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From the expansion of the space of the displacements towards the transformations of the plane

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1 **FROM THE EXPANSION OF THE SPACE OF THE**
2 **DISPLACEMENTS TOWARDS THE TRANSFORMATIONS OF THE**
3 **PLANE***

4 EDERN OLLIVIER†

5 **Abstract.** The odometric model gives the place to a space of displacements. I have been
6 resolving to simulate, integrate, explain and then to validate the odometric model. The model of
7 resolution is to be programmed in the Maple language, with the help of a solver for Maple 2021.
8 An operator with the integral and the imaginary number has been defined to integrate the third
9 equation of the odometric model for the terminal point of it.

10 **Key words.** Odometry, Vehicle location and navigation systems, Mobile positioning systems,
11 Trajectory, Convergence of a model, Control of a vehicle.

12 **AMS subject classifications.** 47J20/45G05

13 **1. Introduction.** As one introduction, I could say that the odometric model is
14 well known now and that everything is done for the space of displacements of a model
15 of automation to be improved with the odometric model. By the non holonomy of
16 the equations of a vehicle, I want to say that the function cannot be integered simply.
17 It is why an odometric model has been developped by the Inria of Rocquencourt in
18 France in order to chase the meaning of the creator of the Cybercars from the Imara
19 team.

20 **2. Main results.** I have done the approximation of the computed odometric
21 model then I can understand the purpose of the space of the displacements : There
22 is the convergence of the odometric model with the absolute convergence until an
23 hypergeometric function with a Gamma, and moreover all this space of displacements
24 has a structure K-completed of a group, with the neutral element that is the rotation
25 of an angle of $2*Pi$ modulo Pi . The formula below is used to be the steering angle of
26 the mathematical function of the computed odometric model.

27
$$\alpha(t) = \tan(t)$$

28 The formula below is used to be the steering angle integered of the mathematical
29 function of the computed odometric model.

30
$$\theta(t) = -\ln(\cos(t))$$

31 Moreover the formula below is used to be the value to integer of the mathematical
32 terminal of the computed odometric model.

33
$$\beta(t) = \int_0^t \cos(x)^I dx$$

34 It deals with the value of the steering angle at the value of $Pi/2$. Normally the speed
35 value of the wheels of the vehicle should always be tangential to the vehicle. Indeed

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36 the wheels are to be rolling in the direction of the vehicle. And when the vehicle is
 37 turning then the direction of the vehicle is changing of the driving direction.

38 Here I state our main result as [Theorem 2.1](#); The proof is deferred to [Proof 2](#).

39 **THEOREM 2.1** (*theta* Integration Value [3]). *There is the absolute convergence*
 40 *to the function integred of the steering angle at the value of integration of t :*

$$41 \quad \theta(t) = -\ln(\cos(t))$$

42 **THEOREM 2.2** (Terminal Point Theorem). *There is the integration of the insta-*
 43 *bility terminal point of the computed odometric model at the value of integration of*
 44 *$Pi/2$:*

$$45 \quad \beta(\Pi/2) = \int_0^{\Pi/2} \cos(x)^I dx = \Gamma(1/2 + I/2) \cdot \sqrt{(\Pi)} / (2 \cdot \Gamma(1 + I/2))$$

46 *In other words, the terminal value is computed to a Gamma.*

$$47 \quad \beta(\Pi/2) = \Gamma(1/2 + I/2) \cdot \sqrt{(\Pi)} / (2 \cdot \Gamma(1 + I/2))$$

48 Observe that [Theorems 2.1](#) and [2.2](#) correctly mix definitions of the value of inte-
 49 gration because of the continuity of the function cosinus.

