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Denis Roegel

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# A reconstruction of Maurolico's tables of sines, tangents and secants (1558)

**Denis Roegel**

**2 September 2021**

**(last version: 2 September 2021)**

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# Introduction

This document gives exact (modern) reconstructions of Francesco Maurolico's trigonometric tables published in 1558 [1] (figures 1, 2, and 3). There are three tables, and each gives the value of a function (sine, tangent, secant) for each degree of the quadrant, and with a radius $R = 100000$. The differences are given for each value, both in decimal and sexagesimal. For instance, the difference given for $\operatorname{Sin} 51^\circ = R \sin 51^\circ$ is 1111, because $\operatorname{Sin} 51^\circ - \operatorname{Sin} 50^\circ = 1111$. And the sexagesimal corresponding value is 18.31, because $1111 = 18 \cdot 60 + 31$.

For the tangents and secants, Maurolico also gives their values for five or six more values at the bottom of the pages. In my survey [2], I describe the context of these tables and how their values have been computed.

TABELLA SINVS RECTI. 65

| g | Sinus. | Dfa | 6m4 | g | Sinus. | Dfa | 6m4 |
|---|---|---|---|---|---|---|---|
| 1 | 1745 | 1745 | 29. 5 | 46 | 71934 | 1223. | 20. 23 |
| 2 | 3490 | 1745 | 29. 5 | 47 | 73135 | 1201. | 20. 1 |
| 3 | 5234 | 1744 | 29. 4 | 48 | 74315 | 1180. | 19. 40 |
| 4 | 6976 | 1742 | 29. 2 | 49 | 75471 | 1156. | 19. 16 |
| 5 | 8716 | 1740 | 29. 0 | 50 | 76604 | 1133. | 18. 53 |
| 6 | 10453 | 1737 | 28. 57 | 51 | 77715 | 1111. | 18. 31 |
| 7 | 12187 | 1734 | 28. 54 | 52 | 78801 | 1086 | 18. 6 |
| 8 | 13917 | 1730 | 28. 50 | 53 | 79864 | 1063 | 17. 43 |
| 9 | 15643 | 1726 | 28. 46 | 54 | 80902 | 1038 | 17. 18 |
| 10 | 17365 | 1722 | 28. 42 | 55 | 81915 | 1013 | 16. 53 |
| 11 | 19081 | 1716 | 28. 36 | 56 | 82904 | 989 | 16. 29 |
