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► To cite this version:

David Gibus, Pierre Gasnier, Adrien Morel, Nicolas Garraud, Adrien Badel. Non-linear losses study in strongly coupled piezoelectric device for broadband energy harvesting. *Mechanical Systems and Signal Processing*, 2022, 165, pp.108370. 10.1016/j.ymssp.2021.108370 . hal-03335253

HAL Id: hal-03335253

<https://hal.science/hal-03335253v1>

Submitted on 6 Sep 2021

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Non-linear losses study in strongly coupled piezoelectric device for broadband energy harvesting

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Abstract

Piezoelectric vibration harvesters with strong electromechanical coupling coefficients have recently been combined with nonlinear electrical techniques capable of tuning their resonant frequency as a solution to provide energy to wireless sensor nodes from wideband vibrations. To be fully competitive, this approach requires piezoelectric generators with strong global electromechanical coupling coefficients k^2 . However, the presence of non-linear dielectric and piezoelectric losses in piezoelectric materials significantly reduces the power harvested when strongly coupled materials are used. This paper presents a non-linear model that allows for a better consideration of losses for such piezoelectric harvesters. An experimental validation is performed with a strongly coupled cantilever based on a PMN-PT material ($k^2=16\%$). The results reveal the importance of considering non-linear dielectric losses. Indeed, we show thanks to the model that neglecting these losses can induce an 18 % overestimation of the harvested power for the presented prototype driven at 0.5 m/s^2 amplitude ambient acceleration.

Nomenclature

M_t	Mass of the proof mass	ω	Vibration pulsation
I_t	Rotary inertia of the proof mass according to its center of gravity	f_{ext}	Vibration frequency
D_t	Distance between the free end of the beam and the center of gravity of the proof mass	$\phi(x)$	Vibration mode shape
J_t	Ratio between the rotation inertia and the mass	k_{31}^2	Material coupling coefficient of the piezoelectric material
L_b	Beam length	k^2	Global electromechanical coupling coefficient of the piezoelectric harvester
L_m	Proof mass length	K	Equivalent linear stiffness
H_m	Proof mass height	B_f	Forcing term
B	Beam and mass width	M	Equivalent mass
\mathcal{V}_p	Substrate volume	Θ	Linear coupling term
\mathcal{V}_p	Volume of piezoelectric material	C_p	Equivalent clamped capacitance
m	Mass per unit length of the bimorph	α	Linear coupling term per unit length
h_p	Piezoelectric patches thickness	$\overline{\theta}_2$	Second order coupling term per unit length
h_s	Substrate thickness	\overline{k}_2	Second order bending stiffness per unit length
e_{31}^{ef}	Linear piezoelectric equivalent coefficient	K_2	Nonlinear stiffness

d_{31}	Coefficient of the piezoelectric constant matrix	Θ_2	Nonlinear coupling term
s_{11}^E	Coefficient of the compliance matrix of the piezoelectric material	R_{load}	Resistive load
ϵ_{33}^T	Coefficient of the free dielectric matrix of the piezoelectric material	b_1	Linear mechanical structural loss coefficient
ϵ_{33}^{ef}	Linear dielectric permittivity equivalent coefficient	b_2	Second order structural loss coefficient
c_{11}^{ef}	Linear elastic equivalent coefficient	d_1	Linear dielectric loss coefficient
c_{111}	Nonlinear elastic constant	d_2	second order dielectric loss coefficient
e_{311}	Nonlinear piezoelectric equivalent constant	a, b	Coefficients that define the beam mode shape expression
Y_s	Young modulus of the substrate	w_{L_b}	Beam end relative velocity displacement
YI	Linear bending stiffness	w_B	Base displacement of the beam
x	Longitudinal spatial coordinate	\mathcal{A}_B	Acceleration amplitude of the beam base
y	Lateral spatial coordinate	$w(x)$	Transverse deflection of the beam
z	Transversal spatial coordinate	H	Nonlinear electric enthalpy density of the piezoelectric material
S_1	Longitudinal strain	L	Lagrangian
E_3	Transverse electric field	W_{ext}	External work
v	Voltage across the piezoelectric electrodes	T	Total kinetic energy
V	Voltage amplitude	F_n	Non-conservative force
r	Generalized spatial coordinate	I_n	Leakage current
R	Amplitude of the generalized spatial coordinate r	$\dot{\square}$	Temporal derivative
λ	Flux linkage	\square'	Spatial derivative according to x coordinate

1. Introduction

Vibration energy harvesting is recognized as a relevant solution to supply electrical power to wireless sensors nodes [1–3]. In order to reach sufficient power density, the use of electromechanical resonators have been shown required for inertial energy harvesting [4,5]. However, the resulting narrow frequency bandwidth of vibration energy harvesters is still an important issue [6,7]. A promising solution to address this problem is to use nonlinear electrical techniques [8,9] and power management circuits [10,11] able to tune the resonant frequency of piezoelectric harvesters. For example, this solution made it possible to expect a relative bandwidth of 43 % with a PZN-PT-based cantilever [9]. In order for this approach to be effective, it is necessary to employ piezoelectric harvesters with very strong global electromechanical coupling coefficients k^2 ($k^2 > 10\%$). Proposals for the design of such harvesters have therefore recently been made in the literature [12,13] and offer promising perspectives for broadband vibration energy harvesting. Nevertheless, the presence of nonlinear dielectric losses in the piezoelectric materials have been shown to considerably reduce the harvested power when strongly coupled materials are used [14]. Such dielectric losses must be taken into account in the models as neglecting them can lead to a significant error in the expected harvested power. Since a model to predict the behavior of the harvester taking into account non-linear dielectric losses is absent from the state of the art, this paper proposes a new nonlinear model corroborated by experimental results for strongly coupled energy harvester.

The linear losses of piezoelectric materials have already been studied in the prior art for various piezoelectric devices [15,16]. As an example, Uchino *et al.* expressed the necessity to consider a piezoelectric loss tangent

coefficient especially for actuator applications [17]. Recently, Wild *et al.* discussed the difficulty to distinguish dissipation origin (mechanical, dielectric and piezoelectric) depending on the piezoelectric constitutive equation form [18]. Concerning energy harvesting applications, dielectric and piezoelectric losses in piezoelectric materials are often neglected [19,20] as they don't have a significant influence for low and moderately coupled devices. The electric field experienced is indeed usually weak in the materials. Nevertheless, the consideration of a dielectric loss coefficient has been shown necessary for devices that include strongly coupled materials [14,21]. While models to consider linear dielectric losses have been proposed in the literature [21], nonlinear dielectric losses can be experienced in the very strongly coupled devices. Indeed, Morel *et al.* demonstrated the increase of the equivalent dielectric loss tangent on a PZN-PT based cantilever under an increasing electric field [14]. They also observed such nonlinear dielectric losses with the increase of the vibration amplitude. They indeed showed an experimental decrease of 30% of the power due to the increasing electric field in the material. Therefore, considering nonlinear dielectric losses is necessary to model the behaviour of vibration energy harvesters in the case of prototypes involving strongly coupled materials.

