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***Abstract*—In this paper, we present a resource allocation mechanism for network slices. We consider a dynamic resource allocation model with multiple independent resource providers. We call our allocation mechanism, the soft-max allocation mechanism, where the slices bid for resources and the resource providers allocate resources such that their revenue is close to the maximum revenue possible. We show that this mechanism translates into a game among the slices with a unique Nash equilibrium. We also show that the network utility obtained at this Nash equilibrium is close to the optimal social utility. We then present a stochastic dual sub-gradient algorithm that provably converges to the unique Nash equilibrium.**

I. Introduction

Traditional mobile communication network was aimed at serving human type communications. In 4G mobile communication, the focus was on bandwidth sharing to improve the spectral efficiency of the radio access network and allocate a fair share of bandwidth to the users. With 5G communications, there is a remarkable shift in focus, more towards machine type communications with Internet of Things (IoT) applications [1] like industrial automation, smart grids, intelligent transportation systems etc. This necessitates 5G communications to support a diverse class of performance and service metrics. In addition to radio resources, some of these applications also require other resources like processing capacity, storage etc. Network slicing is a key to achieve such a goal. A network slice refers to a fragment of the physical infrastructure that can support a particular class of service.

The concept of network slicing involves the following three entities: i) a set of infrastructure providers or resource centres ii) a set of slice tenants and iii) a set of users. A slice tenant gathers multiple resources from different resource centres. The tenants further distribute the gathered resources among the users. The resources need to be shared in an efficient way to ensure optimal use of the available resources. This involves allocating resources from a) resource centres to slices and b) slices to users.

These two problems have contrasting requirements. In the former, the needs of each beneficiary, i.e., each slice is different. Consider for example two slices with slice s_1 requiring only radio resources and slice s_2 needs an equal share of both the radio and processing. Hence, resource allocation for slices is a multi-resource allocation problem. In contrast, a set of users requesting resource from a slice have similar requirements. We can treat the resource allocation to users as a single resource allocation problem. Also, the set of users can be quite large and hence the complexity of allocating resources to the users should be low.

For the ease of implementation, we divide the set of resources into two components, a static component and a dynamic component. The static component ensures a guaranteed performance for the slices. The dynamic component is needed to improve resource utilization. In this paper, we focus on the dynamic component allocation for slices. Our objective is to maximize network utilization and also ensure a fair allocation of resources.

A. Related Literature

The concept of network virtualization [2] enables future communication network to provide a variety of services without modifying the physical infrastructure. In the context of network virtualization, the problem of network embedding is studied in [3] to map the virtual network to the infrastructure network to optimize various metrics like throughput, delay etc., to improve the quality of service (QoS), minimize the network cost, and also provide a resilient network embedding. A fractional relaxation of the network function embedding was considered in [4] and a backpressure based algorithm was proposed to minimize the network cost. In [5], the problem of admission control for slice requests was formulated as a geometric knapsack problem and low-complexity algorithms were developed.

In [6] Leconte et al. modelled the infrastructure comprising of radio and computing resources as a directed graph and proposed an ADMM based utility maximization algorithm to ensure fair resource sharing. Spectrum sharing in the context of network slicing was studied in [7]. Fossati et al. [8] propose a fairness metric for multi-resource allocation called Ordered Weighted Averaging and discuss its properties. A Fisher market mechanism [9] and a modified version of Fisher market [10] was proposed and conditions on the existence of Nash equilibrium was established.

B. Contributions

1) In comparison to prior work, we consider a more realistic network model, where the physical infrastructure belong to discrete set in contrast to the assumption that the infrastructure can be divided infinitesimally small.
2) We propose an allocation mechanism called the soft-max allocation mechanism. Under this mechanism, slices bid for the resources and the resource owners allocate resources such that the allocation results in a revenue close to the maximum possible for the set of bids.

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3) We show that the allocation mechanism results in a game among the slices and the game has a unique Nash equilibrium. Also, we show that the social utility obtained at this Nash equilibrium is close to the optimal social utility.
4) We present a stochastic dual-subgradient algorithm that provably converges to the unique Nash equilibrium.