| 12 | 20791 | 1710 | 28. 30 | 57 | 83867 | 963 | 16. 3 |
| 13 | 22495 | 1704 | 28. 24 | 58 | 84805 | 938 | 15. 38 |
| 14 | 24192 | 1697 | 28. 17 | 59 | 85717 | 912 | 15. 21 |
| 15 | 25882 | 1690 | 28. 10 | 60 | 86603 | 886 | 14. 46 |
| 16 | 27564 | 1682 | 28. 2 | 61 | 87462 | 859 | 14. 19 |
| 17 | 29237 | 1673 | 27. 53 | 62 | 88295 | 833 | 13. 53 |
| 18 | 30902 | 1665 | 27. 45 | 63 | 89101 | 806 | 13. 26 |
| 19 | 32557 | 1655 | 27. 35 | 64 | 89879 | 778 | 12. 58 |
| 20 | 34202 | 1645 | 27. 25 | 65 | 90631 | 752 | 12. 32 |
| 21 | 35837 | 1635 | 27. 15 | 66 | 91355 | 724 | 12. 4 |
| 22 | 37461 | 1624 | 27. 4 | 67 | 92051 | 696 | 11. 36 |
| 23 | 39073 | 1612 | 26. 52 | 68 | 92718 | 667 | 11. 7 |
| 24 | 40674 | 1601 | 26. 41 | 69 | 93358 | 640 | 10. 40 |
| 25 | 42262 | 1588 | 26. 28 | 70 | 93969 | 611 | 10 11 |
| 26 | 43837 | 1575 | 26. 15 | 71 | 94552 | 583 | 9. 43 |
| 27 | 45399 | 1562 | 26. 2 | 72 | 95106 | 554 | 9. 14 |
| 28 | 46947 | 1548 | 25. 48 | 73 | 95631 | 525 | 8. 45 |
| 29 | 48481 | 1534 | 25. 34 | 74 | 96126 | 495 | 8. 15 |
| 30 | 50000 | 1519 | 25. 19 | 75 | 96593 | 467 | 7. 47 |
| 31 | 51504 | 1504 | 25. 4 | 76 | 97030 | 437 | 7. 17 |
| 32 | 52992 | 1488 | 24. 48 | 77 | 97437 | 407 | 6. 47 |
| 33 | 54464 | 1472 | 24. 32 | 78 | 97815 | 378 | 6. 18 |
| 34 | 55919 | 1455 | 24. 15 | 79 | 98163 | 348 | 5. 48 |
| 35 | 57358 | 1439 | 23. 59 | 80 | 98481 | 318 | 5. 18 |
| 36 | 58779 | 1421 | 23. 41 | 81 | 98769 | 288 | 4. 48 |
| 37 | 60182 | 1403 | 23. 23 | 82 | 99027 | 258 | 4. 18 |
| 38 | 61566 | 1384 | 23. 4 | 83 | 99255 | 228 | 3. 48 |
| 39 | 62932 | 1366 | 22. 46 | 84 | 99452 | 197 | 3. 17 |
| 40 | 64279 | 1347 | 22. 27 | 85 | 99620 | 168 | 2. 48 |
| 41 | 65606 | 1327 | 22. 7 | 86 | 99756 | 139 | 2. 16 |
| 42 | 66913 | 1307 | 21. 47 | 87 | 99863 | 107 | 1. 47 |
| 43 | 68200 | 1287 | 21. 27 | 88 | 99939 | 76 | 1. 6 |
| 44 | 69466 | 1266 | 21. 6 | 89 | 99985 | 46 | 0. 46 |
| 45 | 70711 | 1245 | 20. 45 | 90 | 100000 | 15 | 0. 15 |

