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From the expansion of the space of the displacements  
towards the transformations of the plane applied to the  
changing, bifurcating, intersecting and conducting lanes  
for the map matching

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**Abstract**

The odometric model gives the place to a space of displacements. We have been resolving to simulate, integrate, explain and then to validate the odometric model. The model of resolution is to be programmed in the Maple langage, with the help of a solver for Maple 2022.

*Key words:* Odometry, Vehicle location and navigation systems, Mobile positioning systems, Trajectory, Convergence of a model, Control of a vehicle, Example of a model, Application of a model.

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**Introduction**

As one introduction, I could say that the odometric model is well known now and that everything is done for the space of displacements of a model of automation to be improved with the odometric model. Nowadays I can compute the primitive and also tell a bit more about the euclidian space. Indeed, there is the completeness to be proven with respect to the closing of the space of the euclidian plane. Note : The odometric model is given by the following equations of the movements of the vehicle in the euclidian plane :

$$dx(t)/dt = V * \cos(\phi(t))$$

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in the x axis of the euclidian plane,

$$dy(t)/dt = V * \sin(\phi(t))$$

in the y axis of the euclidian plane,

$$\phi(t) = \tan(t)/L$$

in terms of changing of direction of the vehicle, called the heading angle of the vehicle.

### 1. Theorem

Statement of the theorem : Every transformations of the euclidian space are the limit of a convergent serie of displacements. Discussion about the theorem : First of all, there is the convergence of the odometric model with the absolute convergence until an hypergeometric function with a gamma, and moreover all this space of displacements has a structure K-completed of a group, with the neutral element that is the rotation of an angle of  $2*\text{Pi}$  modulo  $\text{Pi}$ . The formula below is used to be the steering angle to be integer in the mathematical function of the computed odometric model.

$$\theta(t) = -\ln(\cos(t))$$

Moreover the formula below is used to be the integrated value of the first and great mathematical terminal of the computed odometric model.

$$\beta(t) = \int_0^t \cos(x)^I dx$$

$$\beta(\text{Pi}/2) = \int_0^{\text{Pi}/2} \cos(x)^I dx = \Gamma(1/2 + I/2) \cdot \sqrt{\text{Pi}} / (2 \cdot \Gamma(1 + I/2))$$

It deals with the value of the steering angle at the value of  $\text{Pi}/2$ . Normally the speed value of the wheels of the vehicle should always be tangential to the vehicle. Indeed the wheels are to be rolling in the direction of the vehicle. And when the vehicle is turning then the direction of the vehicle is changing of the driving direction. Secondly, the primitive give the place of the functions as shown below.

$$x(t) = V * L * (1 + \tan^2(t)) * \sin(\tan(t)/L)$$

$$y(t) = -V * L * (1 + \tan^2(t)) * \cos(\tan(t)/L)$$

with

$$V$$

the speed of the vehicle, and with

$$L$$

the length of the vehicle. Then, the heading angle of the vehicle is given by the function as below,

$$\phi(t) = \tan(t)/L$$

Thirdly, the euclidian space is fully completed with the serie of the displacements converges to the limit of the transformation of the plane. There is the three axioms of Euclide [1] with the condition of Cauchy to complete the euclidian space.

$$|f(y) - f(x)| < K/2 * |y - x|$$

with K the leading coefficient of the function. Note : The first axiom of Euclide says that the two paralells converge at the infinity, the second axiom of Euclide says that two perpendiculars to a same one are paralells between them, and thirdly that the alternate angles of a secant to two paralells are equals. Finally, I may find a medium of closing such a euclidian space by the theorem of Thalès. By the theorem of Gödel, and with these axioms, there is the completness of this euclidian space. And however, according to the first axiom of Euclide, only two lines can meet at the infinity. And so on, it exists commands and controls such that each odometric way can close such an euclidian space. And not at the infinity, because it is too much of length to get a closed loop. Should I use the de Sitter space to close the loop of such an odometric way ? Note : The de Sitter space is the space where it is possible to go from a side to another by the closing of such an euclidian space. Example : The hypercube of the arch of the La Défense at Paris, where there are faces which are meeting at the infinity.

## 2. Proof of the Theorem

On one hand the space of displacements is a subgroup of the space of the transformations of the plane implies that each element of the space of displacements is the limit of a series of elements of this space of displacements.

