Tree Containment with Polytomies

Mathias Weller

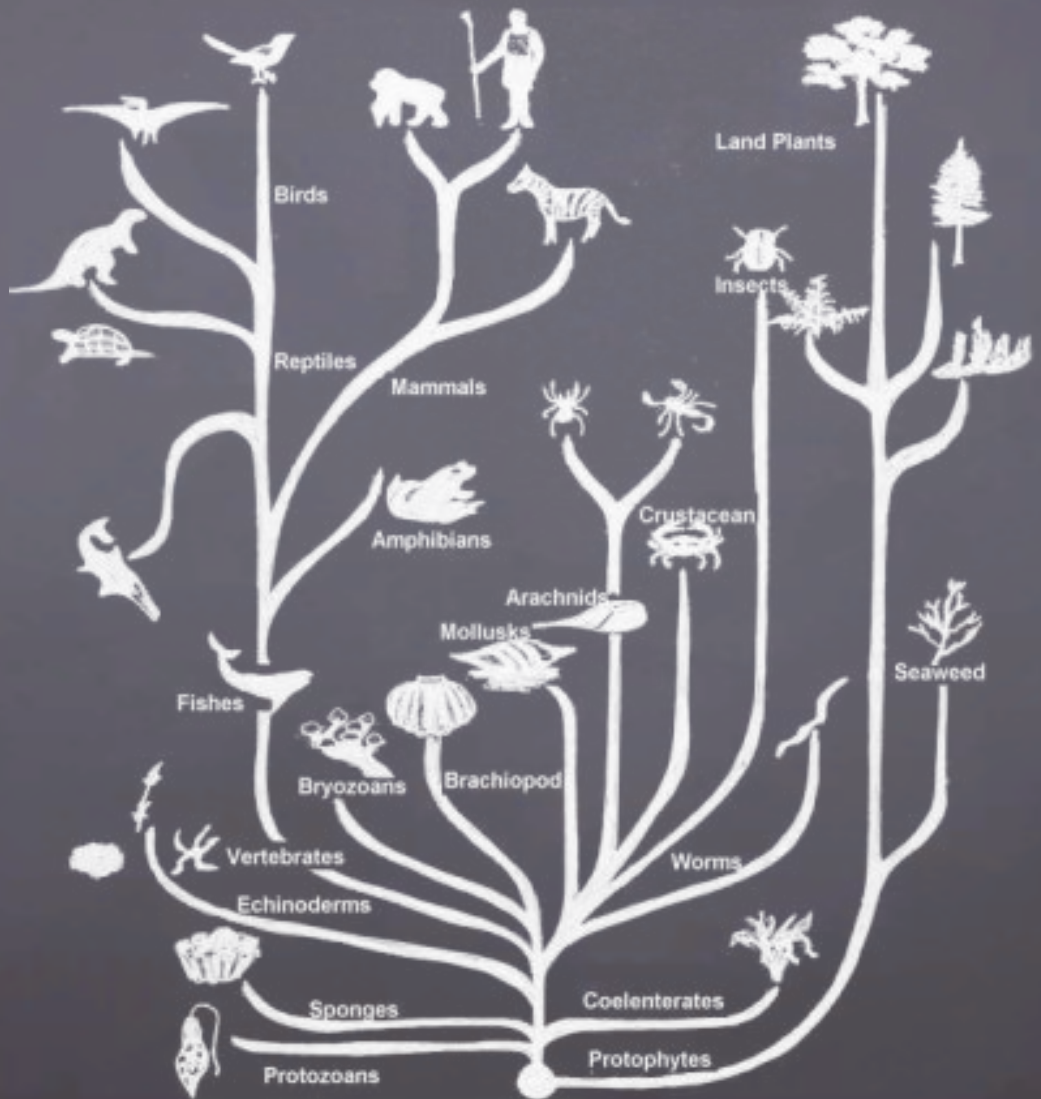
CNRS, LIGM, Paris EST, Marne-la-Vallée

Dagstuhl Seminar:
Algorithms & Complexity in Phylogenetics

Motivation: Tracing Evolution

Evolutionary Tree

= rooted trees with leaf labels ("trees" from now on)



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Problem

Given a network N and a tree T, is N compatible with T?

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Given a network N and a tree T, does N display T?

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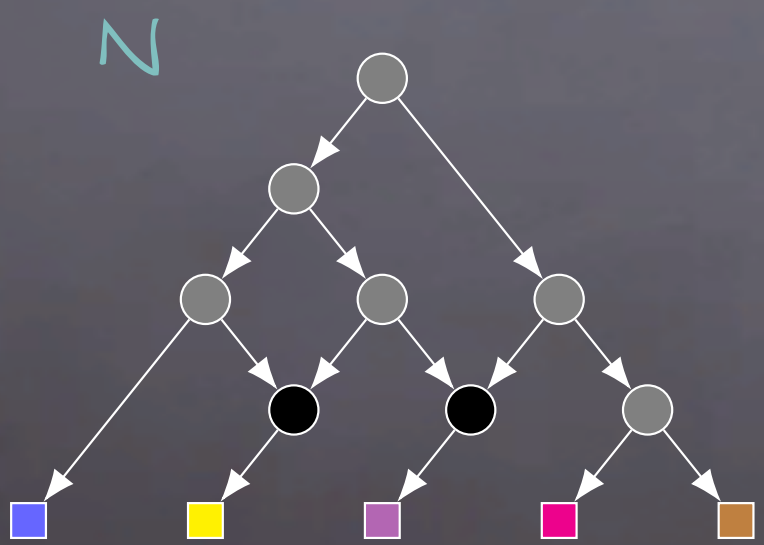

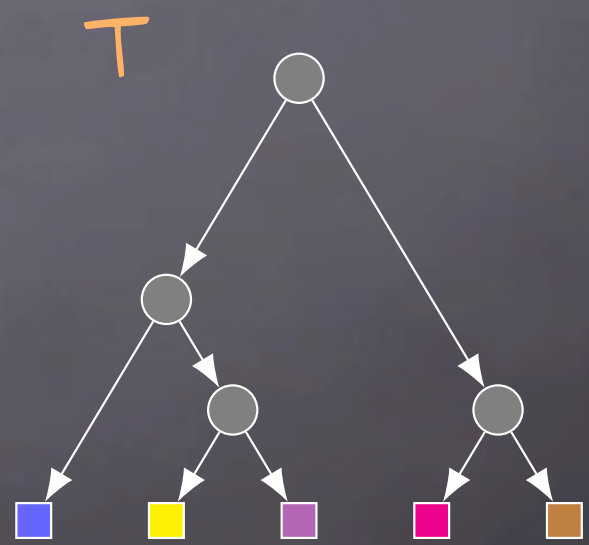

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Polytomies: Hard & Soft

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special events in evolution causing high fan-out

⇝ should be represented as-is

(display = contains subdivision)

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= can be contracted to N (or T)

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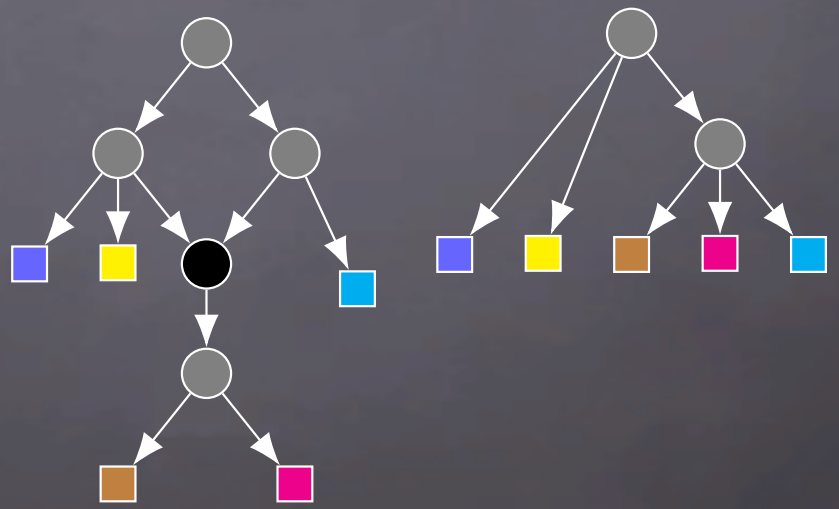

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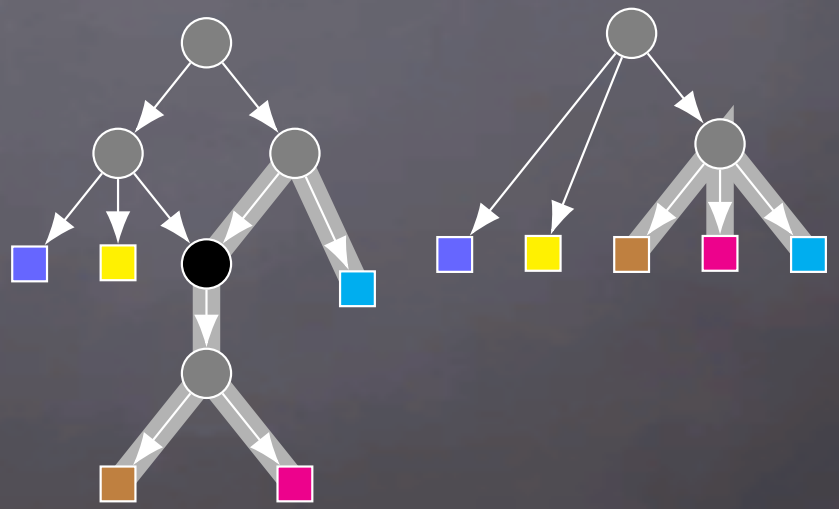

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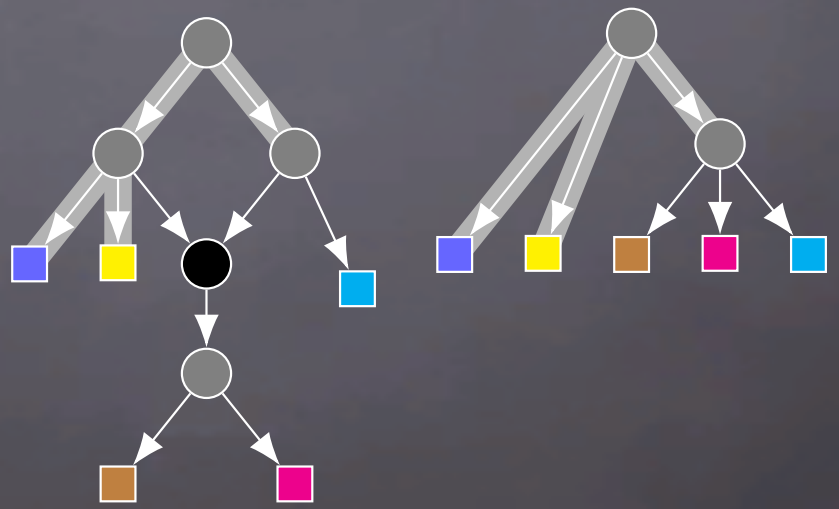

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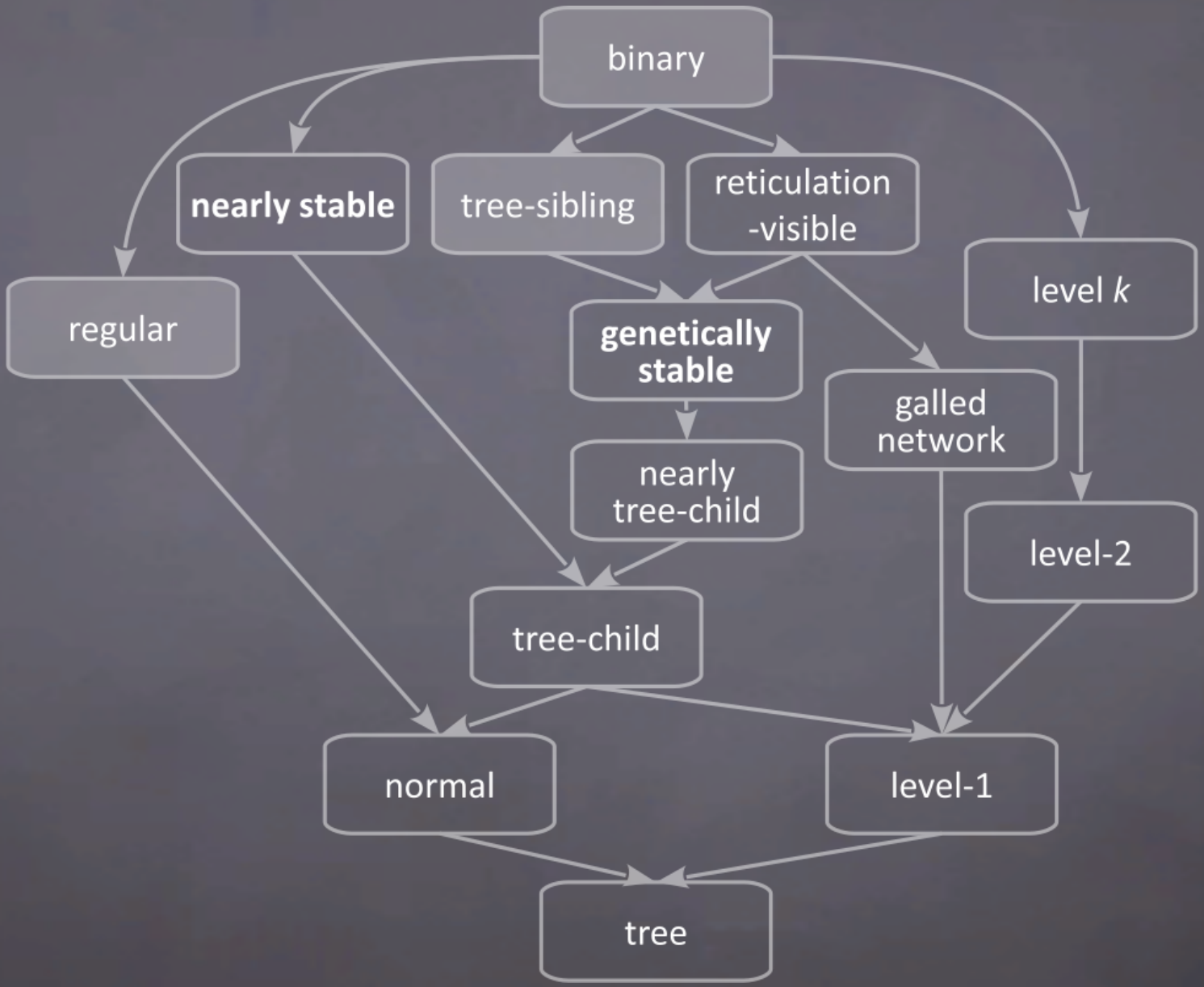
Remark

For binary $N \neq T$, they coincide!

Previous Work



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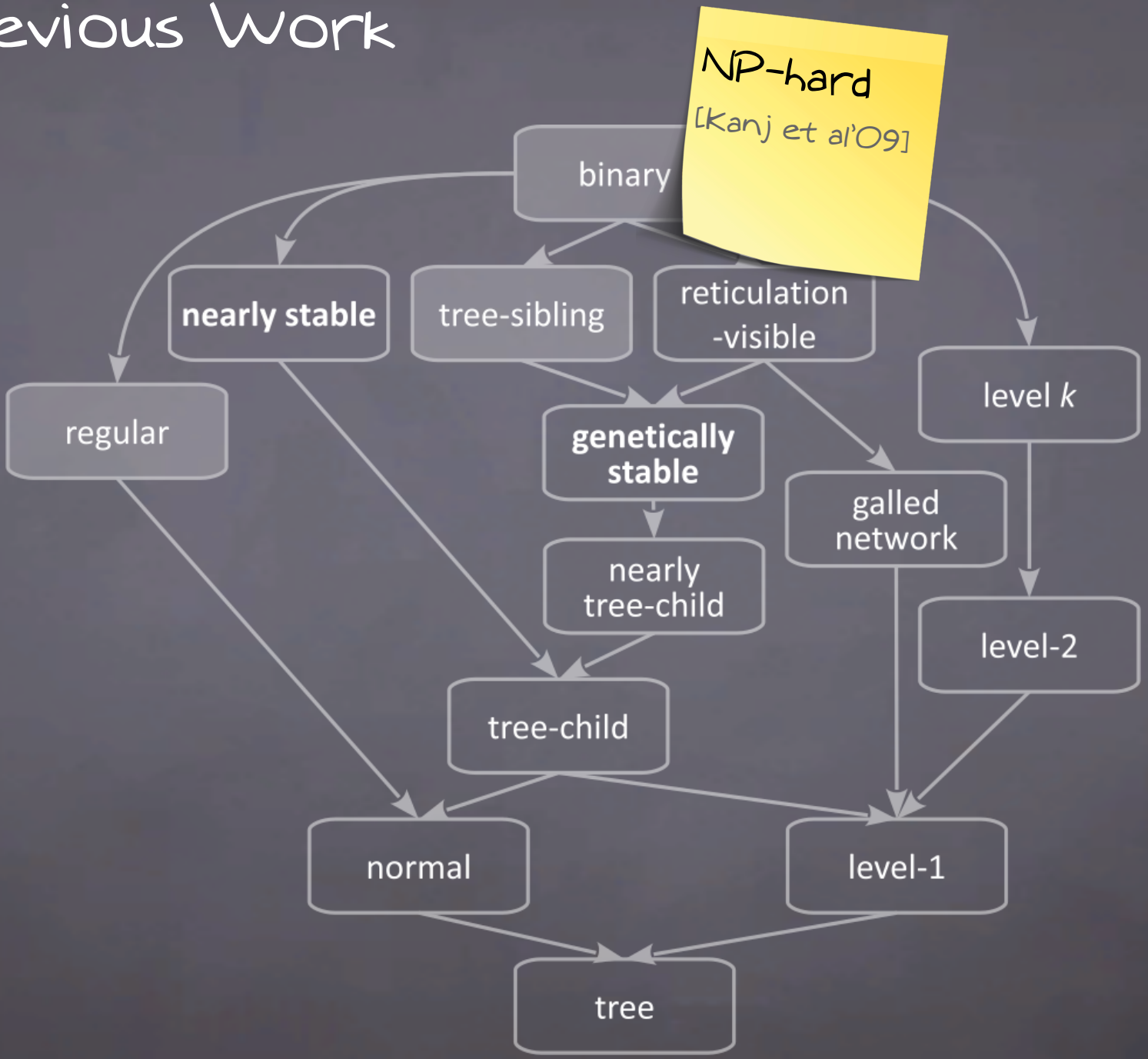
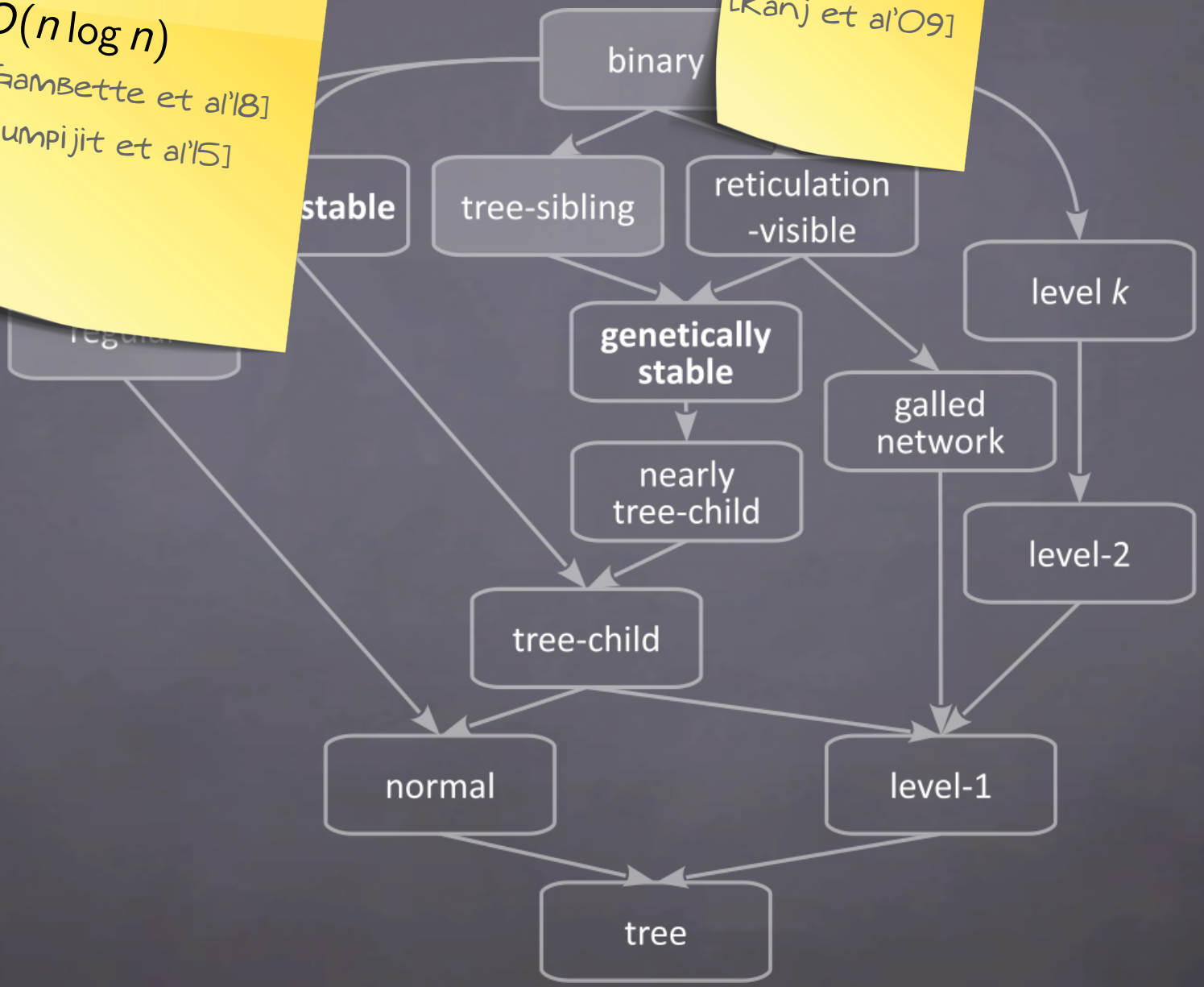


Previous Work

$O(n \log n)$
[Gambette et al'18]
[Kumpijit et al'15]

NP-hard
[Kanj et al'09]



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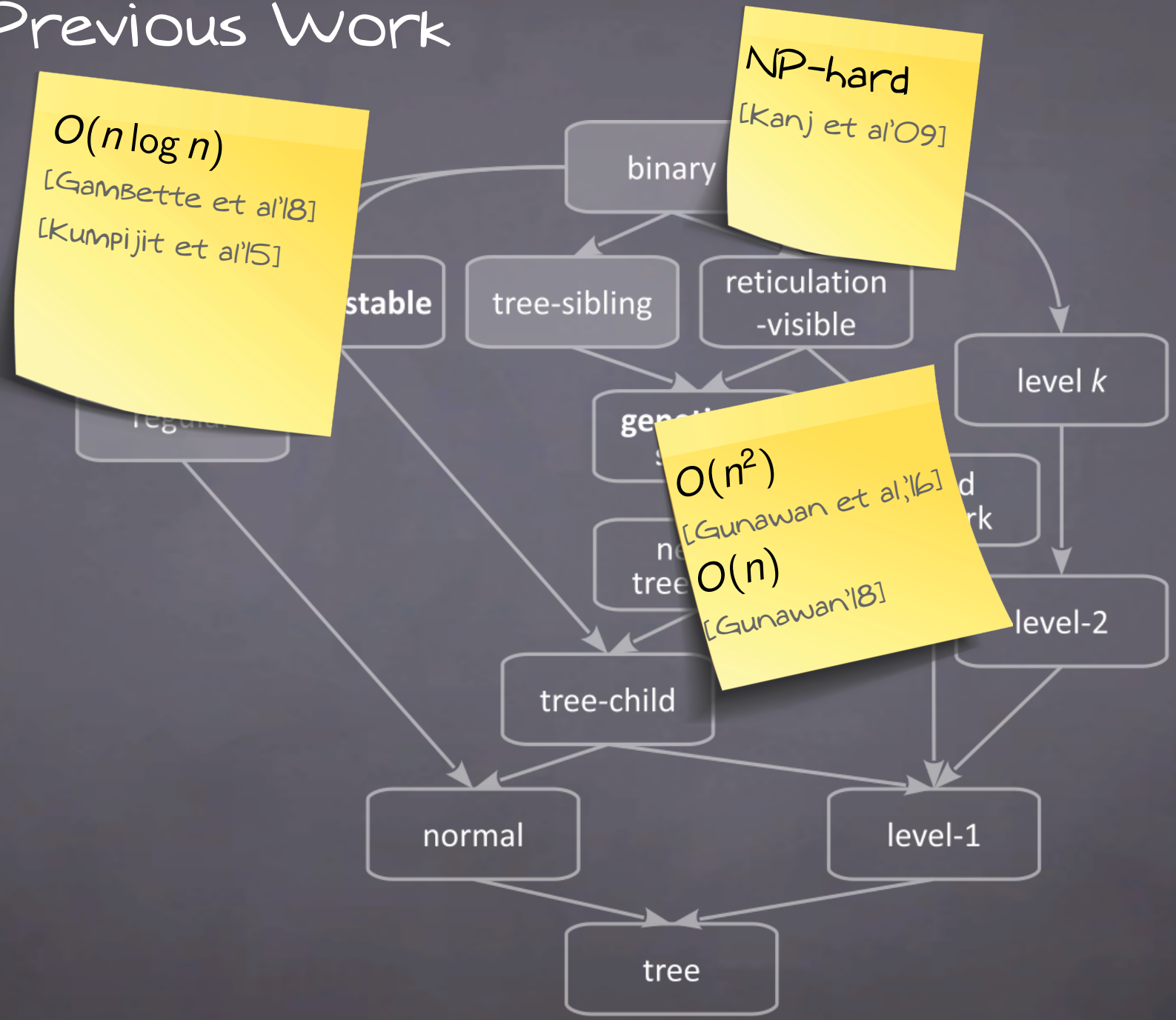

