



# From the expansion of the space of the displacements towards the transformations of the plane

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FROM THE EXPANSION OF THE SPACE OF THE  
DISPLACEMENTS TOWARDS THE TRANSFORMATIONS OF THE  
PLANE\*

EDERN OLLIVIER†

**Abstract.** The odometric model gives the place to a space of displacements. I have been resolving to simulate, integrate, explain and then to validate the odometric model. The model of resolution is to be programmed in the Maple language, with the help of a solver for Maple 2021. An operator with the integral and the imaginary number has been defined to integrate the third equation of the odometric model for the terminal point of it.

**Key words.** Odometry, Vehicle location and navigation systems, Mobile positioning systems, Trajectory, Convergence of a model, Control of a vehicle.

12 AMS subject classifications. 47J20/45G05

13       **1. Introduction.** As one introduction, I could say that the odometric model is  
14 well known now and that everything is done for the space of displacements of a model  
15 of automation to be improved with the odometric model. By the non holonomy of  
16 the equations of a vehicle, I want to say that the function cannot be integrated simply.  
17 It is why an odometric model has been developed by the Inria of Rocquencourt in  
18 France in order to chase the meaning of the creator of the Cybercars from the Imara  
19 team.

**2. Main results.** I have done the approximation of the computed odometric model then I can understand the purpose of the space of the displacements : There is the convergence of the odometric model with the absolute convergence until an hypergeometric function with a Gamma, and moreover all this space of displacements has a structure K-completed of a group, with the neutral element that is the rotation of an angle of  $2^*Pi$  modulo  $Pi$ . The formula below is used to be the steering angle of the mathematical function of the computed odometric model.

$$\alpha(t) = \tan(t)$$

28 The formula below is used to be the steering angle integered of the mathematical  
 29 function of the computed odometric model.

$$\theta(t) = -\ln(\cos(t))$$

31 Moreover the formula below is used to be the value to integer of the mathematical  
32 terminal of the computed odometric model.

$$\beta(t) = \int_0^t \cos(x)^I \, dx$$

It deals with the value of the steering angle at the value of  $Pi/2$ . Normally the speed value of the wheels of the vehicle should always be tangential to the vehicle. Indeed

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36 the wheels are to be rolling in the direction of the vehicle. And when the vehicle is  
 37 turning then the direction of the vehicle is changing of the driving direction.

38 Here I state our main result as [Theorem 2.1](#); The proof is deferred to [Proof 2](#).

39 **THEOREM 2.1** (*theta Integration Value [3]*). *There is the absolute convergence*  
 40 *to the function integered of the steering angle at the value of integration of t :*

$$41 \quad \theta(t) = -\ln(\cos(t))$$

42 **THEOREM 2.2** (Terminal Point Theorem). *There is the integration of the insta-*  
 43 *bility terminal point of the computed odometric model at the value of integration of*  
 44 *Pi/2 :*

$$45 \quad \beta(\Pi/2) = \int_0^{\Pi/2} \cos(x)^I dx = \Gamma(1/2 + I/2) \cdot \sqrt{(\Pi)} / (2 \cdot \Gamma(1 + I/2))$$

46 In other words, the terminal value is computed to a Gamma.

$$47 \quad \beta(\Pi/2) = \Gamma(1/2 + I/2) \cdot \sqrt{(\Pi)} / (2 \cdot \Gamma(1 + I/2))$$

48 Observe that [Theorems 2.1](#) and [2.2](#) correctly mix definitions of the value of inte-  
 49 gration because of the continuity of the function cosinus.