In this paper, we introduce a model that takes material nonlinearities of strongly coupled piezoelectric harvesters into account. The model, based on the Rayleigh method, is introduced in section 2. In order to validate this model, a strongly coupled prototype and the experiments conducted are then presented in section 3. The section 4 is dedicated to results presentation and discussion. In this last section, we discuss the importance of considering a dielectric nonlinear losses coefficient.

2. Modelling

2.1. Discussion about dielectric loss coefficients

This section deals with the loss considerations made in the present work. Losses in piezoelectric materials have been discussed in various ways in the literature [17,18]. It seems important to clearly state the intention of this paper and the definition of the loss coefficients used.

As mentioned by Wild *et al.* [18], the terms “dielectric loss”, “mechanical loss” and “piezoelectric loss” are intended to describe physical contributions that differ depending on the considered form of the piezoelectric constitutive equation (the *e*-form, *d*-form, *h*-form and *g*-form [22]). For the analysis of vibration energy harvesters, the base acceleration is usually fixed and the output voltage on the electrodes is measured, and therefore, the *e*-form is used. For this form of constitutive equation, the energy loss per cycle that increases with the displacement (strain in case of local consideration) at constant voltage (electrical field in case local consideration) is considered in the term of “mechanical loss”. Moreover, the term of “dielectric loss” represents the energy loss per cycle that increases with the output voltage at constant displacement. Finally, the “piezoelectric” term induces an effect that increases with both displacement and electric field. This effect can induce either losses or an increase in the harvested power [23]. The above definitions of “mechanical”, “piezoelectric” and “dielectric” losses will be used in this paper.

According to the work of Leadenham and Erturk [19], a second-order nonlinear mechanical loss coefficient must be considered in order to model the behaviour of PZT based cantilevers. Both linear and non-linear mechanical dissipations must be taken into account in the model. The energy loss per cycle due to linear mechanical loss U_m^{lin} is proportional to the square of the relative displacement of the beam R as expressed in (1) and the energy loss per cycle due to nonlinear mechanical loss U_m^{nonlin} is proportional to the cube of the relative displacement of the beam R as mentioned in (2).

$$U_m^{\text{lin}} \propto R^2 \quad (1)$$

$$U_m^{\text{nonlin}} \propto R^3 \quad (2)$$

The model proposed in [19] did not consider piezoelectric and dielectric losses as this did not seem necessary for the studied PZT-based cantilevers. Considering linear dielectric losses, the energy loss per cycle U_d^{lin} is proportional to the output voltage amplitude V of the harvested as expressed in (3) where V is the voltage amplitude.

$$U_d^{\text{lin}} \propto V^2 \quad (3)$$

Considering a second-order nonlinear dielectric loss coefficient, the energy loss per cycle due to electrical nonlinear loss U_d^{nonlin} is proportional to the cube of the voltage amplitude.

$$U_d^{\text{nonlin}} \propto V^3 \quad (4)$$

Since the experimentally observed dissipations as a function of the voltage are due to both piezoelectric and dielectric hysteresis [24], piezoelectric loss coefficients should be considered in addition to dielectric loss coefficients to finely model the harvesters behavior. However, we show in this paper that only dielectric loss coefficients are sufficient for the phenomenological modelling of the performance of vibration energy harvesters. Dielectric losses and piezoelectric losses have equivalent influences on the mean power harvested by piezoelectric vibration energy harvesters, as long as the vibration frequency remains in the vicinity of the resonant frequency of the harvester. Since off-resonance states are, in most cases, out of interest in energy harvesting applications (because of the low harvested power level), distinguishing these two kind of losses appears unnecessary.

Therefore, we propose a nonlinear model that takes into account linear and nonlinear dielectric losses in addition to linear and nonlinear mechanical losses as proposed by Leadenham and Erturk [19]. The model also considers a nonlinear stiffness coefficient and a nonlinear coupling coefficient in order to take the mechanical softening of the piezoelectric material into account. The device studied is presented in the following section. The model is then introduced in section 2.3.

2.2. Design and parameters of the system

The studied harvester consists of a bimorph cantilever with a bulky proof mass at the tip end (Figure 1.a) and for which the two piezoelectric layers have the same length (L_b) and the same width (B) as the substrate. The piezoelectric layers have the same thicknesses (h_p) and are completely covered with electrodes on the bottom and top surfaces, which are connected in parallel (Figure 2). Figure 1.b shows the problem parameters of the proposed cantilever while bending. The proof mass, symmetrical with respect to the neutral axis of the beam, is modelled with an equivalent point mass defined by a mass M_t and a rotary inertia I_t placed at a distance D_t from the free end of the beam on the neutral axis.

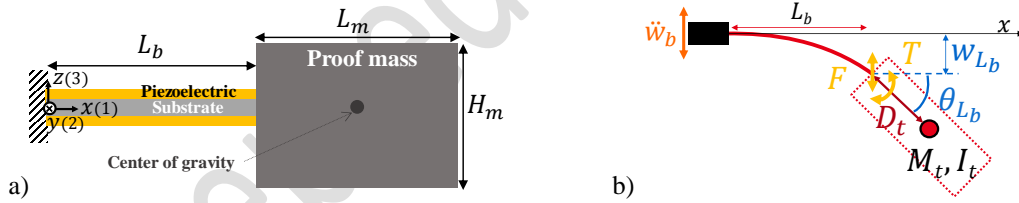


Figure 1: a) Cantilever with proof mass and b) cantilever during bending.

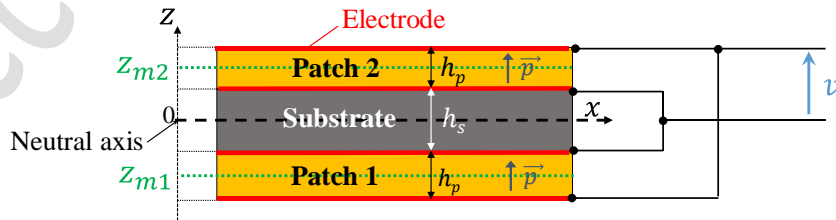


Figure 2: Beam layers and electrode connection. v represents the voltage across the electrodes.

As the beam length is much larger compared to its thickness ($L_b > 10(2h_p + h_s)$), the Euler-Bernoulli theory can be used. Furthermore, as the beam length is much larger than the width ($L_b > 5B$), the plane stress assumption is considered. Finally, due to the electrodes' architecture, the displacement field and the electric field are only considered in the transverse direction (E_3 and D_3 along the z axis).