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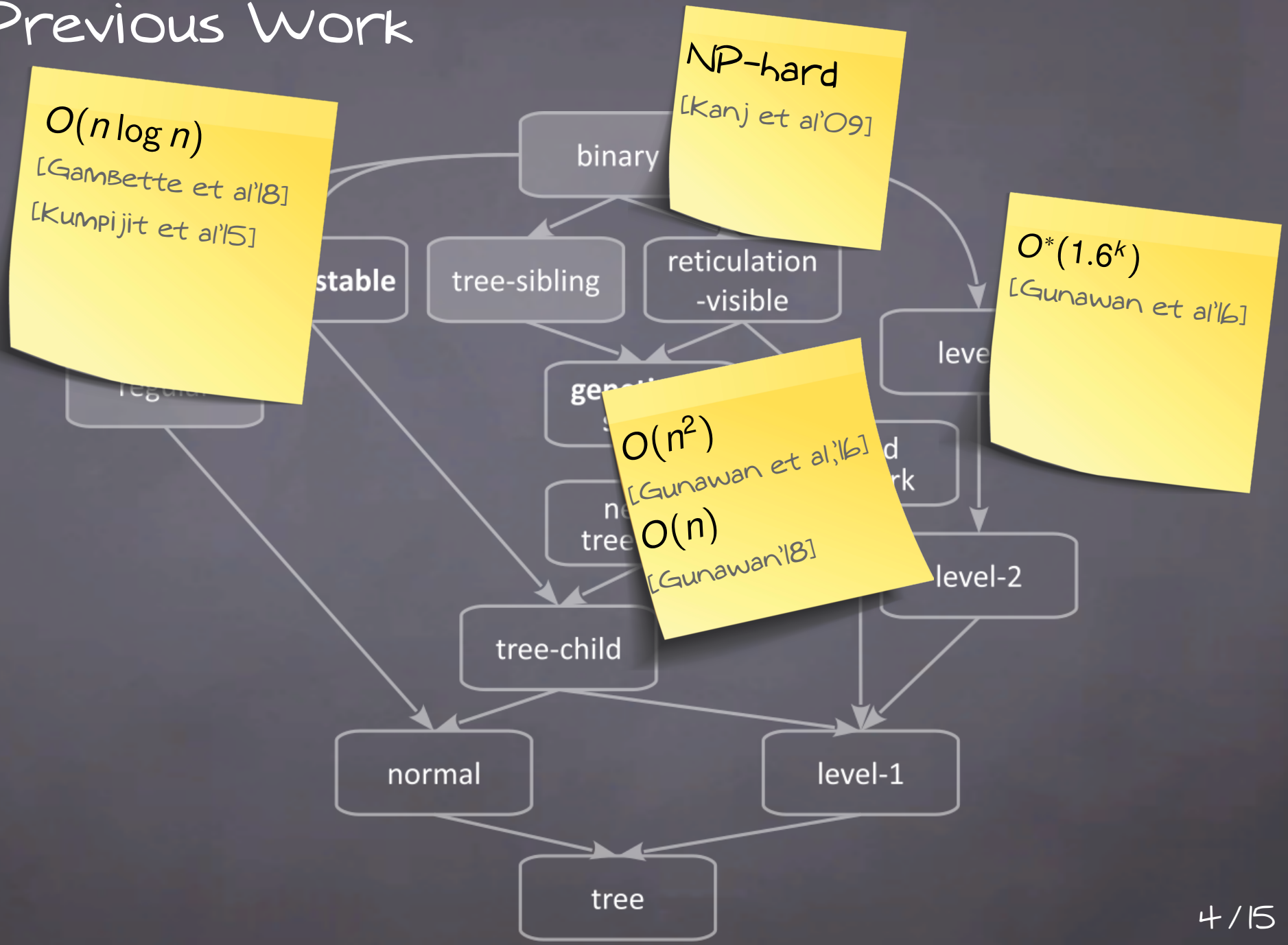

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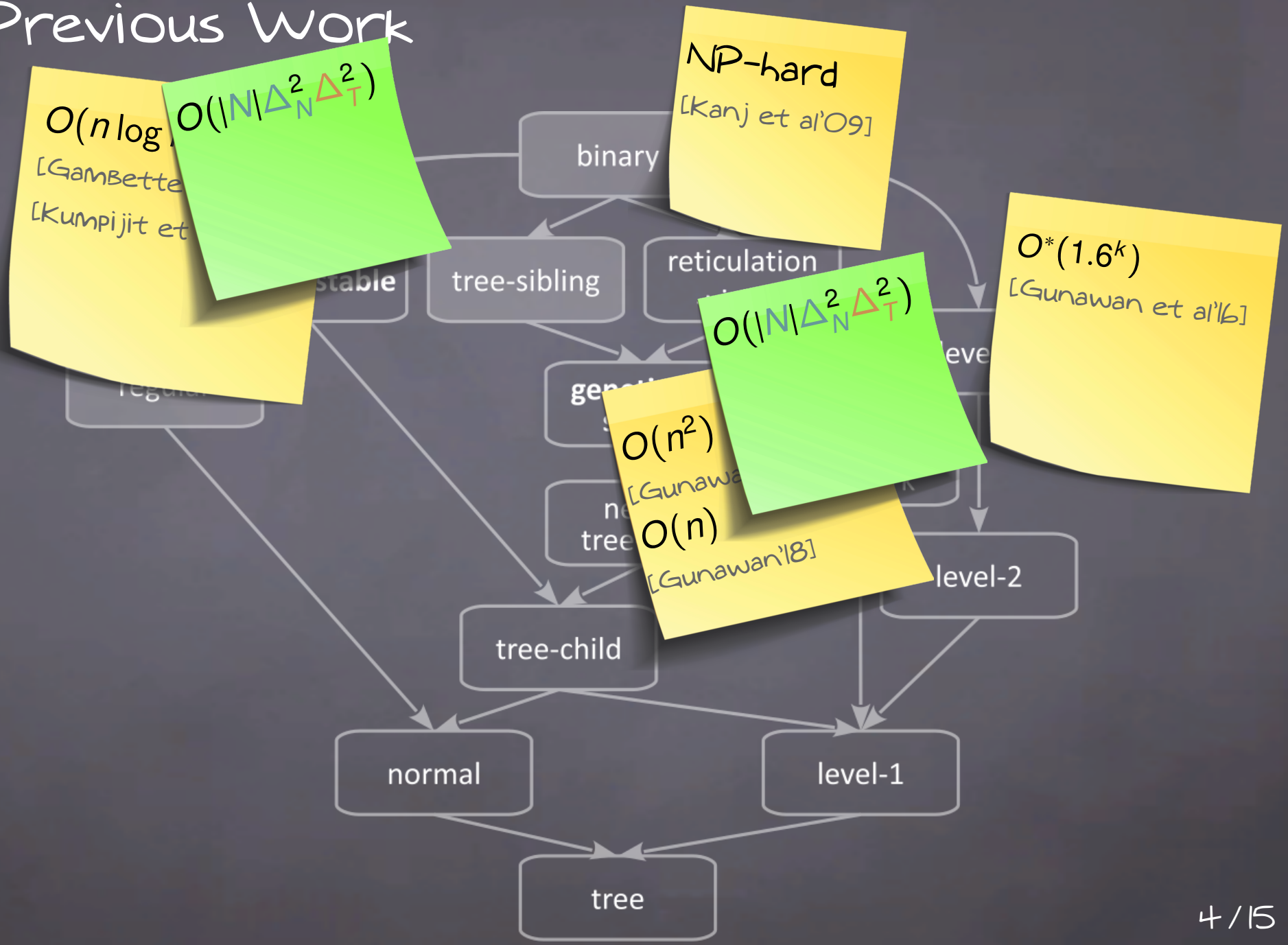

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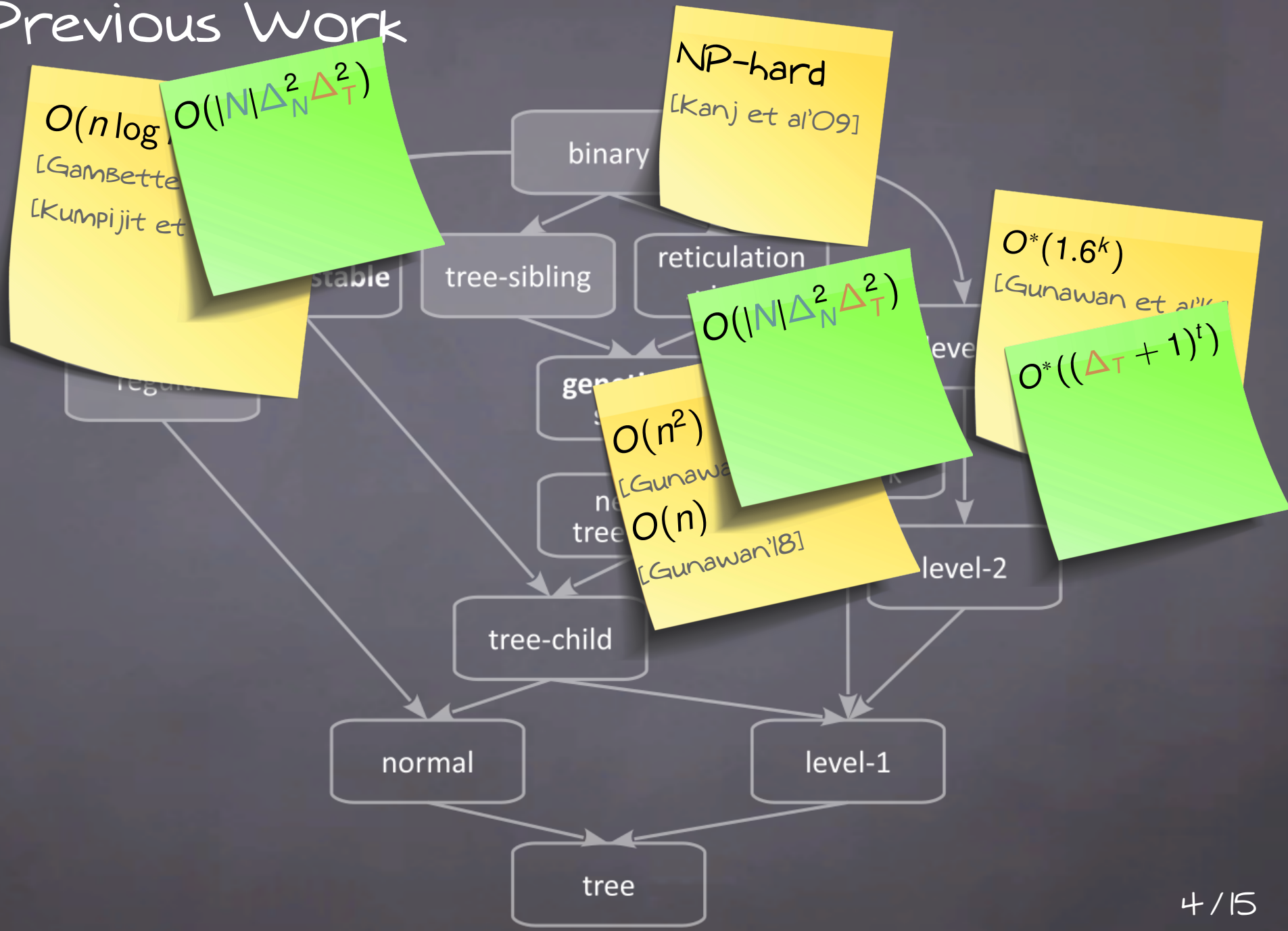

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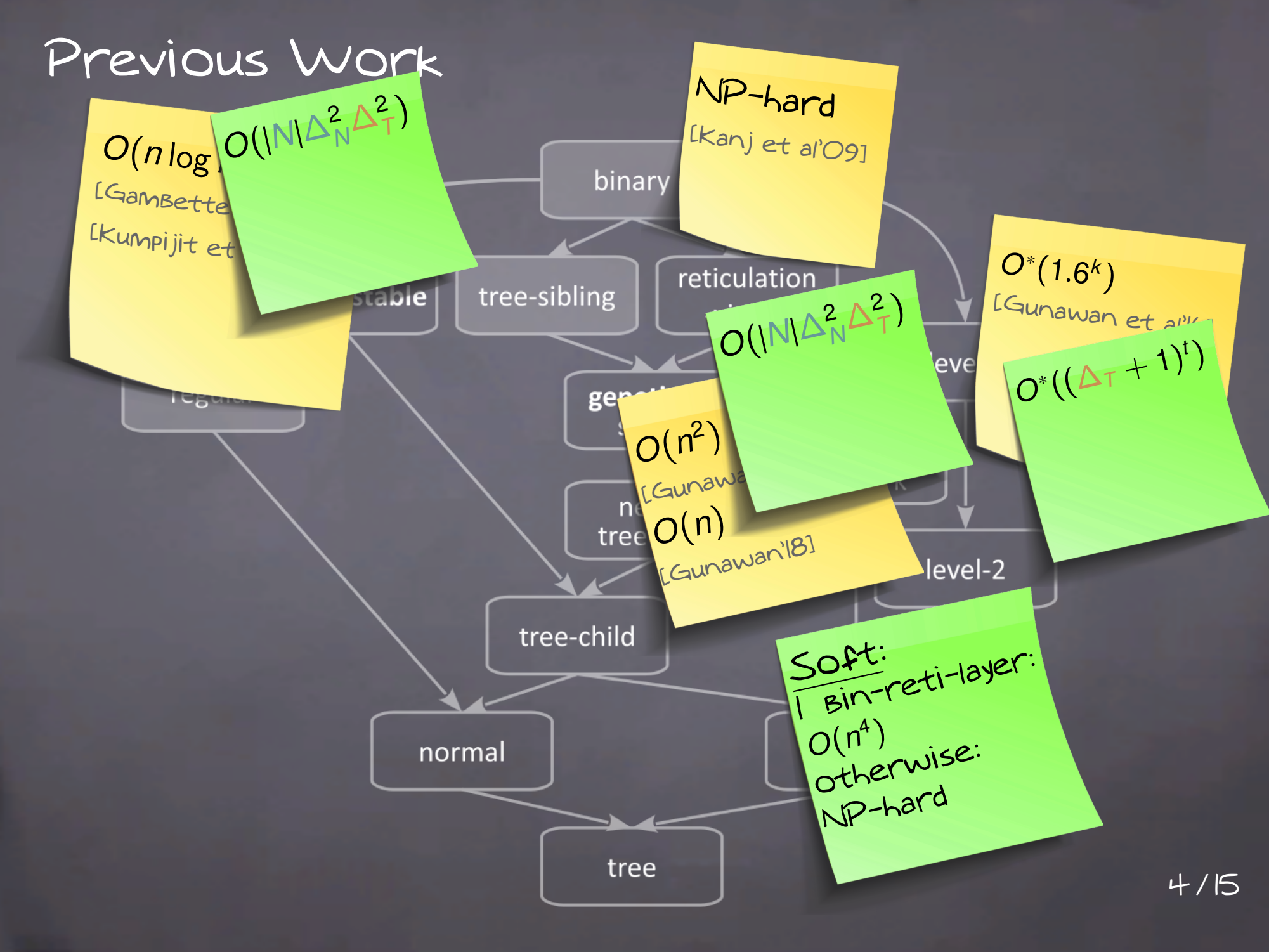

k-labelled Trees

Important "Special Case"

N is a tree, but labels may occur multiple times

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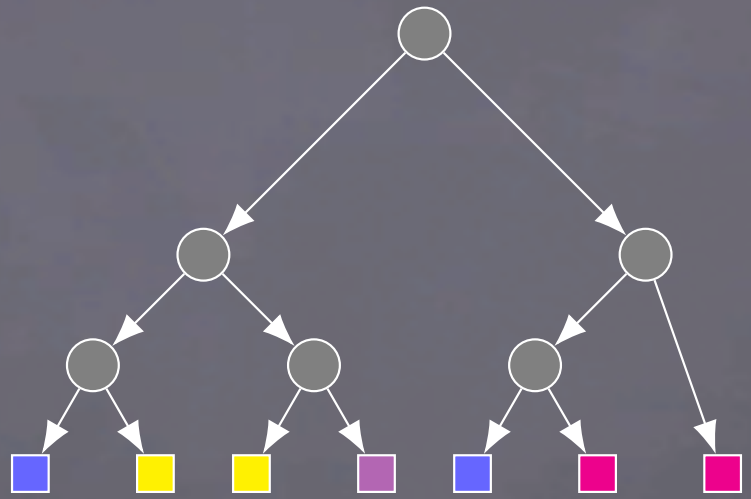

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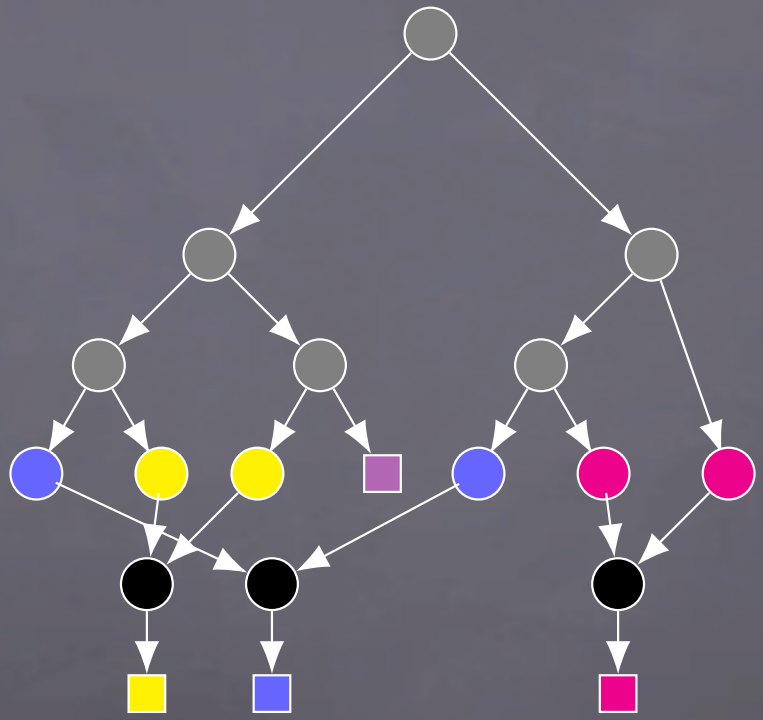

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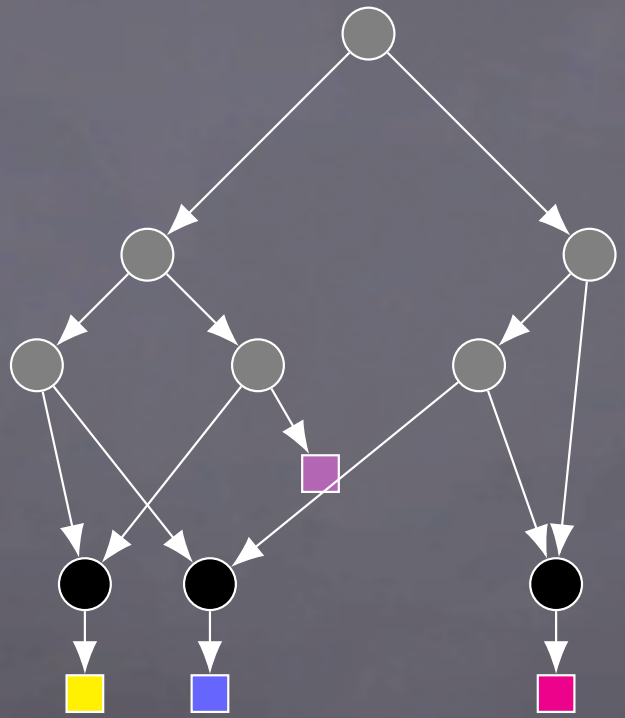

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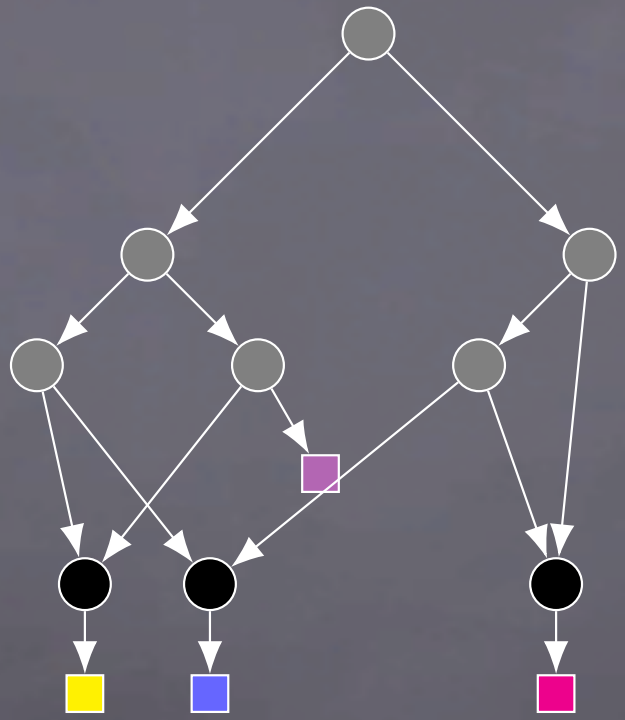

Hard Polytomies $\rightsquigarrow O(|N| \cdot k^2 \Delta_T{}^2)$
Soft Polytomies $\rightsquigarrow O(|N|^4)$ for 2 occurrences, NP-hard for 3

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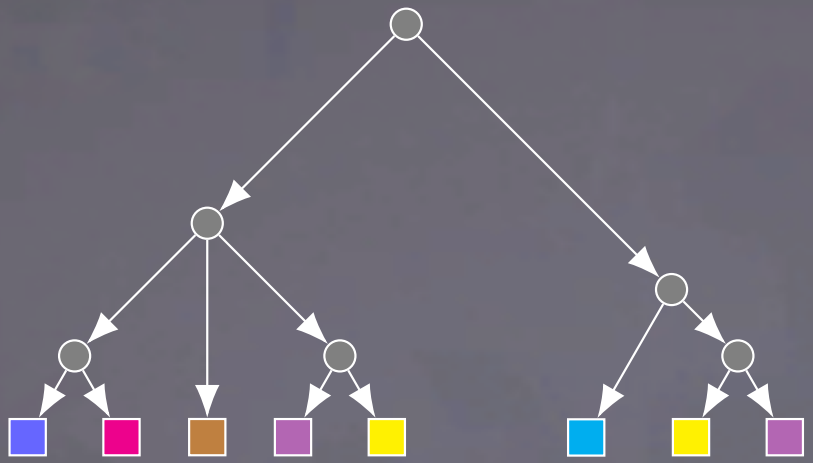