Figure 1: Maurolico's original table of sines [1] (source: Österreichische Nationalbibliothek).

TABELLA FOECVNDA.

| g | Vmbra | Dfa | 6ma | g | Vmbra | Dfa | 6ma |
|---|---|---|---|---|---|---|---|
| 1 | 1745. | 1745 | 29. 5 | 46 | 103551 | 3551. | 59. 11 |
| 2 | 3492 | 1747 | 29. 7 | 47 | 107236 | 3685. | 61. 25 |
| 3 | 5241 | 1749 | 29. 9 | 48 | 111062 | 3826. | 63. 46 |
| 4 | 6992 | 1751 | 29. 11 | 49 | 115037 | 3975. | 66. 15 |
| 5 | 8748 | 1756 | 29. 16 | 50 | 119197 | 4160. | 69. 20 |
| 6 | 10510 | 1762 | 29. 22 | 51 | 123491 | 4294 | 71. 34 |
| 7 | 12278 | 1768 | 29. 28 | 52 | 127994 | 4503 | 75 5 |
| 8 | 14053 | 1775 | 29. 35 | 53 | 132704 | 4710 | 78. 30 |
| 9 | 15838 | 1785 | 29. 45 | 54 | 137639 | 4935 | 82. 15 |
| 10 | 17633 | 1795 | 29. 55 | 55 | 142813 | 5174 | 86. 14 |
| 11 | 19439 | 1806 | 30. 6 | 56 | 148253 | 5440 | 90. 40 |
| 12 | 21256 | 1817 | 30. 17 | 57 | 153987 | 5734 | 95. 34 |
| 13 | 23087 | 1831 | 30. 31 | 58 | 160033 | 6046 | 100.46 |
| 14 | 24932 | 1845 | 30. 45 | 59 | 166429 | 6396 | 106. 36 |
| 15 | 26794 | 1862 | 31. 2 | 60 | 173205 | 6776 | 112. 56 |
| 16 | 28674 | 1880 | 31. 20 | 61 | 180405 | 7200 | 120. 0 |
| 17 | 30573 | 1899 | 31. 39 | 62 | 188073 | 7668 | 127.48 |
| 18 | 32492 | 1919 | 31. 59 | 63 | 196261 | 8188 | 136.28 |
| 19 | 34433 | 1941 | 32. 21 | 64 | 205030 | 8769 | 146. 9 |
| 20 | 36396 | 1963 | 32. 43 | 65 | 214451 | 9421 | 157. 1 |
| 21 | 38387 | 1991 | 33. 11 | 66 | 224603 | 10152 | 169. 12 |
| 22 | 40402 | 2015 | 33. 35 | 67 | 235585 | 10982 | 183. 2 |
| 23 | 42448 | 2046 | 34. 6 | 68 | 247509 | 11924 | 198. 44 |
| 24 | 44522 | 2074 | 34. 34 | 69 | 260509 | 13000 | 216. 40 |
| 25 | 46631 | 2109 | 35. 9 | 70 | 274747 | 14238 | 237. 18 |
| 26 | 48772 | 2141 | 35. 41 | 71 | 290421 | 15674 | 261. 14 |
| 27 | 50952 | 2180 | 36. 20 | 72 | 307768 | 17347 | 289. 7 |
| 28 | 53170 | 2218 | 36. 58 | 73 | 327084 | 19316 | 321. 56 |
| 29 | 55432 | 2262 | 37. 42 | 74 | 348742 | 21658 | 360. 58 |
| 30 | 57735 | 2303 | 38. 23 | 75 | 373205 | 24463 | 407. 43 |
| 31 | 60086 | 2351 | 39. 11 | 76 | 401078 | 27873 | 464. 33 |
| 32 | 62486 | 2400 | 40. 0 | 77 | 433148 | 32070 | 534. 30 |
| 33 | 64940 | 2454 | 40. 54 | 78 | 470453 | 37305 | 621. 45 |
| 34 | 67452 | 2512 | 41. 51 | 79 | 514455 | 44002 | 733. 22 |
| 35 | 70022 | 2570 | 42. 50 | 80 | 567128 | 52673 | 877. 53 |
| 36 | 72654 | 2632 | 43. 52 | 81 | 631375 | 64247 | 1070. 47 |
| 37 | 75356 | 2702 | 45. 2 | 82 | 711537 | 80162 | 1336. 2 |
| 38 | 78129 | 2773 | 46. 13 | 83 | 814435 | 102898 | 1714. 18 |
| 39 | 80978 | 2849 | 47. 29 | 84 | 951436 | 137001 | 2283. 21 |
| 40 | 83909 | 2931 | 48. 51 | 85 | 1143005 | 191569 | 3192. 49 |
| 41 | 86929 | 3020 | 50. 20 | 86 | 1430067 | 287062 | 4784. 22 |
| 42 | 90040 | 3111 | 51. 51 | 87 | 1908113 | 478046 | 7967. 26 |
| 43 | 63252 | 3212 | 53. 52 | 88 | 2863625 | 955512 | 15925. 12 |
| 44 | 96571 | 3319 | 55. 19 | 89 | 5728995 | 2865370 | 47756. 10 |
| 45 | 100000 | 3429 | 57. 9 | 90 | Infinitum. | Infinitum. | Infinitum. |