The model introduced here is derived from the two degree of freedom linear model that we presented in [12], focusing on the first resonant mode. The Hamilton principle and the Rayleigh method are used to determine the equation of motion and the equation of current. Unlike the model in [12], nonlinear terms are introduced in the enthalpy expression to consider the mechanical softening and piezoelectric coefficient nonlinearity. Moreover,

linear and nonlinear losses are considered as an external work by adding non-conservative forces and leakage currents.

The Rayleigh method is detailed, and the equations of motion and current are determined in section 2.3. The expression of the first mode shape is determined in section 2.4 and the equation is solved to find the steady state solution in the case of sinusoidal excitation in section 2.5.

2.3. Hamilton principle

The Hamilton principle applied to the piezoelectric harvester is described in (5) where δ represents virtual variations of the quantities between time t_0 and t_1 . L is the Lagrangian that represents the difference of the kinetic energy T and the total potential energy U as expressed in (6). W_{ext} is the external work that considers the harvested energy and the losses.

$$\int_{t_0}^{t_1} (\delta L + \delta W_{ext}) dt = 0 \quad (5)$$

$$L = T - U \quad (6)$$

The total potential energy U of the piezoelectric bimorph is the sum of the potential energies of the substrate, U_s , and of the piezoelectric material U_p as expressed in (7). The expressions of U_s and U_p are given in (8) and (9) respectively. The equivalent stiffness of the substrate Y_s^{ef} is equal to the Young modulus Y_s of the substrate as the plane stress assumption is considered. S_1 is the longitudinal strain in the material. H represents the enthalpy of the piezoelectric material. \mathcal{V}_p and \mathcal{V}_s represent the volumes of the piezoelectric patches and the substrate respectively.

$$U = U_p + U_s \quad (7)$$

$$U_s = \frac{1}{2} \int_{\mathcal{V}_s} Y_s^{ef} S_1^2 d\mathcal{V}_s \quad (8)$$

$$U_p = \int_{\mathcal{V}_p} H d\mathcal{V}_p \quad (9)$$

According to the work of Leadenham and Erturk [19], the nonlinear electric enthalpy density expression in the piezoelectric material is given by equation (10) where E_3 is the electric field, c_{11}^{ef} and e_{31}^{ef} represent the linear elastic and piezoelectric equivalent coefficients respectively. ϵ_{33}^{ef} is the linear dielectric permittivity equivalent coefficient. c_{111} is the nonlinear elastic constant and e_{311} is the nonlinear piezoelectric constant. The function $\text{sgn}()$ represents the sign function as expressed in (11). As the plane stress assumption is considered, the equivalent linear parameters are given from the matrix coefficients in equations (12) to (14).

$$H = \frac{1}{2} c_{11}^{ef} S_1^2 + \frac{1}{3} c_{111} S_1^3 \text{sgn}(S_1) - e_{31}^{ef} S_1 E_3 - \frac{1}{2} e_{311} S_1^2 \text{sgn}(S_1) E_3 - \frac{1}{2} \epsilon_{33}^{ef} E_3^2 \quad (10)$$

$$\text{sgn}(f) = \begin{cases} 1 & \text{if } f \geq 0 \\ -1 & \text{if } f < 0 \end{cases} \quad (11)$$

$$\epsilon_{33}^{ef} = \epsilon_{33}^T - \frac{d_{31}^2}{S_{11}^E} \quad (12)$$

$$c_{11}^{ef} = \frac{1}{S_{11}^E} \quad (13)$$

$$e_{31}^{ef} = \frac{d_{31}}{S_{11}^E} \quad (14)$$

The longitudinal strain in the beam is given in (15) according to the Euler-Bernoulli theory where $w(x, t)$ is the transverse (in the z direction) deflection of the beam. The apostrophes represent the spatial derivative according to x coordinate. As discussed in [25], the electric field must be considered variable across the patches thicknesses for strongly coupled piezoelectric materials. Then the expression of the transverse electric field is given by (16) where λ is the flux leakage as introduced in [26] and expressed in (17). $z_m = (h_p + h_s)/2$ is the center position of the upper piezoelectric patch. With this definition $-z_m$ is the center position of the lower piezoelectric patch.

$$S_1 = -zw''(x, t) \quad (15)$$

$$E_3 = -\frac{\dot{\lambda}}{h_p} \text{sgn}(z) + \frac{e_{31}^{ef}}{\epsilon_{33}^{ef}} (z - z_m \text{sgn}(z)) w''(x, t) \quad (16)$$

$$\dot{\lambda} = v \quad (17)$$

The total potential energy U can finally be obtained using the equations (8) to (10) and (15) and (16) and is expressed in (18) where YI is the linear bending stiffness expressed in (19). \widetilde{k}_2 is the second order mechanical bending stiffness per unit length expressed in (20), α is the linear coupling term per unit length expressed in (21), $\widetilde{\theta}_2$ is the second order coupling term per unit length expressed in (22), C_p is the equivalent clamped capacitance expressed in (23).

$$U = \frac{1}{2} \int_0^{L_b} \left\{ YI (w'')^2 + \frac{1}{3} \widetilde{k}_2 (w'')^3 \text{sgn}(w'') - [\alpha w'' + \widetilde{\theta}_2 (w'')^2 \text{sgn}(w'')] \dot{\lambda} \right\} dx - \frac{1}{2} C_p \dot{\lambda}^2 \quad (18)$$

$$YI = B \left[Y_s^{ef} \frac{h_s^3}{12} + c_{11}^{ef} \left(\frac{2}{3} \left(h_p + \frac{h_s}{2} \right)^3 - \frac{h_s^3}{12} + k_{e_{31}}^2 \frac{h_p^3}{6} \right) \right] \quad (19)$$

$$\widetilde{k}_2 = B \frac{1}{2} h_p \left[c_{111} (h_p^3 + 4h_p^2 h_s + 3h_p h_s^2 + h_s^3) + \frac{e_{31}^{ef}}{\epsilon_{33}^{ef}} e_{311} (h_p^3 + h_p^2 h_s) \right] \quad (20)$$

$$\alpha = e_{31}^{ef} B (h_p + h_s) \quad (21)$$

$$\widetilde{\theta}_2 = \frac{1}{6} e_{311} B (4h_p^2 + 6h_p h_s + 3h_s^2) \quad (22)$$

$$C_p = \frac{B 2L_b \epsilon_{33}^{ef}}{h_p} \quad (23)$$

The total kinetic energy T considered in the Hamilton principle consists of the kinetic energies of the beam and of the proof mass including its rotary inertia is expressed in (24). m represents the mass per unit length of the bimorph expressed in (25) where ρ_s and ρ_p are the densities of the piezoelectric material and of the substrate respectively. \dot{w}_B is the base (clamped end) velocity, w_{L_b} is the beam end relative velocity, w_{L_b}' is the beam end speed of rotation.