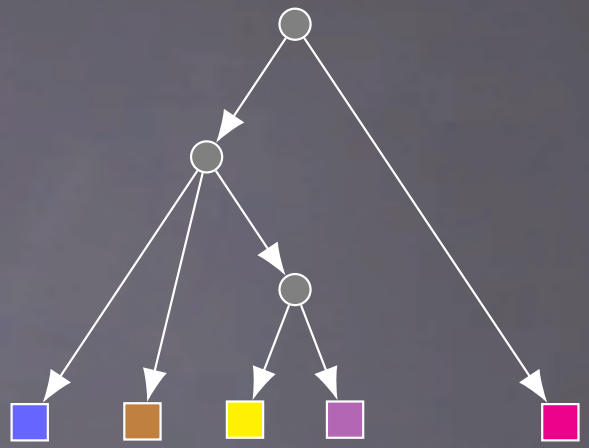

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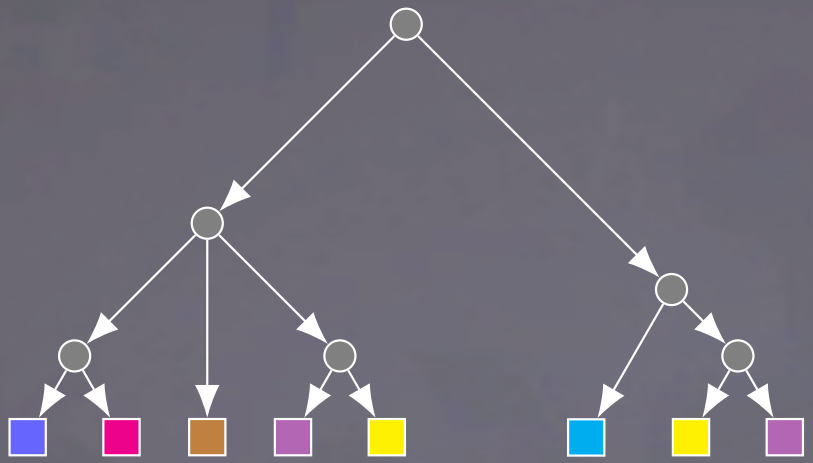

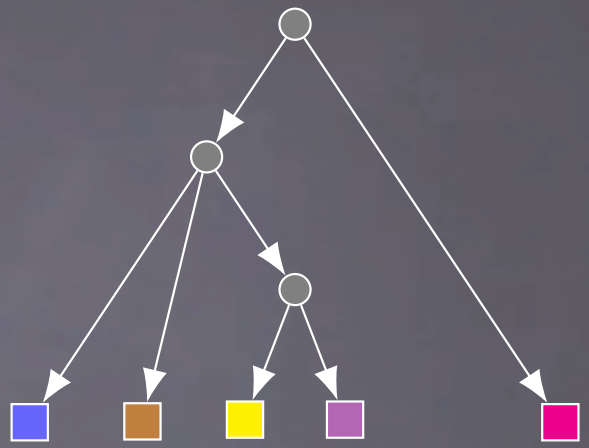

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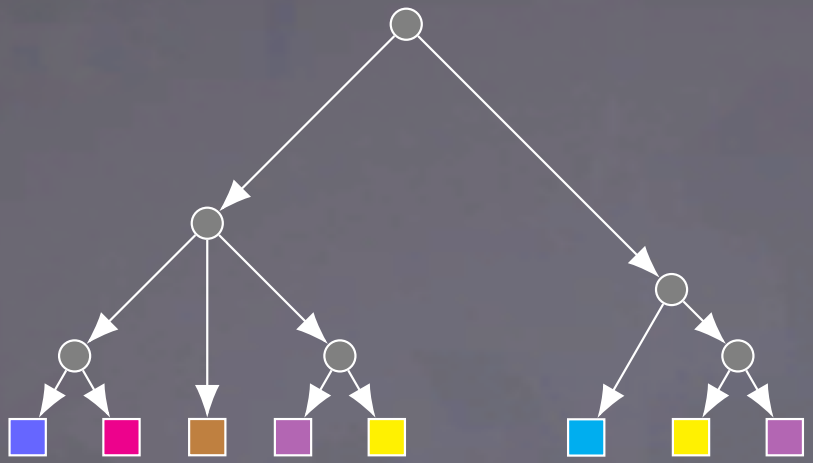

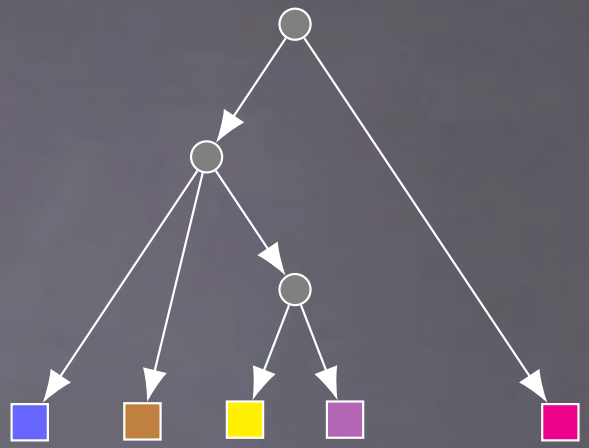

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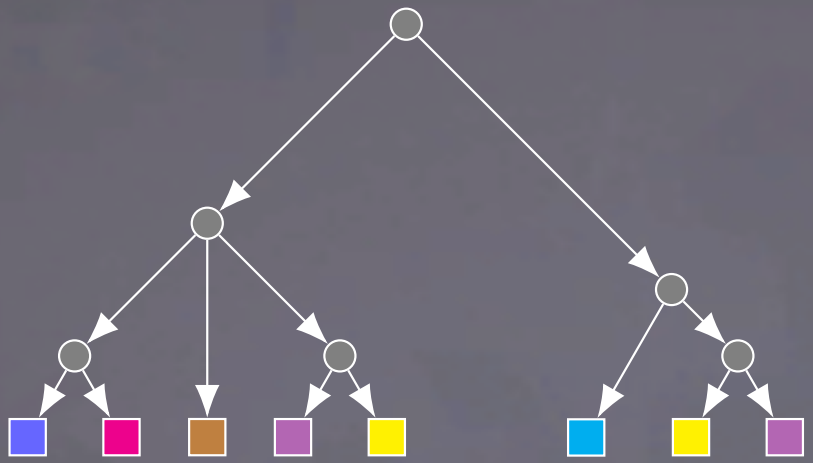

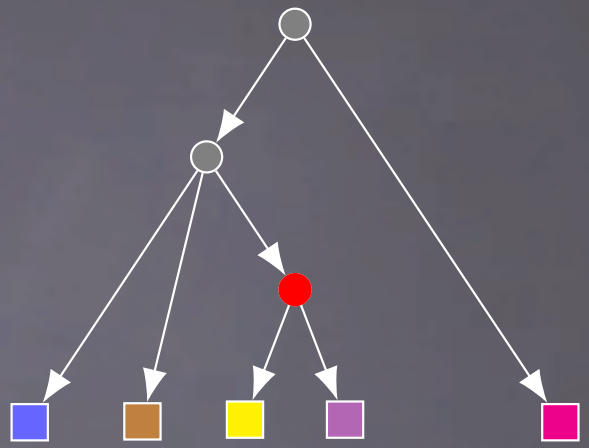

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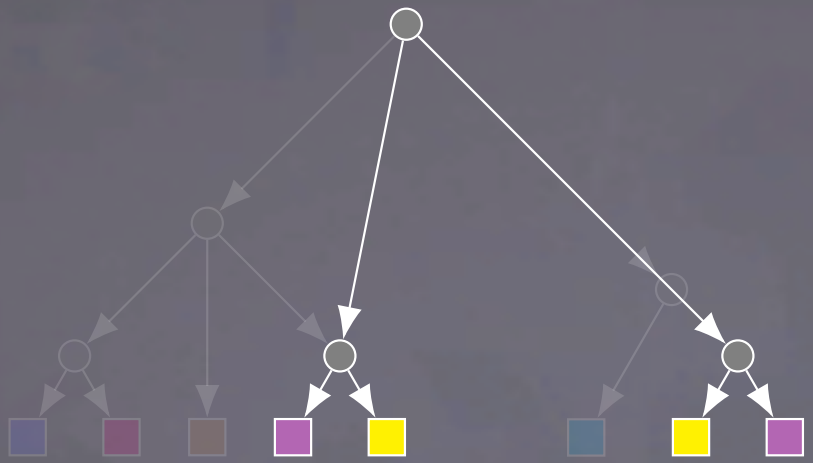

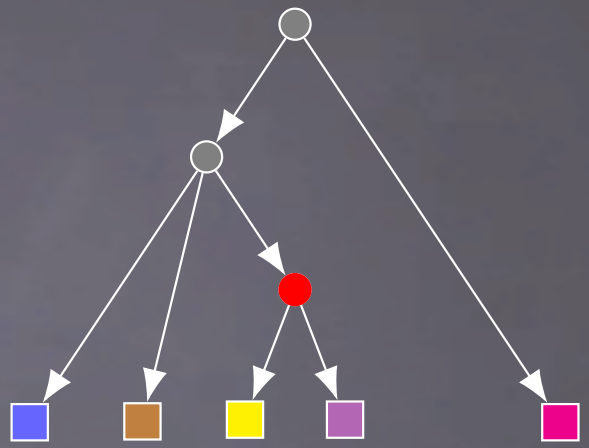

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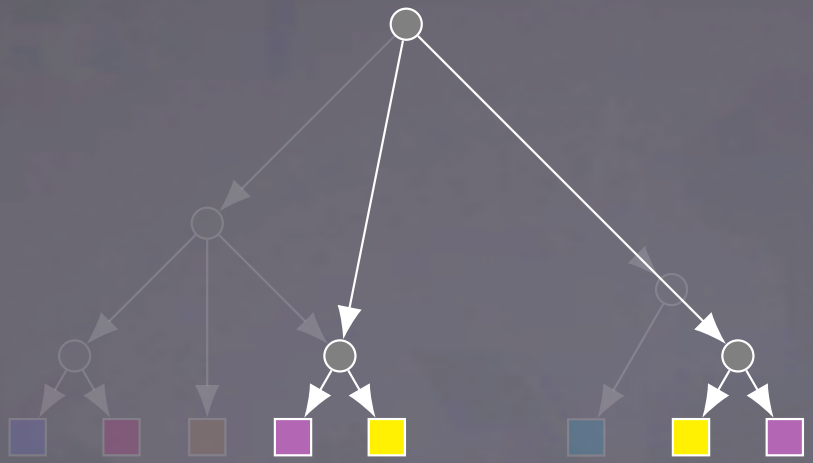

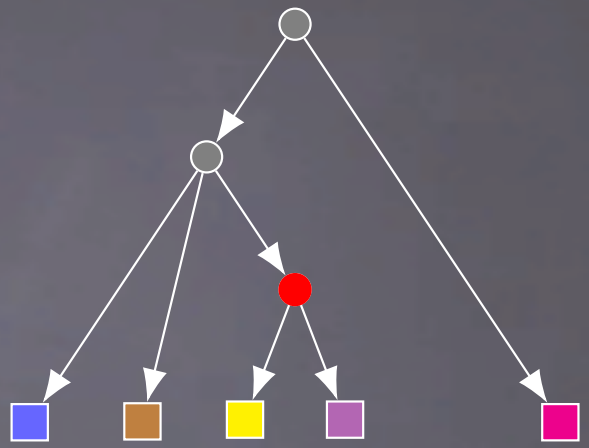

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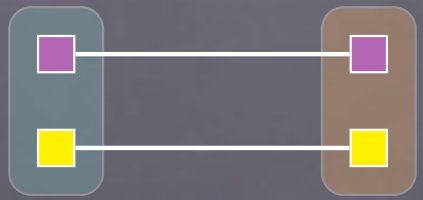

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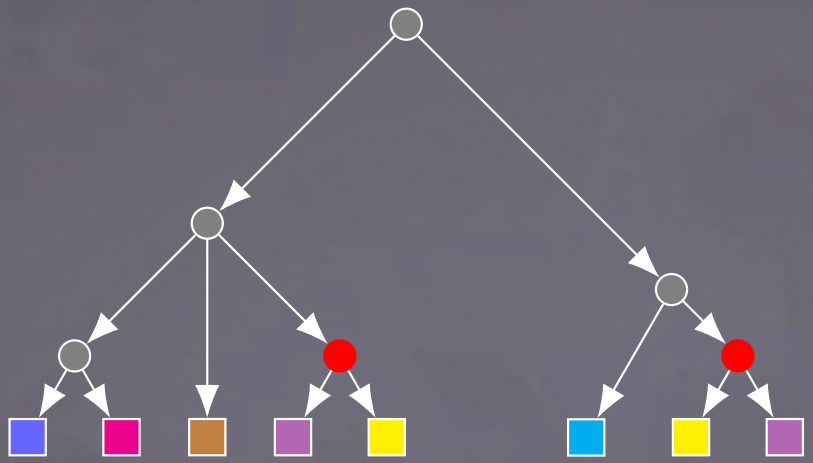

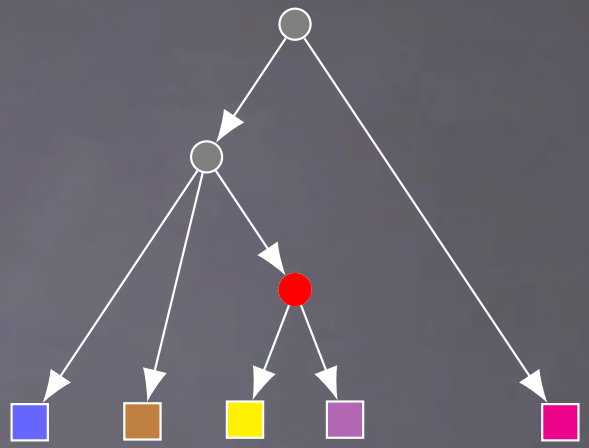

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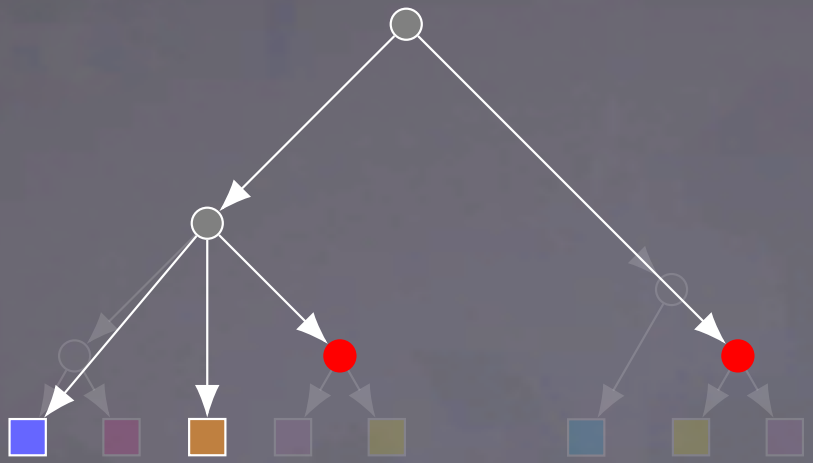

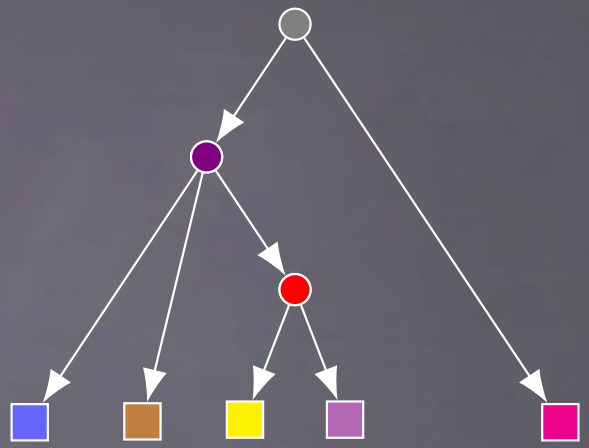

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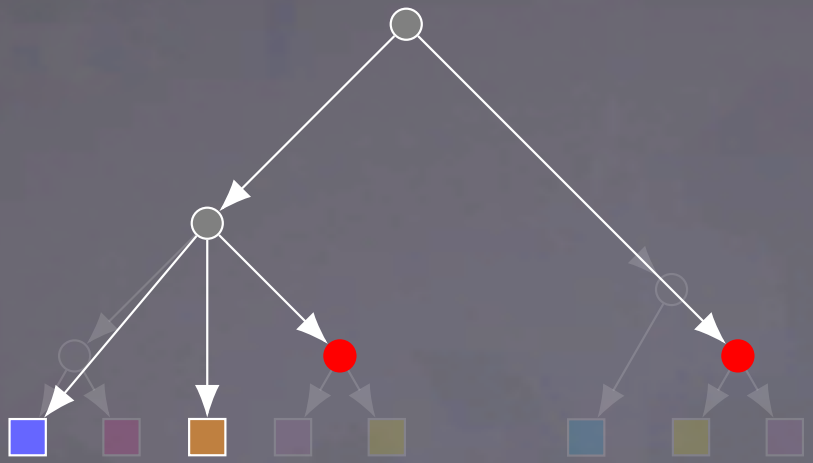

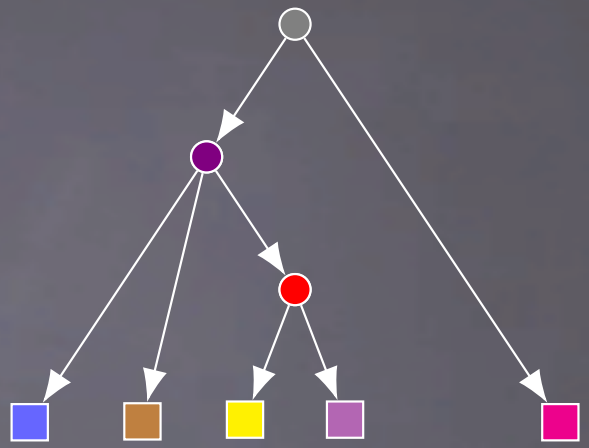

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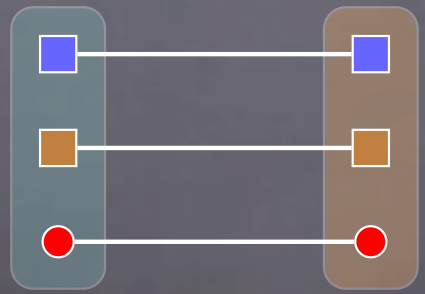

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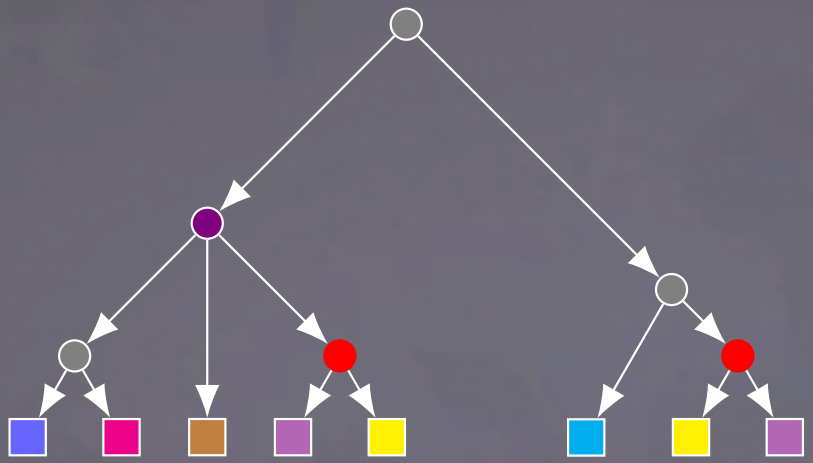

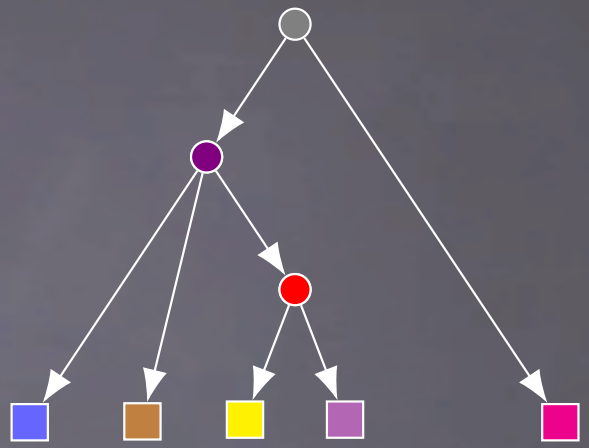

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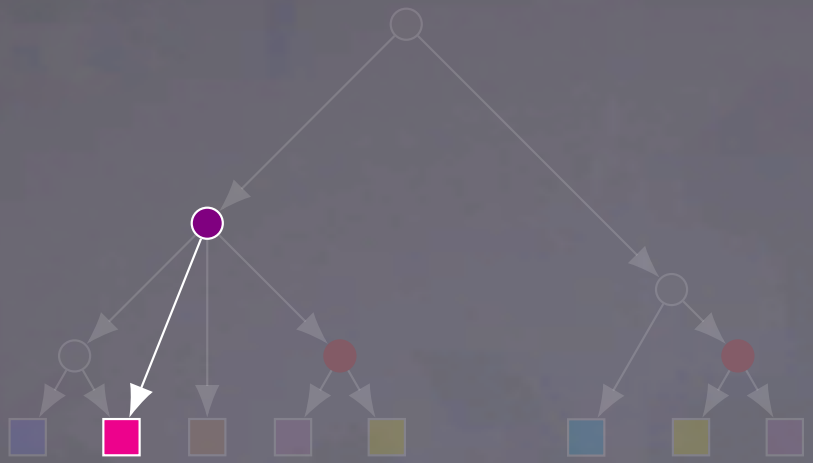

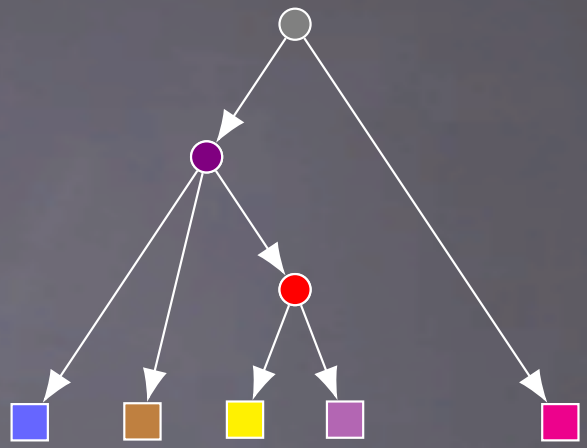

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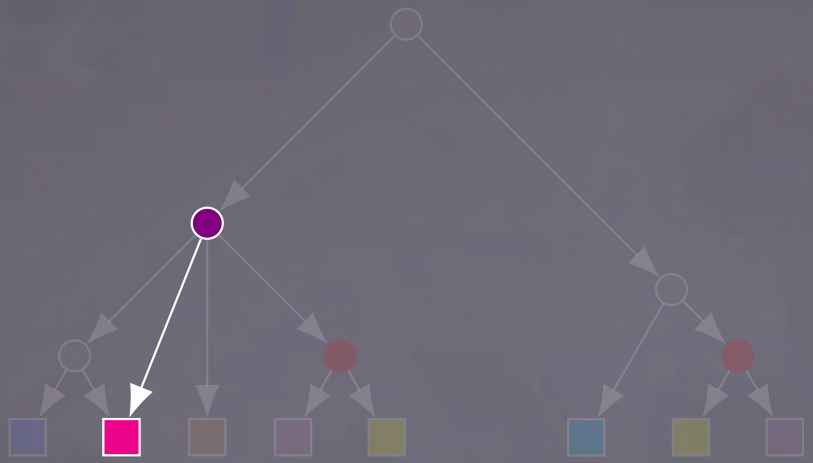
   (b) Max. Bip. Matching

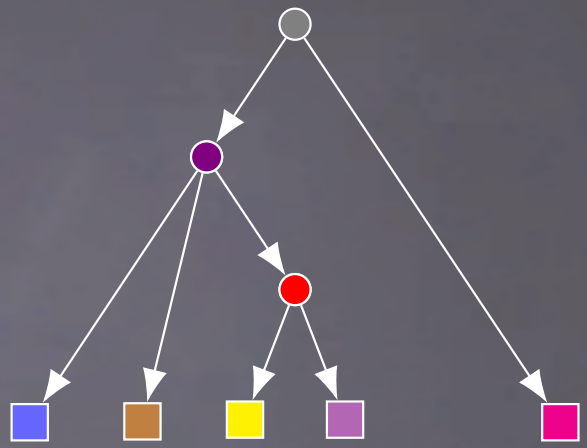
edge $u_j v_i$

$\Leftrightarrow$

$\exists x \leq u_j$ in $M(v_i)$

k-labeled Trees





Definition

$v \in T$: $M(v)$ = lowest vertices u s.t. N_u displays T_v. $\rightsquigarrow |M(v)| \leq k$

for each v bottom-up:
1. Build $Z := \bigcup_i M(v_i) \Downarrow N|_Z$
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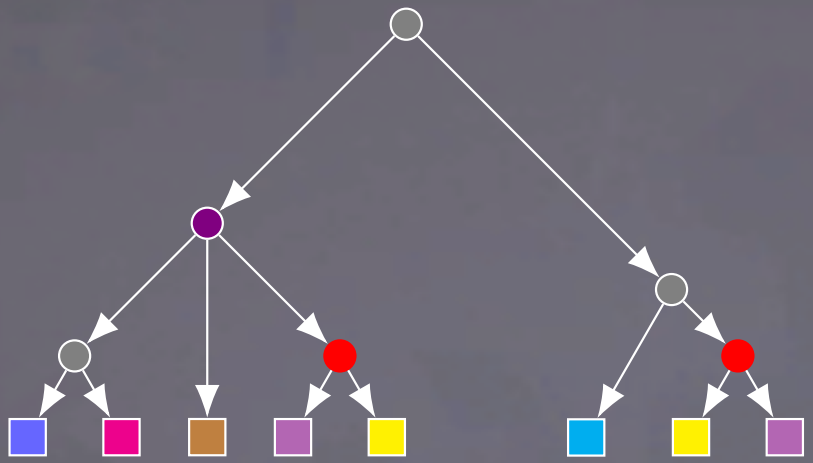