89. 15. . . . . 7638998.
89. 30. . . . 11458872.
89. 45. . . 22918163.
89. 55. . . 68754439.
89. 59. . 343772546.

Figure 2: Maurolico's original table of tangents [1] (source: Österreichische Nationalbibliothek).

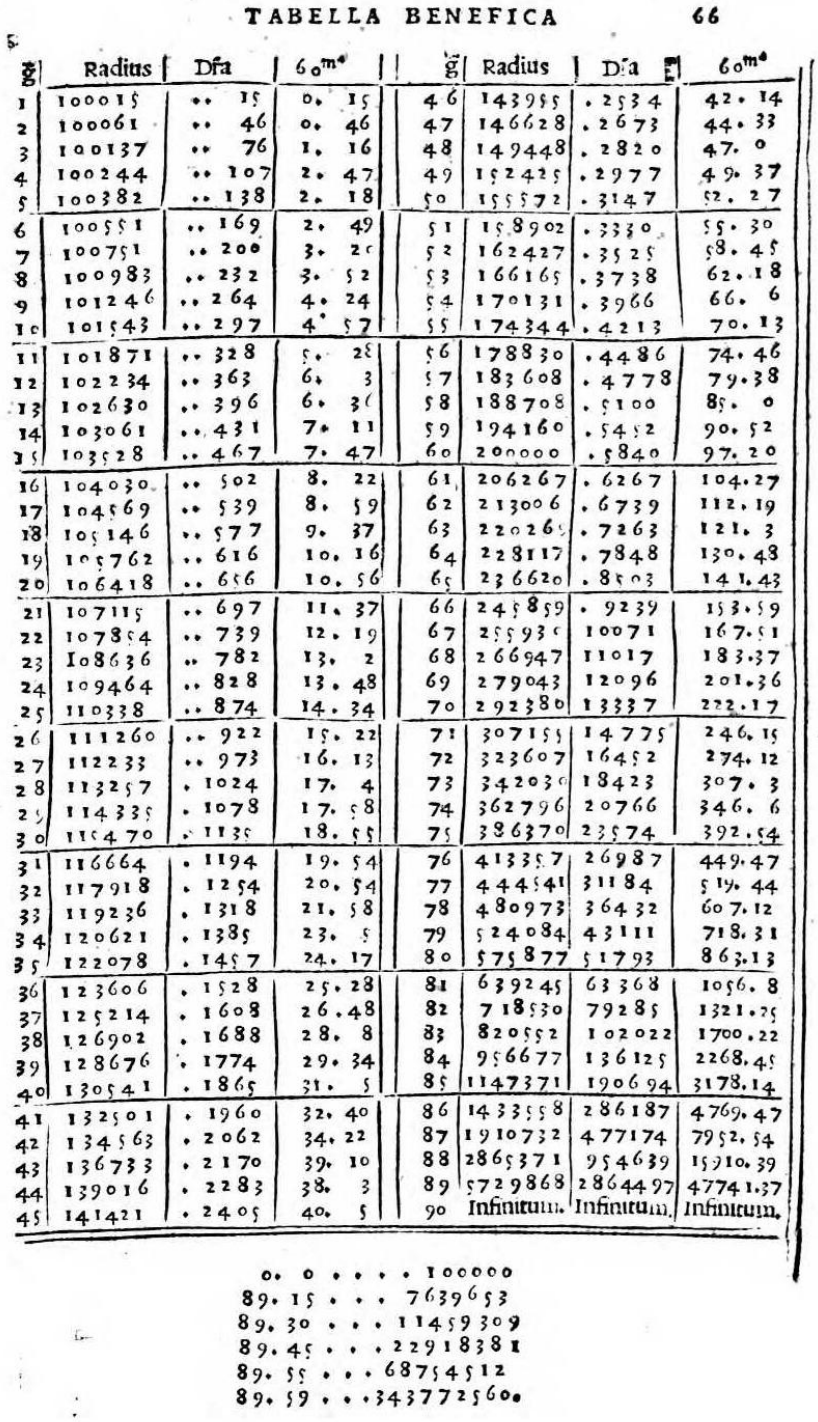

TABELLA BENEFICA 66

| g | Radius | Dfa | 60me | g | Radius | Dfa | 60me |
|---|---|---|---|---|---|---|---|
| 1 | 100015 | .. 15 | 0. 15 | 46 | 143955 | .2534 | 42. 14 |
| 2 | 100061 | .. 46 | 0. 46 | 47 | 146628 | .2673 | 44. 33 |
| 3 | 100137 | .. 76 | 1. 16 | 48 | 149448 | .2820 | 47. 0 |
| 4 | 100244 | .. 107 | 2. 47 | 49 | 152425 | .2977 | 49. 37 |
| 5 | 100382 | .. 138 | 2. 18 | 50 | 155572 | .3147 | 52. 27 |
| 6 | 100551 | .. 169 | 2. 49 | 51 | 158902 | .3330 | 55. 30 |
| 7 | 100751 | .. 200 | 3. 20 | 52 | 162427 | .3525 | 58. 45 |
| 8 | 100983 | .. 232 | 3. 52 | 53 | 166165 | .3738 | 62. 18 |
| 9 | 101246 | .. 264 | 4. 24 | 54 | 170131 | .3966 | 66. 6 |
| 10 | 101543 | .. 297 | 4. 57 | 55 | 174344 | .4213 | 70. 13 |
| 11 | 101871 | .. 328 | 5. 28 | 56 | 178830 | .4486 | 74. 46 |
| 12 | 102234 | .. 363 | 6. 3 | 57 | 183608 | .4778 | 79. 38 |
| 13 | 102630 | .. 396 | 6. 36 | 58 | 188708 | .5100 | 85. 0 |
| 14 | 103061 | .. 431 | 7. 11 | 59 | 194160 | .5452 | 90. 52 |
| 15 | 103528 | .. 467 | 7. 47 | 60 | 200000 | .5840 | 97. 20 |
| 16 | 104030 | .. 502 | 8. 22 | 61 | 206267 | .6267 | 104. 27 |
| 17 | 104569 | .. 539 | 8. 59 | 62 | 213006 | .6739 | 112. 19 |
| 18 | 105146 | .. 577 | 9. 37 | 63 | 220265 | .7263 | 121. 3 |
| 19 | 105762 | .. 616 | 10. 16 | 64 | 228117 | .7848 | 130. 48 |
| 20 | 106418 | .. 656 | 10. 56 | 65 | 236620 | .8503 | 141. 43 |