The rest of the paper is organised as follows: In Section II, we discuss the network model and formulate the social optimal problem. In Section III, we present our allocation mechanism and discuss the game induced by the mechanism. In Section IV, we present a stochastic dual-subgradient algorithm that converges to the Nash equilibrium of the game. Finally, in Section V, we discuss our concluding remarks.

II. Resource Allocation for Slices

We consider a network with a set of $\mathcal{R} = \{1, \ldots, R\}$ resource centers. Resource centers are entities that own physical infrastructure like bandwidth, memory, processing capacity etc. They can lease the resources owned by them to the slice tenants. We assume that each resource center is an independent entity; hence, allocates its resources independently. In theory, resources like bandwidth and processing capacity can be divided infinitesimally small; however, this is seldom true in reality. Consistent with this understanding, we denote the resources that belong to resource center r by a finite set $\mathcal{C}^r = \{1, 2, \ldots, C_r\}$. We consider a set $\mathcal{S} = \{1, \ldots, S\}$ of slice tenants, where each slice corresponds to a service with specific resource requirements. To simplify the analysis, we assume a time slotted system with timescales corresponding to few hours or days.

At each time slot, each of these resources are assigned to one of the slices, thereby a resource $c_r \in \mathcal{C}_r$ cannot be assigned to more than one slice at a time. We denote the assignment of the resources by resource center r to slices by a $C_r \times S$ matrix $\boldsymbol{A}^r$. Here, $\boldsymbol{A}^r(i,j) = 1$ if resource i is assigned to slice j, else $\boldsymbol{A}^r(i,j) = 0$. Note that each row adds to one i.e., for all i, $\sum_j \boldsymbol{A}^r(i,j) = 1$. Let $\mathcal{A}^r$ denote the set of matrices satisfying the above conditions. We denote the columns of $\boldsymbol{A}^r$ by $\boldsymbol{a}_s^r,\ r \in \mathcal{R},\ s \in \mathcal{S}$. Column $\boldsymbol{a}_s^r$ indicates the resources assigned to slice s from resource centre r.

Let $\boldsymbol{A}^r(t)$ denote the assignment matrix of resource r at time t. At time t, let $x_s^r(t)$ denote the benefit attained by slice s from resource type r. The benefit $x_s^r(t)$ is a function depending on the resources of type r allocated to slice s at time t and the network state $\omega(t) \in \Omega$ i.e.,

$$x_s^r(t) = f_s^r(\boldsymbol{a}_s^r(t), \omega(t)),$$

where $\boldsymbol{a}_s^r(t)$ is the resources of type r allocated to slice s and the network state $\omega(t)$ captures the wireless channel in the case of radio resource and slice demand variations. We assume that the benefit is bounded by B, i.e., $x_s^r(t) < B$ for all $r \in \mathcal{R},\ s \in \mathcal{S}$ and at all times. Also, we assume that the state is an ergodic random process taking values in a finite set (Ω is finite) with distribution μ i.e.,

$$\mu(\omega) = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{1}_{\omega(t)=\omega}.$$

The time average benefit attained by slice s from resource centre r is then given by,

$$\overline{x}_s^r = \liminf_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} x_s^r(t)$$

We denote the vector of time average benefit attained by slice s from all the resource centers by $\overline{\boldsymbol{x}}_s = (\overline{x}_s^r,\ r \in \mathcal{R})$. We assume that each slice has a utility $U_s(\cdot)$, which is a function of the time average benefit $\overline{\boldsymbol{x}}_s$ attained by slice s. We assume that $U_s(\cdot)$ is increasing and strictly concave.

