for monitoring the coefficient of variation in short production runs. *European Journal of Industrial Engineering*, *10*(5), 639-663.

Amdouni, A., Castagliola, P., Taleb, H., & Celano, G. (2017). A variable sampling interval Shewhart control chart for monitoring the coefficient of variation in short production runs. *International Journal of Production Research*, *55*(19), 5521-5536.

Aslam, M., Gadde, S. R., Aldosari, M. S., & Jun, C. H. (2019). A hybrid EWMA chart using coefficient of variation. *International Journal of Quality & Reliability Management*, *36* (4), 587-600.

Ayyoub, H. N., Khoo, M. B. C., Saha, S., & Castagliola, P. (2020a). Multivariate coefficient of variation charts with measurement errors. *Computers & Industrial Engineering*, *147*, 106633.

Ayyoub, H. N., Khoo, M. B. C., Saha, S., & Lee, M. H. (2020b). Variable sampling interval EWMA chart for multivariate coefficient of variation. Published online in *Communications in Statistics - Theory and Methods*, 1-21.

Calzada, M. E., & Scariano, S. M. (2013). A synthetic control chart for the coefficient of variation. *Journal of Statistical Computation and Simulation*, *83*(5), 853-867.

Castagliola, P., Celano, G., & Psarakis, S. (2011). Monitoring the coefficient of variation using EWMA charts. *Journal of Quality Technology*, *43*(3), 249-265.

Castagliola, P., Achouri, A., Taleb, H., Celano, G., & Psarakis, S. (2013a). Monitoring the coefficient of variation using a variable sampling interval control chart. *Quality and Reliability Engineering International*, *29*(8), 1135–1149.

Castagliola, P., Achouri, A., Taleb, H., Celano, G., & Psarakis, S. (2013b). Monitoring the coefficient of variation using control charts with run rules. *Quality Technology & Quantitative Management*, *10*(1), 75–94.

Castagliola, P., Achouri, A., Taleb, H., Celano, G., & Psarakis, S. (2015a). Monitoring the coefficient of variation using a variable sample size control chart. *The International Journal of Advanced Manufacturing Technology*, *80*(9-12), 1561-1576.

Castagliola, P., Amdouni, A., Taleb, H., & Celano, G. (2015b). One-sided Shewhart-type charts for monitoring the coefficient of variation in short production runs. *Quality Technology & Quantitative Management*, *12*(1), 53-67.

Centore, P. (2016). The coefficient of variation as a measure of spectrophotometric repeatability. *Color Research & Application*, *41*(6), 571-579.

Chen, R., Li, Z., & Zhang, J. (2019). A generally weighted moving average control chart for monitoring the coefficient of variation. *Applied Mathematical Modelling*, *70*, 190-205.

Chew, X. Y., Khoo, M. B. C., Khaw, K. W., Yeong, W. C., & Chong, Z. L. (2019). A proposed variable parameter control chart for monito-ring the multivariate coefficient of variation. *Quality and Reliability Engineering International*, *35*(7), 2442-2461.

Chew, X. Y., Khaw, K. W., & Yeong, W. C. (2020a). The efficiency of run rules schemes for the multivariate coefficient of variation: a Markov chain approach. *Journal of Applied Statistics*, *47*(3), 460-480.

Chew, X. Y., & Khaw, K. W. (2020). One-sided downward control chart for monitoring the multivariate coefficient of variation with VSSI strategy. *Journal of Mathematical and Fundamental Sciences*, *52*(1), 112-130.

Chew, Y. Y., Khoo, M. B. C., Khaw, K. W., & Yeong, W. C. (2020b). Economic and economic-statistical designs of variable sample size and sampling interval coefficient of variation chart. Published online in *Communications in Statistics - Theory and Methods*, doi: 10.1080/03610926.2020.1769670.

Chun, D. J., & Chung, Y. B. (2016). Design of the robust CV control chart using location parameter. *Journal of the Society of Korea Industrial and Systems Engineering*, *39*(1), 116-122.

Davis, R.B., Homer, A., & Woodall, W.H. (1990). Performance of the zone control chart. Communications in Statistics – Theory and Methods, 19, 1581-1587.

Dawod, A. B., Abbasi, S. A., & Al-Momani, M. (2018). On the performance of coefficient of variation control charts in Phase I. *Quality and Reliability Engineering International*, *34*(6), 1029-1040.

Du Nguyen, H., Nguyen, Q. T., Tran, K. P., & Ho, D. P. (2019). On the performance of VSI Shewhart control chart for monitoring the coefficient of variation in the presence of measurement errors. *The International Journal of Advanced Manufacturing Technology*, *104*(1-4), 211-243.

Eleftheriou, M. (2019). A change-point model for monitoring the coefficient of variation based on squared ranks test. *Computers & Industrial Engineering*, *136*, 366-372.

Gong, J., & Li, Y. (1999). Relationship between the estimated Weibull modulus and the coefficient of variation of the measured strength for ceramics. *Journal of the American Ceramic Society*, *82*(2), 449-452.

Guo, B., & Wang, B. X. (2018). Control charts for the coefficient of variation. *Statistical Papers*, *59*(3), 933-955.

Giner-Bosch, V., Tran, K. P., Castagliola, P., & Khoo, M. B. C. (2019). An EWMA control chart for the multivariate coefficient of variation. *Quality and Reliability Engineering International*, *35*(6), 1515-1541.

Haq, A., & Khoo, M. B. C. (2019). New adaptive EWMA control charts for monitoring univariate and multivariate coefficient of variation. *Computers & Industrial Engineering*, *131*, 28-40.

He, X., & Oyadiji, S. O. (2001). Application of coefficient of variation in reliability-based mechanical design and manufacture. *Journal of Materials Processing Technology*, *119*(1-3), 374-378.

Hendricks, W. and Robey, W. (1936). The sampling distribution of the coefficient of variation. *Annals of Mathematical Statistics*, 7(3), 129-132.

Hong E. P, Kang C. W., Baek J. W., & Kang H. W. (2008). Development of CV control chart using EWMA technique. *Journal of the Society of Korea Industrial and Systems Engineering*; *31*:114–120.

Hong, E. P., Kang, H. W., & Kang, C. W. (2011b). DEWMA control chart for the coefficient of variation. In *Advanced Materials Research*, *201*, 1682-1688. Trans Tech Publications Ltd.

Hong, E. P., Kang, H. W., Kang, C. W., & Baik, J. W. (2011a). CV control chart using GWMA technique. In *Advanced Materials Research*, *337*, 247-254. Trans Tech Publications Ltd.

Hong, E. P., Kang, C. W., & Kang, H. W. (2012). Extension of CV-GWMA Control Chart with Fast Initial Response Feature. *Advanced Science Letters*, *14*(1), 471-476.

Iglewicz, B., Myers, R. & Howe, R. (1968). On the percentage points of the sample coefficient of variation. *Biometrika*, 55(3), 580-581.