| 21 | 107115 | .. 697 | 11. 37 | 66 | 245859 | .9239 | 153. 59 |
| 22 | 107854 | .. 739 | 12. 19 | 67 | 255935 | 10071 | 167. 51 |
| 23 | 108636 | .. 782 | 13. 2 | 68 | 266947 | 11017 | 183. 37 |
| 24 | 109464 | .. 828 | 13. 48 | 69 | 279043 | 12096 | 201. 36 |
| 25 | 110338 | .. 874 | 14. 34 | 70 | 292380 | 13337 | 222. 17 |
| 26 | 111260 | .. 922 | 15. 22 | 71 | 307155 | 14775 | 246. 15 |
| 27 | 112233 | .. 973 | 16. 13 | 72 | 323607 | 16452 | 274. 12 |
| 28 | 113257 | . 1024 | 17. 4 | 73 | 342030 | 18423 | 307. 3 |
| 29 | 114335 | . 1078 | 17. 58 | 74 | 362796 | 20766 | 346. 6 |
| 30 | 115470 | . 1135 | 18. 55 | 75 | 386370 | 23574 | 392. 54 |
| 31 | 116664 | . 1194 | 19. 54 | 76 | 413357 | 26987 | 449. 47 |
| 32 | 117918 | . 1254 | 20. 54 | 77 | 444541 | 31184 | 519. 44 |
| 33 | 119236 | . 1318 | 21. 58 | 78 | 480973 | 36432 | 607. 12 |
| 34 | 120621 | . 1385 | 23. 5 | 79 | 524084 | 43111 | 718. 31 |
| 35 | 122078 | . 1457 | 24. 17 | 80 | 575877 | 51793 | 863. 13 |
| 36 | 123606 | . 1528 | 25. 28 | 81 | 639245 | 63368 | 1056. 8 |
| 37 | 125214 | . 1608 | 26. 48 | 82 | 718530 | 79285 | 1321. 25 |
| 38 | 126902 | . 1688 | 28. 8 | 83 | 820552 | 102022 | 1700. 22 |
| 39 | 128676 | . 1774 | 29. 34 | 84 | 956677 | 136125 | 2268. 45 |
| 40 | 130541 | . 1865 | 31. 5 | 85 | 1147371 | 190694 | 3178. 14 |
| 41 | 132501 | . 1960 | 32. 40 | 86 | 1433558 | 286187 | 4769. 47 |
| 42 | 134563 | . 2062 | 34. 22 | 87 | 1910732 | 477174 | 7952. 54 |
| 43 | 136733 | . 2170 | 39. 10 | 88 | 2865371 | 954639 | 15910. 39 |
| 44 | 139016 | . 2283 | 38. 3 | 89 | 5729868 | 2864497 | 47741. 37 |
| 45 | 141421 | . 2405 | 40. 5 | 90 | Infinitum. | Infinitum. | Infinitum. |