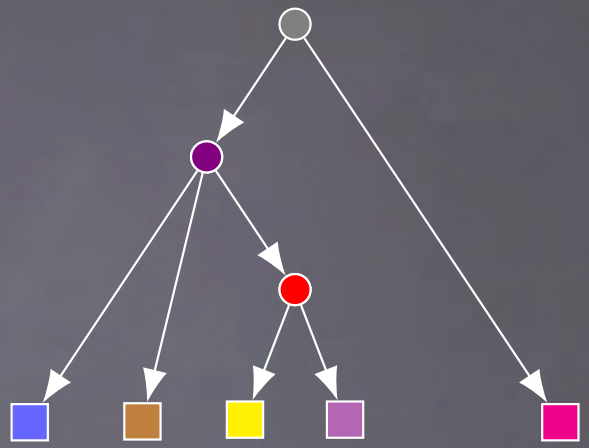

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$\rightsquigarrow O(|N| \cdot (k\Delta_T)^2)$ time

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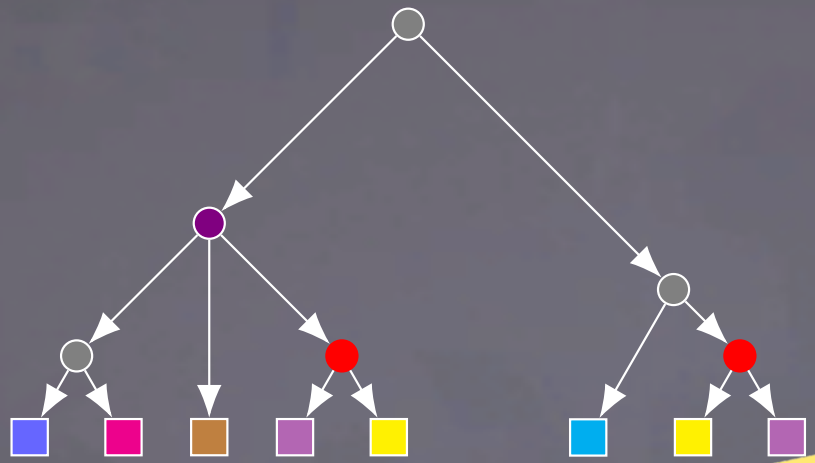
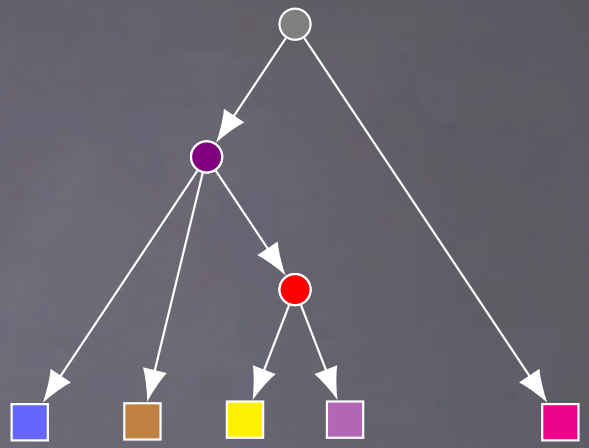
k-labeled Trees





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   (a) Build Bip. Graph
   (b) Max. Bip. Matching

$\rightsquigarrow O(|N| \cdot (k\Delta_T)^2)$ time

Theorem
We get highest v s.t. N displays T_v in $O(|N| \cdot (k\Delta_T)^2)$ time.

Note: these T_v are leaf-disjoint

edge $u_j v_i$
$\Leftrightarrow$
$\exists x \le u_j$ in $M(v_i)$

For Networks...

Observation

[Gunawan et al., '16]

$N - R$ = forest $\rightsquigarrow$ N = DAG of tree components + R

For Networks...

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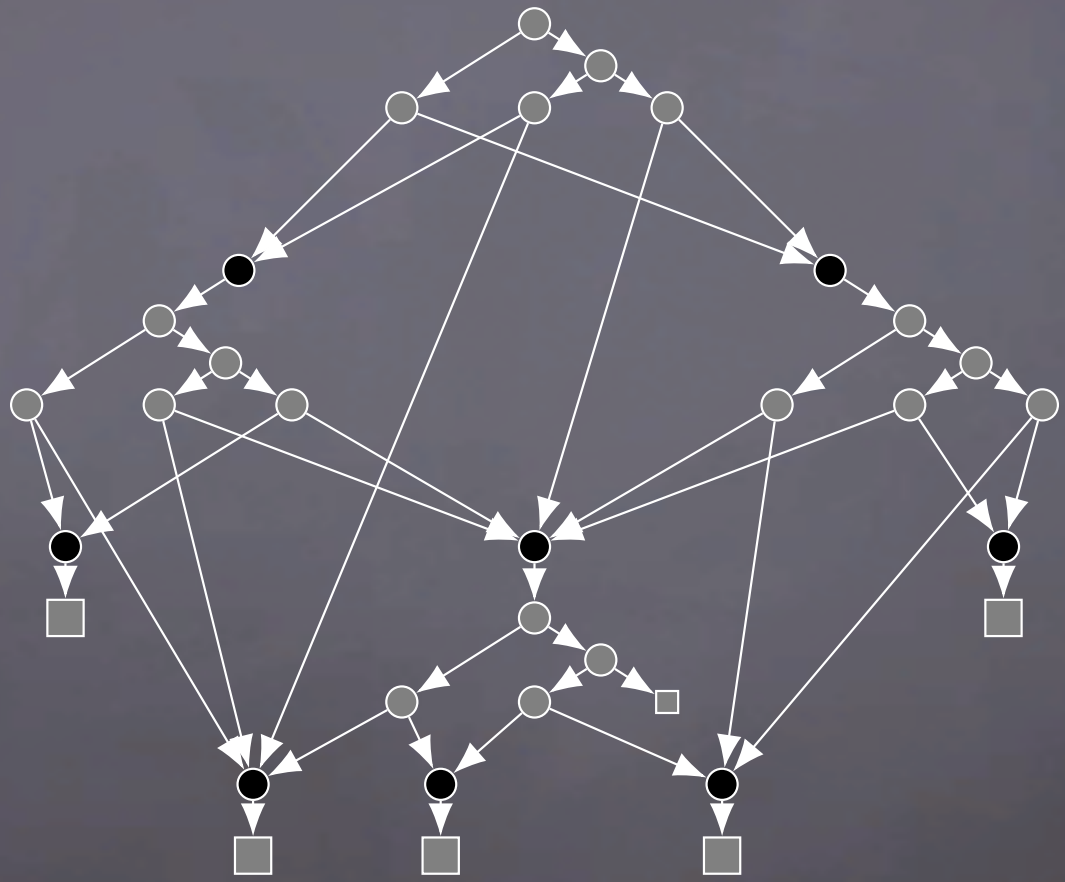

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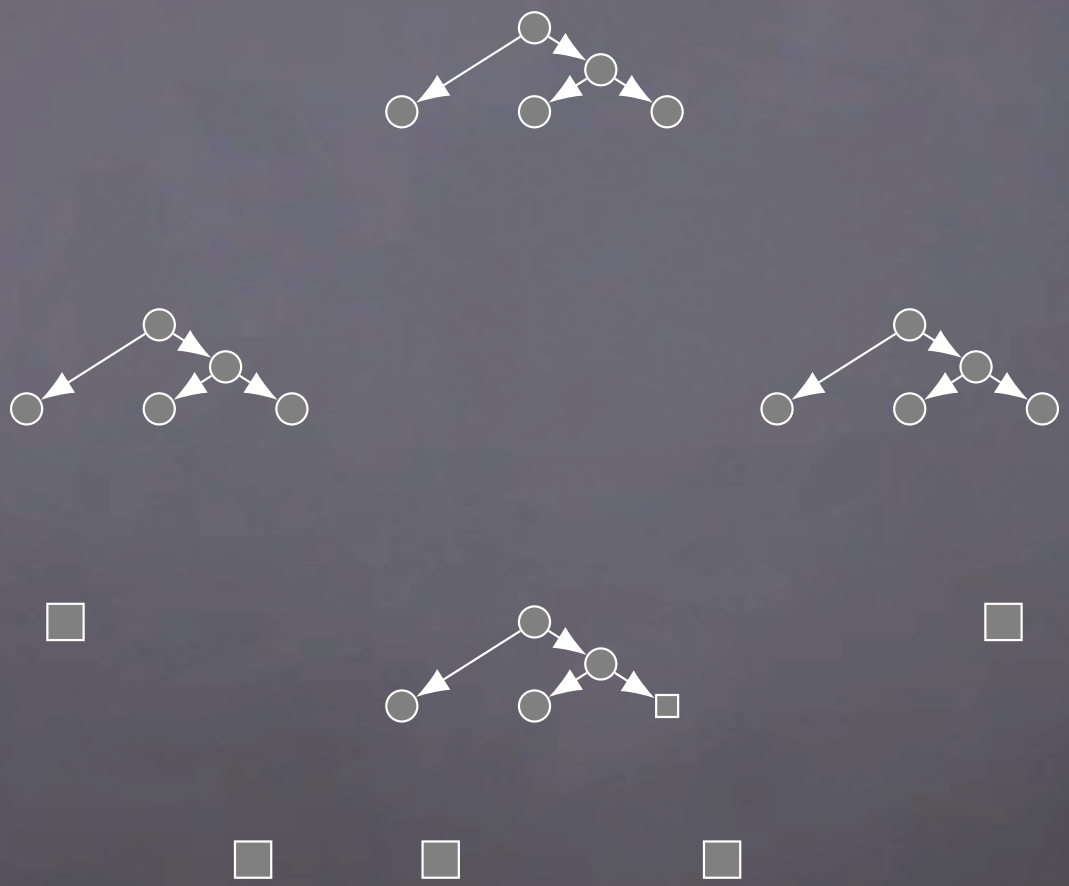

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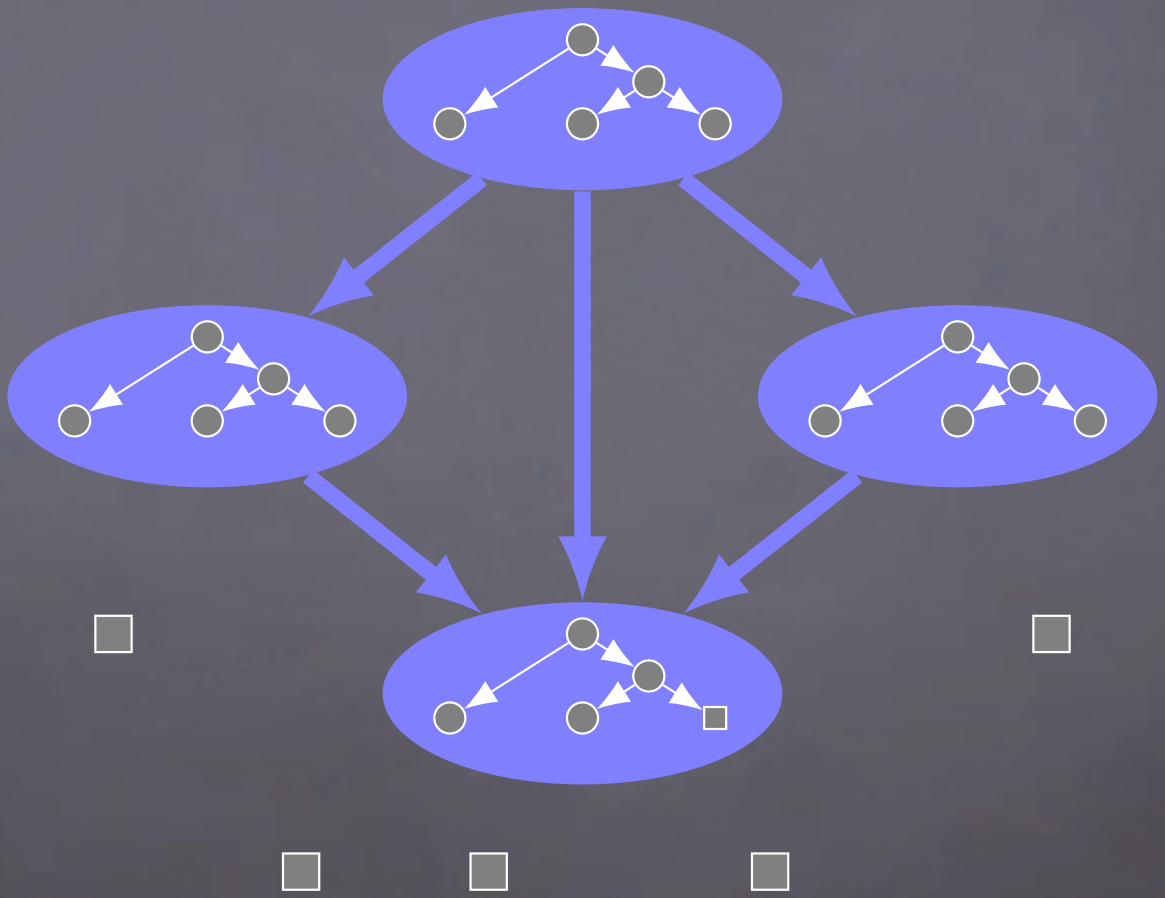

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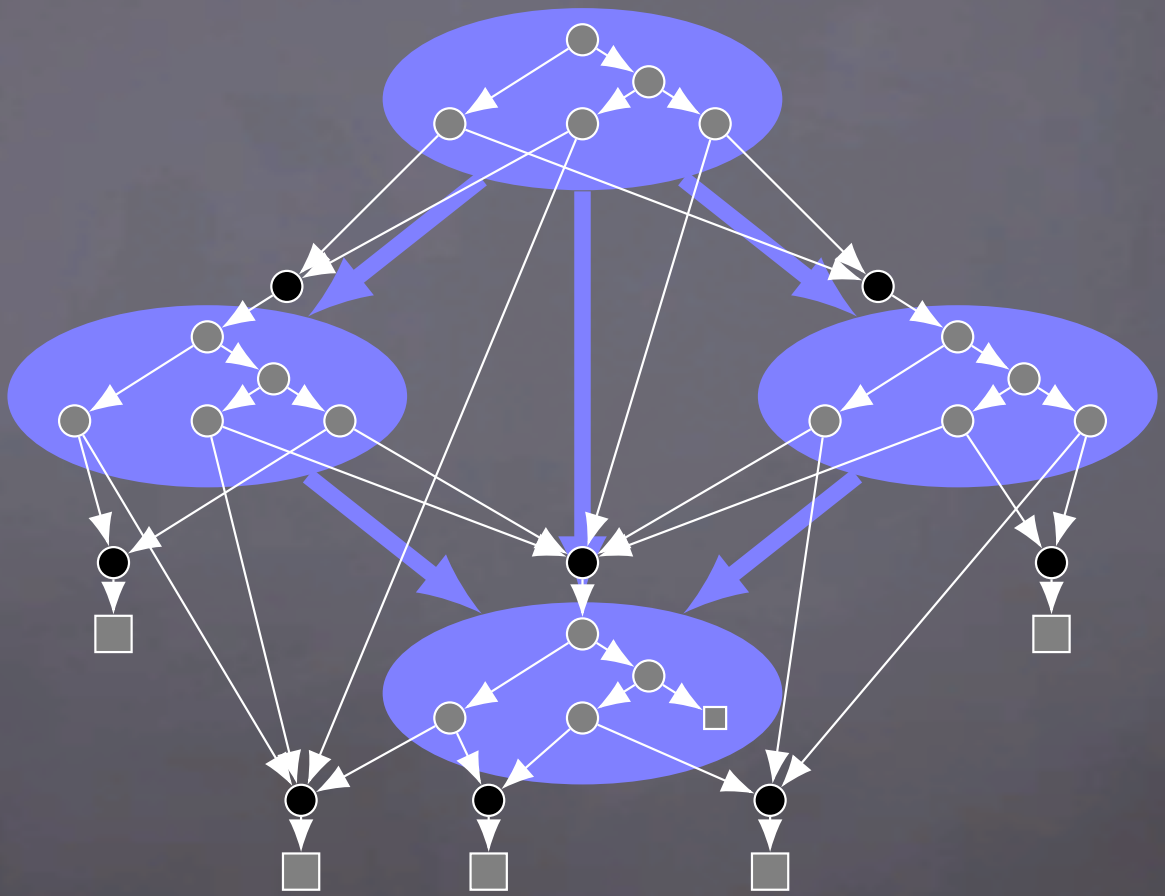

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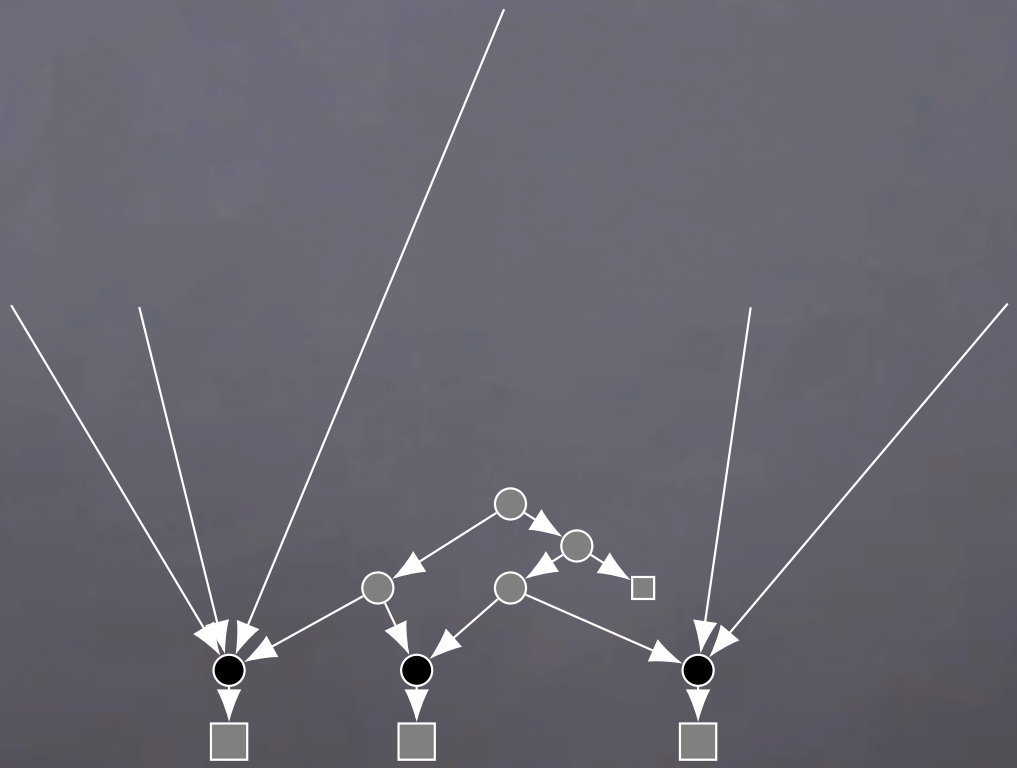

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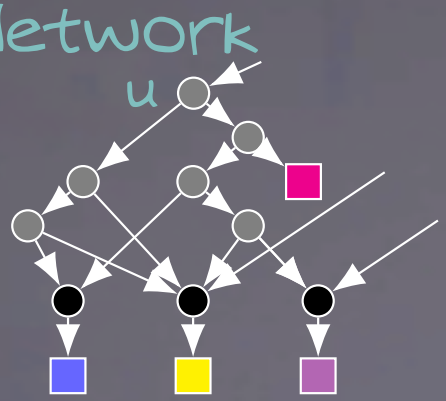
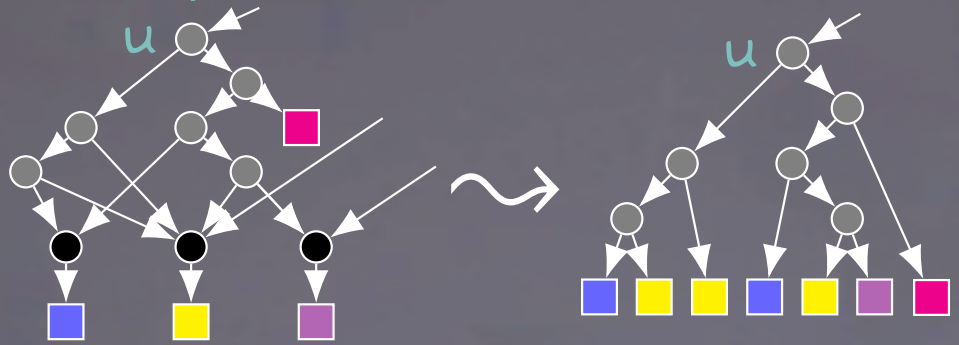


For Networks...

Network



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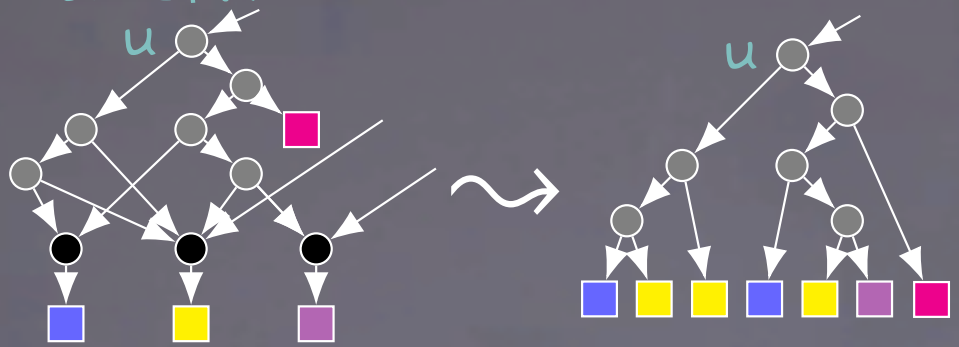
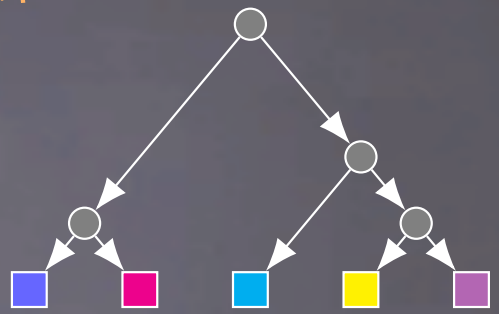


Tree



For Networks...

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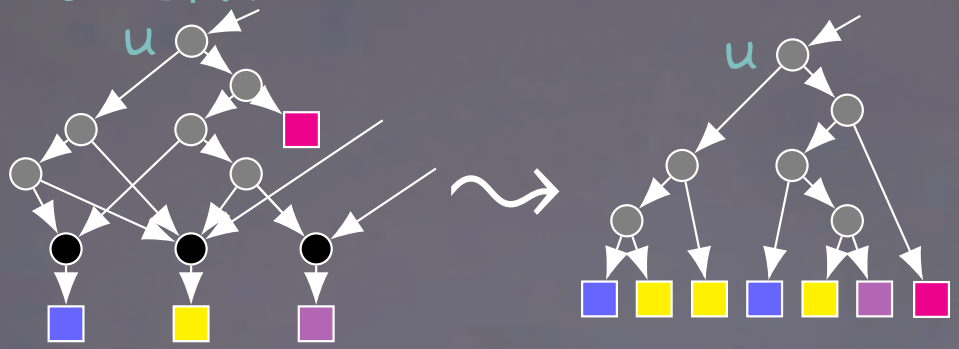

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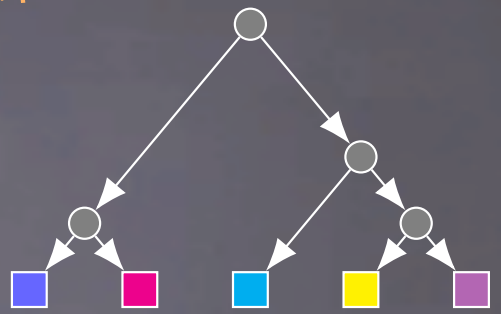
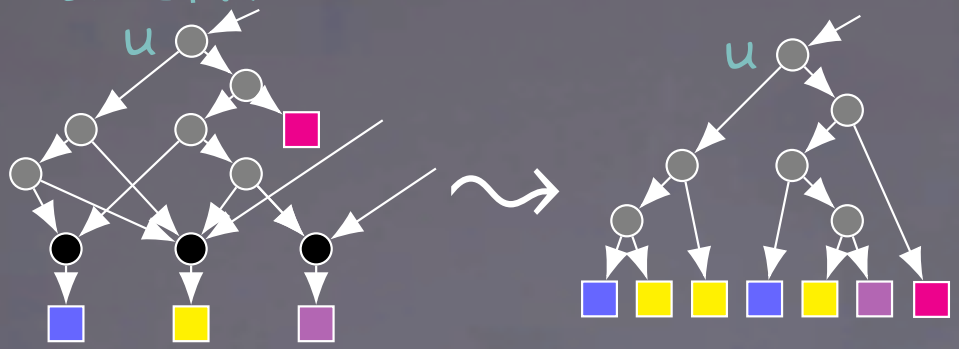


Recall:

Can get highest T_v displayed by N_u in $O(|N_u| \cdot (\Delta_N \Delta_T)^2)$

For Networks...