Iglewicz, B. & Myers, R. (1970). Comparisons of approximations to the percentage points of the sample coefficient of variation. *Technometrics*, 12(1), 166-169.

Ismail, D. J., Saleh, D. M., & Ahmed, K. H. (2017). Construction of new coefficient of variation control chart based on median absolute deviation. *Qalaai Zanist Journal*, *2*(5), 765-776.

Kang, C. W., Lee, M. S., Seong, Y. J., & Hawkins, D. M. (2007). A control chart for the coefficient of variation. *Journal of Quality Technology*, *39*(2), 151-158.

Khatun, M., Khoo, M. B. C., Lee, M. H., & Castagliola, P. (2019). One-sided control charts for monitoring the multivariate coefficient of variation in short production runs. *Transactions of the Institute of Measurement and Control*, *41*(6), 1712-1728.

Khaw, K. W., Khoo, M. B. C., Yeong, W. C., & Wu, Z. (2017). Monitoring the coefficient of variation using a variable sample size and sampling interval control chart. *Communications in Statistics - Simulation and Computation*, *46*(7), 5772-5794.

Khaw, K. W., Khoo, M. B. C., Castagliola, P., & Rahim, M. A. (2018). New adaptive control charts for monitoring the multivariate coefficient of variation. *Computers & Industrial Engineering, 126*, 595–610.

Khaw, K. W., & Chew, X. Y. (2019). A re-evaluation of the run rules control charts for monitoring the coefficient of variation. *Statistics, Optimization & Information Computing*, *7*(4), 716-730.

Khaw, K. W., Chew, X. Y., Yeong, W. C., & Lim, S. L. (2019). Optimal design of the synthetic control chart for monitoring the multivariate coefficient of variation. *Chemometrics and Intelligent Laboratory Systems*, *186*, 33-40.

Knight, J., & Satchell, S. (2005). A Re-Examination of Sharpe's Ratio for Log-Normal Prices. *Applied Mathematical Finance*, *12*(1), 87-100.

Kolbe, R. H., & Burnett, M. S. (1991). Content-analysis research: An examination of applications with directives for improving research reliability and objectivity. *Journal of Consumer Research, 18*(2), 243–250.

Lee, J. H., Kang, H. W., Hong, E. P., & Kang, C. W. (2010). Statistical design of CV-CUSUM control chart using fast initial response. *Journal of the Korean Society for Quality Management*, *38*(3), 313-321.

Lee, M. H., Khoo, M. B. C., Chew, X. Y., & Then, P. H. (2020). Effect of measurement errors on the performance of coefficient of variation chart with short production runs. *IEEE Access*, *8*, 72216-72228.

Li, Z., & Qiu, P. (2014). Statistical process control using a dynamic sampling scheme. *Technometrics*, *56*(3), 325-335.

Lim, A. J. X., Khoo, M. B. C., Teoh, W. L., & Haq, A. (2017). Run sum chart for monitoring multivariate coefficient of variation. *Computers & Industrial Engineering, 109*, 84–95.

Lim, S. L., Yeong, W. C., Khoo, M. B. C., Chong, Z. L., & Khaw, K. W. (2019). An alternative design for the variable sample size coefficient of variation chart based on the median run length and expected median run length. *International Journal of Industrial Engineering*, *26*(2), 199-220.

Lorenzen, T.J. and Vance, L.C. (1986). The economic design of control charts: a unified approach, *Technometrics*, 28(1), 3–10.

Maleki, M. R., Castagliola, P., Amiri, A. (2017). Measurement errors in statistical process monitoring: A literature review. *Computers & Industrial Engineering*, *103*, 316-329.

Mahmoudvand, R. & Hassani, H. (2009). Two new confidence intervals for the coefficient of variation in a normal distribution. *Journal of Applied Statistics*, 36(4), 429-442.

Menzefricke, U. (2010). Control charts for the variance and coefficient of variation based on their predictive distribution. *Communications in Statistics - Theory and Methods*, 39(16), 2930–2941.

Muhammad, A. N. B., Yeong, W. C., Chong, Z. L., Lim, S. L., & Khoo, M. B. C. (2018). Monitoring the coefficient of variation using a variable sample size EWMA chart. *Computers & Industrial Engineering*, *126*, 378-398.

Nguyen, Q. T., Tran, K. P., Heuchenne, H. L., Nguyen, T. H., & Nguyen, H. D. (2019). Variable sampling interval Shewhart control charts for monitoring the multivariate coefficient of variation. *Applied Stochastic Models in Business and Industry*, *35*(5), 1253-1268.

Noor-ul-Amin, M., Tariq, S., & Hanif, M. (2019). Control charts for simultaneously monitoring of process mean and coefficient of variation with and without auxiliary information. *Quality and Reliability Engineering International*, *35*(8), 2639-2656.

Noor-ul-Amin, M., & Riaz, A. (2020). EWMA control chart for coefficient of variation using log-normal transformation under ranked set sampling. *Iranian Journal of Science and Technology, Transactions A: Science*, *44*(1), 155-165.

Page, E. S. (1954). Continuous inspection schemes. *Biometrika*, *41*(1/2), 100-115.

Park, Y. S., Hong, E. P., Park, K. Y., & Shon, W. H. (2014). Performance improvement study of CV-EWMA control chart to detect small shifts of CV. In *Advanced Materials Research*, *1051*,1016-1022. Trans Tech Publications Ltd.

Reh, W. & Scheffler, B. (1996). Significance tests and confidence intervals for coefficients of variation. *Computational Statistics & Data Analysis*, 22(4), 449-452.

Riaz, A., Noor-ul-Amin, M., Khan, H. (2020). Auxiliary information based exponentially weighted moving co-efficient of variation control chart: An application to monitor electric conductivity for water system, *Communications in Statistics: Case Studies, Data Analysis and Applications*, 1-18.

Riaz, A., & Noor-ul-Amin, M. (2020). Improved simultaneous monitoring of mean and coefficient of variation under ranked set sampling schemes. *Communications in Statistics-Simulation and Computation*, 1-17.

Roberts, S.W. (1959). Control chart tests based on geometric moving averages. *Technometrics*, 1, 239–250.

Saha, S., Khoo, M. B. C., Castagliola, P., & Haq, A. (2021). Side-sensitive modified group runs charts with and without measurement errors for monitoring the coefficient of variation. *Quality and Reliability Engineering International*, *37*(2), 598-617.

Saniga, E.M. (1989). Economic statistical control chart with an application to X-bar and R charts, *Technometrics*, 31(3), 313–320.

Sharpe, W. F. (1994). The sharpe ratio. *Journal of Portfolio Management*, *21*(1), 49-58.

Shechtman, O. (2013). The coefficient of variation as an index of measurement reliability. In *Methods of Clinical Epidemiology*, Springer, Berlin, Heidelberg, 39-49.

