



# A reconstruction of Maurolico's tables of sines, tangents and secants (1558)

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# A reconstruction of Maurolico's tables of sines, tangents and secants (1558)

Denis Roegel

2 September 2021

(last version: 2 September 2021)



## Introduction

This document gives exact (modern) reconstructions of Francesco Maurolico's trigonometric tables published in 1558 [1] (figures 1, 2, and 3). There are three tables, and each gives the value of a function (sine, tangent, secant) for each degree of the quadrant, and with a radius  $R = 100000$ . The differences are given for each value, both in decimal and sexagesimal. For instance, the difference given for  $\text{Sin } 51^\circ = R \sin 51^\circ$  is 1111, because  $\text{Sin } 51^\circ - \text{Sin } 50^\circ = 1111$ . And the sexagesimal corresponding value is 18.31, because  $1111 = 18 \cdot 60 + 31$ .

For the tangents and secants, Maurolico also gives their values for five or six more values at the bottom of the pages. In my survey [2], I describe the context of these tables and how their values have been computed.

TABELLA SINVS RECTI.

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$\bar{g}$	Sinus.	Dfa	$6^{\text{ma}}$	$\bar{g}$	Sinus.	Dfa	$6^{\text{ma}}$
1	1745	1745	29. 5	46	71934	1223.	20. 23
2	3490	1745	29. 5	47	73135	1201.	20. 1
3	5234	1744	29. 4	48	74315	1180.	19. 40
4	6976	1742	29. 2	49	75471	1156.	19. 16
5	8716	1740	29. 0	50	76604	1133.	18. 53
6	10453	1737	28. 57	51	77715	1111.	18. 31
7	12187	1734	28. 54	52	78801	1086	18. 6
8	13917	1730	28. 50	53	79864	1063	17. 43
9	15643	1726	28. 46	54	80902	1038	17. 18
10	17365	1722	28. 42	55	81915	1013	16. 53
11	19081	1716	28. 36	56	82904	989	16. 29
12	20791	1710	28. 30	57	83867	963	16. 3
13	22495	1704	28. 24	58	84805	938	15. 38
14	24192	1697	28. 17	59	85717	912	15. 21
15	25882	1690	28. 10	60	86603	886	14. 46
16	27564	1682	28. 2	61	87462	859	14. 19
17	29237	1673	27. 53	62	88295	833	13. 53
18	30902	1665	27. 45	63	89101	806	13. 26
19	32557	1655	27. 35	64	89879	778	12. 58
20	34202	1645	27. 25	65	90631	752	12. 32
21	35837	1635	27. 15	66	91355	724	12. 4
22	37461	1624	27. 4	67	92051	696	11. 36
23	39073	1612	26. 52	68	92718	667	11. 7
24	40674	1601	26. 41	69	93358	640	10. 40
25	42262	1588	26. 28	70	93969	611	10. 11
26	43837	1575	26. 15	71	94552	583	9. 43
27	45399	1562	26. 2	72	95106	554	9. 14
28	46947	1548	25. 48	73	95631	525	8. 45
29	48481	1534	25. 34	74	96126	495	8. 15
30	50000	1519	25. 19	75	96593	467	7. 47
31	51504	1504	25. 4	76	97030	437	7. 17
32	52992	1488	24. 48	77	97437	407	6. 47
33	54464	1472	24. 32	78	97815	378	6. 18
34	55919	1455	24. 15	79	98163	348	5. 48
35	57358	1439	23. 59	80	98481	318	5. 18
36	58779	1421	23. 41	81	98769	288	4. 48
37	60182	1403	23. 23	82	99027	258	4. 18
38	61566	1384	23. 4	83	99255	228	3. 48
39	62932	1366	22. 46	84	99452	197	3. 17
40	64279	1347	22. 27	85	99620	168	2. 48
41	65606	1327	22. 7	86	99756	139	2. 16
42	66913	1307	21. 47	87	99863	107	1. 47
43	68200	1287	21. 27	88	99939	76	1. 6
44	69466	1266	21. 6	89	99985	46	0. 46
45	70711	1245	20. 45	90	100000	15	0. 15

Figure 1: Maurolico's original table of sines [1] (source: Österreichische Nationalbibliothek).

TABELLA FOECVNDA.