Network



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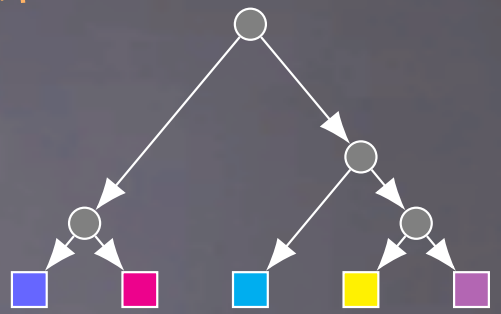

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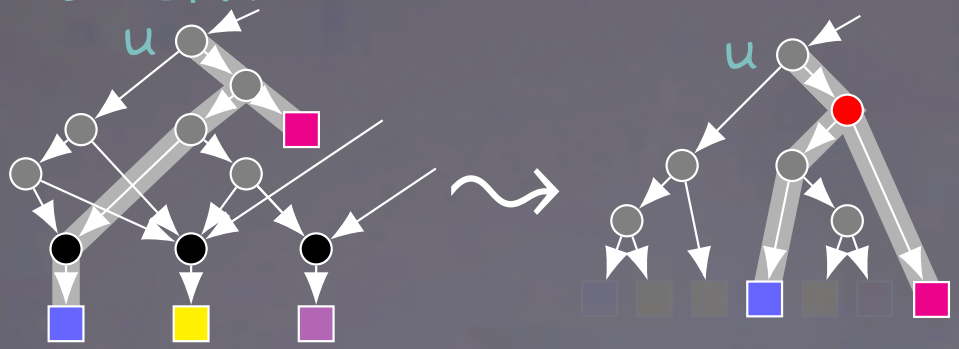
Problem
Which T_v to choose?

For Networks...

Network



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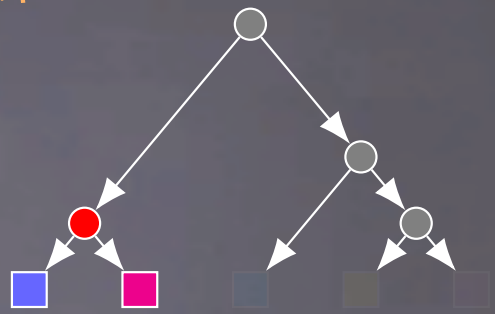

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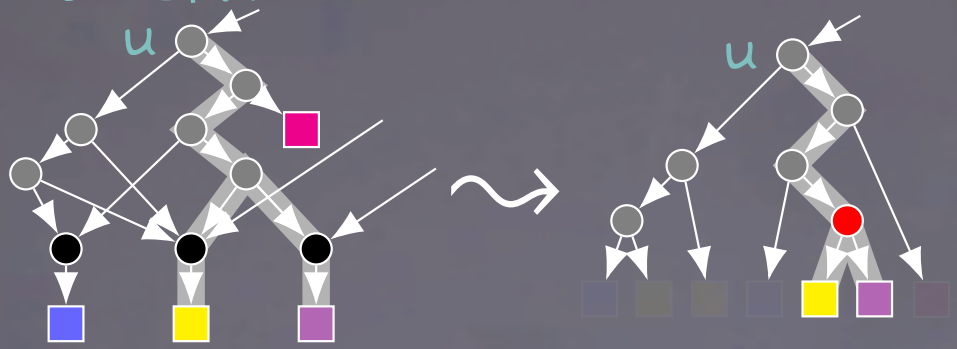

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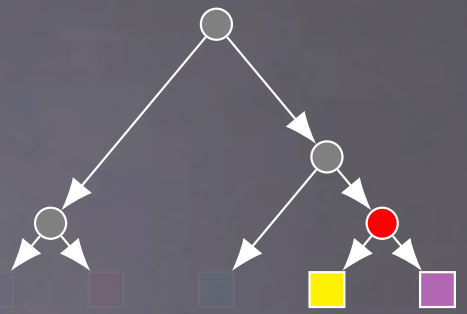

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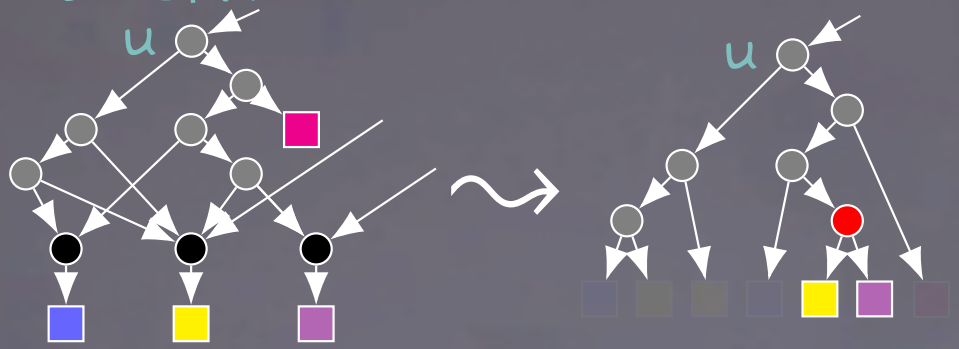

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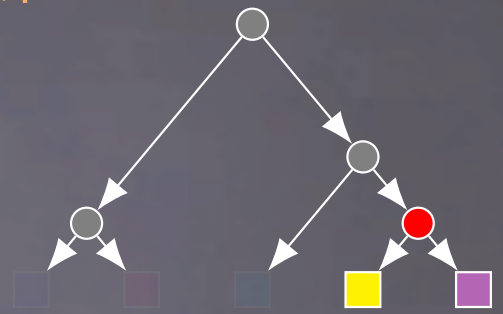


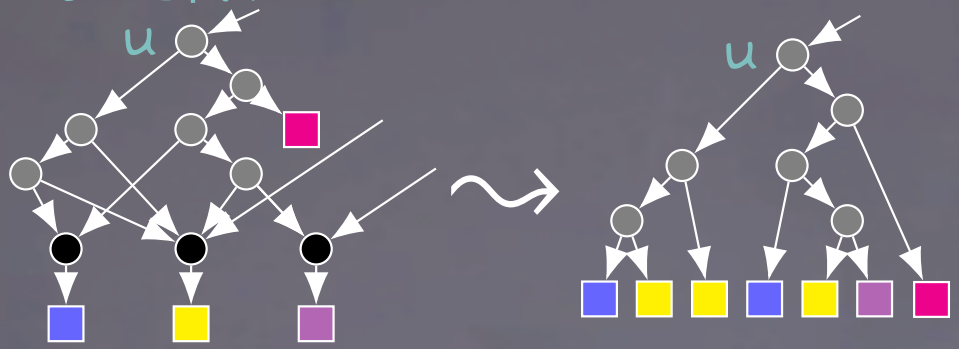
Recall:

Can get highest T_v displayed by N_u in $O(|N_u| \cdot (\Delta_N \Delta_T)^2)$

Choose the T_v containing the leaves that are accessible only via u!

For Networks...

Network



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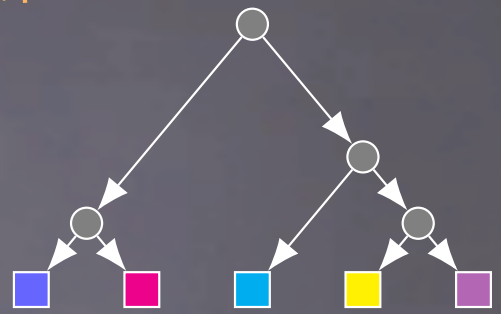

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Definition

u is visible on ■ $\Leftrightarrow$ all root-■-paths contain u

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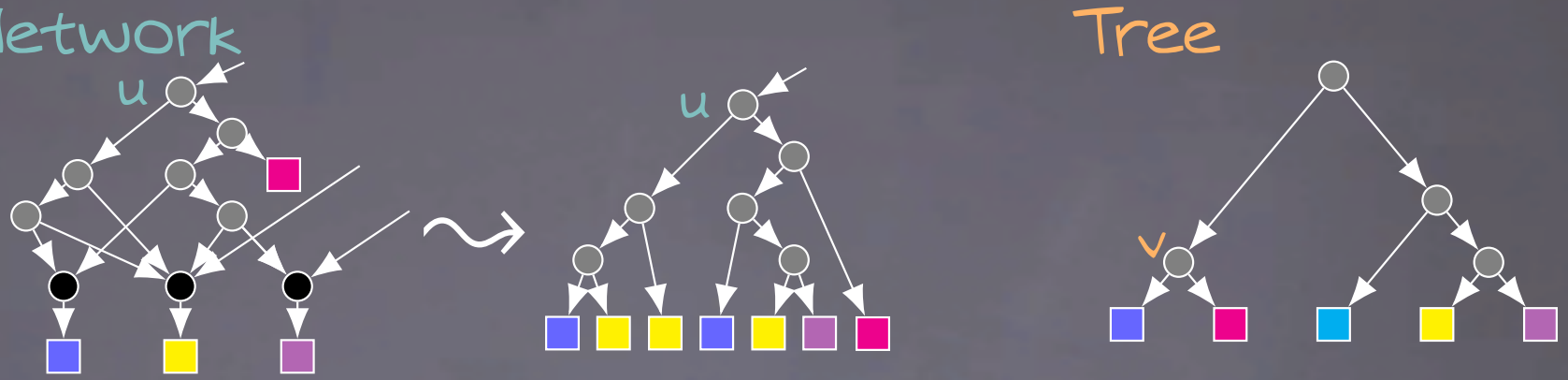
For Networks...



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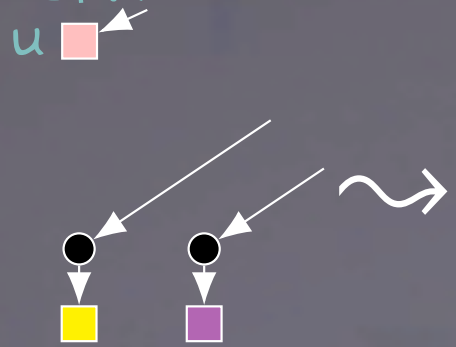
Reduction Rule

Root u of lowest tree-component N_u in N is visible on ■ $\rightsquigarrow$ collapse $N_u \notin$ the highest T_v with ■ $\in T_v$ that N_u displays.

Choose the T_v containing the leaves that are accessible only via u!

For Networks...

Network



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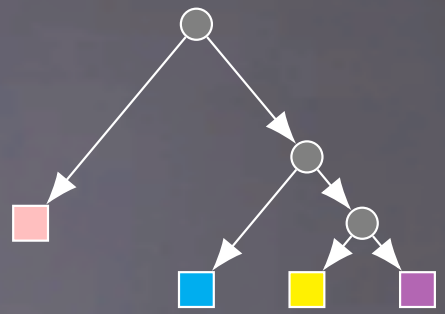

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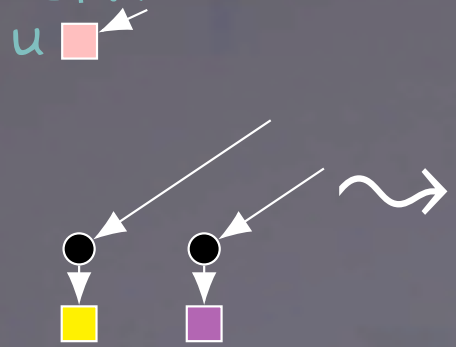

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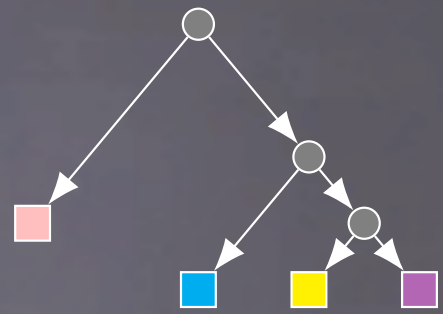

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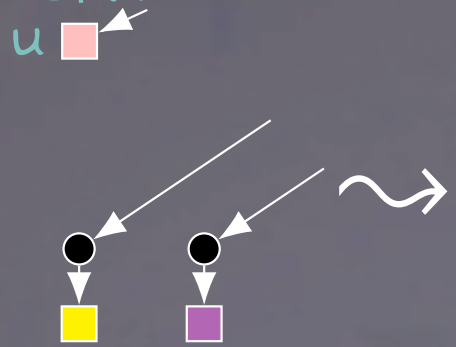
ree-component N_u
collapse N_u ⇜ the
that N_u displays.

Theorem
all tree-component-roots are visible
⇝ solve TC in $O(|N|(c\Delta_N\Delta_T)^2)$
where c = longest reticulation chain

Choose the T_v containing the leaves that are accessible only via u!

For Networks...

Network



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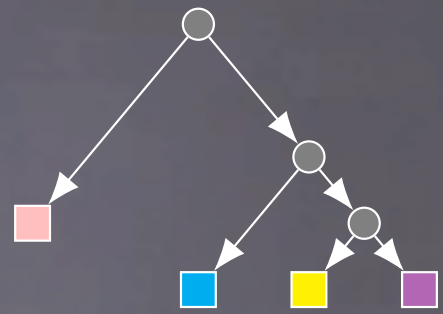

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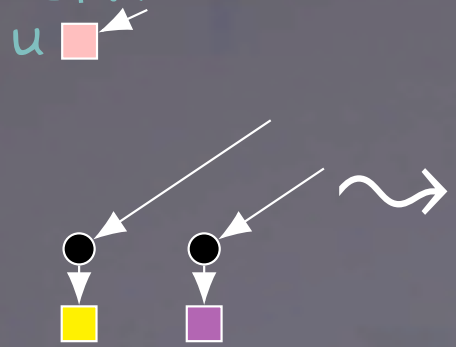
Corollary
solve Bin. TC in $O(|N|)$ on
(a) reticulation visible
(all reti. visible)
$\not\Leftarrow$
(b) nearly-stable
(invisible $\Rightarrow$ visible parents)

Choose the T_v containing the leaves that are accessible only via u!

For Networks...

Network



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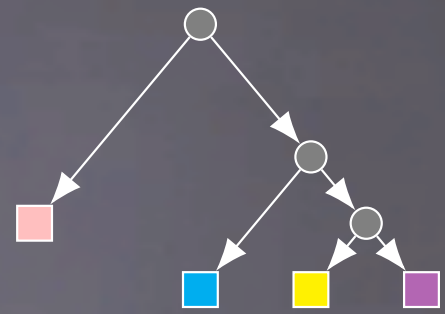

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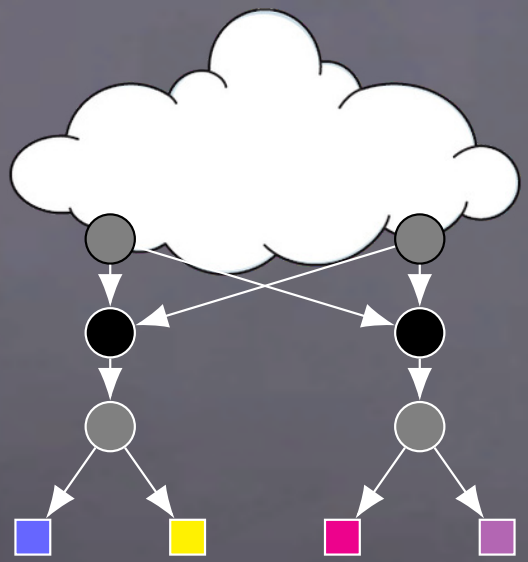
Corollary
TC can be solved in poly time on networks N in which all tree-vertices with reticulation parents are visible.

General Networks

Idea: $u \in M(v)$

$\Leftrightarrow$ N_u displays T_v

Network



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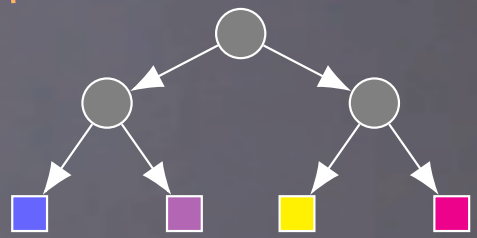

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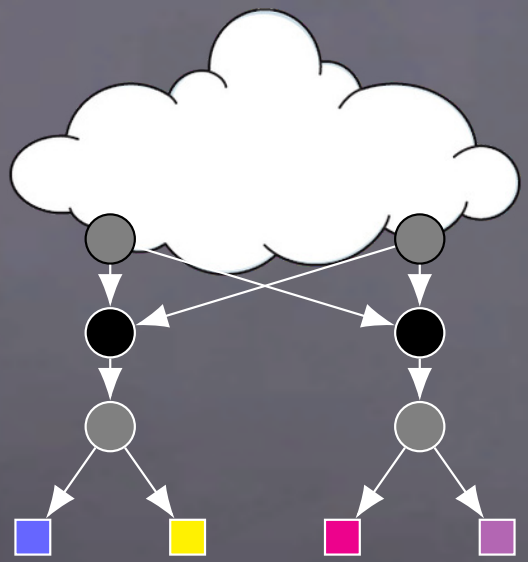
Problem:
Avoid using tree-components multiple times!

Network



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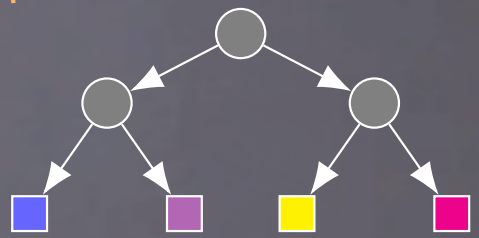

General Networks

Idea: $u \in M(v, R)$

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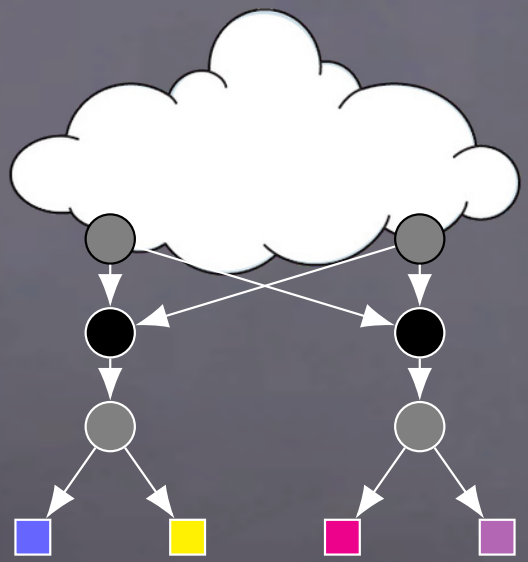
Network



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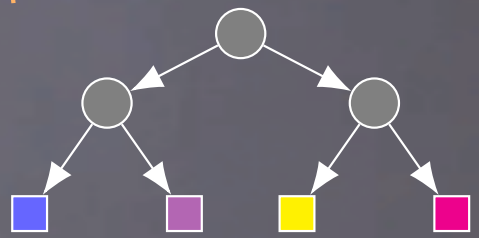

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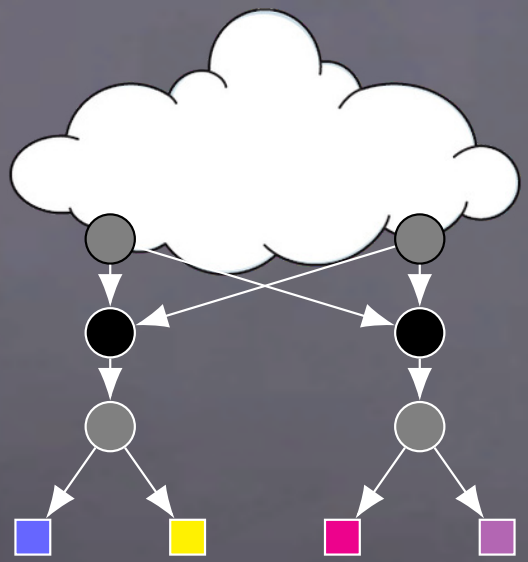
$\Leftrightarrow$ R can be partitoned into $R_1 \uplus R_2 \uplus \ldots \uplus R_{\deg(v)}$ s.t. bipartite graph with edge set $\{u_i v_j \mid u_i \in M(v_j, R_j)\}$ has matching of size $\deg(v)$

Problem: Avoid using tree-components multiple times!

Network



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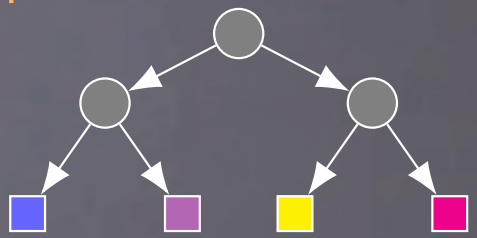

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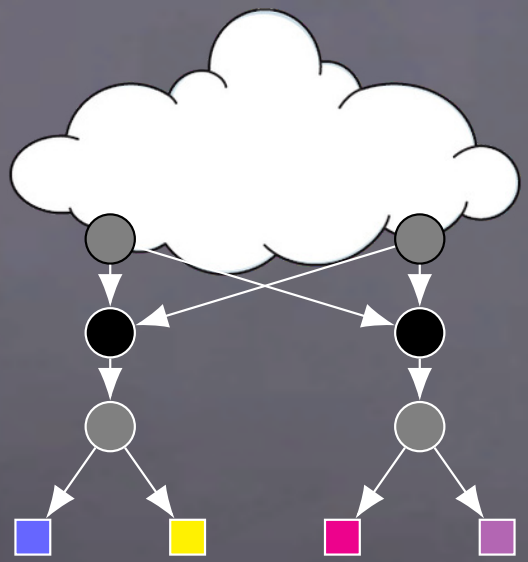
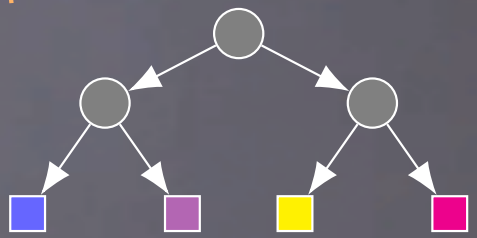
Problem: Avoid using tree-components multiple times!