Singh, D. P., & Singh, J. R. (2012). Power of mean chart under second order auto-correlation with known coefficient of variation. *International Journal of Scientific and Research Publications*, *2*(12), 325-335.

Teoh, W. L., Khoo, M. B. C., Yeong, W. C., & Teh, S. Y. (2016). A comparison between the performances of synthetic and EWMA charts for monitoring the coefficient of variation. *Journal of Scientific Research and Development*, *3*(1), 16-20.

Teoh, W. L., Khoo, M. B. C., Castagliola, P., Yeong, W. C., & Teh, S. Y. (2017). Run-sum control charts for monitoring the coefficient of variation. *European Journal of Operational Research*, *257*(1), 144-158.

Teoh, W. L., Lim, J. Y., Khoo, M. B. C., Chong, Z. L., & Yeong, W. C. (2018). Optimal designs of EWMA charts for monitoring the coefficient of variation based on median run length and expected median run length. *Journal of Testing and Evaluation*, *47*(1), 459-479.

Tian, L. (2005). Inferences on the common coefficient of variation. *Statistics in Medicine*, 24(14), 2213-2220.

Tran, P. H., & Tran, K. P. (2016). The efficiency of CUSUM schemes for monitoring the coefficient of variation. *Applied Stochastic Models in Business and Industry*, *32*(6), 870-881.

Tran, P. H., & Heuchenne, C. (2019). CUSUM control charts for the coefficient of variation with measurement errors. *IFAC-PapersOnLine*, *52*(13), 1884-1889.

Tran, K. P., Du Nguyen, H., Tran, P. H., & Heuchenne, C. (2019a). On the performance of CUSUM control charts for monitoring the coefficient of variation with measurement errors. *The International Journal of Advanced Manufacturing Technology*, *104*(5-8), 1903-1917.

Tran, K. P., Heuchenne, C., & Balakrishnan, N. (2019b). On the performance of coefficient of variation charts in the presence of measurement errors. *Quality and Reliability Engineering International*, *35*(1), 329-350.

Tran, P. H., Heuchenne, C., Nguyen, H. D., & Marie, H. (2020). Monitoring coefficient of variation using one-sided run rules control charts in the presence of measurement errors. *Journal of Applied Statistics*, doi.org/10.1080/02664763.2020.1787356.

Vangel, M. (1996). Confidence intervals for a normal coefficient of variation. *The American Statistician*, 15(1), 21-26.

Van Zyl, R., & Van Der Merwe, A. J. (2017). A Bayesian control chart for a common coefficient of variation. *Communications in Statistics-Theory and Methods*, *46*(12), 5795-5811.

Verrill, S. and Johnson, R. (2007). Confidence bounds and hypothesis tests for normal distribution coefficients of variation. *Communications in Statistics – Theory and Methods*, 36(12), 2187-2206.

Warren, W. (1982). On the adequacy of the chi-squared approximation for the coefficient of variation. *Communications in Statistics – Simulation and Computation*, 11(6), 659-666.

Wijsman, R. A. (1957). Random orthogonal transformations and their use in some classical distribution problems in multivariate analysis. *The Annals of Mathematical Statistics*, 28, 415–423.

Woodall, W. H. (1985). The statistical design of quality control charts. *The Statistician*, 34(2), 155–160.

Wu, Z., & Spedding, T. A. (2000). A synthetic control chart for detecting small shifts in the process mean. *Journal of Quality Technology*, *32*(1), 32-38.

Ye, J., Feng, P., Xu, C., Ma, Y., & Huang, S. (2018). A novel approach for chatter online monitoring using coefficient of variation in machining process. The International Journal of Advanced Manufacturing Technology, 96(1–4), 287–297.

Yeong, W. C., Chuah, M. E., Teoh, W. L., Khoo, M. B. C., & Lim, S. L. (2015). The Economic and Economic-Statistical Designs of the Coefficient of Variation Chart. *Academic Journal of Science*, *4*(3), 57-72.

Yeong, W. C., Khoo, M. B. C., Lim, S. L., Teoh, W. L., & Ng, P. S. (2016a). Effects of misspecification on the economic-statistical design of the coefficient of variation chart. *Journal of Engineering and Applied Sciences*, *11*(7), 1655-1659.

Yeong, W. C., Khoo, M. B. C., Teoh, W. L., & Castagliola, P. (2016b). A control chart for the multivariate coefficient of variation. *Quality and Reliability Engineering International*, *32*(3), 1213-1225.

Yeong, W. C., Khoo, M. B. C., Lim, S. L., & Lee, M. H. (2017a). A direct procedure for monitoring the coefficient of variation using a variable sample size scheme. *Communications in Statistics-Simulation and Computation*, *46*(6), 4210-4225.

Yeong, W. C., Lim, S. L., Khoo, M. B. C., Chuah, M. H., & Lim, A. X. J. (2017b). The economic and economic-statistical designs of the synthetic chart for the coefficient of variation. *Journal of Testing and Evaluation*, *46*(3), 1175-1195.

Yeong, W. C., Khoo, M. B. C., Tham, L. K., Teoh, W. L., & Rahim, M. A. (2017c). Monitoring the coefficient of variation using a variable sampling interval EWMA chart. *Journal of Quality Technology*, *49*(4), 380-401.

Yeong, W. C., Khoo, M. B. C., Lim, S. L., & Teoh, W. L. (2017d). The coefficient of variation chart with measurement error. *Quality Technology & Quantitative Management*, *14*(4), 353-377.

Yeong, W. C., Lim, S. L., Khoo, M. B. C., & Castagliola, P. (2018). Monitoring the coefficient of variation using a variable parameters chart. *Quality Engineering*, *30*(2), 212-235.

You, H. W., Khoo, M. B. C., Castagliola, P., & Haq, A. (2016). Monitoring the coefficient of variation using the side sensitive group runs chart. *Quality and Reliability Engineering International*, *32*(5), 1913-1927.

Zhang, J., Li, Z., Chen, B., & Wang, Z. (2014). A new exponentially weighted moving average control chart for monitoring the coefficient of variation. *Computers & Industrial Engineering*, *78*, 205-212.

Zhang, J., Li, Z., & Wang, Z. (2018). Control chart for monitoring the coefficient of variation with an exponentially weighted moving average procedure. *Quality and Reliability Engineering International*, *34*(2), 188-202.