$\bar{g}$	Vimbra	Dfa	$6^m$	$\bar{g}$	Vimbra	Dfa	$6^m$
1	1745.	1745	29. 5	46	103551	3551.	59. 11
2	3492	1747	29. 7	47	107236	3685.	61. 25
3	5241	1749	29. 9	48	111062	3826.	63. 46
4	6992	1751	29. 11	49	115037	3975.	66. 15
5	8748	1756	29. 16	50	119197	4160.	69. 20
6	10510	1762	29. 22	51	123491	4294	71. 34
7	12278	1768	29. 28	52	127994	4593	75. 5
8	14053	1775	29. 35	53	132704	4710	78. 30
9	15838	1785	29. 45	54	137639	4935	82. 15
10	17633	1795	29. 55	55	142813	5174	86. 14
11	19439	1806	30. 6	56	148253	5440	90. 40
12	21256	1817	30. 17	57	153987	5734	95. 34
13	23087	1831	30. 31	58	160033	6046	100. 46
14	24932	1845	30. 45	59	166429	6396	106. 36
15	26794	1862	31. 2	60	173205	6776	112. 56
16	28674	1880	31. 20	61	180405	7200	120. 0
17	30573	1899	31. 39	62	188073	7668	127. 48
18	32492	1919	31. 59	63	196261	8188	136. 28
19	34433	1941	32. 21	64	205030	8769	146. 9
20	36396	1963	32. 43	65	214451	1421	157. 1
21	38387	1991	33. 11	66	224603	10152	169. 12
22	40402	2015	33. 35	67	235585	10982	183. 2
23	42448	2046	34. 6	68	247509	11924	198. 44
24	44522	2074	34. 34	69	260509	13000	216. 40
25	46631	2109	35. 9	70	274747	14238	237. 18
26	48772	2141	35. 41	71	290421	15674	261. 14
27	50952	2180	36. 20	72	307768	17347	289. 7
28	53170	2218	36. 58	73	327084	19316	321. 56
29	55432	2262	37. 42	74	348742	21658	360. 58
30	57735	2303	38. 23	75	373205	24463	407. 43
31	60086	2351	39. 11	76	401078	27873	464. 33
32	62486	2400	40. 0	77	433148	32070	534. 30
33	64940	2454	40. 54	78	470453	37305	621. 45
34	67452	2512	41. 51	79	514455	44002	733. 22
35	70022	2570	42. 50	80	567128	52673	877. 53
36	72654	2632	43. 52	81	631375	64247	1070. 47
37	75356	2702	45. 2	82	711537	86162	1336. 2
38	78129	2773	46. 13	83	814435	102898	1714. 18
39	80978	2849	47. 29	84	951456	127601	2285. 21
40	83909	2931	48. 51	85	1143005	191569	3192. 49
41	86929	3020	50. 20	86	1430067	287062	4784. 22
42	90040	3118	51. 51	87	1908113	478046	7967. 26
43	63252	3212	53. 52	88	2863625	955512	15925. 12
44	96571	3319	55. 19	89	5728995	2865370	47756. 10
45	100000	3429	57. 9	90	Infinitum.	Infinitum.	Infinitum.

89. 15. . . . . 7638998.

89. 30. . . . . 11458872.

89. 45. . . . 22918163.

89. 55. . . . 68754439.

89. 59. . . 343772546.

Figure 2: Maurolico's original table of tangents [1] (source: Österreichische Nationalbibliothek).