$$T = \frac{1}{2} \int_0^{L_b} m [\dot{w} + \dot{w}_B(t)]^2 dx_1 + \left[M_t (w_{L_b} + \dot{w}_B)^2 + M_t D_t \left((w_{L_b} + \dot{w}_B) w_{L_b}' + w_{L_b}' (w_{L_b} + \dot{w}_B) \right) + (M_t D_t^2 + I_t) w_{L_b}'^2 \right] \quad (24)$$

$$m = B (h_s \rho_s + 2h_p \rho_p) \quad (25)$$

Using the Rayleigh approach, $w(x, t)$ can be expressed by the mode shape $\phi(x)$ multiplied by a generalized mechanical coordinate $r(t)$ as given in (26). $\phi(x)$ is the first resonant frequency mode shape and $r(t)$ describes the displacement of a point on the beam as a function of time. Consequently, we can define two generalized coordinates that are the displacement at the end of the beam $r(t)$ and the flux leakage $\lambda(t)$. The variation of the Lagrangian can then be expressed as (27).

$$w(x, t) = \phi(x) r(t) \quad (26)$$

$$\delta L(r, \dot{r}, \lambda) = \frac{\partial L}{\partial \dot{r}} \delta \dot{r} + \frac{\partial L}{\partial r} \delta r + \frac{\partial L}{\partial \dot{\lambda}} \delta \dot{\lambda} \quad (27)$$

As discussed in section 2.2, the mechanical losses and dielectric losses are considered by a non-conservative force F_n and a leakage current I_n that are expressed in (28) and (29) respectively, where b_1 is the linear mechanical

structural loss coefficient, b_2 is a second order structural loss coefficient, d_1 is a linear dielectric loss coefficient, d_2 is a second order dielectric loss coefficient. Considering that the harvester is connected to a resistive load R_{load} , the variation of the external work is expressed by equation (30).

$$F_n = -[b_1 r \operatorname{sgn}(r) + b_2 r^2] \operatorname{sgn}(\dot{r}) \quad (28)$$

$$I_n = -[d_1 \dot{v} \operatorname{sgn}(v) + d_2 v \dot{v}] \operatorname{sgn}(\dot{v}) \quad (29)$$

$$\delta W_{ext} = -\frac{\dot{\lambda}}{R_{load}} \delta \lambda - [b_1 r \operatorname{sgn}(r) + b_2 r^2] \operatorname{sgn}(\dot{r}) \delta r - [d_1 \dot{\lambda} \operatorname{sgn}(\dot{\lambda}) + d_2 \dot{\lambda} \ddot{\lambda}] \operatorname{sgn}(\ddot{\lambda}) \delta \lambda \quad (30)$$

Integrating by parts the Lagrangian variation in (27) and using the work expression in (30), the Hamilton principle given in (5) becomes the equation (31).

$$\int_{t_0}^{t_1} \left\{ \left[-\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial r} \right) + \frac{\partial L}{\partial r} - [b_1 r \operatorname{sgn}(r) + b_2 r^2] \operatorname{sgn}(\dot{r}) \right] \delta r + \left[-\frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{\lambda}} - \frac{\dot{\lambda}}{R} - [d_1 \dot{\lambda} \operatorname{sgn}(\dot{\lambda}) + d_2 \dot{\lambda} \ddot{\lambda}] \operatorname{sgn}(\ddot{\lambda}) \right] \delta \lambda \right\} dt = 0 \quad (31)$$

Replacing the Lagrangian with the expressions of the potential energy (18) and of the kinetic energy (24) in equation (31) allows to deduce equations (32) and (33), where the equivalent mass M , the linear stiffness K , the linear coupling term Θ , the forcing term B_f , the nonlinear stiffness K_2 and the nonlinear coupling term Θ_2 are expressed in equations (34) to (39).

$$M \ddot{r} + Kr + K_2 r^2 \operatorname{sgn}(r) - \Theta v - \Theta_2 v r \operatorname{sgn}(r) + (b_1 r \operatorname{sgn}(r) + b_2 r^2) \operatorname{sgn}(\dot{r}) = -B_f \ddot{w}_B \quad (32)$$

$$\Theta \dot{r} + \Theta_2 r \dot{r} \operatorname{sgn}(r) + C_p \dot{v} + \frac{v}{R_{load}} + (d_1 \dot{v} \operatorname{sgn}(v) + d_2 v \dot{v}) \operatorname{sgn}(\dot{v}) = 0 \quad (33)$$

$$M = m \int_0^{L_b} \phi(x)^2 dx + M_t [\phi(L_b)]^2 + 2M_t D_t \phi'(L_b) \phi(L_b) + (I_t + M_t D_t^2) [\phi'(L_b)]^2 \quad (34)$$

$$K = YI \int_0^{L_b} (\phi''(x))^2 dx \quad (35)$$

$$\Theta = \alpha \phi'(L_b) \quad (36)$$

$$B_f = m \int_0^{L_b} \phi(x) dx + M_t \phi(L_b) + D_t M_t \phi'(L_b) \quad (37)$$

$$K_2 = \tilde{k}_2 \int_0^{L_b} \operatorname{sgn}(\phi''(x)) (\phi''(x))^3 dx \quad (38)$$

$$\Theta_2 = \tilde{\theta}_2 \int_0^{L_b} \operatorname{sgn}(\phi''(x)) (\phi''(x))^2 dx \quad (39)$$

2.4. Beam mode shape expression

The mode shape $\phi(x)$ determination of the first bending resonant mode of a cantilever with proof mass was previously detailed in [12]. As the beam mass is considered small compared to the mass of the proof mass, the mode shape can be expressed analytically as a function of the proof mass parameters and the beam length. If the expression of the mode shape is normalized by the beam end displacement ($\phi(L_b) = 1$), $r(t)$ represents the beam displacement at $x = L_b$ and $\phi(x)$ is expressed by (40). a and b are coefficients that are expressed in the (41) and (42) respectively. β is the rotation amplitude to deflection amplitude ratio as introduced in [12] and expressed in (43), where J_t is the ratio between the rotation inertia and the mass defined as $J_t = I_t/M_t$.

$$\phi(x) = \frac{x^2 \left(3b - a \frac{x}{L_b} + 3a \right)}{6} \quad (40)$$

$$a = \frac{6(2 - L_b \beta)}{L_b^2} \quad (41)$$

$$b = \frac{2(2L_b \beta - 3)}{L_b^2} \quad (42)$$

$$\beta = \frac{3(2D_t + L_b)}{3J_t - (9D_t^4 + 18D_t^3L_b + 15D_t^2L_b^2 + 18D_t^2J_t + 6D_tL_b^3 + 18D_tL_bJ_t + L_b^4 + 3L_b^2J_t + 9J_t^2)^{\frac{1}{2}} + 3D_t^2 - L_b^2} \quad (43)$$