0. 0 . . . . . 100000
89. 15 . . . 7639653
89. 30 . . . 11459309
89. 45 . . . 22918381
89. 55 . . . 68754512
89. 59 . . . 343772560.

Figure 3: Maurolico's original table of secants [1] (source: Österreichische Nationalbibliothek).

# References

[1] Francesco Maurolico, editor. *Theodosii Sphaericorvm Elementorvm Libri. III, etc.* Messina: Petrus Spira, 1558.

[2] Denis Roegel. A survey of the main fundamental European trigonometric tables printed in the 15th and 16th centuries. Technical report, LORIA, Nancy, 2021.

TABELLA SINVS RECTI.

| $\tilde{g}$ | Sinus. | Dfa | $6^{ma}$ | $\tilde{g}$ | Sinus. | Dfa | $6^{ma}$ |
|---|---|---|---|---|---|---|---|
| 1 | 1745 | 1745 | 29. 5 | 46 | 71934 | 1223 | 20.23 |
| 2 | 3490 | 1745 | 29. 5 | 47 | 73135 | 1201 | 20. 1 |
| 3 | 5234 | 1744 | 29. 4 | 48 | 74314 | 1179 | 19.39 |
| 4 | 6976 | 1742 | 29. 2 | 49 | 75471 | 1157 | 19.17 |
| 5 | 8716 | 1740 | 29. 0 | 50 | 76604 | 1133 | 18.53 |
| 6 | 10453 | 1737 | 28.57 | 51 | 77715 | 1111 | 18.31 |
| 7 | 12187 | 1734 | 28.54 | 52 | 78801 | 1086 | 18. 6 |
| 8 | 13917 | 1730 | 28.50 | 53 | 79864 | 1063 | 17.43 |
| 9 | 15643 | 1726 | 28.46 | 54 | 80902 | 1038 | 17.18 |
| 10 | 17365 | 1722 | 28.42 | 55 | 81915 | 1013 | 16.53 |
| 11 | 19081 | 1716 | 28.36 | 56 | 82904 | 989 | 16.29 |
| 12 | 20791 | 1710 | 28.30 | 57 | 83867 | 963 | 16. 3 |
| 13 | 22495 | 1704 | 28.24 | 58 | 84805 | 938 | 15.38 |
| 14 | 24192 | 1697 | 28.17 | 59 | 85717 | 912 | 15.12 |
| 15 | 25882 | 1690 | 28.10 | 60 | 86603 | 886 | 14.46 |
| 16 | 27564 | 1682 | 28. 2 | 61 | 87462 | 859 | 14.19 |
| 17 | 29237 | 1673 | 27.53 | 62 | 88295 | 833 | 13.53 |
| 18 | 30902 | 1665 | 27.45 | 63 | 89101 | 806 | 13.26 |
| 19 | 32557 | 1655 | 27.35 | 64 | 89879 | 778 | 12.58 |
| 20 | 34202 | 1645 | 27.25 | 65 | 90631 | 752 | 12.32 |
| 21 | 35837 | 1635 | 27.15 | 66 | 91355 | 724 | 12. 4 |
| 22 | 37461 | 1624 | 27. 4 | 67 | 92050 | 695 | 11.35 |
| 23 | 39073 | 1612 | 26.52 | 68 | 92718 | 668 | 11. 8 |
| 24 | 40674 | 1601 | 26.41 | 69 | 93358 | 640 | 10.40 |
| 25 | 42262 | 1588 | 26.28 | 70 | 93969 | 611 | 10.11 |
| 26 | 43837 | 1575 | 26.15 | 71 | 94552 | 583 | 9.43 |
| 27 | 45399 | 1562 | 26. 2 | 72 | 95106 | 554 | 9.14 |
| 28 | 46947 | 1548 | 25.48 | 73 | 95630 | 524 | 8.44 |
| 29 | 48481 | 1534 | 25.34 | 74 | 96126 | 496 | 8.16 |
| 30 | 50000 | 1519 | 25.19 | 75 | 96593 | 467 | 7.47 |
| 31 | 51504 | 1504 | 25. 4 | 76 | 97030 | 437 | 7.17 |
| 32 | 52992 | 1488 | 24.48 | 77 | 97437 | 407 | 6.47 |
| 33 | 54464 | 1472 | 24.32 | 78 | 97815 | 378 | 6.18 |
| 34 | 55919 | 1455 | 24.15 | 79 | 98163 | 348 | 5.48 |
| 35 | 57358 | 1439 | 23.59 | 80 | 98481 | 318 | 5.18 |
| 36 | 58779 | 1421 | 23.41 | 81 | 98769 | 288 | 4.48 |
| 37 | 60182 | 1403 | 23.23 | 82 | 99027 | 258 | 4.18 |
| 38 | 61566 | 1384 | 23. 4 | 83 | 99255 | 228 | 3.48 |
| 39 | 62932 | 1366 | 22.46 | 84 | 99452 | 197 | 3.17 |
| 40 | 64279 | 1347 | 22.27 | 85 | 99619 | 167 | 2.47 |
| 41 | 65606 | 1327 | 22. 7 | 86 | 99756 | 137 | 2.17 |
| 42 | 66913 | 1307 | 21.47 | 87 | 99863 | 107 | 1.47 |
| 43 | 68200 | 1287 | 21.27 | 88 | 99939 | 76 | 1.16 |
| 44 | 69466 | 1266 | 21. 6 | 89 | 99985 | 46 | 0.46 |
| 45 | 70711 | 1245 | 20.45 | 90 | 100000 | 15 | 0.15 |

TABELLA FOECVNDA.