Table 1. Comparison of CV control charts in the literature

Authors	Proposed control chart	Competing control chart	Shift size	Best method
Guo and Wang (2018)	Synthetic	Shewhart	Small and large	Synthetic
Muhammad et al. (2018)	VSS EWMA	Shewhart, EWMA, Synthetic Shewhart, RR Shewhart and VSS Shewhart	Small and large	VSS EWMA
Khaw et al. (2018)	VSSI Shewhart	VSI Shewhart, VSS Shewhart, Shewhart	Small and large	VSSI Shewhart
Yeong et al. (2018)	VP Shewhart	VSSI Shewhart, VSI Shewhart, VSS Shewhart, EWMA , Synthetic Shewhart, Shewhart	Small and large	VP Shewhart
Khaw et al. (2019)	Synthetic	Run sum, Shewhart	Small and large	Synthetic
Chew et al. (2019)	VP Shewhart	VSSI Shewhart, Synthetic	Small and large	VP Shewhart
Haq and Khoo (2019)	AEWMA	Shewhart	small	AEWMA
			large	Shewhart
Du Nguyen et al. (2019)	VSI Shewhart	FSI Shewhart	Small and large	VSI
Chen et al. (2019)	OSRG (GWMA)	RES, MOSE, OSE(EWMA)	Small and large	OSRG
Tran et al. (2019a)	CUSUM	EWMA	Small and large	CUSUM
Tran et al. (2019b)	two- sided EWMA-γ^2	Shewhart	Small and large	Two- sided EWMA-γ^2
Aslam et al. (2019)	Hybrid EWMA	ECV, MCV	Small and large	Hybrid EWMA
Khaw and Chew (2019)	ORR Shewhart RR Shewhart	Shewhart	Small and large	ORR Shewhart
Nguyen et al. (2019)	VSI Shewhart	FSI Shewhart	Small and large	VSI Shewhart
Noor-ul-Amin et al. (2019)	AIB-Max EWMA	Max EWMA	Small and large	AIB-Max EWMA
Chew et al. (2020a)	RR Shewhart	Run sum	Small and large	Run sum
Noor-ul-Amin and Riaz (2020)	SEWMCV REWMCV	EWMCV	small & moderate	REWMCV
			large	SEWMCV

Tran et al. (2020)	RR-γ^2	VSI-γ^2	small	RR-γ^2
			large	VSI-γ^2
Ayyoub et al. (2020b)	VSI EWMA-$\boldsymbol{\gamma^2}$	Shewhart ,VP Shewhart, EWMA-$\boldsymbol{\gamma^2}$	Small and large	VSI EWMA-$\boldsymbol{\gamma^2}$
Riaz et al. (2020)	AIB-EWMA	EWMA	small	EWMA
			large	AIB-EWMA
Riaz and Noor-ul-Amin (2020)	Max-EWMAS	Max-EWMAR, Max-EWMAM, Max-EWMAE	small	Max-EWMAR, Max-EWMAM, Max-EWMAE
			large	Max-EWMAS
Chew et al. (2020b)	VSSI Shewhart	Shewhart	Small and large	VSSI Shewhart

Table 2. Best CV control chart under different shift sizes

Authors	Considered sample sizes	Control chart	Performance Criteria	Best sample size
Guo and Wang (2018)	5, 10, 15	Shewhart	ARL, SDRL	15
Muhammad et al. (2018)	5,7, 10, 15	EWMA	ARL, RARL, EARL	15
Dawod et al. (2018)	5, 10, 15	$\overline{W}$ $\widetilde{W}$ W_p	MSE, PTS	15
Abbasi and Adegoke (2018)	5, 10, 15	Shewhart	PTS	15
Khaw et al. (2018)	5, 10	Shewhart	ATS, SDTS, EATS	10
Zhang et al. (2018)	5, 10	OSE, MOSE, SRR, SSGR, RES, EWMA	ARL, SDRL	10
Yeong et al. (2018)	5,7, 10, 15	Shewhart	ATS, SDTS, EATS	15
Khaw et al. (2019)	5, 10, 15	Synthetic	ARL, SDRL,EARL	15
Giner-Bosch et al. (2019)	5, 10, 15, 20	MEWMA	ARL, SDRL	20
Chew et al. (2019)	7, 10, 15	Shewhart	SDTS, EATS,ATS	15
Haq and Khoo (2019)	5,7, 10, 15	EWMA	ARL, MRL, SDRL	15
Du Nguyen et al. (2019)	5, 15	Shewhart	ARL, ATS	15
Khatun et al. (2019)	5,7, 10, 15	Shewhart	TARL,TSDRL	15
Tran and Heuchenne (2019)	5,7, 10, 15	one-sided CUSUM	ARL, EARL	15
Chen et al. (2019)	5, 10, 15	GWMA	ARL	15
Tran et al. (2019a)	5,7, 10, 15	CUSUM	ARL, EARL	15

Tran et al. (2019b)	5,7, 10, 15	Shewhart, two- sided EWMA-γ^2	ARL	15
Aslam et al. (2019)	5,7, 10, 15	Hybrid EWMA	ARL	15
Eleftheriou (2019)	10	SRT model	$\bar{\hat{\tau}}, SD(\hat{\tau})$	10
Khaw and Chew (2019)	5, 10, 15	Shewhart	ARL, EARL,SDRL	15
Nguyen et al. (2019)	5, 10, 15	Shewhart	ATS, SDTS, ASI	15
Noor-ul-Amin et al. (2019)	5	Max EWMA AIB-Max EWMA	ARL,SDRL	5
Chew et al. (2020a)	5, 10, 15	Shewhart	ARL, SDRL,EARL	15
Noor-ul-Amin and Riaz (2020)	5,8,12	SEWMCV REWMCV	ARL,SDRL	12
Abbasi (2020)	5,7, 10, 12, 15	Shewhart	Power	15
Lee et al.(2020)	5, 10, 15	Shewhart	TARL, ETARL	15
Tran et al. (2020)	5,15	Shewhart	ARL, SDRL	15
Ayyoub et al. (2020a)	5,7	Shewhart	ARL	7
Ayyoub et al. (2020b)	7, 10, 15, 20	EWMA	ATS, EATS, SDTS	20
Riaz et al. (2020)	5, 10	EWMA	ARL, SDRL	10
Riaz and Noor-ul-Amin (2020)	10, 15	EWMA	ARL, SDRL	15
Chew et al. (2020b)	5, 10, 15	Shewhart	ARLSDRL, EARL	15

Table 3. Questions and feasible answers

1. Which field of SPM is the paper related to? (1.1) statistical design of control charts (1.2) economic design/economic-statistical design
2. Which kind of quality characteristic is considered in the paper? (2.1) univariate CV (2.2) multivariate CV
3. Type of shift size? (3.1) Known (3.2) Unknown
4. Type of control chart? (4.1) Shewhart-type control chart (4.2) Memory-based control chart (4.3) Adaptive control chart (4.3.1) Variable Parameters (VP) (4.3.2) Variable Sampling Interval (VSI) (4.3.3) Variable Sample Size (VSS)

(4.3.4) Variable Sample Size and Sampling Interval (VSSI) (4.3.5) Fixed parameters (FP) (4.3) Other control chart
5. Which phase does the article consider? (5.1) Phase I (5.2) Phase II
6. Which criteria are used to evaluate the performance? (6.1) ARL and SDRL for statistical designs (6.2) ARL and expected value of cost for economic and/or economic-statistical design (6.3) $\bar{\hat{\tau}}, SD(\hat{\tau})$ for change point estimation
7. Sampling method? (7.1) Simple Random Sampling (SRS) (7.2) Ranked Set Sampling (RSS)
8. Measurement errors? (8.1) Yes (8.2) No