2.5. Model resolution

As done in [19], the method of harmonic balance is used to solve the nonlinear differential equations (32) and (33). For an harmonic base acceleration with constant amplitude ($\ddot{w}_B(t) = \mathcal{A}_B \cos(\Omega t)$), we assume that the displacement x and the voltage v are harmonic and we consider the first harmonic terms of the Fourier series R_1 , R_2 , V_1 and V_2 as expressed in (44) and (45).

$$r(t) = R_1 \cos(\omega t) + R_2 \sin(\omega t) \quad (44)$$

$$v(t) = V_1 \cos(\omega t) + V_2 \sin(\omega t) \quad (45)$$

Replacing $r(t)$ and $v(t)$ in equations (32) and (33) and considering the first harmonic terms in the application of the harmonic balance results to equations (46) to (49) where R and V are the displacement and voltage amplitudes given by $\sqrt{R_1^2 + R_2^2}$ and $\sqrt{V_1^2 + V_2^2}$ respectively.

$$-M\omega^2 R_1 + \left(\frac{2}{\pi} b_1 + \frac{4}{3\pi} b_2 R\right) R_2 + \left(K + \frac{8}{3\pi} K_2 R\right) R_1 - \theta V_1 - \frac{4}{3\pi} \theta_2 \left[\frac{(2R_1^2 + R_2^2)V_1 + R_1 R_2 V_2}{R} \right] + B_f \mathcal{A}_B = 0 \quad (46)$$

$$-M\omega^2 R_2 - \left(\frac{2}{\pi} b_1 + \frac{4}{3\pi} b_2 R\right) R_1 + \left(K + \frac{8}{3\pi} K_2 R\right) R_2 - \theta V_2 - \frac{4}{3\pi} \theta_2 \left[\frac{(2R_2^2 + R_1^2)V_2 + R_1 R_2 V_1}{R} \right] = 0 \quad (47)$$

$$C_p \omega V_2 + \frac{1}{R_{load}} V_1 + \left(\theta + \frac{4}{3\pi} \theta_2 R\right) \omega R_2 + \frac{2}{\pi} d_1 \omega V_1 + d_2 \frac{4}{3\pi} V \omega V_1 = 0 \quad (48)$$

$$C_p \omega V_1 - \frac{1}{R_{load}} V_2 + \left(\theta + \frac{4}{3\pi} \theta_2 R\right) \omega R_1 - \frac{2}{\pi} d_1 \omega V_2 - d_2 \frac{4}{3\pi} V \omega V_2 = 0 \quad (49)$$

Equations (46) to (49) are then solved thanks to Newton-Raphson algorithm in order to determine the voltage amplitudes (V_1 and V_2) and relative displacement amplitudes (R_1 and R_2) at given excitation pulsation ω , resistive load R_{load} and base acceleration amplitude \mathcal{A}_B . With this resolution, we are interested in determining the amplitudes of the beam displacement $r(t)$ and the voltage $v(t)$. The experimental validation of the model is detailed in the next section.

3. Experiments and results

3.1. Device introduction

The experimental validation was carried out on the strongly coupled cantilever based on PMN-PT single crystals shown in Figure 3. The PMN-PT material we used is the [001] poled PMN-0.29PT produced by TRS ceramics (TRS X2B). The PMN-PT patches have been cut to size by the manufacturer and glued in our laboratory on a steel beam with epoxy glue (Epotecny E505). The proof mass is made from two steel sheets bonded on the substrates with a 3M[®] epoxy glue. The geometrical parameters are given in Table 1 and the material parameters are given in Table 2.

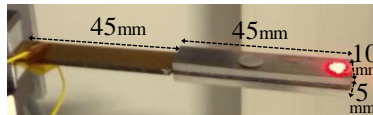


Figure 3 : Picture of the assembled prototype.

Material	PMN-PT
Beam length L_b	45 mm
Mass length L_m	45 mm
Height of mass H_m	5 mm
Beam and mass width B	10 mm
Substrate thickness h_s	0.5 mm
Piezoelectric thickness h_p	0.5 mm

d_{31}	-699 pm.V ⁻¹
s_{11}^E	52.1×10^{-12} Pa ⁻¹
ϵ_{33}^T	$5400 \epsilon_0$ F.m ⁻¹
ρ_p	7750 kg.m ⁻³

Based on the material parameters given in Table 2, the expected values of the linear global parameters can be determined from equations (34) to (39) and are given in Table 3.

Table 3 : Coefficients deduced from the model and the material parameters.

Name	Coefficient	Value
Equivalent mass	M	35.4 g
Forcing term	B_f	25.4 g
Linear stiffness	K	1 358 N/m
Coupling term	Θ	-3.70×10^{-3} N/V
Capacitance	C_p	69.1×10^{-9} F

3.2. Measurements under vibration and results

The realized experiments consist in measuring the output power and the displacement amplitude under harmonic vibration excitation for various accelerations levels and various resistive loads. The experimental setup, depicted in Figure 4, is composed of an electromagnetic shaker (K2075E-HT), an accelerometer (PCB Piezotronics 356A17) are a programmable electrical resistance that are controlled from a computer thanks to a dSpace board. A dedicated Matlab script defines the acceleration level, frequency and control the resistive loads. The experiments have been done for 5 acceleration levels between 0.019 m/s² and 0.5 m/s² and 60 resistive loads R_{load} logarithmically spaced between 1 k Ω and 10 M Ω over 95 excitation frequencies between 27 Hz and 33 Hz.

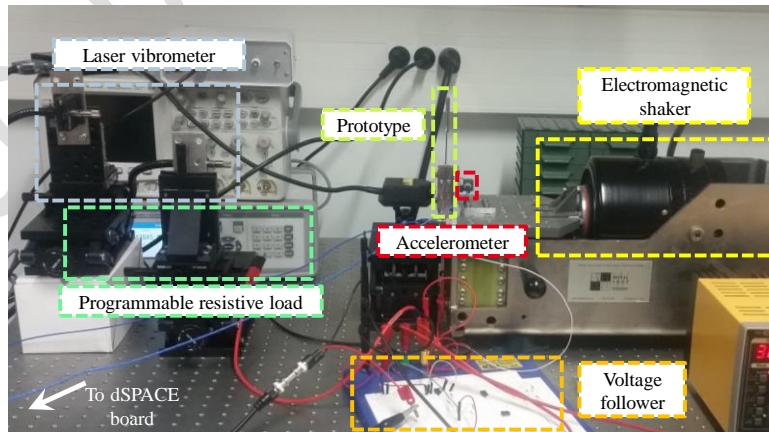


Figure 4 : Experimental setup for characterization under vibrations.