## TABELLA BENEFICA

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$\frac{g}{g}$	Radius	Dra	$60^m^a$	$\frac{g}{g}$	Radius	Dra	$60^m^a$
1	100015	.. 15	0. 15	46	143985	. 2534	42. 14
2	100061	.. 46	0. 46	47	146628	. 2673	44. 33
3	100137	.. 76	1. 16	48	149448	. 2820	47. 0
4	100244	.. 107	2. 47	49	152425	. 2977	49. 37
5	100382	.. 138	2. 18	50	155572	. 3147	52. 27
6	100551	.. 169	2. 49	51	158902	. 3330	55. 30
7	100751	.. 200	3. 20	52	162427	. 3525	58. 45
8	100983	.. 232	3. 52	53	166165	. 3738	62. 18
9	101246	.. 264	4. 24	54	170131	. 3966	66. 6
10	101543	.. 297	4. 57	55	174344	. 4213	70. 13
11	101871	.. 328	5. 28	56	178830	. 4486	74. 46
12	102234	.. 363	6. 3	57	183608	. 4778	79. 38
13	102630	.. 396	6. 31	58	188708	. 5100	83. 0
14	103061	.. 431	7. 11	59	194160	. 5452	90. 52
15	103528	.. 467	7. 47	60	200000	. 5840	97. 20
16	104030	.. 502	8. 22	61	206267	. 6267	104. 27
17	104569	.. 539	8. 59	62	213006	. 6739	112. 19
18	105146	.. 577	9. 37	63	220265	. 7263	121. 3
19	105762	.. 616	10. 16	64	228117	. 7848	130. 43
20	106418	.. 656	10. 56	65	236620	. 8503	141. 43
21	107115	.. 697	11. 37	66	245859	. 9239	153. 59
22	107854	.. 739	12. 19	67	255930	10071	167. 51
23	108636	.. 782	13. 2	68	266947	11017	183. 37
24	109464	.. 828	13. 48	69	279043	12096	201. 36
25	110338	.. 874	14. 34	70	292380	13337	222. 17
26	111260	.. 922	15. 22	71	307155	14775	246. 15
27	112233	.. 973	16. 13	72	323607	16452	274. 12
28	113257	.. 1024	17. 4	73	342030	18423	307. 3
29	114335	.. 1078	17. 58	74	362796	20766	346. 6
30	115470	.. 1130	18. 55	75	386370	23574	392. 54
31	116664	.. 1194	19. 54	76	413357	26987	449. 47
32	117918	.. 1254	20. 54	77	444541	31184	516. 44
33	119236	.. 1318	21. 58	78	480973	36432	607. 12
34	120621	.. 1385	23. 5	79	524084	43111	718. 31
35	122078	.. 1457	24. 17	80	575877	51793	863. 13
36	123606	.. 1528	25. 28	81	639245	63368	1056. 8
37	125214	.. 1608	26. 48	82	718530	79285	1321. 75
38	126902	.. 1688	28. 8	83	820552	102022	1700. 22
39	128676	.. 1774	29. 34	84	956677	136125	2268. 45
40	130541	.. 1865	31. 5	85	1147371	190694	3178. 14
41	132501	.. 1960	32. 40	86	143358	286187	4769. 47
42	134563	.. 2062	34. 22	87	1910732	477174	7952. 54
43	136733	.. 2170	39. 10	88	2865371	954639	1510. 39
44	139016	.. 2283	38. 3	89	5729868	2864497	4774. 137
45	141421	.. 2405	40. 5	90	Infinatum.	Infinatum.	Infinatum.

$$\begin{aligned}
 & 0. 0 . . . . 100000 \\
 & 89. 15 . . . . 7639653 \\
 & 89. 30 . . . . 11459309 \\
 & 89. 45 . . . . 22918381 \\
 & 89. 55 . . . . 68754512 \\
 & 89. 59 . . . . 343772560
 \end{aligned}$$

Figure 3: Maurolico's original table of secants [1] (source: Österreichische Nationalbibliothek).

## References

- [1] Francesco Maurolico, editor. *Theodosii Sphaericorum Elementorum Libri. III, etc.* Messina: Petrus Spira, 1558.
- [2] Denis Roegel. A survey of the main fundamental European trigonometric tables printed in the 15th and 16th centuries. Technical report, LORIA, Nancy, 2021.

T A B E L L A S I N V S R E C T I .