Network



Tree



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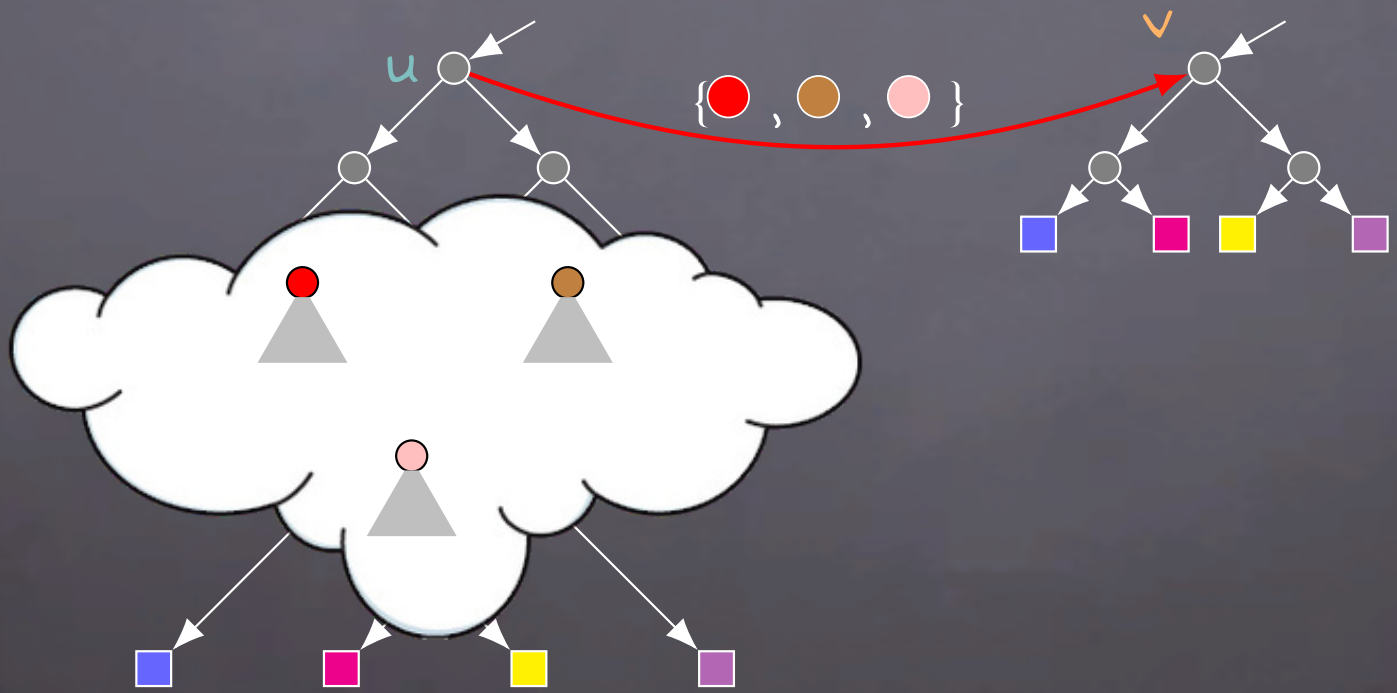

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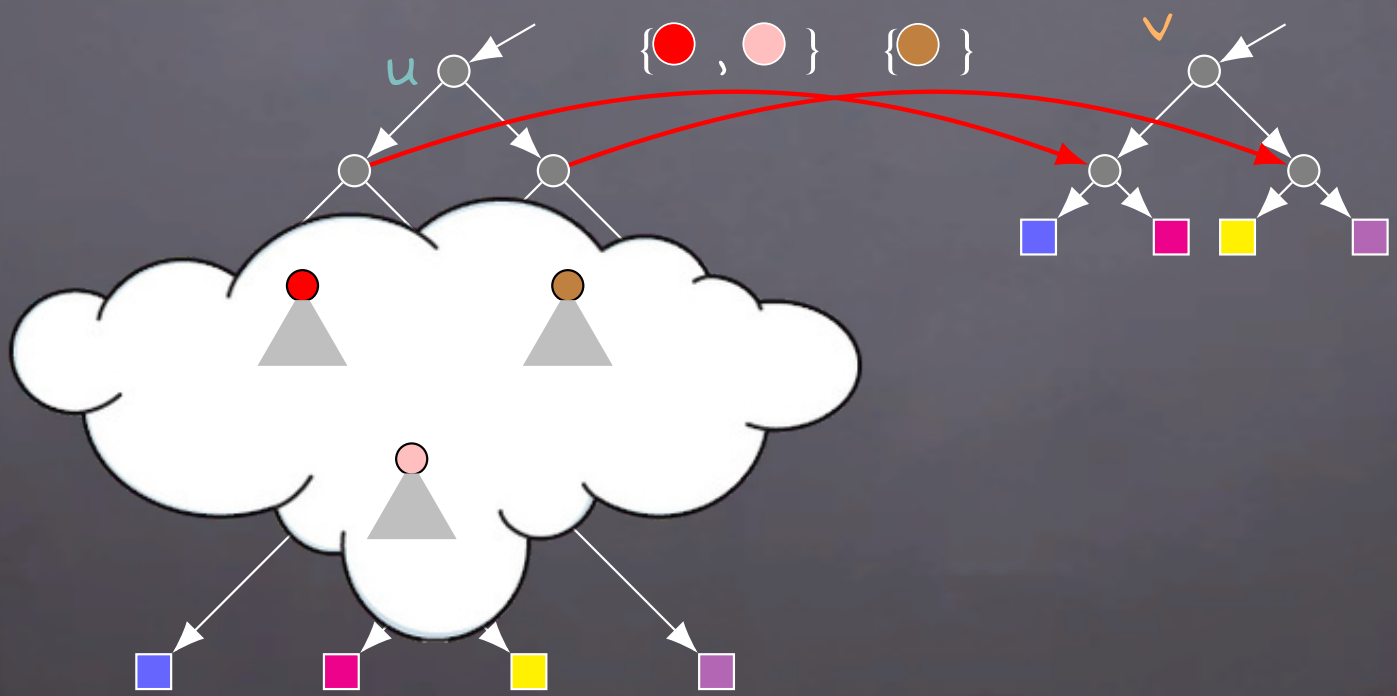

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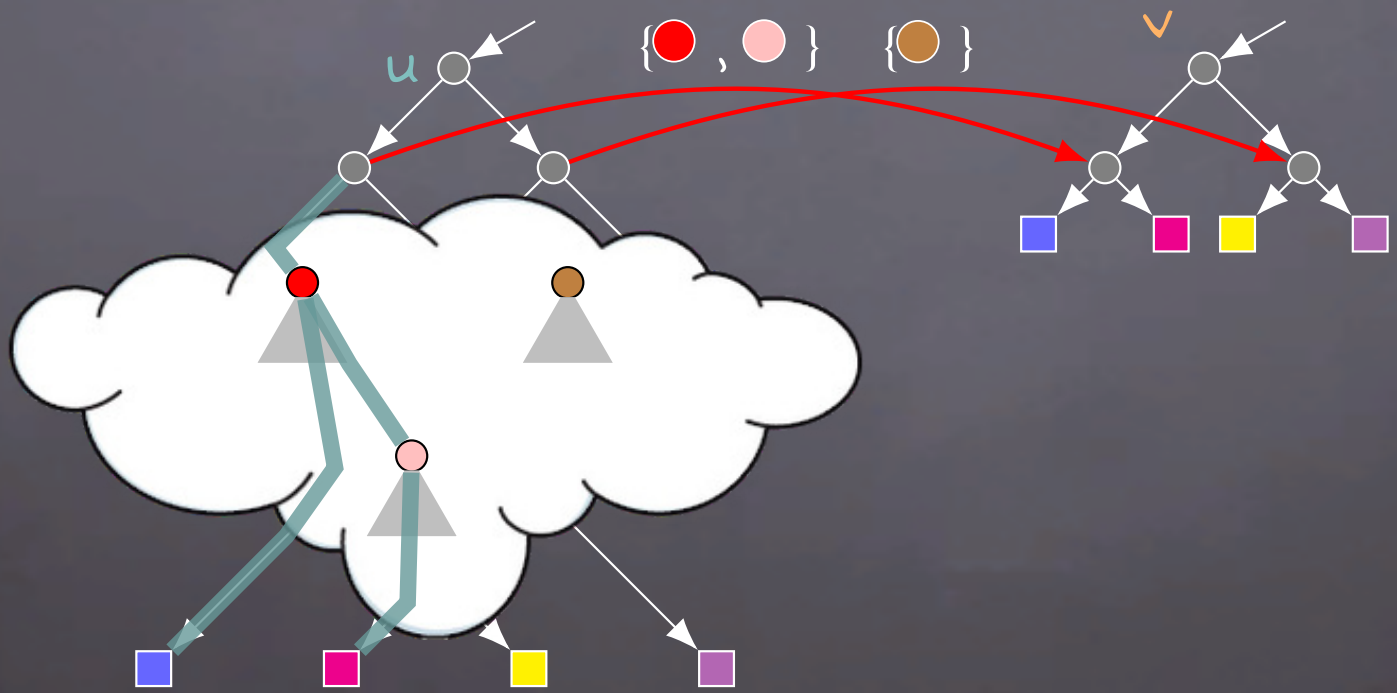

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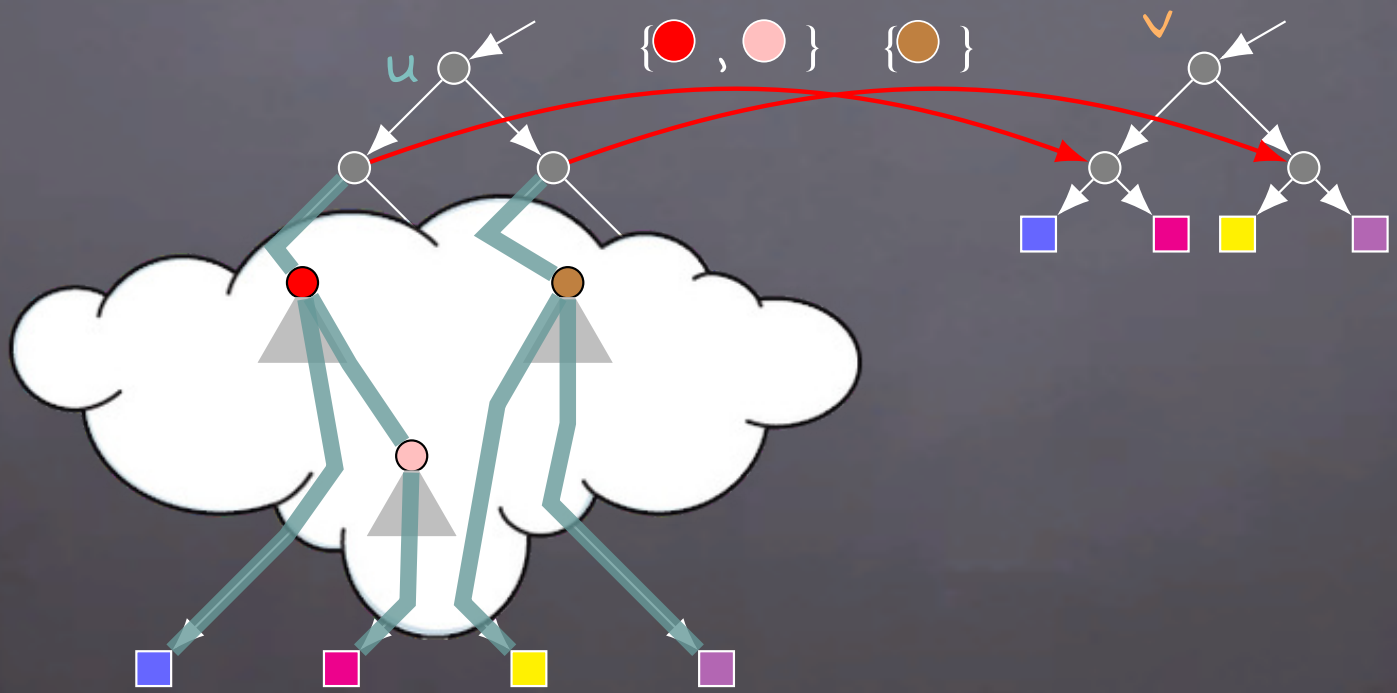

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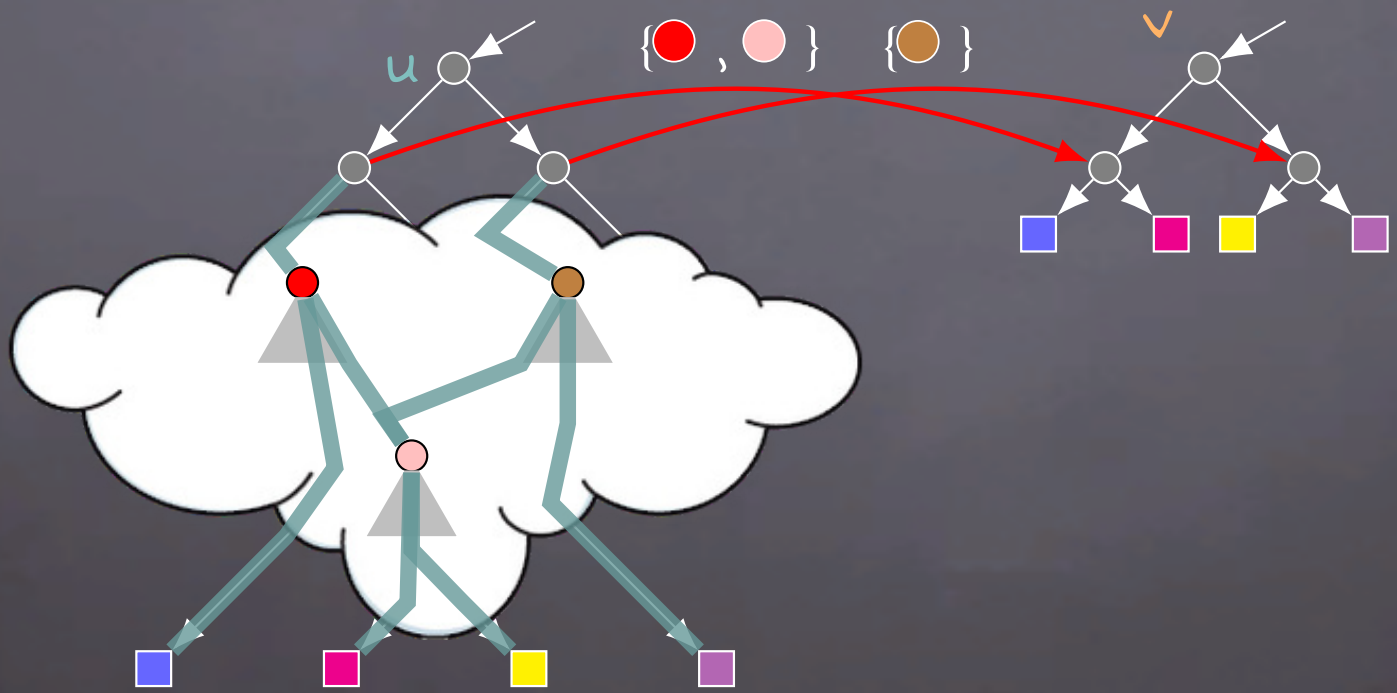

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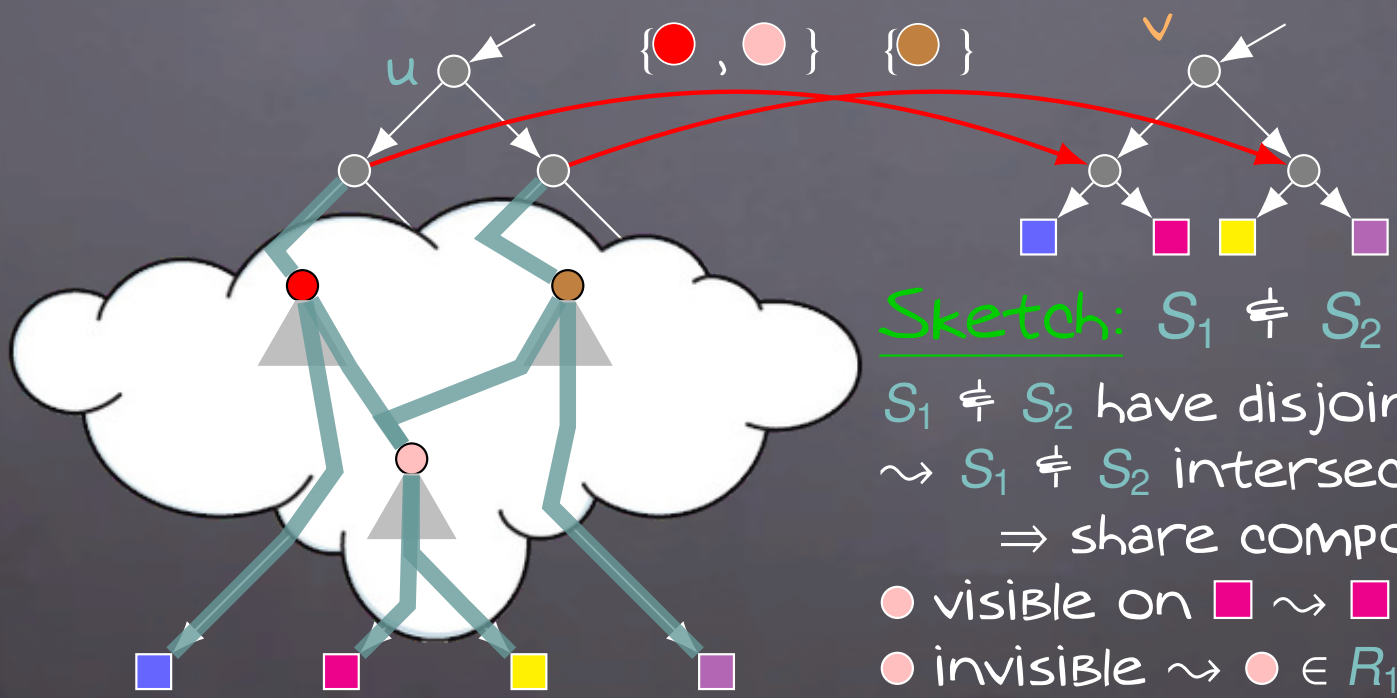
Problem: Avoid using tree-components multiple times!

Definition

Subdivision S of T_v is nice $\Leftrightarrow$
for each $u \in S$, S contains all leaves that u is visible on.



Sketch: $S_1 \lneq S_2$ disjoint

$S_1 \lneq S_2$ have disjoint leaf-sets
$\rightsquigarrow$ $S_1 \lneq S_2$ intersect
$\Rightarrow$ share component-root ●
● visible on ■ $\rightsquigarrow$ ■ $\in S_1 \lneq S_2$ ↯
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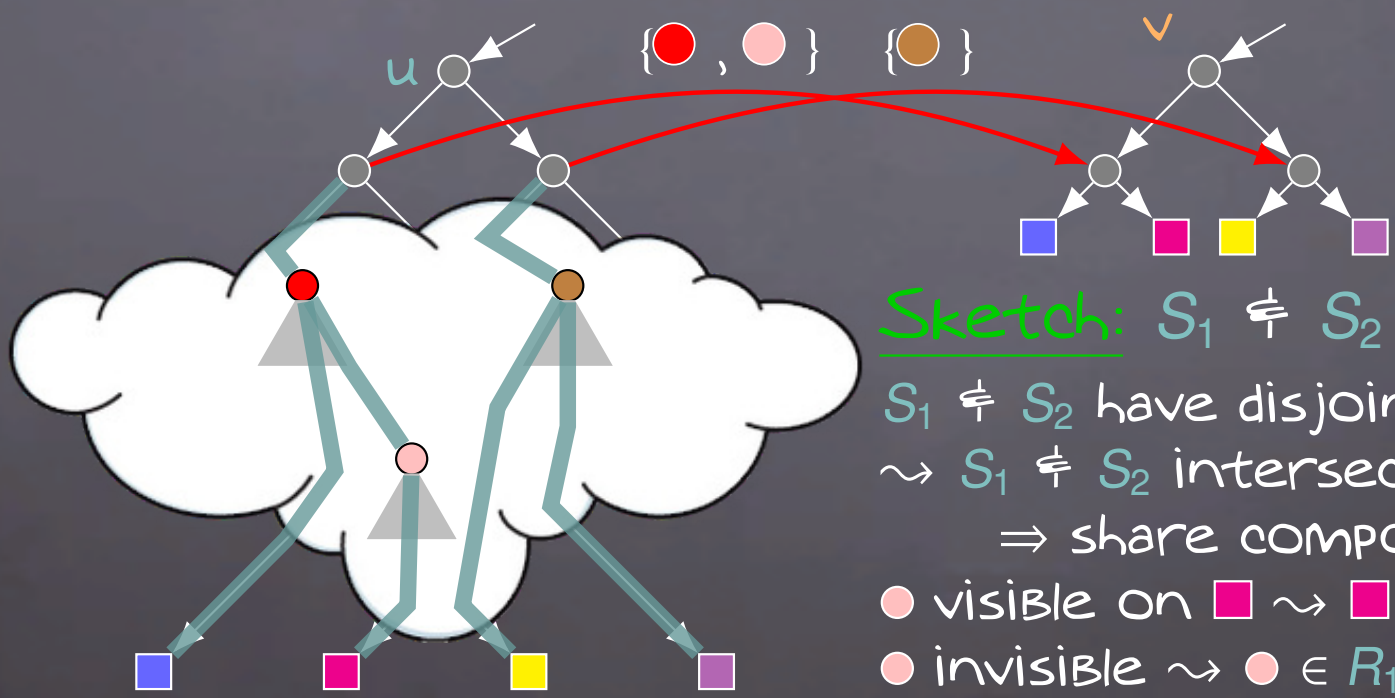

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Part II:
Soft
Poly-
tomies

Recall:
Bin. resolution of N displays
Bin. resolution of T

Hardness For MUL-Trees

2-Union Independent Set

Input: collection $G = (V, E_1)$ of disjoint P_3s,
collection $H = (V, E_2)$ of ℓ disjoint cliques

Question: Does $G + H$ have a size-ℓ independent set?

Hardness For MUL-Trees

k-SAT:
≤ k-cliques

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2-Union Independent Set

Input: collection $G = (V, E_1)$ of disjoint P_3s,
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Question: Does G have a size-ℓ colorful independent set?

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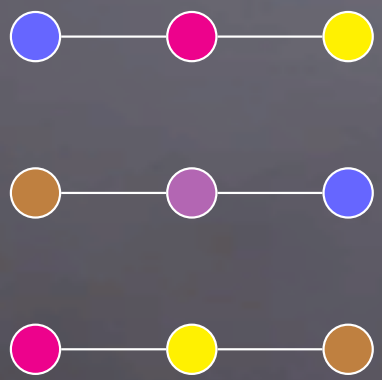

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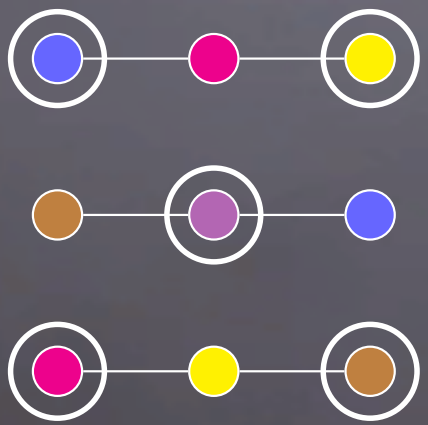

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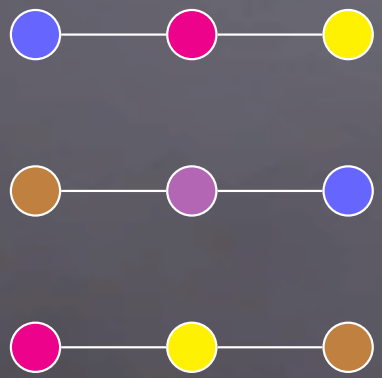

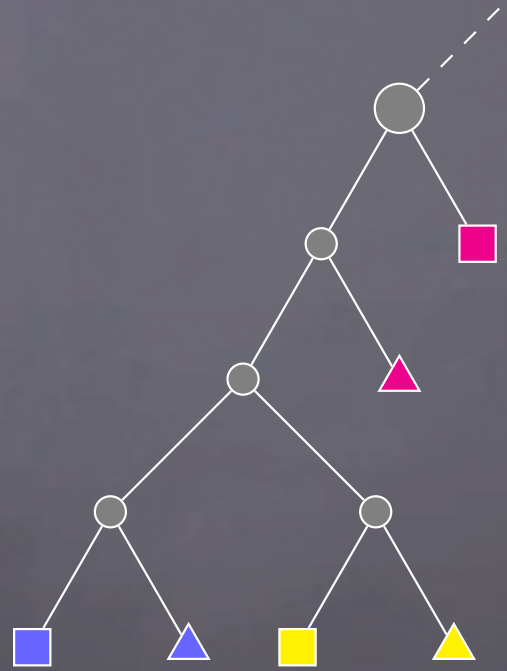

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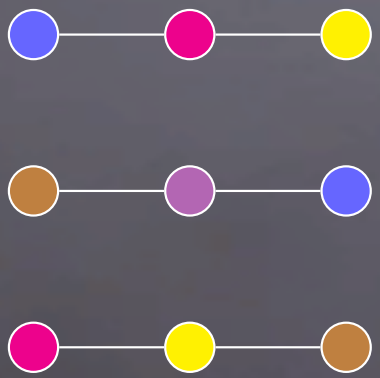

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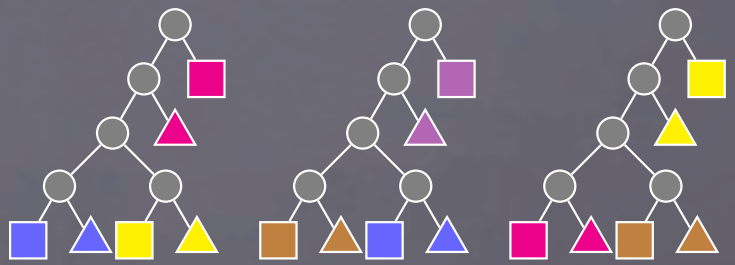

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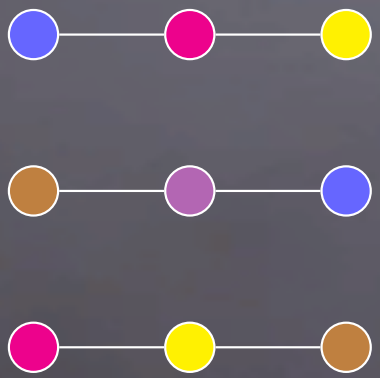