50 **COROLLARY 2.3.** *Let f(x) be continuous and differentiable everywhere. If f(x)<sup>I</sup>*  
 51 *can be integered, then f(x)<sup>I</sup> has an integered value to the terminal point of it.*

52 *Proof.* Let a and b be two distinct terminal points of f with a < b. By [Theo-](#)  
 53 [rem 2.2](#), there exists a number b such that

$$54 \quad \beta(b) = \int_a^b f(x)^I dx \quad \square$$

55 *Proof.* First of all I wish a pleasant retirement to the professors of maths at  
 56 school for their support along the courses of maths : Paris, Payssant, Girard, Gomez,  
 57 Landès, Ruaud, Borel, Barrère and al. They have been founding the explanations  
 58 to better understand the great mathematic theorems as Pythagore, Thalès, Euler,  
 59 Leibnitz, Rolle, d'Alembert, Pascal, Newton, Gauss and al. They have been founding  
 60 the solving's method to exercice the thinkings to the resolutions of the roots of the  
 61 equations of the first and second's degree polynoms, with the Gupta's theory of the  
 62 numbers, and also to give the understanding's solutions of the differential's equations  
 63 of the first and second's order with or without a second member. The formula below  
 64 is used to be the angle Pi integered by the exponential function.

$$65 \quad \exp(I * \Pi) = -1$$

66 And with the Euler's notation :

$$67 \quad \cos(\Pi) + I * \sin(\Pi) = -1$$

68 On one hand the space of displacements is a subgroup of the space of the transfor-  
 69 mations of the plane involves that each element of the space of displacements is the  
 70 limit of a series of elements of this space of displacements. And another hand for each  
 71 element of this space of displacements [Proof 2](#) there exists a bounded sequence for  
 72 any displacements, cf. The work of Guillaume Artus on the slavings to be given to

73 the computer to make a robot go from one position to another in the plane [1], in the  
 74 space of the displacements such that the limit belongs to this space of the displace-  
 75 ments of the plane involves that the space of the displacements is completed in the  
 76 body of the transformations of the plane. Finally the subgroup of the displacements  
 77 of the plane is inside the body of the transformations of the plane which is a group  
 78 from the SO(2), the group of the orthogonal symmetries of the dimension two, and also  
 79 the neutral element belongs to it. It is more than enough to argue that the space of  
 80 the displacements is a group.

81 Note about the explanations on the first page of calculus : The theorem has  
 82 been fully proven so that now I can access the datas from the first series with the  
 83 linearized functions in the matrix of state and measured, given by the chinese team  
 84 in a paper published by the IEEE editor, I have done the primary effort to the  
 85 state matrix of functions, nevertheless they did the last one to ensure the compati-  
 86 bility of the odometric model with the mathematical background of mine and also  
 87 to get the values with the vehicle [2]. All the datas have been computed in a ta-  
 88 ble by a program under Ubuntu 20.04 with the GCC and also the GLIBC software,  
 89 it used to be accessed on the DataPort of the IEEE website with the odometric key  
 90 word : <https://ieee-dataport.org/open-access/odometric-datas-simulation-odometric-model>  
 91 to give an example from an linear approximation of the odometric model  
 92 with GCC. For the simulation of the odometric model please see at : <https://ieee-dataport.org/documents/new-figure-simulation-odometric-model> to give an example  
 93 from a simulation of the odometric model with Scilab.  
 94

95 Note about the example on the first page of calculus : The best effort example  
 96 is the half turn like maneuver. In the space of the displacements there are two arcs  
 97 of circle to do for instance, instead of a rotation of  $\Pi$  in the space of the transfor-  
 98 mations in the plane, and usually its can be done for more figures of maneuver with  
 99 the homothety, or even with the translation. It is usually appointed the transverse  
 100 function in the space of the displacements. □

101 **3. Algorithm.** Our analysis leads to the algorithm in [Algorithm 1](#).