$\tilde{g}$	Sinus.	Dfa	$6^m^a$	$\tilde{g}$	Sinus.	Dfa	$6^m^a$
1	1745	1745	29. 5	46	71934	1223	20.23
2	3490	1745	29. 5	47	73135	1201	20. 1
3	5234	1744	29. 4	48	74314	1179	19.39
4	6976	1742	29. 2	49	75471	1157	19.17
5	8716	1740	29. 0	50	76604	1133	18.53
6	10453	1737	28.57	51	77715	1111	18.31
7	12187	1734	28.54	52	78801	1086	18. 6
8	13917	1730	28.50	53	79864	1063	17.43
9	15643	1726	28.46	54	80902	1038	17.18
10	17365	1722	28.42	55	81915	1013	16.53
11	19081	1716	28.36	56	82904	989	16.29
12	20791	1710	28.30	57	83867	963	16. 3
13	22495	1704	28.24	58	84805	938	15.38
14	24192	1697	28.17	59	85717	912	15.12
15	25882	1690	28.10	60	86603	886	14.46
16	27564	1682	28. 2	61	87462	859	14.19
17	29237	1673	27.53	62	88295	833	13.53
18	30902	1665	27.45	63	89101	806	13.26
19	32557	1655	27.35	64	89879	778	12.58
20	34202	1645	27.25	65	90631	752	12.32
21	35837	1635	27.15	66	91355	724	12. 4
22	37461	1624	27. 4	67	92050	695	11.35
23	39073	1612	26.52	68	92718	668	11. 8
24	40674	1601	26.41	69	93358	640	10.40
25	42262	1588	26.28	70	93969	611	10.11
26	43837	1575	26.15	71	94552	583	9.43
27	45399	1562	26. 2	72	95106	554	9.14
28	46947	1548	25.48	73	95630	524	8.44
29	48481	1534	25.34	74	96126	496	8.16
30	50000	1519	25.19	75	96593	467	7.47
31	51504	1504	25. 4	76	97030	437	7.17
32	52992	1488	24.48	77	97437	407	6.47
33	54464	1472	24.32	78	97815	378	6.18
34	55919	1455	24.15	79	98163	348	5.48
35	57358	1439	23.59	80	98481	318	5.18
36	58779	1421	23.41	81	98769	288	4.48
37	60182	1403	23.23	82	99027	258	4.18
38	61566	1384	23. 4	83	99255	228	3.48
39	62932	1366	22.46	84	99452	197	3.17
40	64279	1347	22.27	85	99619	167	2.47
41	65606	1327	22. 7	86	99756	137	2.17
42	66913	1307	21.47	87	99863	107	1.47
43	68200	1287	21.27	88	99939	76	1.16
44	69466	1266	21. 6	89	99985	46	0.46
45	70711	1245	20.45	90	100000	15	0.15

## T A B E L L A F O E C V N D A.

$\tilde{g}$	Vmbra	Dfa	$6^m$ <sup>a</sup>	$\tilde{g}$	Vmbra	Dfa	$6^m$ <sup>a</sup>
1	1746	1746	29. 6	46	103553	3553	59.13
2	3492	1746	29. 6	47	107237	3684	61.24
3	5241	1749	29. 9	48	111061	3824	63.44
4	6993	1752	29.12	49	115037	3976	66.16
5	8749	1756	29.16	50	119175	4138	68.58
6	10510	1761	29.21	51	123490	4315	71.55
7	12278	1768	29.28	52	127994	4504	75. 4
8	14054	1776	29.36	53	132704	4710	78.30
9	15838	1784	29.44	54	137638	4934	82.14
10	17633	1795	29.55	55	142815	5177	86.17
11	19438	1805	30. 5	56	148256	5441	90.41
12	21256	1818	30.18	57	153986	5730	95.30
13	23087	1831	30.31	58	160033	6047	100.47
14	24933	1846	30.46	59	166428	6395	106.35
15	26795	1862	31. 2	60	173205	6777	112.57
16	28675	1880	31.20	61	180405	7200	120. 0
17	30573	1898	31.38	62	188073	7668	127.48
18	32492	1919	31.59	63	196261	8188	136.28
19	34433	1941	32.21	64	205030	8769	146. 9
20	36397	1964	32.44	65	214451	9421	157. 1
21	38386	1989	33. 9	66	224604	10153	169.13
22	40403	2017	33.37	67	235585	10981	183. 1
23	42447	2044	34. 4	68	247509	11924	198.44
24	44523	2076	34.36	69	260509	13000	216.40
25	46631	2108	35. 8	70	274748	14239	237.19
26	48773	2142	35.42	71	290421	15673	261.13
27	50953	2180	36.20	72	307768	17347	289. 7
28	53171	2218	36.58	73	327085	19317	321.57
29	55431	2260	37.40	74	348741	21656	360.56
30	57735	2304	38.24	75	373205	24464	407.44
31	60086	2351	39.11	76	401078	27873	464.33
32	62487	2401	40. 1	77	433148	32070	534.30
33	64941	2454	40.54	78	470463	37315	621.55
34	67451	2510	41.50	79	514455	43992	733.12
35	70021	2570	42.50	80	567128	52673	877.53
36	72654	2633	43.53	81	631375	64247	1070.47
37	75355	2701	45. 1	82	711537	80162	1336. 2
38	78129	2774	46.14	83	814435	102898	1714.58
39	80978	2849	47.29	84	951436	137001	2283.21
40	83910	2932	48.52	85	1143005	191569	3192.49
41	86929	3019	50.19	86	1430067	287062	4784.22
42	90040	3111	51.51	87	1908114	478047	7967.27
43	93252	3212	53.32	88	2863625	955511	15925.11
44	96569	3317	55.17	89	5728996	2865371	47756.11
45	100000	3431	57.11	90	Infinitum.	Infinitum.	Infinitum.