The harvested power is represented in Figure 5 as a function of the resistive load R_{load} and the excitation frequency f_{ext} for the 5 acceleration amplitudes \mathcal{A}_B . As the proposed prototype has a strong global electromechanical coupling coefficient k^2 and a high quality factor Q_m ($\left[\frac{k^2}{1-k^2}\right] Q_m > 9$ as $k^2 = 16\%$ and $Q_m > 50$), two power peaks are observed. One power peak is reached near the short-circuit resonant frequency (about

29 Hz) for a small resistive load (about 10 kΩ), the second peak is reached near the open-circuit resonant frequency (about 31.5 Hz) for a high resistive load (about 1.5 MΩ). As the prototype exhibits mechanical softening, the power peaks occur at lower frequencies with increasing acceleration level.

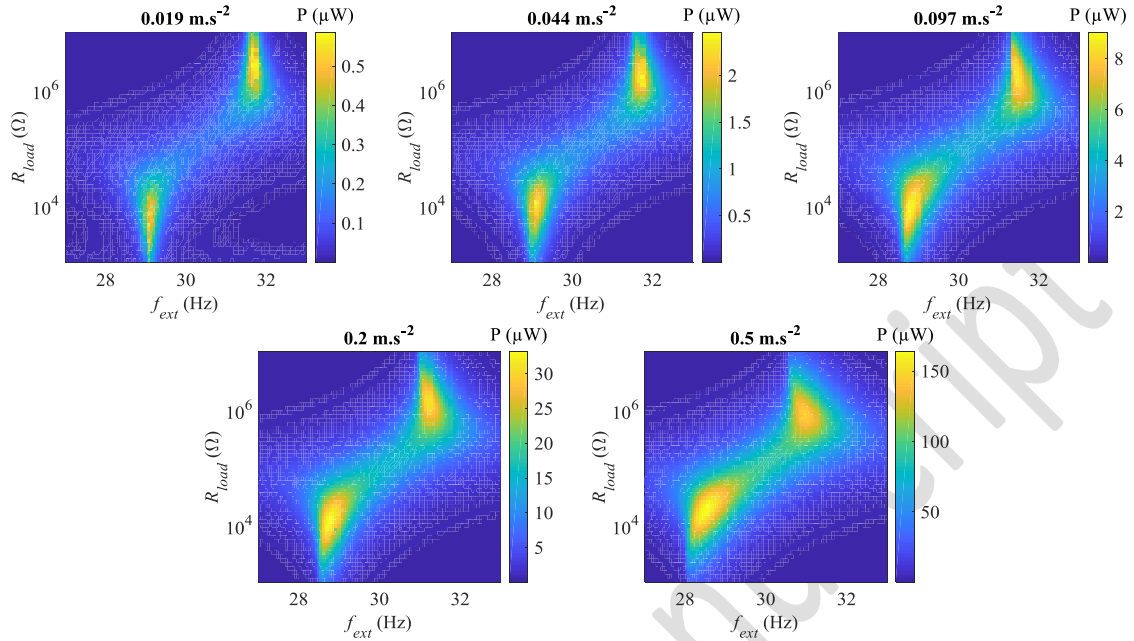


Figure 5 : Measured mean power as function of frequency and resistive load for 5 acceleration amplitudes \mathcal{A}_B . The mean power is given by $P = V^2/R_{load}$.

The experimental equivalent parameters of the equations (32) and (33) have been determined by fitting the power and displacement amplitude deduced from the model to the measured values. For this purpose, the amplitude error between the model mean power and the experiment was minimised by varying the equivalent parameters. The measured power and displacement for several resistive loads are represented in Figure 6 and Figure 7 respectively. The parameters derived from the fit are given in Table 4.

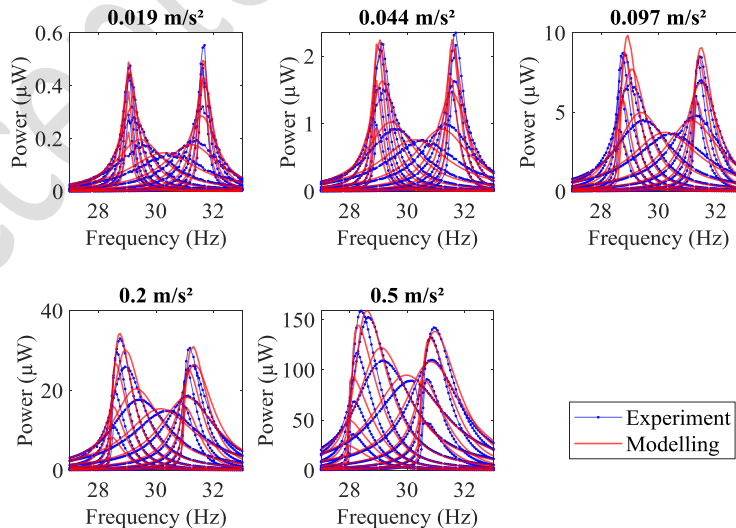


Figure 6 : Output mean power as a function of the excitation frequency for resistive loads of 1.2 kΩ, 2.6 kΩ, 5.6 kΩ, 12 kΩ, 27 kΩ, 57 kΩ, 126 kΩ, 276 kΩ, 602 kΩ, 1.3 MΩ, 2.9 MΩ and 6.3 MΩ: experimental results and model results with fitted coefficients.

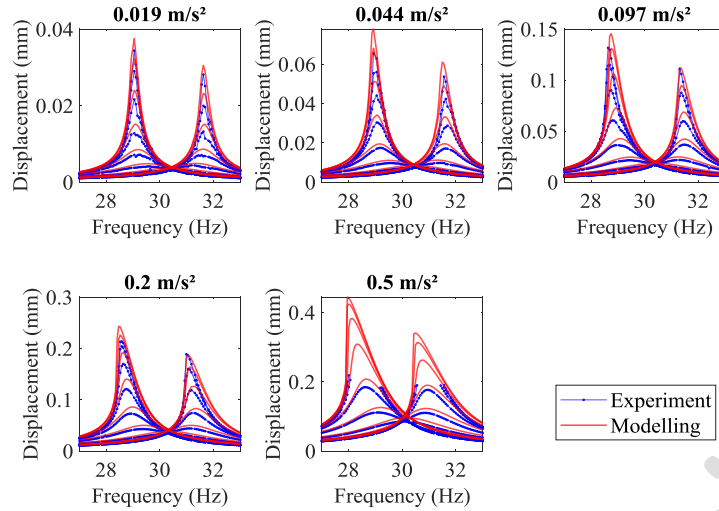


Figure 7 : Displacement amplitude R as a function of the excitation frequency for resistive loads of 1.2 k Ω , 2.6 k Ω , 5.6 k Ω , 12 k Ω , 27 k Ω , 57 k Ω , 126 k Ω , 276 k Ω , 602 k Ω , 1.3 M Ω , 2.9 M Ω and 6.3 M Ω : experimental results and model results with fitted coefficients. At 0.5 m/s², points are missing from the displacement measurement as the displacement could be only measured in the ± 0.2 mm range.