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**Algorithm 1** Build spiral

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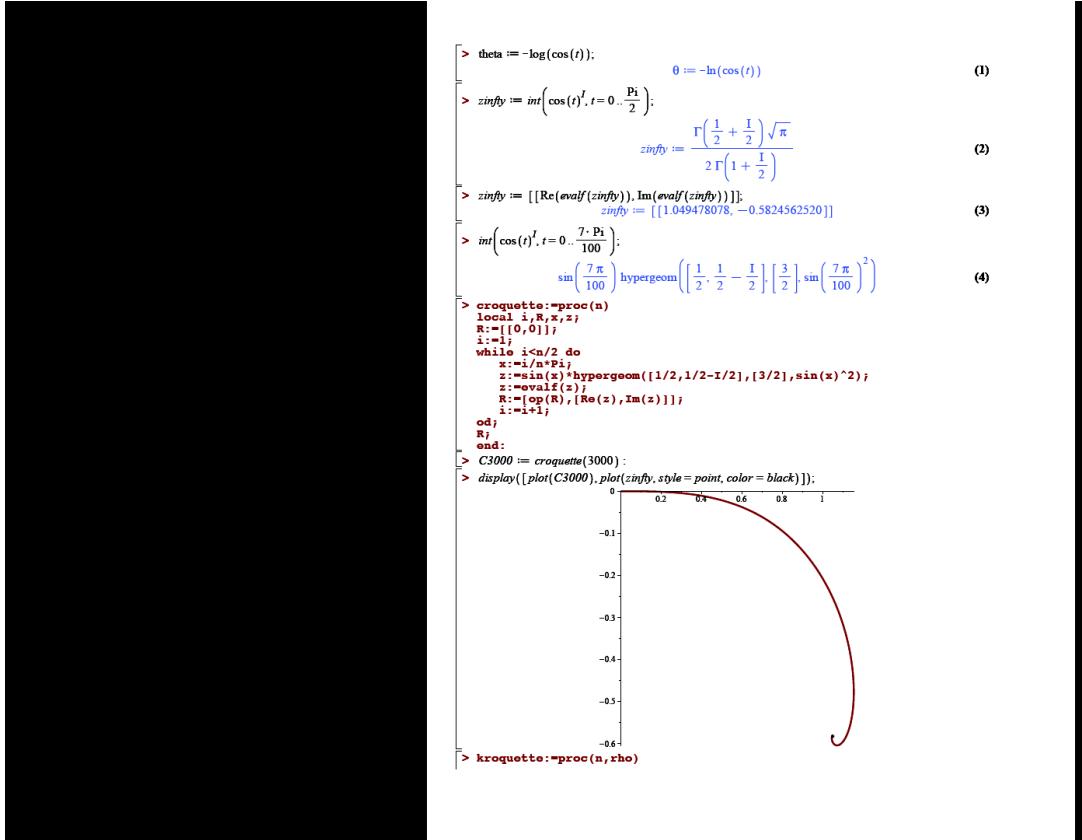
```

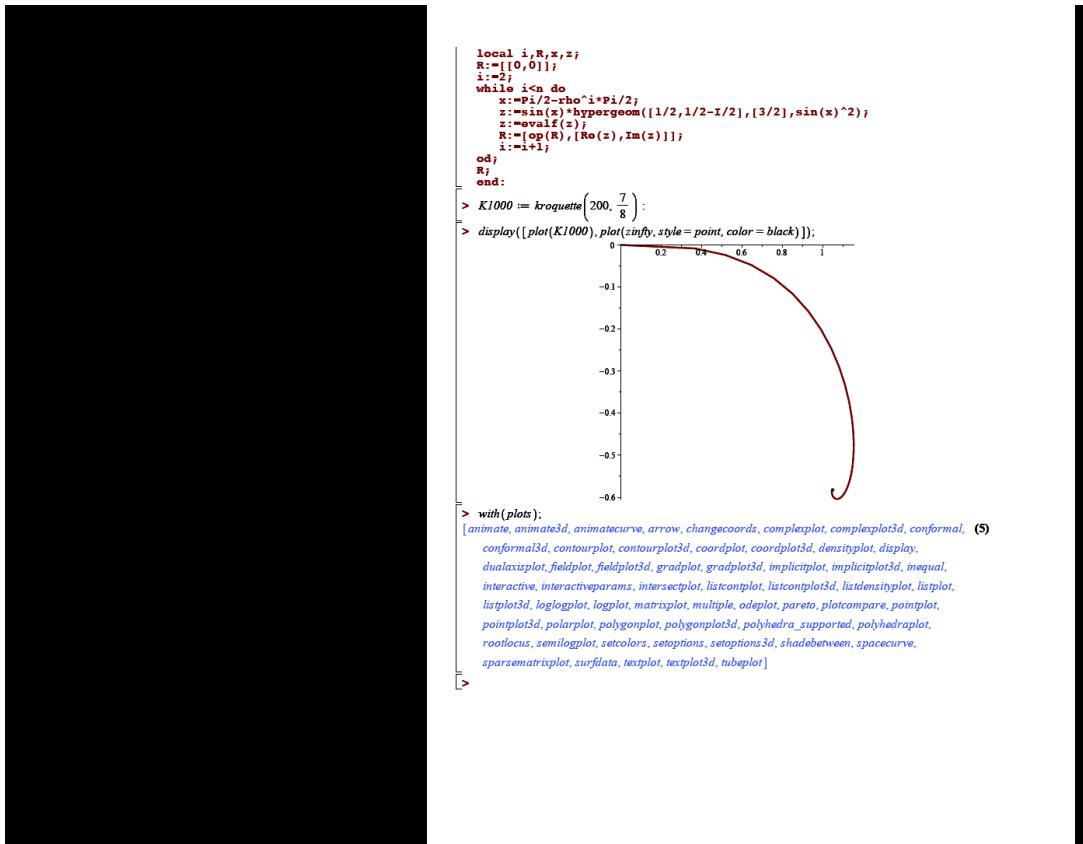
Define croquette := proc(n)
Define local i, R, x, z
Define R := [[0, 0]]
Define i := 1
while i < n/2 do
    Define x := i/n *  $\Pi$ 
    Define z := sin(x) * hypergeom([1/2, 1/2 - I/2], [3/2], sin(x)2)
    Update z := evalf(z)
    Update R := [op(R), [Re(z), Im(z)]]
    Update i := i + 1
end while
return R
Define end