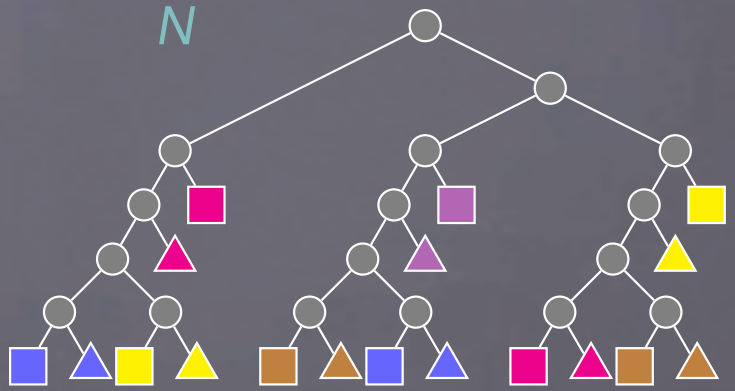

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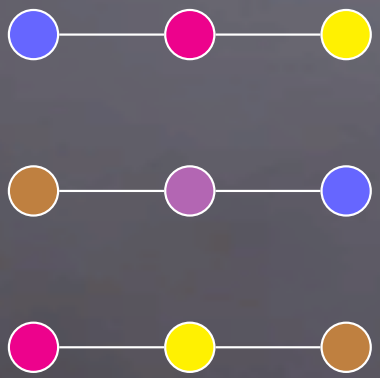

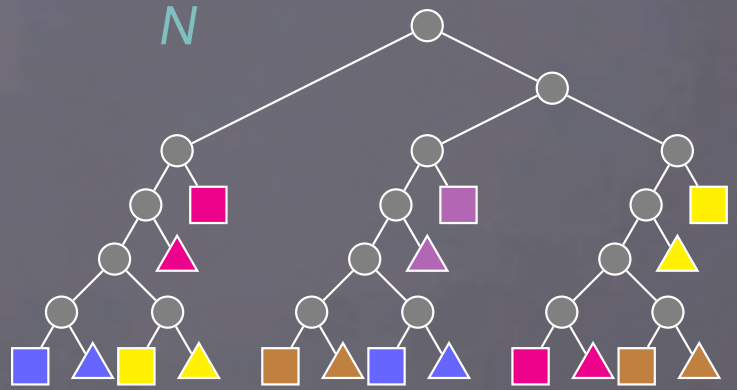

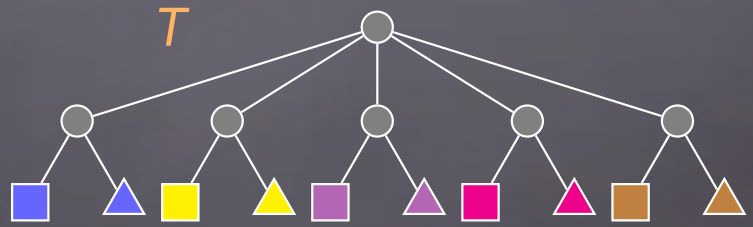

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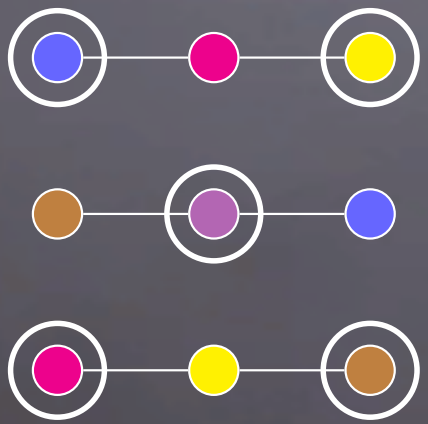

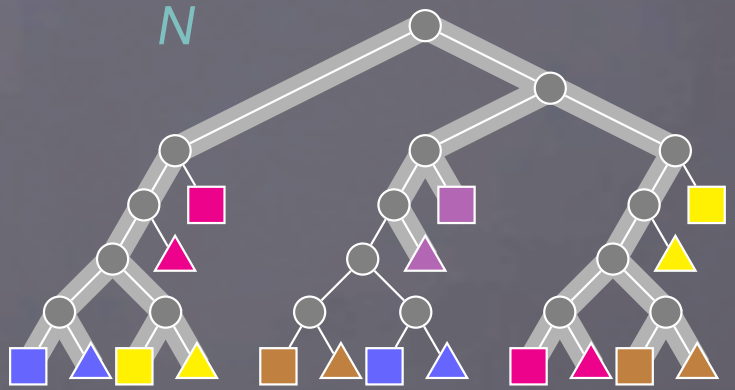

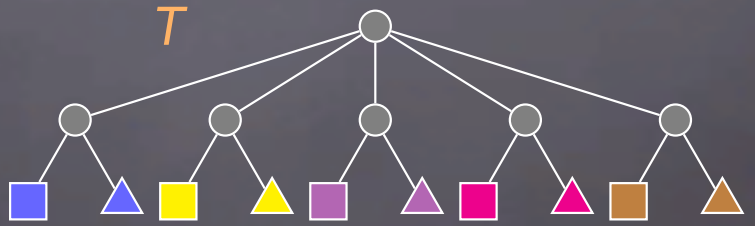

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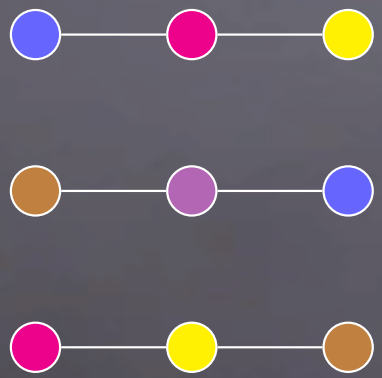

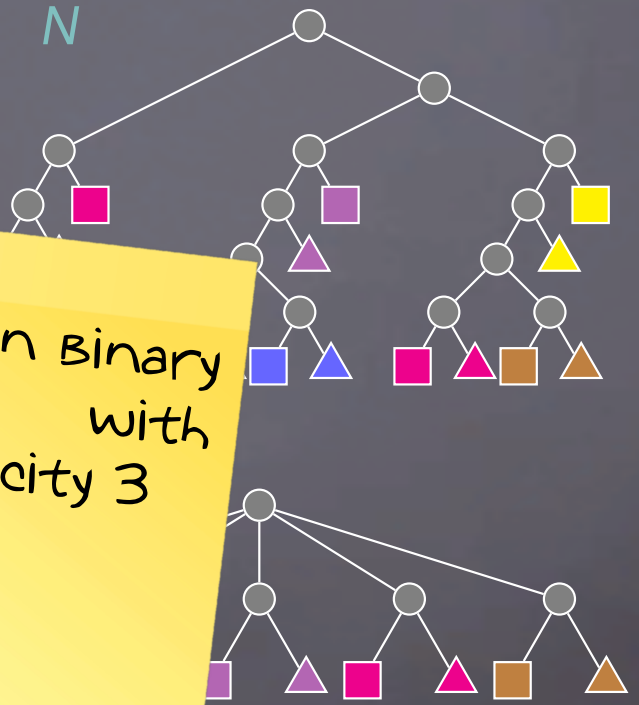

Theorem
Soft TC on binary mul-trees with label-multiplicity 3
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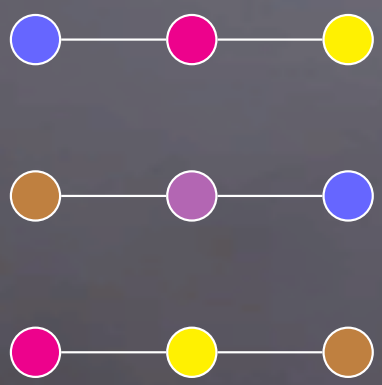

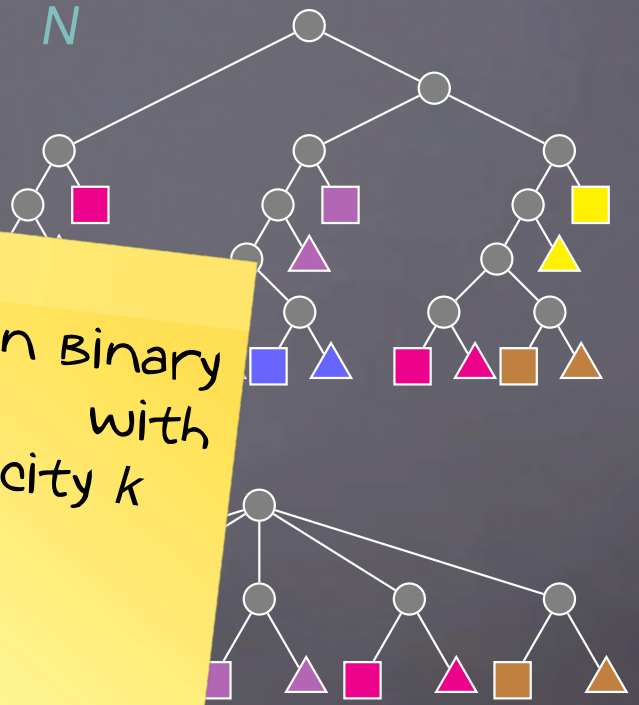

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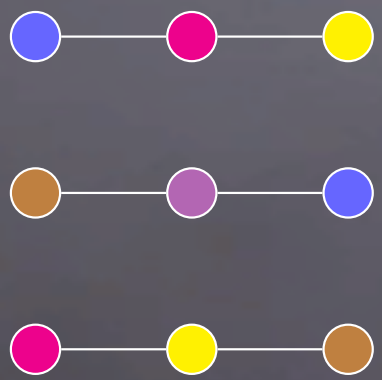

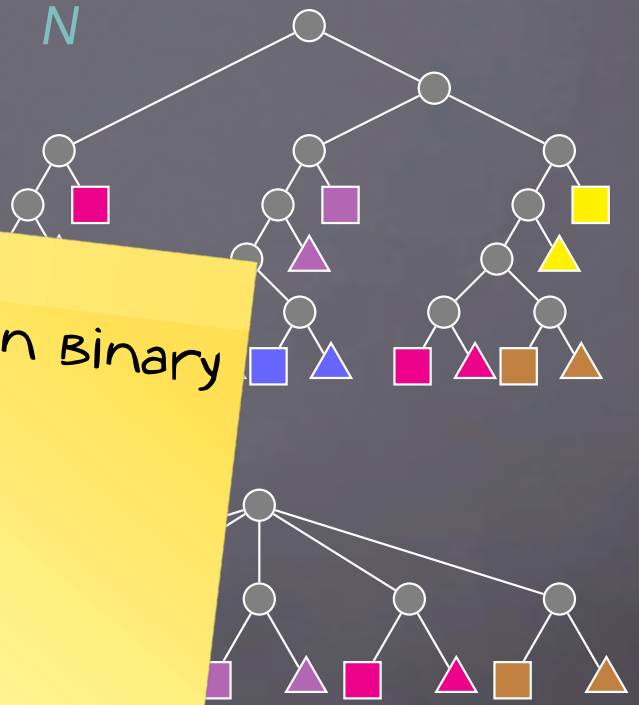

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CNF-SAT

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Observation

"solution" $S \rightsquigarrow$ map "$\rightarrow_S$" with $v \rightarrow_S \mathrm{LCA}_S(\mathcal{L}(v))$

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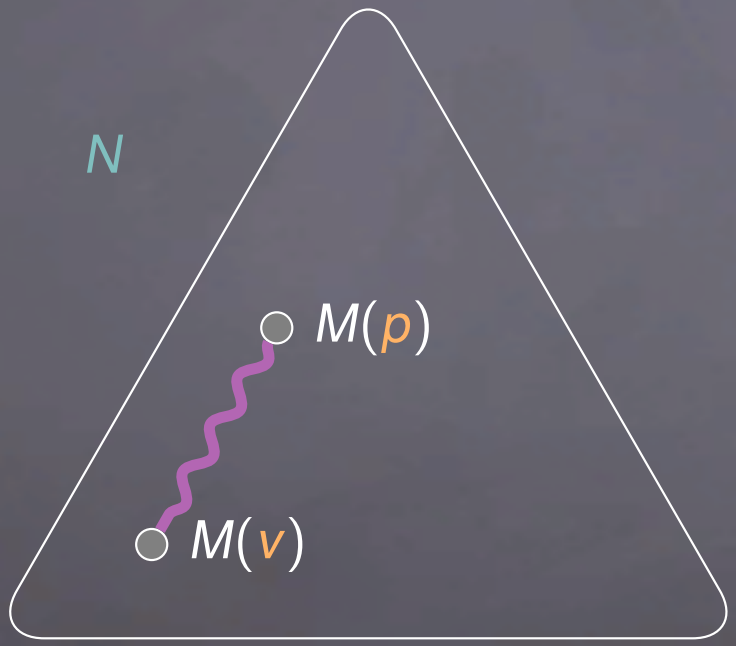

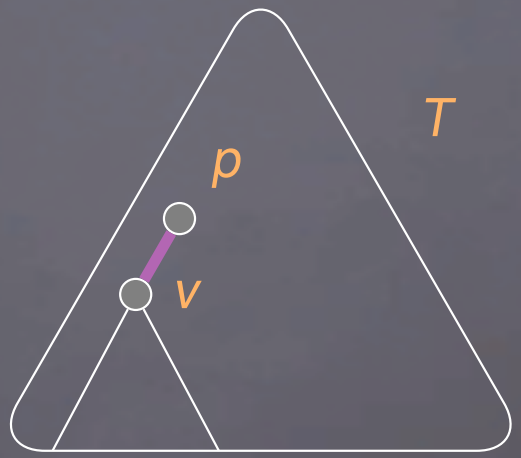

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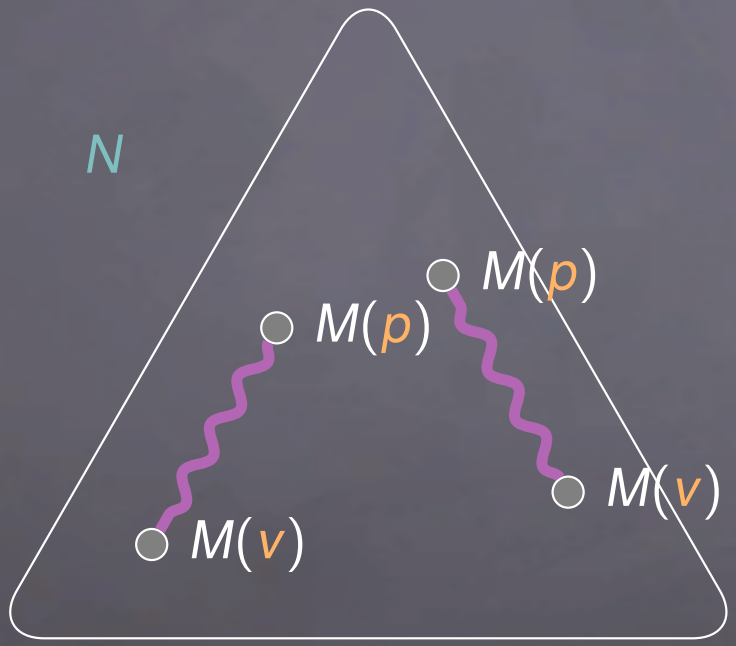

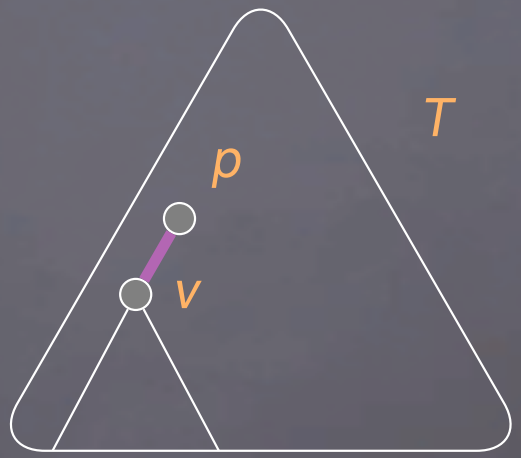

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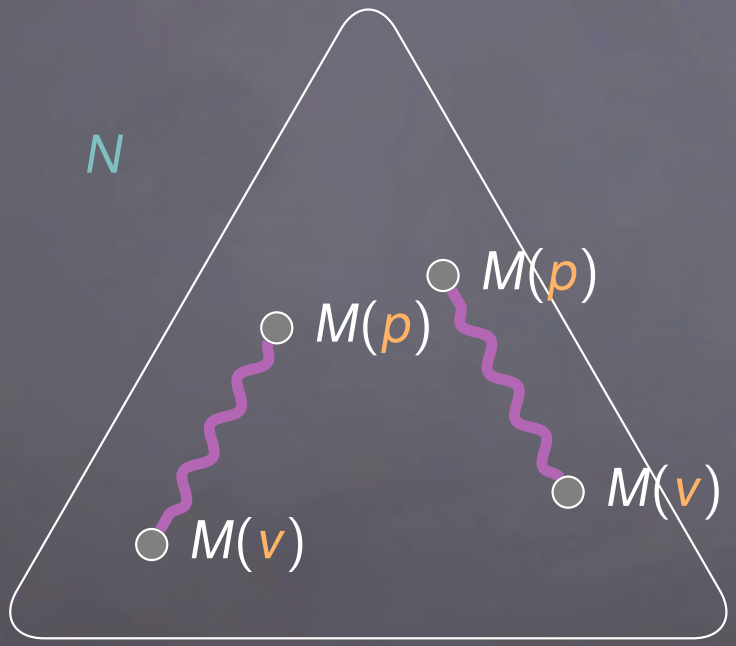

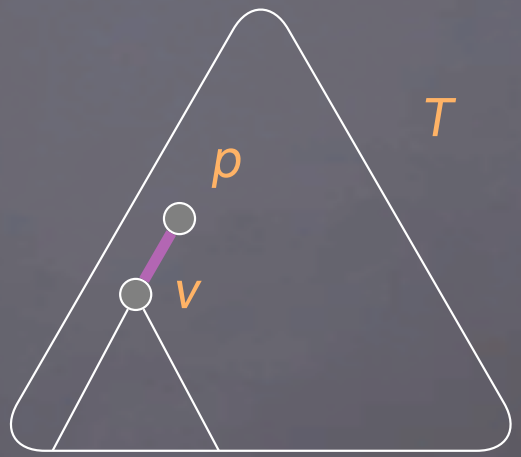

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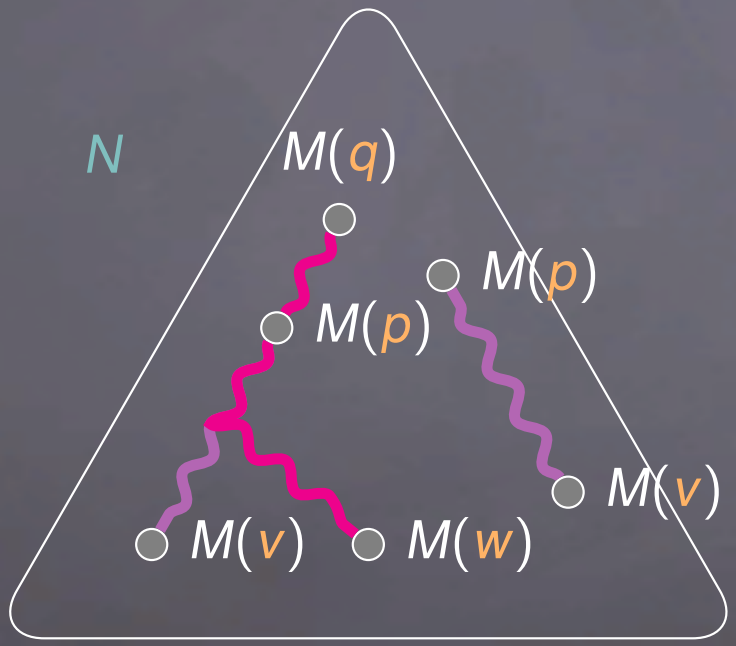

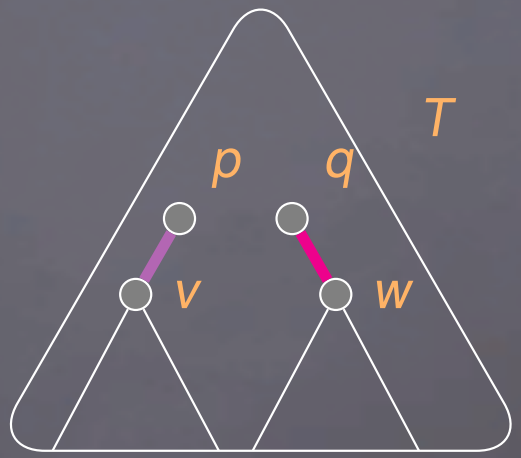

The Polynomial Case: 2-Labeled Trees

Observation

"solution" $S \rightsquigarrow$ map "$\to_S$" with $v \to_S \mathrm{LCA}_S(\mathcal{L}(v)) \notin \mathrm{LCA}_S(\mathcal{L}(v)) \in M(v)$

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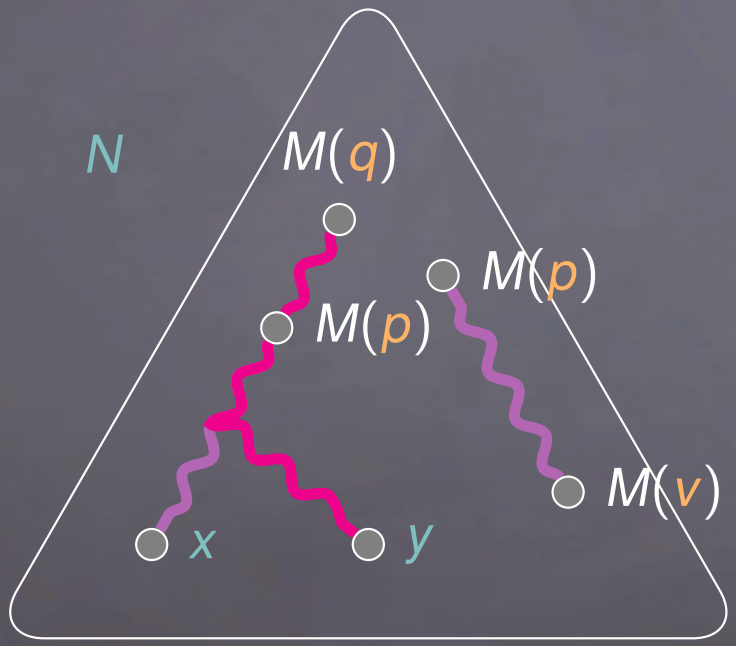

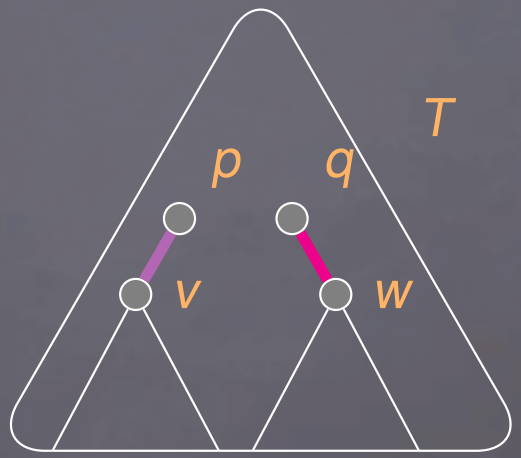

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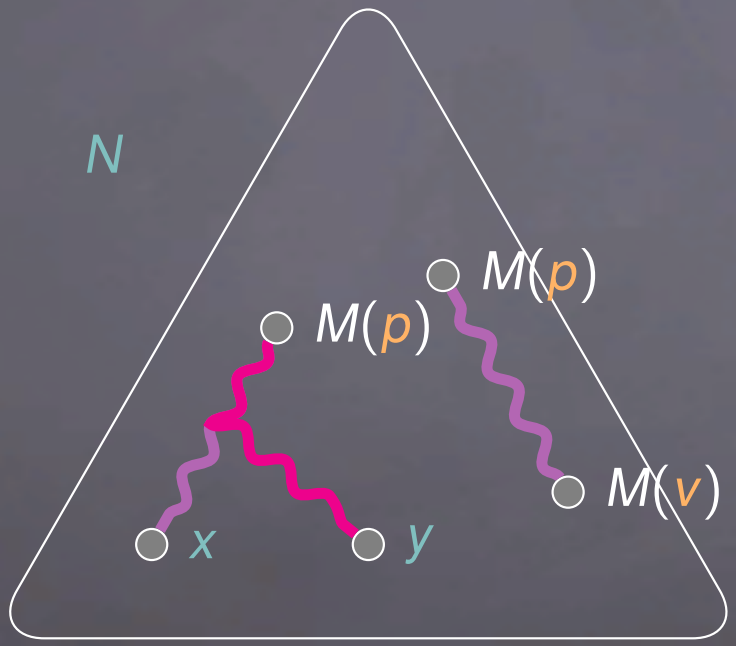