Table 4 : Coefficients deduced from the experiments.

Name	Coefficient	Value
Equivalent mass	M	35.2 g
Forcing term	B_f	23.1 g
Linear stiffness	K	1 178 N/m
Coupling term	Θ	-3.05×10^{-3} N/V
Capacitance	C_p	41.9×10^{-9} F
Nonlinear stiffness	K_2	-2.44×10^5 N/m ²
Nonlinear coupling term	Θ_2	1,22 N/Vm
Linear mechanical loss	b_1	12.8 N/m
Nonlinear mechanical loss	b_2	$8,57 \times 10^4$ N/m ²
Linear dielectric loss	d_1	0 F
Nonlinear dielectric loss	d_2	$2,03 \times 10^{-10}$ F/V

The average error between the model and experiment is calculated using equation (50) where n_{acc} , n_{res} and n_{frq} are the numbers of accelerations, resistances and frequencies which are equal to 5, 60 and 95 respectively. P_{exp} is the experimental mean power and P_{th} is the mean power derived from the model.

$$\frac{1}{n_{acc} n_{res} n_{frq}} \sum \sum \sum \left(\frac{|P_{th} - P_{exp}|}{P_{exp}} \right) \quad (50)$$

The average error the model and experiment is equal to 9.5 % for power evaluation. The average error for the displacement evaluation is equal to 14.4 % for the 4 exploitable acceleration levels (i.e. measurements the 0.5 m/s² is not considered due to missing points).

4. Discussions

4.1. Discussion on model validation

It can be seen from Figure 6 and Figure 7 that the proposed model provides a good evaluation of the experimental behaviour of the cantilever-type generator. The presented model is therefore able to predict the nonlinear behaviour of strongly coupled harvesters subjected to various acceleration levels.

Furthermore, the linear parameter deduced from model fitting (Table 3) are close to the linear parameters deduced from the material parameters of the material supplier (Table 4), which means that the model is also able to approximately predict the performance of a harvester in a design phase from the parameters given a priori. The differences, especially on the linear stiffness K and linear coupling term Θ can be explained by uncertainties in material parameters, manufacturing and clamping conditions as discussed in previous work [12].

4.2. Dielectric losses consideration

It can be noted that the linear dielectric loss coefficient d_1 is taken as zero in the model fitting (Table 4). The nonlinear dielectric loss coefficient seems to be more relevant to consider the harvester behavior than the linear one. In order to analyze the influence of the nonlinear dielectric loss coefficient on the harvested power, the power at optimal resistive loads is represented in Figure 8 as a function of the frequency for 3 acceleration levels. The harvested power is shown with the consideration of the dielectric loss coefficient d_2 and without its consideration. Not taking the nonlinear dielectric loss coefficient into account is equivalent to using the model presented in [19].

The power for optimal pairs of parallel capacitive and resistive loads is also plotted to deduce the expected results when the resonant frequency of the harvester is electrically tuned. Indeed, non-linear electrical interfaces, such as those based on Synchronous Electric Charge Extraction (SECE) techniques, are able to emulate combinations of resistive and capacitive loads in order to tune the resonant frequency of strongly coupled piezoelectric harvesters [28]. The harvested power for resistive and capacitive loads is a relevant indicator that allows us to know the expected power using a non-linear electrical interface. As an example, the model predicts that the frequency bandwidths defined at half of the maximal power are equal to 3.1 Hz and 3.7 Hz for optimal resistive and capacitive loads and acceleration amplitudes \mathcal{A}_B of 0.019 m/s² and 0.5 m/s² respectively (Figure 8). This represents relative bandwidths of 10 % and 12.3 % of the center frequencies respectively. This bandwidths corresponds to the minimum bandwidth expected when using the short-circuit synchronous electric charge extraction (SC-SECE) technique [9]. The expected behavior of a strongly coupled harvester with material nonlinearities has been validated experimentally with combinations of resistive and capacitive loads in a previous work [29].

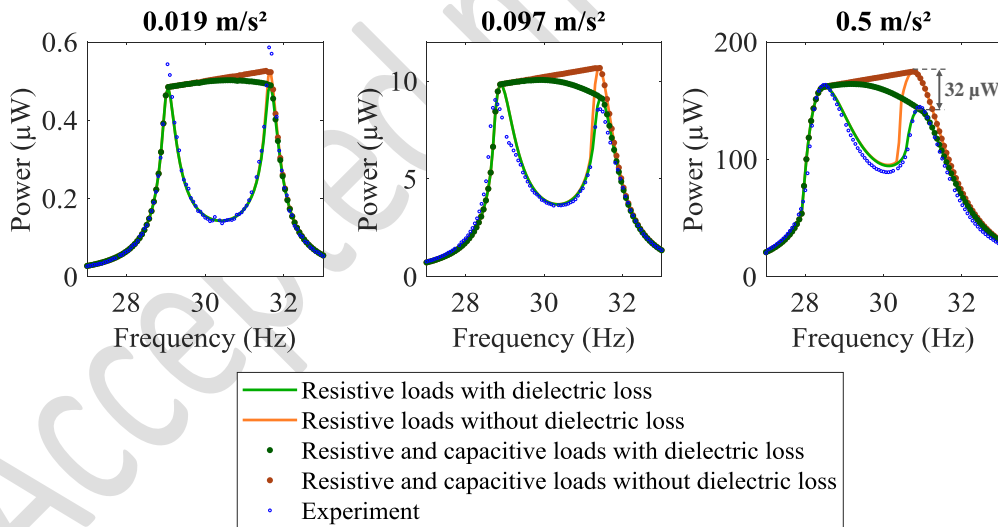


Figure 8: Model-derived power harvested by the proposed piezoelectric cantilever versus frequency for optimal resistive loads and optimal pairs of parallel capacitive and resistive loads.

The average errors between the models and experiment on power with optimal resistive loads are calculated for the three acceleration levels and are given in Table 5.

Table 5 : Average error between the models and experiment on power with optimal resistive loads

Acceleration amplitude	0.019 m/s ²	0.097 m/s ²	0.5 m/s ²
Average error of the model with dielectric losses	3,6%	7,1%	6,5%
Average error of the model without dielectric losses	3,4%	8,6%	12,3%

The proposed model offers a better accuracy on the power evaluation than the literature model with the increase of the acceleration amplitude. From the model (Figure 8), we can see that the nonlinear dielectric loss coefficient has a significant influence on the power harvested near the open-circuit resonant frequency (around 31.5 Hz). This is because the load impedance magnitudes required to maximize the power are higher near the open-circuit resonant frequency and lower near to the short-circuit resonant frequency. This means that the optimal voltage close to 31.5 Hz is higher than the optimal voltage close to 29 Hz, as shown in Figure 9. Therefore, since the energy loss per cycle increase with the voltage amplitude when we take into account dielectric losses, lower output power is expected around 31.5 Hz compared to the case when these losses are not taken into account.