50 **COROLLARY 2.3.** *Let $f(x)$ be continuous and differentiable everywhere. If $f(x)^I$*
 51 *can be integred, then $f(x)^I$ has an integred value to the terminal point of it.*

52 *Proof.* Let a and b be two distinct terminal points of f with $a < b$. By [Theo-](#)
 53 [rem 2.2](#), there exists a number b such that

$$54 \quad \beta(b) = \int_a^b f(x)^I dx \quad \square$$

55 *Proof.* First of all I wish a pleasant retirement to the professors of maths at
 56 school for their support along the courses of maths : Paris, Payssant, Girard, Gomez,
 57 Landès, Ruaud, Borel, Barrère and al. They have been founding the explanations
 58 to better understand the great mathematic theorems as Pythagore, Thalès, Euler,
 59 Leibnitz, Rolle, d'Alembert, Pascal, Newton, Gauss and al. They have been founding
 60 the solving's method to exercice the thinkings to the resolutions of the roots of the
 61 equations of the first and second's degree polynoms, with the Gupta's theory of the
 62 numbers, and also to give the understanding's solutions of the differential's equations
 63 of the first and second's order with or without a second member. The formula below
 64 is used to be the angle Pi integred by the exponential function.

$$65 \quad \exp(I * \Pi) = -1$$

66 And with the Euler's notation :

$$67 \quad \cos(\Pi) + I * \sin(\Pi) = -1$$

68 On one hand the space of displacements is a subgroup of the space of the transfor-
 69 mations of the plane involves that each element of the space of displacements is the
 70 limit of a series of elements of this space of displacements. And another hand for each
 71 element of this space of displacements [Proof 2](#) there exists a bounded sequence for
 72 any displacements, cf. The work of Guillaume Artus on the slavings to be given to

73 the computer to make a robot go from one position to another in the plane [1], in the
 74 space of the displacements such that the limit belongs to this space of the displace-
 75 ments of the plane involves that the space of the displacements is completed in the
 76 body of the transformations of the plane. Finally the subgroup of the displacements
 77 of the plane is inside the body of the transformations of the plane which is a group
 78 from the $SO(2)$, the group of the orthogonal symetries of the dimension two, and also
 79 the neutral element belongs to it. It is more than enough to argue that the space of
 80 the displacements is a group.

81 Note about the explanations on the first page of calculus : The theorem has
 82 been fully proven so that now I can access the datas from the first series with the
 83 linearized functions in the matrix of state and measured, given by the chinese team
 84 in a paper published by the IEEE editor, I have done the primary effort to the
 85 state matrix of functions, nevertheless they did the last one to ensure the compat-
 86 ibility of the odometric model with the mathematical background of mine and also
 87 to get the values with the vehicle [2]. All the datas have been computed in a ta-
 88 ble by a program under Ubuntu 20.04 with the GCC and also the GLIBC software,
 89 it used to be accessed on the DataPort of the IEEE website with the odometric key
 90 word : [https://ieeedataport.org/open-access/odometric-datas-simulation-odometric-](https://ieeedataport.org/open-access/odometric-datas-simulation-odometric-model)
 91 [model](https://ieeedataport.org/open-access/odometric-datas-simulation-odometric-model) to give an example from an linear approximation of the odometric model
 92 with GCC. For the simulation of the odometric model please see at : <https://ieeedataport.org/documents/new-figure-simulation-odometric-model> to give an example
 93 from a simulation of the odometric model with Scilab.

94 Note about the example on the first page of calculus : The best effort example
 95 is the half turn like maneuver. In the space of the displacements there are two arcs
 96 of circle to do for instance, instead of a rotation of Π in the space of the transfor-
 97 mations in the plane, and usually its can be done for more figures of maneuver with
 98 the homothety, or even with the translation. It is usually appointed the transverse
 99 function in the space of the displacements. □

101 **3. Algorithm.** Our analysis leads to the algorithm in [Algorithm 1](#).

Algorithm 1 Build spiral

```

Define croquette := proc(n)
Define local i, R, x, z
Define R := [[0, 0]]
Define i := 1
while i < n/2 do
  Define x := i/n * Pi
  Define z := sin(x) * hypergeom([1/2, 1/2 - I/2], [3/2], sin(x)2)
  Update z := evalf(z)
  Update R := [op(R), [Re(z), Im(z))]
  Update i := i + 1
end while
return R
Define end

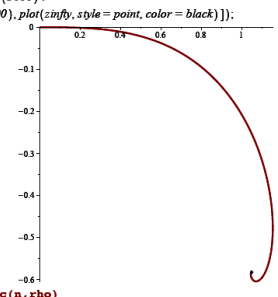
```