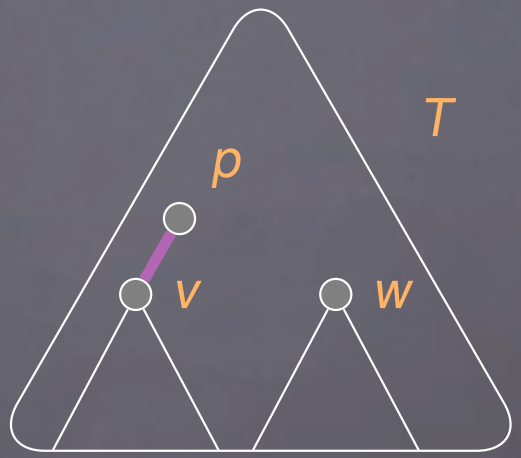

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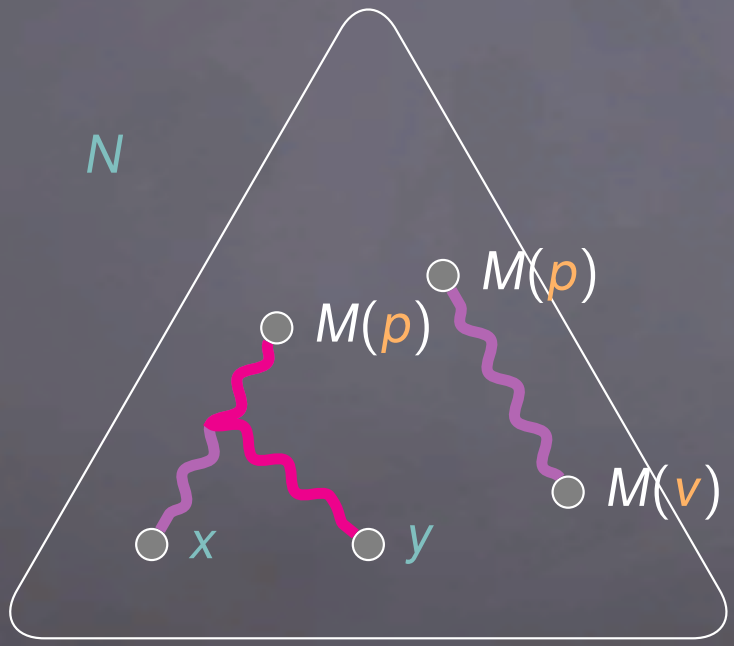

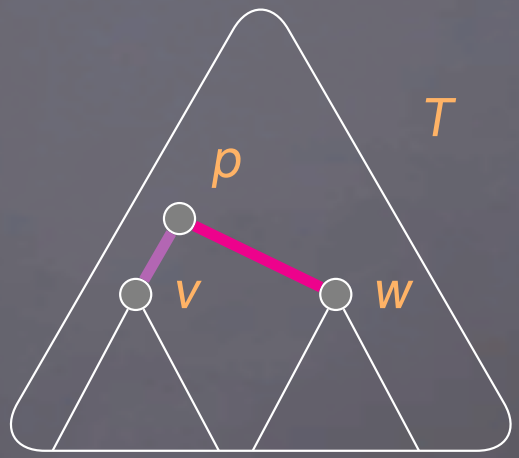

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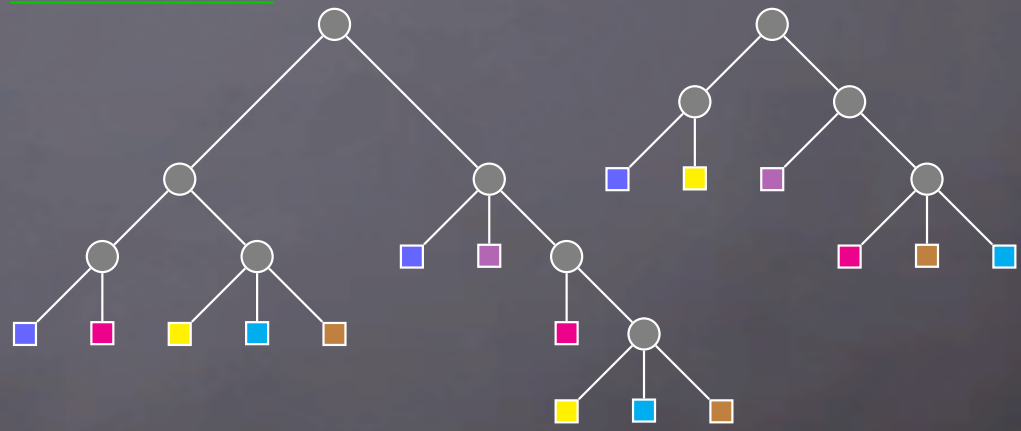

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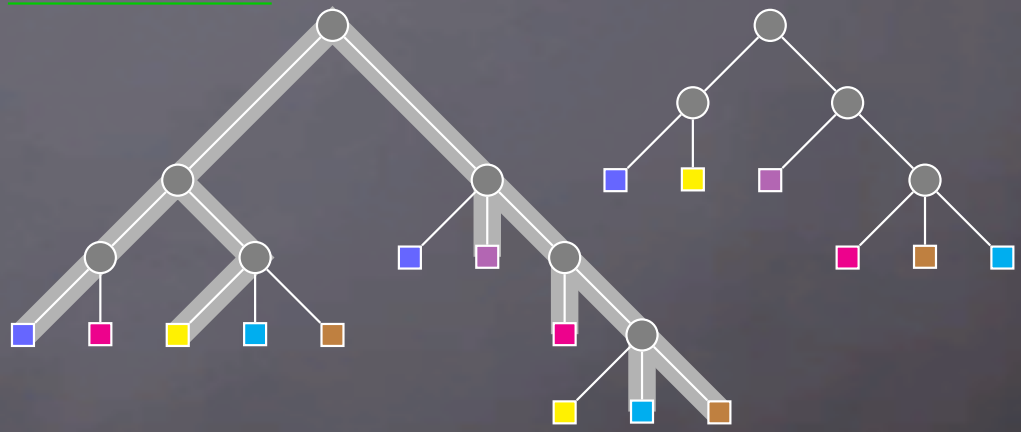

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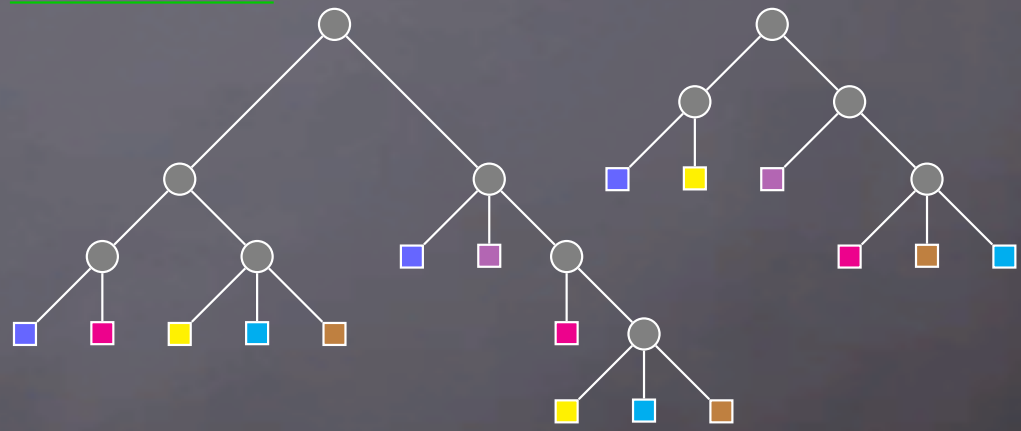

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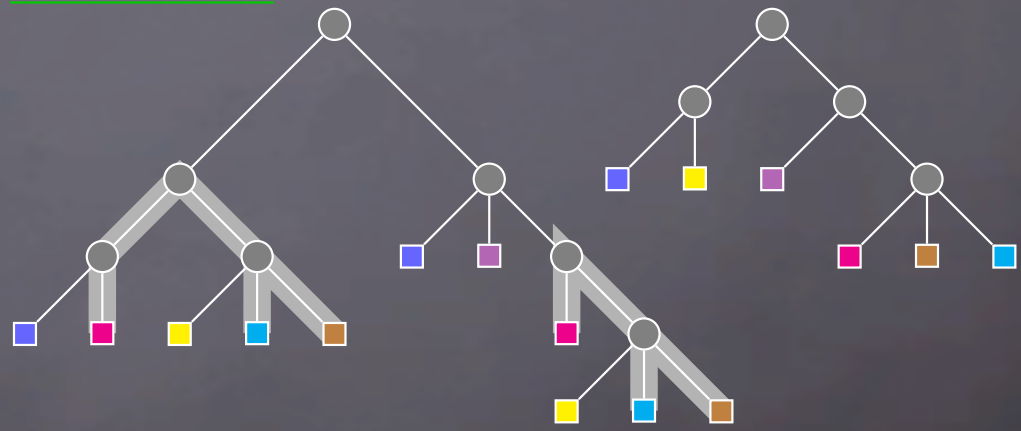

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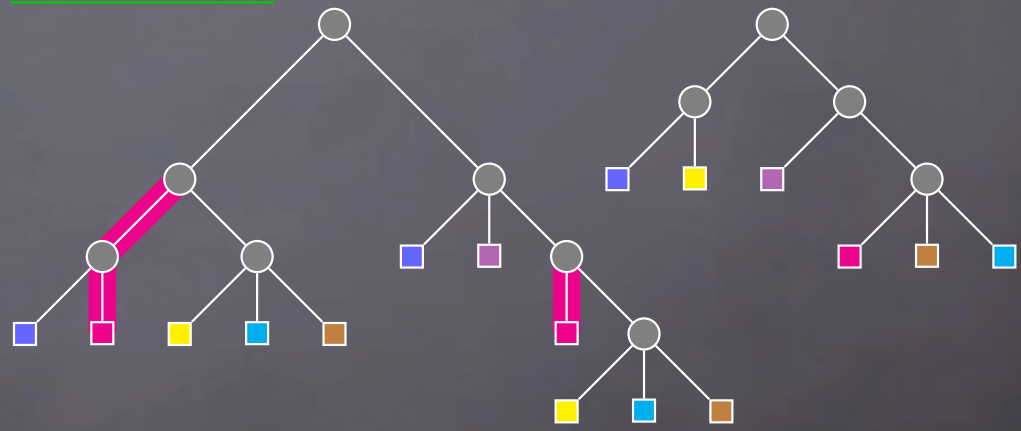

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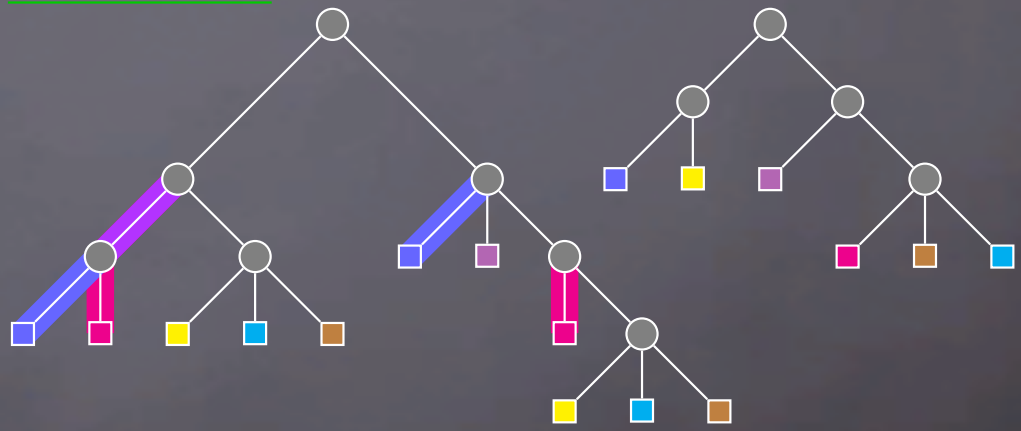

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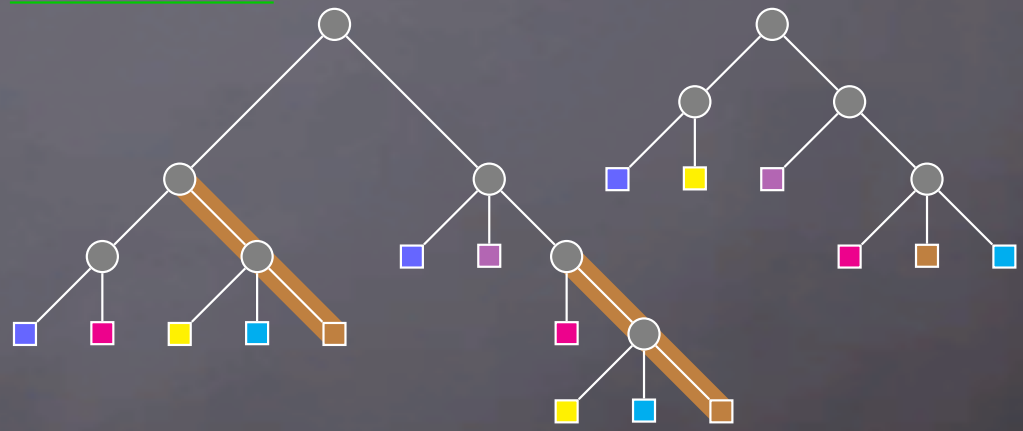

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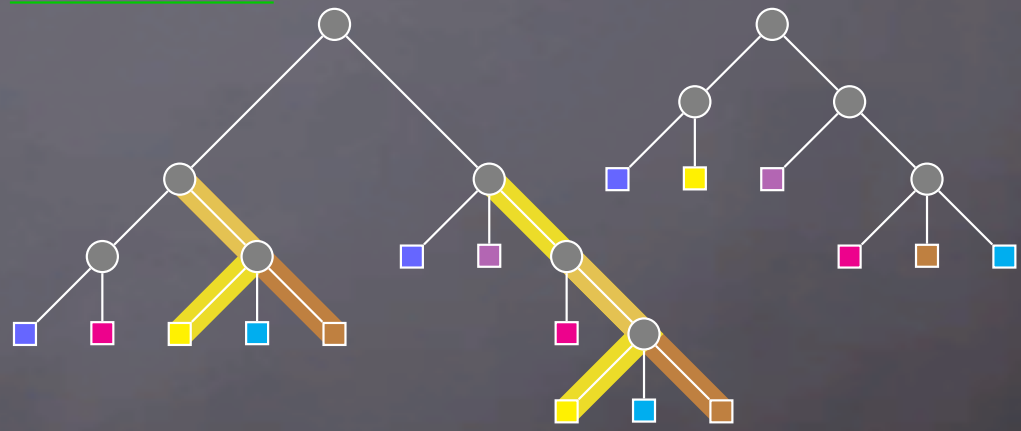

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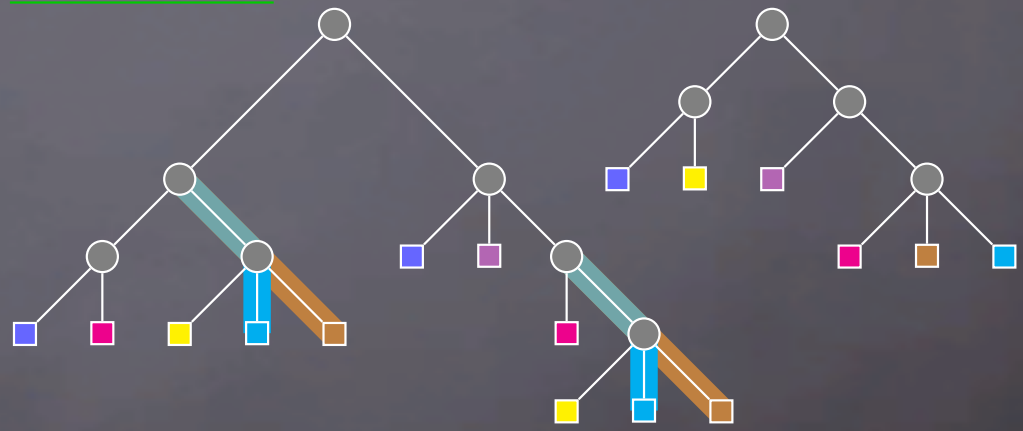

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2. ...maps v above children
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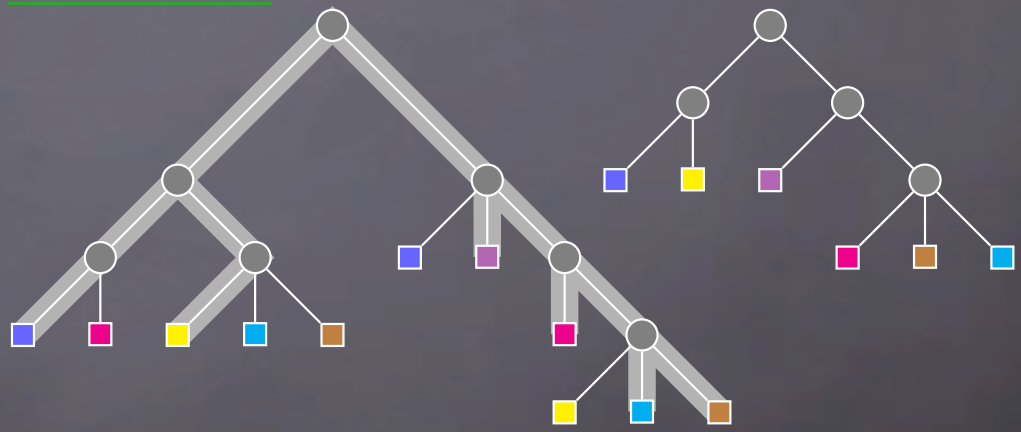

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sufficient!

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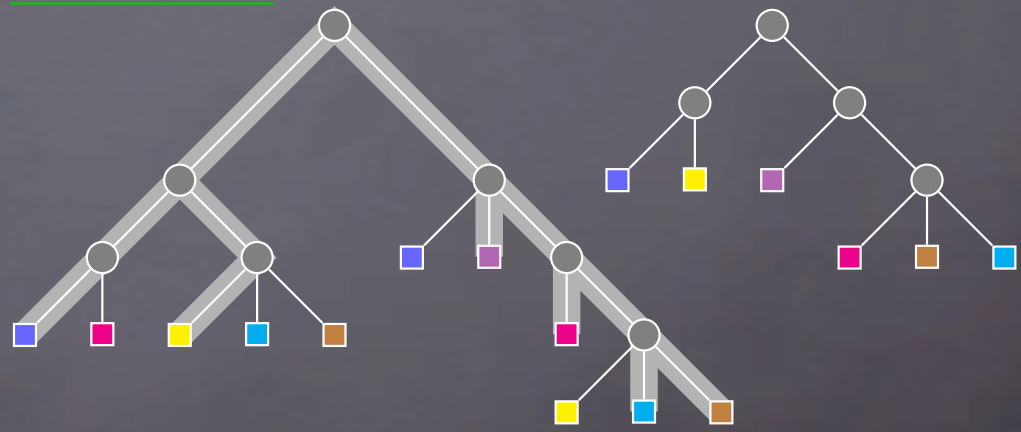

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The Polynomial Case: 2-Labeled Trees

Bottom-up dynamic programming: decide if N_u displays T_ρ

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For each $v < \rho$, we know $M(v)$

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Construction

construct 2-CNF $\varphi_{\rho \to u} := \bigwedge_{v \in T_\rho} \varphi_v$:

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$\varphi_{\rho \to u}$ can be constructed in $O(|N_u| \cdot |T_\rho|^2)$ time.

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Theorem

Soft TC on 2-labeled trees in $O(|N|^2 |T|^2)$

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Conclusion

What we Saw...

- hard TC in $O(|N|\Delta_N^2\Delta_T^2)$ on...
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Parameters

t = #invisible comp.-roots in N

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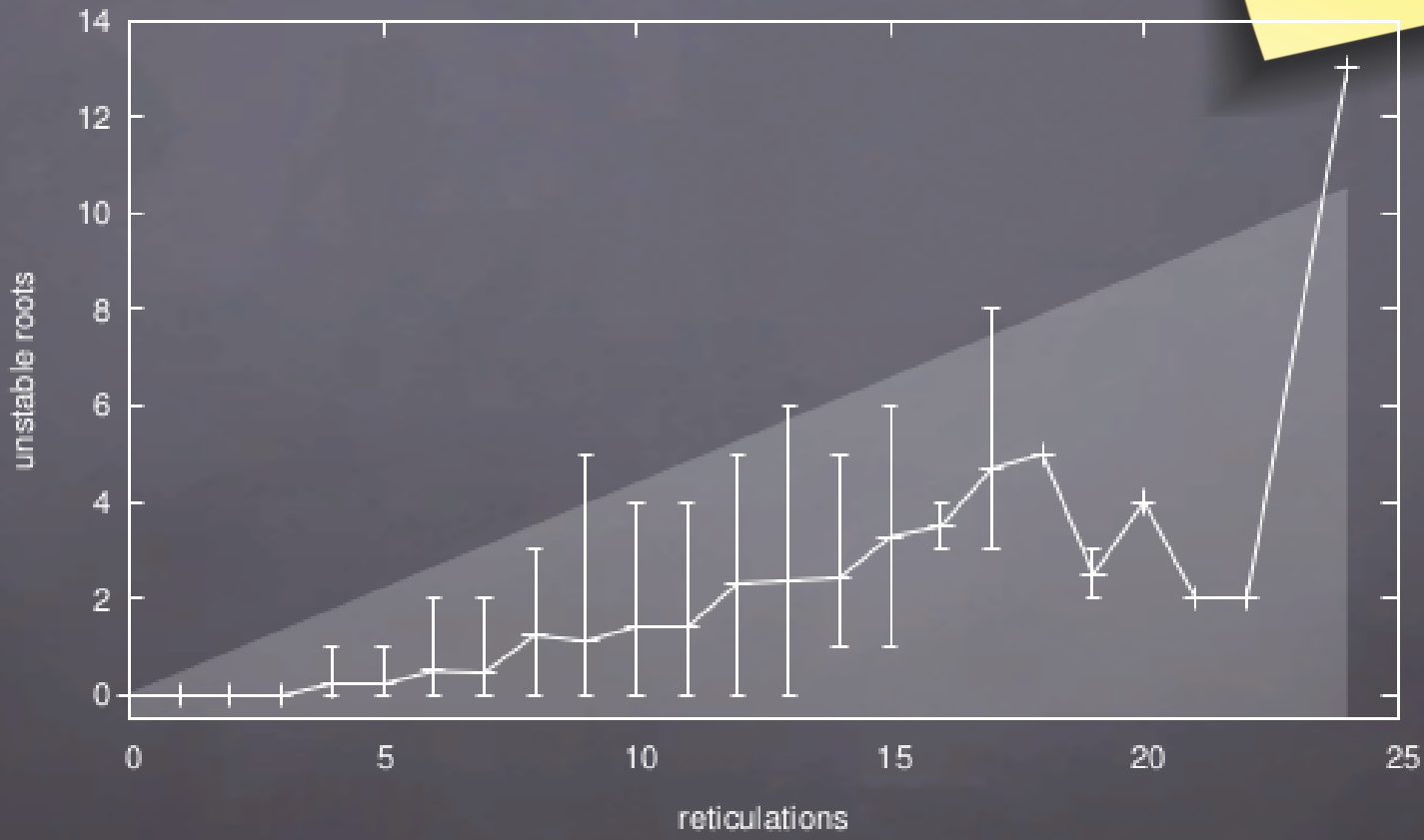

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Parameters

t = #invisible comp.-roots in N

t^* = max. #invisible comp.-roots / block

Also in the Paper

- solve biconnected components "independently" ($t \to t^*$)
- no poly kernel for t^* (already for weaker "level")

Conclusion

What we Saw...

- hard TC in $O(|N|\Delta_N^2\Delta_T^2)$ on...
  - ...reticulation visible
  - ...nearly stable
  - ...more
- hard TC in $O^*((\Delta_T+1)^t)$ time
- soft TC somewhat more difficult than hard TC
  $\rightsquigarrow$ NP-hard in 3-labeled mul-trees
  $\rightsquigarrow$ NP-hard in reticulation visible with in-degree ≥ 3
- poly in 2-labelled mul-trees

Parameters

t = #invisible comp.-roots in N

t^* = max. #invisible comp.-roots / block

Also in the Paper

- solve biconnected components "independently" ($t \to t^*$)
- no poly kernel for t^* (already for weaker "level")

Ongoing Work

- parameterized cpx for soft TC
- other types of networks (tree-based, ...)
- parameterize more by t^*
- soft TC $\neq$ Cluster Containment
- mix soft $\neq$ hard polytomies


The End