And another hand for each element of this space of displacements there exists a bounded sequence for any displacements, cf. The work of Guillaume Artus on the slavings to be given to the computer to make a robot go from one position to another in the plane [2], in the space of the displacements such that the limit belongs to this space of the displacements of the plane implies that the space of the displacements is completed in the body of the transformations of the plane. For every functions in the euclidian space, like for instance the equations of the odometric model, I can apply the theorem of Pythagore for two points of the domain of definition of the function.

$$|f(y) - f(x)| < l1 + l2 < K * l1$$

with

$$l1$$

and

$$l2$$

the two projections of

$$f(y) - f(x)$$

onto the two euclidian axis.

$$|f(y) - f(x)| < K * |y|$$

Then, I have two inequalities for each term of x and y, with K the leading coefficient of the function.

$$|f(y) - f(x)| < K * |y|$$

$$|f(x) - f(y)| < K * |x|$$

Indeed, I can substract the two equations and have the formula of the condition of Cauchy. Moreover and actually, I have been proven the Cauchy condition by the way. For the differential point of view I have done the following, as well as in the definition of the limit of a function :  $\lim_{h \rightarrow 0} (f(x + h, y) - f(x, y))/h$  is equal to the next,  $\lim_{h \rightarrow 0} (f(x, y + h) - f(x, y))/h$  and if the function is plain, then the Schwartz equality has been proven for the partial derivation of the plain functions, like in the books or in the courses.

### **3. Explanations about the Theorem**

The theorem has been fully proven so that now we can access the datas from the first series with the linearized functions in the matrix of state and measured, given by the chinese team in a paper published by the IEEE editor, I have done the primary effort to the state matrix of functions, nevertheless they did the last one to ensure the compatibility of the odometric model with the mathematical background of mine and also to get the values with the vehicle [3]. All the datas have been computed in a table by a program under Ubuntu 20.04 with the GCC and also the GLIBC software, it used to be accessed on the DataPort of the IEEE website with the odometric key word : <https://iee-dataport.org/open-access/odometric-datas-simulation-odometric-model> to give an example from a linear approximation of the odometric model with GCC. For the simulation of the odometric model please see at : <https://iee-dataport.org/documents/new-figure-simulation-odometric-model> to give an example from a simulation of the odometric model with Scilab.

### **4. Example about the Theorem**

The best effort example is the half turn like maneuver. In the space of the displacements there are two arcs of circle to do for instance, instead of a rotation of Pi in the space of the transformations in the plane, and usually it can be done for more figures of maneuver with the homothety, or even with the translation. It is usually appointed the transverse function in the space of the displacements.

### **5. Application about the Theorem**

The best effort application is the map matching of the vehicle inside the road of the map to get the proper trajectory. It is to argue to go from a place to a destination with the help of the geometry of the roads. Not evident for a Cybercar to go to the different configurations of the vehicle based roads of destinations. I suggest to use the algorithm of going doors to doors to the following of the route given by the matrix of routing of the vehicle. First of all init the position of the vehicle by detecting the two points of the segment given by the middle of it as a position of intialization. Secondly define the next targetted position of the segment of position and try to go

to its middle point of repositionning. Then I have tried to increment the position of definition of the target positionning of position by detecting the left and right points of the segment which are defining the targetted position in the middle. Finally move to the targetted position in the middle. And Also stop the vehicle as soon as there are no more segments for repositionning the vehicle.

## 6. Conclusion

As one conclusion, I could say that the odometric model is well known now and that everything is done for the commands of a model of automation to be validated with the odometric model [4]. I should also emphase that a thesis can be done with the first issue of mine about the so called linearization of the equations of the odometric model given back by the issue of the chinese team of researchers [3]. Secondly, the map matching was ended with COSE SARL at the table of a bar of Rocquencourt. Thirdly, I do suggest to approve the limit of the odometric model. Finally, the mathematical and technical issue are over. ————— 1. ”[https://fr.wikipedia.org/wiki/Éléments \(Euclide\)](https://fr.wikipedia.org/wiki/Éléments_(Euclide))” 2. Guillaume Artus, Pascal Morin, Claude Samson. Control of a maneuvering mobile robot by the transverse function approach : control design and simulation results. [Research Report] RR-5155, INRIA. 2004, pp.16. <inria-00077042>; 3. Xiaocong Su, Chaoyong Zhang, Duanfeng Chu, Shidong Liu, Deng, and Chaozhong Wu, Trajectory Planning and Tracking for Autonomous Vehicle Based On State Lattice And Model Predictive Lattice, IEEE Intelligent Transportation Systems Magazine (Volume 11, Issue 2, Summer 2019 ); 4. Edern Ollivier. Simulation, integration and explanation of the odometric model for the purpose of the numerical analysis: A pool of trajectories for the engineers of the automation of the fair and automated vehicle. 2019. <hal-02162501v10>. —————