Table 4. Number and percentage of publications in each journal

Journal Name	Number	Percentage
Quality and Reliability Engineering International	10	14.08
Computers & Industrial Engineering	7	9.86
The International Journal of Advanced Manufacturing Technology	5	7.04
Communications in Statistics - Theory and Methods	3	4.23
Journal of Applied Statistics	3	4.23
Quality Technology & Quantitative Management	3	4.23
Journal of Quality Technology	3	4.23
Advanced Materials Research	3	4.23
Communications in Statistics - Simulation and Computation	3	4.23
Journal of the Society of Korea Industrial and Systems Engineering	2	2.82
Journal of Testing and Evaluation	2	2.82
IEEE Access	2	2.82
European Journal of Operational Research	2	2.82
International Journal of Production Research	1	1.41
International Journal of Scientific and Research Publications	1	1.41
Journal of Engineering and Applied Sciences	1	1.41
Journal of the Korean Society for Quality Management	1	1.41
Advanced Science Letters	1	1.41
Quality Engineering	1	1.41

Iranian Journal of Science and Technology, Transactions A: Science	1	1.41
Applied Stochastic Models in Business and Industry	1	1.41
Statistics, Optimization & Information Computing	1	1.41
Applied Mathematical Modelling	1	1.41
Transactions of the Institute of Measurement and Control	1	1.41
Chemometrics and Intelligent Laboratory Systems	1	1.41
Journal of Statistical Computation and Simulation	1	1.41
Journal of Scientific Research and Development	1	1.41
Qalaai Zanist Journal	1	1.41
Statistical Papers	1	1.41
IFAC-PapersOnLine	1	1.41
International Journal of Quality & Reliability Management	1	1.41
International Journal of Industrial Engineering	1	1.41
Applied Stochastic Models in Business and Industry	1	1.41
Journal of Mathematical and Fundamental Sciences	1	1.41
Academic Journal of Science	1	1.41
Communications in Statistics: Case Studies, Data Analysis and Applications	1	1.41

Table 5. Authors contributing to the area of monitoring the coefficient of variation

Researcher	Affiliation/country	Number of papers
Khoo, M. B. C.	School of Mathematical Sciences, Universiti Sains Malaysia, Penang, Malaysia.	25
Yeong, W. C.	Department of Physical and Mathematical Sciences, Faculty of Science, Universiti Tunku Abdul Rahman, Perak, Malaysia.	18
Castagliola, P.	Université de Nantes & LS2N UMR CNRS 6004, Nantes, France.	16
Lim, S. L.	Faculty of Science, Institute of Mathematical Sciences, Universiti Malaya, Kuala lumpur, Malaysia.	9
Teoh, W. L.	Department of Physical and Mathematical Sciences, Faculty of Science, Universiti Tunku Abdul Rahman, Perak, Malaysia.	9
Khaw, K. W.	School of Management, Universiti Sains Malaysia, Penang, Malaysia.	9
Celano, G.	Department of Industrial Engineering, University of Catania, Catania, Italy.	8
Taleb, H.	Higher Institute of Business Administration of Gafsa, University of Gafsa, Gafsa, Tunisia.	8
Chew, X. Y.	School of Computer Sciences, Universiti Sains Malaysia, Penang, Malaysia.	7
Kang, C. W.	Department of Information and Industrial Engineering,Hanyang University, Ansan, Korea.	6
Tran, K. P.	GEMTEX Laboratory, Ecole Nationale Supérieure des Arts et Industries Textiles, Roubaix, France.	6

Hong, E. P.	Department of Industrial and Management Engineering, Hanyang University, Korea.	6
Amdouni, A.	Institut Supérieur de Gestion, Université de Tunis, Tunisia.	5
Tran, P. H.	Management School, University of Li`ege, Li`ege, Belgium.	4
Heuchenne, C.	HEC- Management School, University of Liege, Liege, Belguim.	4
Chong, Z. L.	Department of Physical and Mathematical Science, Faculty of Science, Universiti Tunku Abdul Rahman, 31900 Perak, Malaysia.	4
Lee, M. H.	Swinburne University of Technology Sarawak Campus, Sarawak, Malaysia.	4
Riaz, M.	Department of Mathematics and Statistics, King Fahad University of Petroleum and Minerals, Dhahran, 31261 Saudi Arabia.	3
Zhang, J	Department of Mathematics, Liaoning University, Shenyang 110036, PR China.	3
Psarakis, S.	Athens University of Economics and Business, Athens, Greece.	3
Achouri, A.	Institut Suprieur de Gestion, Universit de Tunis, Tunis, Tunisia.	3
Abbasi, S. A.	Department of Mathematics, Statistics, and Physics, Qatar University, Doha, Qatar.	3
Haq, A.	Department of Statistics, Quaid-i-Azam University, Islamabad, Pakistan.	3
You, H. W.	School of Mathematical Sciences, Universiti Sains Malaysia, 11800 Penang, Malaysia.	3

Table 6: Categorization of published papers for monitoring the CV

Papers	Areas of SPM		Types of quality characteristics		Types of shifts		Types of control charts	Phase		Performance criteria	Sampling method	Measurement errors	Adaptive
	Statistical design	E/E-S design	Univariate	Multivariate	Known	Unknown		I	II				
Kang et al. (2007)	*		*		*		Shewhart		*	ARL	SRS	-	Non-adaptive
Hong et al. (2008)	*		*		*		EWMA		*	ARL	SRS	-	Non-adaptive
Lee et al. (2010)	*		*		*		CUSUM		*	ARL	SRS	-	Non-adaptive
Menzefricke (2010)	*		*		*		Shewhart		*	ARL	SRS	-	Non-adaptive
Hong et al. (2011a)	*		*		*		GWMA		*	ARL, SDRL	SRS	-	Non-adaptive
Hong et al. (2011b)	*		*		*		DEWMA		*	ARL, SDRL	SRS	-	Non-adaptive
Castagliola et al. (2011)	*		*		*	*	EWMA-γ^2		*	ARL	SRS	-	Non-adaptive
Singh and Singh (2012)	*		*		*		$\bar{X}$	*		Power	SRS	-	Non-adaptive
Hong et al. (2012)	*		*		*		FIR CV-GWMA		*	ARL	SRS	-	Non-adaptive
Castagliola et al. (2013a)	*		*		*		Shewhart		*	EATS, SDTS	SRS	-	VSI
Castagliola et al. (2013b)	*		*		*		Shewhart		*	ARL, SDRL	SRS	-	Non-adaptive
Calzada and Scariano (2013)	*		*		*		Synthetic		*	ARL,SDRL	SRS	-	Non-adaptive
Zhang et al. (2014)	*		*		*		EWMA		*	ARL	SRS	-	Non-adaptive
Park et al. (2014)	*		*		*		EWMA		*	ARL, SDRL	SRS	-	Non-adaptive
Yeong et al. (2015)		*	*		*		Shewhart		*	ARL	SRS	-	Non-adaptive