| g̃ | Vmbra | Dfa | 6ma | g̃ | Vmbra | Dfa | 6ma |
|---|---|---|---|---|---|---|---|
| 1 | 1746 | 1746 | 29. 6 | 46 | 103553 | 3553 | 59.13 |
| 2 | 3492 | 1746 | 29. 6 | 47 | 107237 | 3684 | 61.24 |
| 3 | 5241 | 1749 | 29. 9 | 48 | 111061 | 3824 | 63.44 |
| 4 | 6993 | 1752 | 29.12 | 49 | 115037 | 3976 | 66.16 |
| 5 | 8749 | 1756 | 29.16 | 50 | 119175 | 4138 | 68.58 |
| 6 | 10510 | 1761 | 29.21 | 51 | 123490 | 4315 | 71.55 |
| 7 | 12278 | 1768 | 29.28 | 52 | 127994 | 4504 | 75. 4 |
| 8 | 14054 | 1776 | 29.36 | 53 | 132704 | 4710 | 78.30 |
| 9 | 15838 | 1784 | 29.44 | 54 | 137638 | 4934 | 82.14 |
| 10 | 17633 | 1795 | 29.55 | 55 | 142815 | 5177 | 86.17 |
| 11 | 19438 | 1805 | 30. 5 | 56 | 148256 | 5441 | 90.41 |
| 12 | 21256 | 1818 | 30.18 | 57 | 153986 | 5730 | 95.30 |
| 13 | 23087 | 1831 | 30.31 | 58 | 160033 | 6047 | 100.47 |
| 14 | 24933 | 1846 | 30.46 | 59 | 166428 | 6395 | 106.35 |
| 15 | 26795 | 1862 | 31. 2 | 60 | 173205 | 6777 | 112.57 |
| 16 | 28675 | 1880 | 31.20 | 61 | 180405 | 7200 | 120. 0 |
| 17 | 30573 | 1898 | 31.38 | 62 | 188073 | 7668 | 127.48 |
| 18 | 32492 | 1919 | 31.59 | 63 | 196261 | 8188 | 136.28 |
| 19 | 34433 | 1941 | 32.21 | 64 | 205030 | 8769 | 146. 9 |
| 20 | 36397 | 1964 | 32.44 | 65 | 214451 | 9421 | 157. 1 |
| 21 | 38386 | 1989 | 33. 9 | 66 | 224604 | 10153 | 169.13 |
| 22 | 40403 | 2017 | 33.37 | 67 | 235585 | 10981 | 183. 1 |
| 23 | 42447 | 2044 | 34. 4 | 68 | 247509 | 11924 | 198.44 |
| 24 | 44523 | 2076 | 34.36 | 69 | 260509 | 13000 | 216.40 |
| 25 | 46631 | 2108 | 35. 8 | 70 | 274748 | 14239 | 237.19 |
| 26 | 48773 | 2142 | 35.42 | 71 | 290421 | 15673 | 261.13 |
| 27 | 50953 | 2180 | 36.20 | 72 | 307768 | 17347 | 289. 7 |
| 28 | 53171 | 2218 | 36.58 | 73 | 327085 | 19317 | 321.57 |
| 29 | 55431 | 2260 | 37.40 | 74 | 348741 | 21656 | 360.56 |
| 30 | 57735 | 2304 | 38.24 | 75 | 373205 | 24464 | 407.44 |
| 31 | 60086 | 2351 | 39.11 | 76 | 401078 | 27873 | 464.33 |
| 32 | 62487 | 2401 | 40. 1 | 77 | 433148 | 32070 | 534.30 |
| 33 | 64941 | 2454 | 40.54 | 78 | 470463 | 37315 | 621.55 |
| 34 | 67451 | 2510 | 41.50 | 79 | 514455 | 43992 | 733.12 |
| 35 | 70021 | 2570 | 42.50 | 80 | 567128 | 52673 | 877.53 |
| 36 | 72654 | 2633 | 43.53 | 81 | 631375 | 64247 | 1070.47 |
| 37 | 75355 | 2701 | 45. 1 | 82 | 711537 | 80162 | 1336. 2 |
| 38 | 78129 | 2774 | 46.14 | 83 | 814435 | 102898 | 1714.58 |
| 39 | 80978 | 2849 | 47.29 | 84 | 951436 | 137001 | 2283.21 |
| 40 | 83910 | 2932 | 48.52 | 85 | 1143005 | 191569 | 3192.49 |
| 41 | 86929 | 3019 | 50.19 | 86 | 1430067 | 287062 | 4784.22 |
| 42 | 90040 | 3111 | 51.51 | 87 | 1908114 | 478047 | 7967.27 |
| 43 | 93252 | 3212 | 53.32 | 88 | 2863625 | 955511 | 15925.11 |
| 44 | 96569 | 3317 | 55.17 | 89 | 5728996 | 2865371 | 47756.11 |
| 45 | 100000 | 3431 | 57.11 | 90 | Infinitum. | Infinitum. | Infinitum. |

89.15 . . . . . 7639001
89.30 . . . . 11458865
89.45 . . . . 22918166
89.55 . . . . 68754887
89.59 . . . 343774662

TABELLA BENEFICA.