89.15 . . . . . 7639001  
 89.30 . . . . . 11458865  
 89.45 . . . . . 22918166  
 89.55 . . . . . 68754887  
 89.59 . . . . . 343774662

## T A B E L L A B E N E F I C A .

$\tilde{g}$	Radius	Dfa	$60^{\text{m}^a}$	$\tilde{g}$	Radius	Dfa	$60^{\text{m}^a}$
1	100015	15	0.15	46	143956	2535	42.15
2	100061	46	0.46	47	146628	2672	44.32
3	100137	76	1.16	48	149448	2820	47. 0
4	100244	107	1.47	49	152425	2977	49.37
5	100382	138	2.18	50	155572	3147	52.27
6	100551	169	2.49	51	158902	3330	55.30
7	100751	200	3.20	52	162427	3525	58.45
8	100983	232	3.52	53	166164	3737	62.17
9	101247	264	4.24	54	170130	3966	66. 6
10	101543	296	4.56	55	174345	4215	70.15
11	101872	329	5.29	56	178829	4484	74.44
12	102234	362	6. 2	57	183608	4779	79.39
13	102630	396	6.36	58	188708	5100	85. 0
14	103061	431	7.11	59	194160	5452	90.52
15	103528	467	7.47	60	200000	5840	97.20
16	104030	502	8.22	61	206267	6267	104.27
17	104569	539	8.59	62	213005	6738	112.18
18	105146	577	9.37	63	220269	7264	121. 4
19	105762	616	10.16	64	228117	7848	130.48
20	106418	656	10.56	65	236620	8503	141.43
21	107114	696	11.36	66	245859	9239	153.59
22	107853	739	12.19	67	255930	10071	167.51
23	108636	783	13. 3	68	266947	11017	183.37
24	109464	828	13.48	69	279043	12096	201.36
25	110338	874	14.34	70	292380	13337	222.17
26	111260	922	15.22	71	307155	14775	246.15
27	112233	973	16.13	72	323607	16452	274.12
28	113257	1024	17. 4	73	342030	18423	307. 3
29	114335	1078	17.58	74	362796	20766	346. 6
30	115470	1135	18.55	75	386370	23574	392.54
31	116663	1193	19.53	76	413357	26987	449.47
32	117918	1255	20.55	77	444541	31184	519.44
33	119236	1318	21.58	78	480973	36432	607.12
34	120622	1386	23. 6	79	524084	43111	718.31
35	122077	1455	24.15	80	575877	51793	863.13
36	123607	1530	25.30	81	639245	63368	1056. 8
37	125214	1607	26.47	82	718530	79285	1321.25
38	126902	1688	28. 8	83	820551	102021	1700.21
39	128676	1774	29.34	84	956677	136126	2268.46
40	130541	1865	31. 5	85	1147371	190694	3178.14
41	132501	1960	32.40	86	1433559	286188	4769.48
42	134563	2062	34.22	87	1910732	477173	7952.53
43	136733	2170	36.10	88	2865371	954639	15910.39
44	139016	2283	38. 3	89	5729869	2864498	47741.38
45	141421	2405	40. 5	90	Infinitum.	Infinitum.	Infinitum.

0. 0 . . . . . 100000  
 89.15 . . . . . 7639655  
 89.30 . . . . . 11459301  
 89.45 . . . . . 22918385  
 89.55 . . . . . 68754959  
 89.59 . . . . . 343774676