Castagliola et al. (2015a)	*		*		*		Shewhart		*	ASS, ARL, SDRL	SRS	-	VSS
Castagliola et al. (2015b)	*		*		*		One-Sided Shewhart		*	TARL, TSDRL,TRL	SRS	-	Non-adaptive
Amdouni et al. (2015)	*		*		*		Shewhart		*	ASS, TARL	SRS	-	VSS
Yeong et al. (2016a)		*	*		*		Shewhart		*	ARL	SRS	-	Non-adaptive
Chun and Chang (2016)	*		*			*	Shewhart		*	ARL	SRS	-	Non-adaptive
Teoh et al. (2016)	*		*		*		EWMA, Synthetic EWMA		*	ARL, SDRL	SRS	-	Non-adaptive
Amdouni et al. (2016)	*		*		*		Shewhart		*	TARL,TSDRL	SRS	-	Non-adaptive
You et al. (2016)	*		*		*	*	Shewhart		*	ARL, EARL,SDRL	SRS	-	Non-adaptive
Tran and Tran (2016)	*		*		*	*	CUSUM		*	ARL, SDRL	SRS	-	Non-adaptive
Yeong et al. (2016b)	*			*	*	*	Shewhart		*	ARL, EARL, SDRL	SRS	-	Non-adaptive
Amdouni et al. (2017)	*		*		*		Shewhart		*	TARL, TSDRL	SRS	-	VSI
Ismail et al. (2017)	*		*		*		Shewhart	*		Signal Probability ARL	SRS	-	Non-adaptive
Yeong et al. (2017a)	*		*		*		Shewhart		*	ARL, SDRL, ASS	SRS	-	VSS
Yeong et al. (2017b)		*	*		*	*	Synthetic		*	EARL	SRS	-	Non-adaptive
Van Zyl and Van Der Merwe (2017)	*		*			*	Shewhart	*		95% Bayesian confidence intervals ($\alpha = 0.05$)	SRS	-	Non-adaptive
Lim et al. (2017)	*			*	*	*	Run Sum		*	ARL, EARL, SDRL	SRS	-	Non-adaptive
Yeong et al. (2017c)	*		*		*	*	EWMA		*	ATS, SDTS, EATS	SRS	-	VSI

Khaw et al. (2017)	*		*		*	*	Shewhart		*	ATS, SDTS, EATS	SRS	-	VSSI
Teoh et al. (2017)	*		*		*	*	Run Sum		*	ARL, SDRL,EARL	SRS	-	Non-adaptive
Yeong et al. (2017d)	*		*		*		Shewhart		*	ARL	SRS	*	Non-adaptive
Guo and Wang (2018)	*		*		*		Shewhart		*	ARL, SDRL	SRS	-	Non-adaptive
Muhammad et al. (2018)	*		*		*	*	EWMA		*	ARL, RARL, EARL	SRS	-	VSS
Teoh et al. (2018)	*		*		*	*	EWMA		*	MRL, EMRL	SRS	-	Non-adaptive
Dawod et al. (2018)	*		*		*		$\overline{W}$ $\widetilde{W}$ W_p	*		MSE	SRS	-	Non-adaptive
Abbasi and Adegoke (2018)	*			*	*		Shewhart	*		PTS	SRS	-	Non-adaptive
Khaw et al. (2018)	*			*	*		Shewhart		*	ATS, SDTS, EATS	SRS	-	VSSI
Zhang et al. (2018)	*		*		*		OSE, MOSE, SRR, SSGR, RES, EWMA		*	ARL, SDRL	SRS	-	Non-adaptive
Yeong et al. (2018)	*		*		*	*	Shewhart		*	ATS, SDTS, EATS	SRS	-	VP
Khaw et al. (2019)	*			*	*	*	Synthetic		*	ARL, SDRL,EARL	SRS	-	Non-adaptive
Giner-Bosch et al. (2019)	*			*		*	MEWMA		*	ARL, SDRL	SRS	-	Non-adaptive
Chew et al. (2019)	*			*	*		Shewhart		*	SDTS, EATS,ATS	SRS	-	VP
Haq and Khoo (2019)	*		*	*	*		EWMA		*	ARL, MRL, SDRL	SRS	-	VSSI
Du Nguyen et al. (2019)	*		*		*		Shewhart		*	ARL, ATS	SRS	*	VSI
Khatun et al. (2019)	*			*	*	*	Shewhart		*	TARL,TSDR L	SRS	-	Non-adaptive
Tran and Heuchenne (2019)	*		*		*		one-sided CUSUM		*	ARL, EARL	SRS	*	Non-adaptive
Chen et al. (2019)	*			*	*		GWMA		*	ARL	SRS	-	Non-adaptive

Tran et al. (2019a)	*		*			*	CUSUM		*	ARL, EARL	SRS	*	Non-adaptive
Tran et al. (2019b)	*		*		*		Shewhart, two-sided EWMA-γ^2		*	ARL	SRS	*	Non-adaptive
Aslam et al. (2019)	*		*			*	Hybrid EWMA		*	ARL	SRS	-	Non-adaptive
Eleftheriou (2019)	*		*		*		SRT model		*	$\bar{\hat{\tau}}, SD(\hat{\tau})$	SRS	-	Non-adaptive
Khaw and Chew (2019)	*			*	*	*	Shewhart		*	ARL, EARL,SDRL	SRS	-	Non-adaptive
Nguyen et al. (2019)	*			*		*	Shewhart		*	ATS, SDTS, ASI	SRS	-	VSI
Noor-ul-Amin et al. (2019)	*		*		*		Max EWMA AIB-Max EWMA		*	ARL,SDRL	SRS	-	Non-adaptive
Lim et al. (2019)	*		*		*		Shewhart		*	MRL, EMRL	SRS	-	VSS
Chew et al. (2020a)	*			*	*		Shewhart		*	ARL, SDRL,EARL	SRS	-	Non-adaptive
Noor-ul-Amin and Riaz (2020)	*		*		*		SEWMCV REWMCV		*	ARL,SDRL	RSS SRS	-	Non-adaptive
Abbasi (2020)	*		*		*		Shewhart	*		Power	SRS	-	Non-adaptive
Lee et al.(2020)	*		*		*		Shewhart		*	TARL, ETARL	SRS	*	Non-adaptive
Chew and Khaw (2020)	*			*	*		Shewhart		*	ATS, EATS	SRS	-	VSSI
Tran et al. (2020)	*		*		*		Shewhart		*	ARL, SDRL	SRS	*	Non-adaptive
Ayyoub et al. (2020a)	*			*	*		Shewhart		*	ARL	SRS	*	Non-adaptive
Ayyoub et al. (2020b)	*			*	*		EWMA		*	ATS, EATS, SDTS	SRS	-	VSI
Chew et al. (2020b)		*	*		*		Shewhart		*	ATS, ASI	SRS	-	VSSI
Riaz et al. (2020)	*		*		*		EWMA		*	ARL, SDRL	SRS	-	Non-adaptive
Riaz and Noor-ul-Amin (2020)	*		*		*		EWMA		*	ARL, SDRL	RSS	-	Non-adaptive