| g̃ | Radius | Dfa | 60$^{ma}$ | g̃ | Radius | Dfa | 60$^{ma}$ |
|---|---|---|---|---|---|---|---|
| 1 | 100015 | 15 | 0.15 | 46 | 143956 | 2535 | 42.15 |
| 2 | 100061 | 46 | 0.46 | 47 | 146628 | 2672 | 44.32 |
| 3 | 100137 | 76 | 1.16 | 48 | 149448 | 2820 | 47. 0 |
| 4 | 100244 | 107 | 1.47 | 49 | 152425 | 2977 | 49.37 |
| 5 | 100382 | 138 | 2.18 | 50 | 155572 | 3147 | 52.27 |
| 6 | 100551 | 169 | 2.49 | 51 | 158902 | 3330 | 55.30 |
| 7 | 100751 | 200 | 3.20 | 52 | 162427 | 3525 | 58.45 |
| 8 | 100983 | 232 | 3.52 | 53 | 166164 | 3737 | 62.17 |
| 9 | 101247 | 264 | 4.24 | 54 | 170130 | 3966 | 66. 6 |
| 10 | 101543 | 296 | 4.56 | 55 | 174345 | 4215 | 70.15 |
| 11 | 101872 | 329 | 5.29 | 56 | 178829 | 4484 | 74.44 |
| 12 | 102234 | 362 | 6. 2 | 57 | 183608 | 4779 | 79.39 |
| 13 | 102630 | 396 | 6.36 | 58 | 188708 | 5100 | 85. 0 |
| 14 | 103061 | 431 | 7.11 | 59 | 194160 | 5452 | 90.52 |
| 15 | 103528 | 467 | 7.47 | 60 | 200000 | 5840 | 97.20 |
| 16 | 104030 | 502 | 8.22 | 61 | 206267 | 6267 | 104.27 |
| 17 | 104569 | 539 | 8.59 | 62 | 213005 | 6738 | 112.18 |
| 18 | 105146 | 577 | 9.37 | 63 | 220269 | 7264 | 121. 4 |
| 19 | 105762 | 616 | 10.16 | 64 | 228117 | 7848 | 130.48 |
| 20 | 106418 | 656 | 10.56 | 65 | 236620 | 8503 | 141.43 |
| 21 | 107114 | 696 | 11.36 | 66 | 245859 | 9239 | 153.59 |
| 22 | 107853 | 739 | 12.19 | 67 | 255930 | 10071 | 167.51 |
| 23 | 108636 | 783 | 13. 3 | 68 | 266947 | 11017 | 183.37 |
| 24 | 109464 | 828 | 13.48 | 69 | 279043 | 12096 | 201.36 |
| 25 | 110338 | 874 | 14.34 | 70 | 292380 | 13337 | 222.17 |
| 26 | 111260 | 922 | 15.22 | 71 | 307155 | 14775 | 246.15 |
| 27 | 112233 | 973 | 16.13 | 72 | 323607 | 16452 | 274.12 |
| 28 | 113257 | 1024 | 17. 4 | 73 | 342030 | 18423 | 307. 3 |
| 29 | 114335 | 1078 | 17.58 | 74 | 362796 | 20766 | 346. 6 |
| 30 | 115470 | 1135 | 18.55 | 75 | 386370 | 23574 | 392.54 |
| 31 | 116663 | 1193 | 19.53 | 76 | 413357 | 26987 | 449.47 |
| 32 | 117918 | 1255 | 20.55 | 77 | 444541 | 31184 | 519.44 |
| 33 | 119236 | 1318 | 21.58 | 78 | 480973 | 36432 | 607.12 |
| 34 | 120622 | 1386 | 23. 6 | 79 | 524084 | 43111 | 718.31 |
| 35 | 122077 | 1455 | 24.15 | 80 | 575877 | 51793 | 863.13 |
| 36 | 123607 | 1530 | 25.30 | 81 | 639245 | 63368 | 1056. 8 |
| 37 | 125214 | 1607 | 26.47 | 82 | 718530 | 79285 | 1321.25 |
| 38 | 126902 | 1688 | 28. 8 | 83 | 820551 | 102021 | 1700.21 |
| 39 | 128676 | 1774 | 29.34 | 84 | 956677 | 136126 | 2268.46 |
| 40 | 130541 | 1865 | 31. 5 | 85 | 1147371 | 190694 | 3178.14 |
| 41 | 132501 | 1960 | 32.40 | 86 | 1433559 | 286188 | 4769.48 |
| 42 | 134563 | 2062 | 34.22 | 87 | 1910732 | 477173 | 7952.53 |
| 43 | 136733 | 2170 | 36.10 | 88 | 2865371 | 954639 | 15910.39 |
| 44 | 139016 | 2283 | 38. 3 | 89 | 5729869 | 2864498 | 47741.38 |
| 45 | 141421 | 2405 | 40. 5 | 90 | Infinitum. | Infinitum. | Infinitum. |

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89.15 . . . . . 7639655
89.30 . . . . 11459301
89.45 . . . . 22918385
89.55 . . . . 68754959
89.59 . . . 343774676