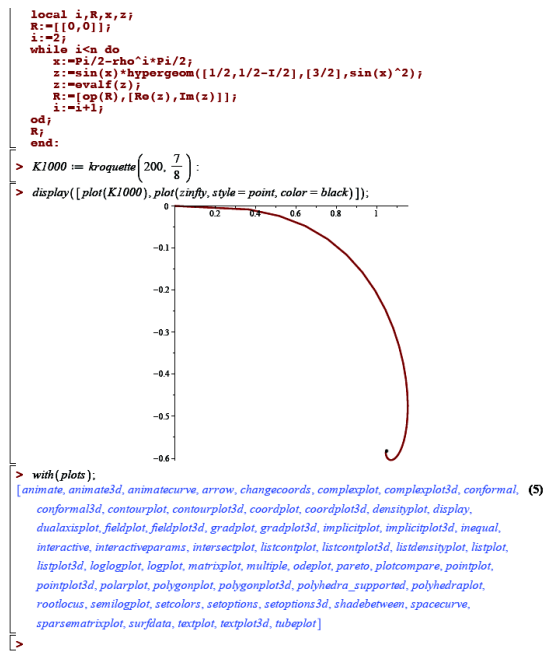
102 **4. Experimental results.** [Figure 4.1](#) shows an example result of the integra-
 103 tion. [Figure 4.2](#) shows again an example result of the integration.

```

> theta := -log(cos(t));
                                0 := -ln(cos(t))
=
> zinfy := int(cos(t)^t, t=0..Pi/2);
                                zinfy :=  $\frac{\Gamma\left(\frac{1}{2} + \frac{1}{2}\right)\sqrt{\pi}}{2\Gamma\left(1 + \frac{1}{2}\right)}$ 
=
> zinfy := [[Re(evalf(zinfy)), Im(evalf(zinfy))]];
                                zinfy := [[1.049478078, -0.5824562520]]
=
> int(cos(t)^t, t=0..7*Pi/100);
                                sin $\left(\frac{7\pi}{100}\right)$  hypergeom $\left(\left[\frac{1}{2}, \frac{1}{2} - \frac{1}{2}\right], \left[\frac{3}{2}\right], \sin\left(\frac{7\pi}{100}\right)^2\right)$ 
=
> croquette:=proc(n)
local i, R, x, z;
R:=[[0,0]];
i:=1;
while i<n/2 do
x:=i/n*Pi;
z:=sin(x)*hypergeom([1/2, 1/2-1/2], [3/2], sin(x)^2);
z:=evalf(z);
R:=[op(R), [Re(z), Im(z)]];
i:=i+1;
od;
R;
end;
> C3000 := croquette(3000);
> display([plot(C3000), plot(zinfy, style=point, color=black)]);
=
> kroquette:=proc(n, xho)

```





104 **5. Discussion of $z = [\text{Re}(z), \text{Im}(z)]$.** As one discussion, I could say that the
 105 values of the integrated computed odometric model are computed in the given figures
 106 with a procedure recording the complex values at every steps of the procedure given
 107 by a volunteer from the Inria of Saclay in France.

108 **6. Conclusions.** As one conclusion, I could say that the odometric model is well
 109 known now and that everything is done for the commands of a model of automation to
 110 be validated with the odometric model [3]. I should also emphasize that a thesis can be
 111 done with the first issue of mine about the so called linearization of the equations of the
 112 odometric model given back by the issue of the chinese team of researchers [1]. Here
 113 and now I am convinced that the simulation can improve the way of the automation
 114 of the Cybercars or such an electrical automated guided vehicle. I complain that
 115 the framework has now been fully completed and I am waiting for the datas from
 116 the industrial opponents of such a task of automation because as when as I could
 117 robotize such a project of automation by a control command I could be adopted by
 118 the community of the robotic and automation counterparts. I am still registered on
 119 the SMF/SMAI to be a valuable person for the control and command task of such a
 120 deployment of the automated guided vehicle in the cities of France.

121 **Appendix A. An appendix for the modulo Pi .**

122 LEMMA A.1. *Idem with the $3*Pi/2$ value modulo Pi .*

123 **Acknowledgments.** I would like to acknowledge the assistance of a volunteer
 124 of the Inria of Saclay in France in solving together these figures.

125

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