```

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102 **4. Experimental results.** [Figure 4.1](#) shows an example result of the integra-  
 103 tion. [Figure 4.2](#) shows again an example result of the integration.





**5. Discussion of  $z = [Re(z), Im(z)]$ .** As one discussion, I could say that the values of the integrated computed odometric model are computed in the given figures with a procedure recording the complex values at every steps of the procedure given by a volunteer from the Inria of Saclay in France.

108     **6. Conclusions.** As one conclusion, I could say that the odometric model is well  
109 known now and that everything is done for the commands of a model of automation to  
110 be validated with the odometric model [3]. I should also emphasize that a thesis can be  
111 done with the first issue of mine about the so called linearization of the equations of the  
112 odometric model given back by the issue of the chinese team of researchers [1]. Here  
113 and now I am convinced that the simulation can improve the way of the automation  
114 of the Cybercars or such an electrical automated guided vehicle. I complain that  
115 the framework has now been fully completed and I am waiting for the datas from  
116 the industrial opponents of such a task of automation because as when as I could  
117 robotize such a project of automation by a control command I could be adopted by  
118 the community of the robotic and automation counterparts. I am still registered on  
119 the SMF/SMAI to be a valuable person for the control and command task of such a  
120 deployment of the automated guided vehicle in the cities of France.

## Appendix A. An appendix for the modulo $P_i$ .

LEMMA A.1. *Idem with the  $3^*Pi/2$  value modulo  $Pi$ .*

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[2] Xiaocong Su, Chaoyong Zhang, Duanfeng Chu, Shidong Liu, Deng, and Chaozhong Wu, Trajectory Planning and Tracking for Autonomous Vehicle Based On State Lattice And Model Predictive Lattice, IEEE Intelligent Transportation Systems Magazine (Volume 11, Issue 2, Summer 2019 );

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