Saha et al. (2021)	*		*		*		Side-sensitive modified group runs charts		*	ARL, SDRL	SRS	*	Non-adaptive

The abbreviations in Table 6 are as follows:

FIR: Fast initial response
PTS: Probability to signal
OSE: one-sided EWMA chart
MOSE: modified one-sided EWMA CV
SEWMCV: Simple Random Sampling EWMA chart for CV
REWMCV: Ranked Set Sampling EWMA chart for CV

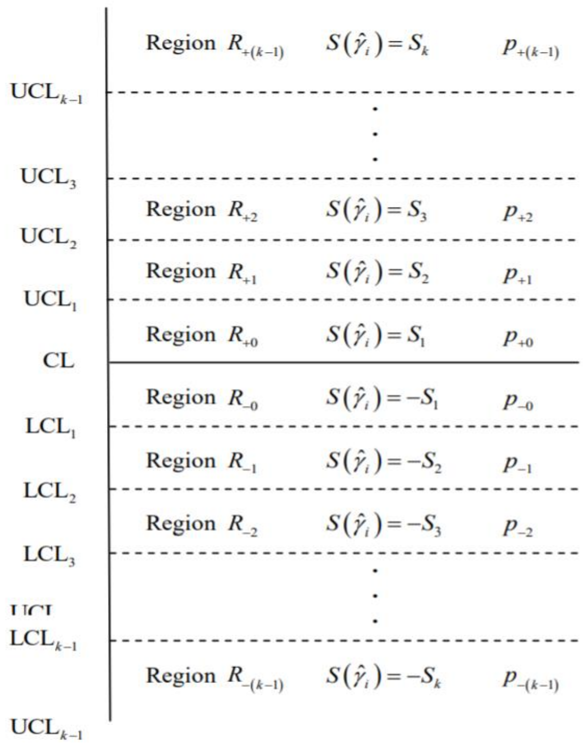

Fig 1. ***k*** **regions above and below the center line of the RS-γ chart (Teoh et al., 2016)**