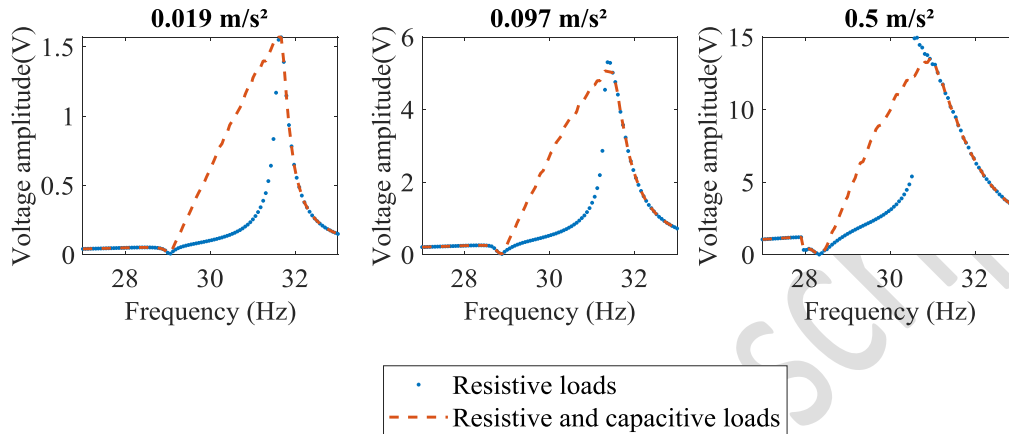


Figure 9 : Voltage magnitude derived from the model as a function of frequency for optimal resistive loads and optimal pairs of parallel capacitors and resistors.

The model (Figure 8) also shows that the influence of dielectric losses increases with the acceleration level. While the influence is hardly noticeable at low level (0.019 m/s^2), a strong influence is noticed at high acceleration level (0.5 m/s^2). Considering the dielectric losses, the maximum power harvested at the high optimal resistive load is $32 \mu\text{W}$ lower than the maximum power that would be harvested without dielectric losses at the optimal reactive load at 0.5 m/s^2 (Figure 8). This represents a relative decrease of more than 18 % of the expected power at the optimal resistive load. Neglecting nonlinear dielectric losses when modelling strongly coupled vibration energy harvesters can therefore lead to a significant overestimation of the harvested power.

Figure 10 represents the evolution of the calculated power at the second power peak on the optimal resistive load (the peaks around 31.5 Hz) as a function of the acceleration level when the dielectric loss coefficient d_2 is taken into account and not taken into account. The relative decrease when considering the dielectric loss coefficient is also shown. This figure shows the increasing influence of the dielectric loss with the acceleration level. The relative power loss due to dielectric loss increases rapidly (15 % decrease over 0.12 m/s^2) and tends to saturate below 20 % above 0.5 m/s^2 . As this relative power decrease is not constant with the acceleration level variation, a linear dielectric loss coefficient cannot be sufficient to consider the increasing loss with voltage. Non-linear dielectric loss modelling is thus necessary to adequately account for the behavior of strongly coupled harvesters based on strongly coupled materials. As the behavior of strongly coupled harvester may differ depending on the piezoelectric material used, an analysis using this model on different types of materials should be carried out.

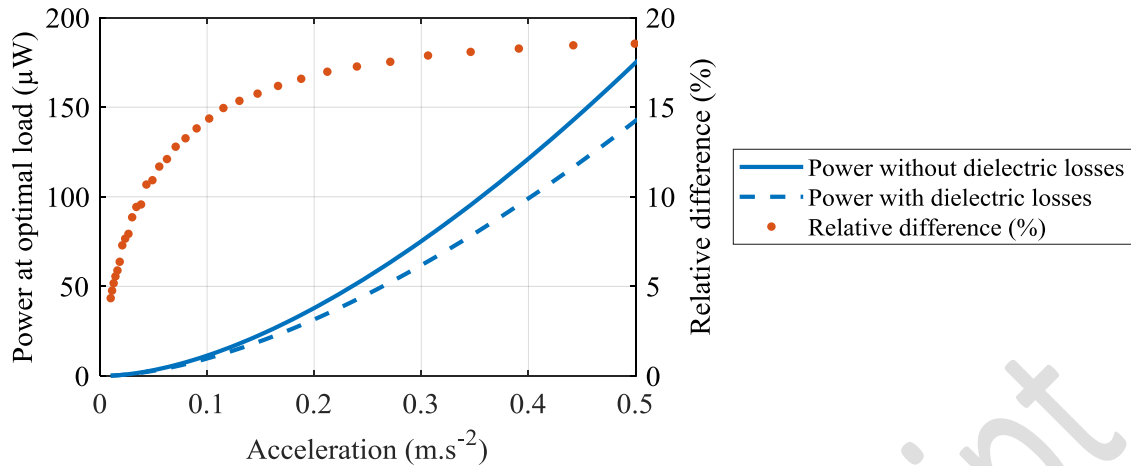


Figure 10 : Harvested power deduced from the model at the second power peak (around 31.5 Hz) as a function of the acceleration amplitude \mathcal{A}_B . The relative difference is given by $|P_{\text{without}} - P_{\text{with}}|/P_{\text{with}}$ where P_{without} is the harvested power without the consideration of dielectric losses and P_{with} is the harvested power with the consideration of dielectric losses

5. Conclusions

This paper reports on the experimental validation of a new model that considers nonlinear material losses in piezoelectric harvesters. For this purpose, it takes into account a second order dielectric loss term in addition to a mechanical second order mechanical loss term. The presented model allows a better consideration of the harvested power under vibration than previous models and is proven to be necessary for accurately predicting the performance of strongly coupled harvesters. An experimental validation is carried out with a strongly coupled cantilever based on PMN-PT material. The results reveal the importance of considering a nonlinear dielectric loss coefficient. Indeed, we show thanks to the proposed model that neglecting this coefficient can induce an error up to 18% between the measured power and the predicted power for the presented harvester.

As it has been shown that strongly coupled vibrational energy harvesters are needed to extend the performance of resonant frequency tuning by electrical methods, the use of the present model will be necessary for the study of broadband vibration harvesters. In future work, the effect of nonlinear dielectric loss on the power harvested by strongly coupled energy harvesters combined with advanced energy extraction circuit able to tune the generator's resonant frequency will be studied (e.g. SC-SECE).

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Accepted manuscript