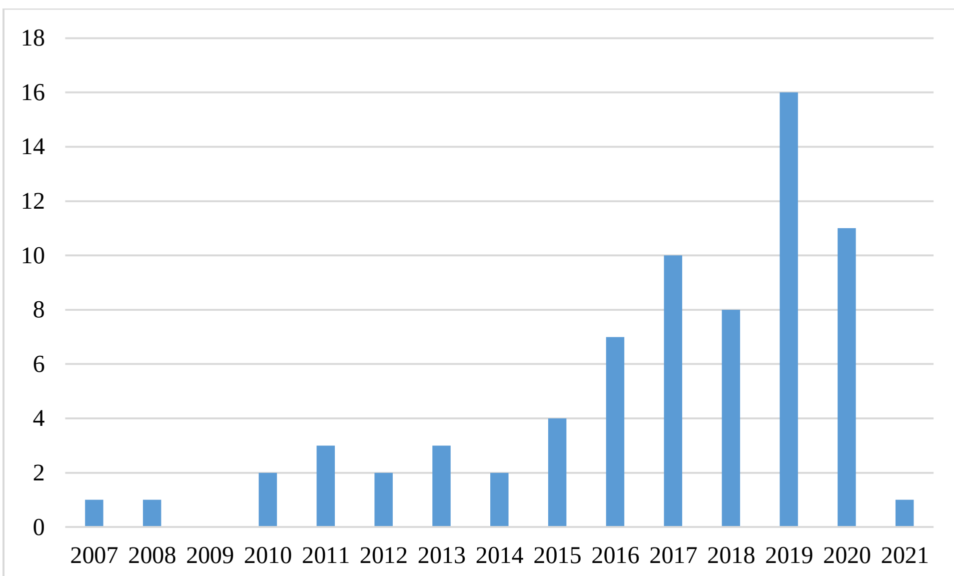

Fig 2.Trend of relevant papers from 2007 to 2020

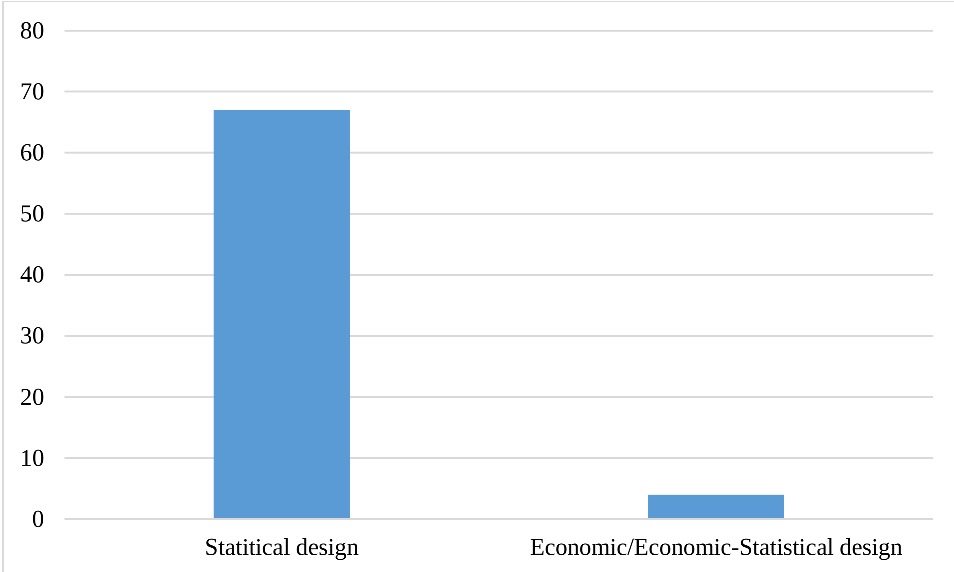

Fig 3.Percentage of published papers in different areas of SPM

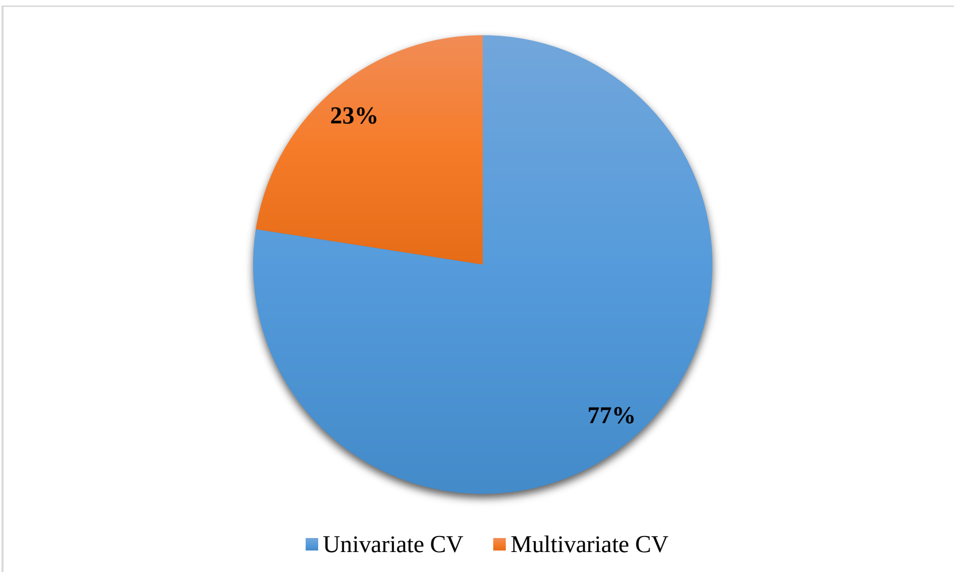

Fig 4.Percentage of papers based on the types of quality characteristics

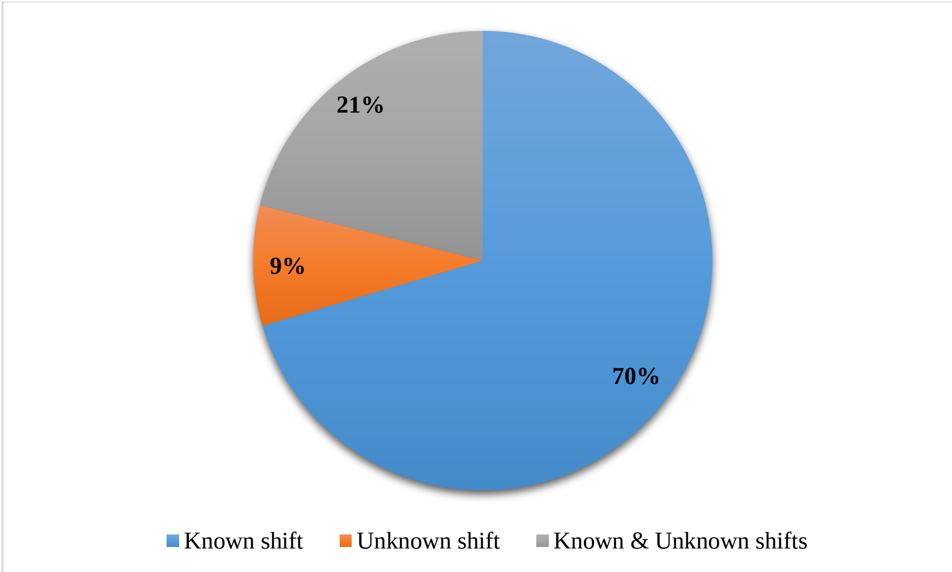

Fig 5. Percentage of papers based on the types of shift sizes

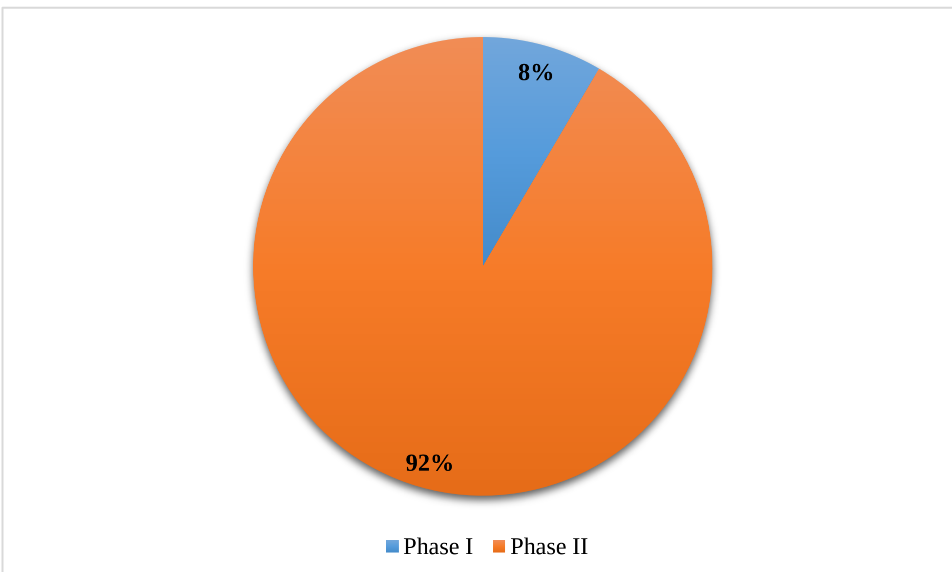

Fig 6.Percentage of reviewed papers based on the analysis phase

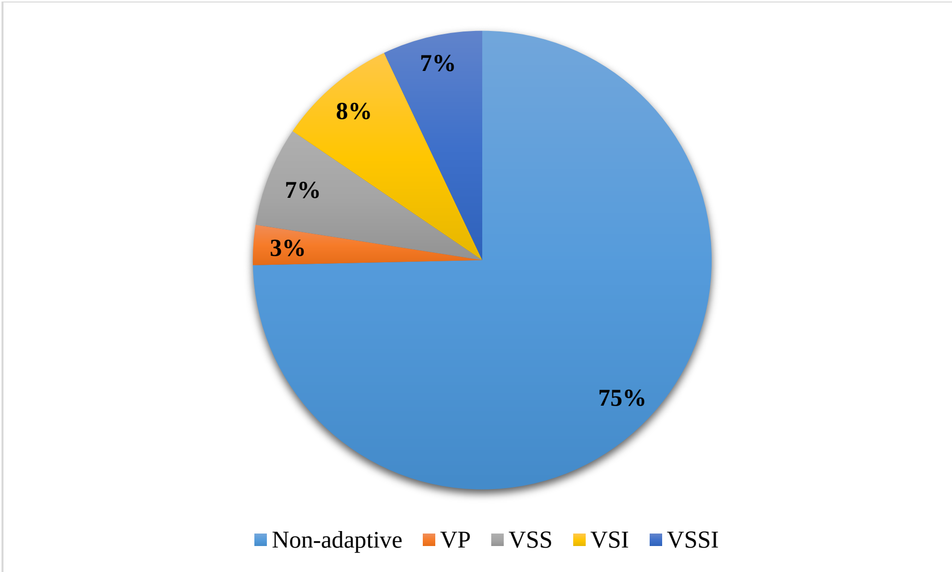

Fig 7.Percentage of papers based on the types of adaptive control charts