The set of average benefits that can be attained by the slices by time sharing the set of resources is known as the rate region of the network. We denote the rate region by $\mathcal{X}$. Formally, the rate region is defined as:

$$\mathcal{X} = \left\{ (\overline{\boldsymbol{x}}_s) \;\middle|\; \begin{array}{c} s \in \mathcal{S},\ \overline{\boldsymbol{x}}_s = (\overline{x}_s^r,\ r \in \mathcal{R}), \\ \boldsymbol{x}_s^r = \displaystyle\sum_{\omega \in \Omega} \mu(\omega) \sum_{A^r \in \mathcal{A}^r} p(A^r, \omega) f_s^r(\boldsymbol{a}_s^r, \omega), \\ p(A^r, \omega) \geq 0,\ \displaystyle\sum_{A^r \in \mathcal{A}} p(A^r, \omega) = 1,\ \forall \omega \in \Omega \end{array} \right\}.$$

A. Social Optimal allocation

We say that an allocation is socially optimal, if the average benefit maximize the sum of the utilities of the slices, i.e.,

$$\begin{aligned} \text{Maximize: } & \sum_{s \in \mathcal{S}} U_s(\overline{\boldsymbol{x}}_s) \\ \text{such that: } & \overline{\boldsymbol{x}}_s \in \mathcal{X} \end{aligned} \tag{1}$$

This choice of maximizing the sum of utilities over the average benefit as the social optimal choice is reminiscent of the utility maximization to attain fair solution [11]. We assume that the slices do not interact with each other i.e., they do not know each others utilities. Our objective is to employ a bidding mechanism such that the social optimal is achieved. In the next section, we shall present our bidding mechanism, the Soft-Max Allocation (SMA) mechanism.

III. Soft-Max Allocation Mechanism

Now we shall present Soft-Max Allocation (SMA) mechanism, a bidding mechanism to allocate resources among slices.

In SMA, each slice has to submit independent bids to the resource centers. The bid submitted to the resource center is also regarded as the cost per unit average benefit received by the slice. The resources are then allocated to the slices by the resource center based on the bids. Let $b_{s,r}$ denote the bid submitted by slice s to resource center $r \in \mathcal{R}$ and let x_s^r be the average benefit received by slice s from resource center r. Then, resource center r receives a payment of $b_{s,r} x_s^r$ from slice s, i.e., each slice pays the resource center the bid amount weighted by the average benefit received for that resource. Note that the average benefit x_s^r will depend on the allocation mechanism that we will propose below.

In the SMA mechanism, for a given set of bids $b_{s,r}$ resource center r allocates its resources to maximize the following:

$$\begin{aligned}\underset{p}{\text{Maximize:}}\ \sum_{\omega\in\Omega}\mu(\omega)\Bigg(&\beta\sum_{A^r}p(A^r,\omega)\sum_{s}b_{s,r}f_s^r(\boldsymbol{a}_s^r,\omega)\\&-\sum_{A^r}p(A^r,\omega)\log(p(A^r,\omega))\\&-p(O,\omega)\log(p(O,\omega))\Bigg),\\ \text{such that: } p(A^r,\omega)\geq 0,\ &\sum_{A^r\in\mathcal{A}}p(A^r,\omega)=1-p(O,\omega).\end{aligned}\tag{2}$$

Here, $p(O)$ denotes the fraction of time the resources are retained by the resource center and are allocated to none of the slices. This is done to ensure a certain minimum price for the resources.

In the following lemma, we shall establish that, for any bid $\boldsymbol{b}$ the allocation mechanism in (2) leads to a unique allocation.

Lemma 1. *Let $\boldsymbol{b}=\{b_{s,r},\ s\in\mathcal{S}\}$ be the set of bids at resource center r. Then SMA allocation in* (2) *is uniquely given by:*

$$p(A^r,\omega;\boldsymbol{b})=\frac{e^{\beta\sum_s b_{s,r}f_s^r(\boldsymbol{a}_s^r,\omega)}}{1+Z(\omega)},$$

where Z is the following normalizing constant

$$Z(\omega)=\sum_{A^r\in\mathcal{A}^r}e^{\beta\sum_s b_{s,r}f_s^r(\boldsymbol{a}_s^r,\omega)}.$$

Also, the corresponding benefit for slice s is given by:

$$\overline{x}_s^r(\boldsymbol{b})=\sum_{\omega\in\Omega}\mu(\omega)\frac{e^{\beta\sum_s b_{s,r}f_s^r(\boldsymbol{a}_s^r,\omega)}}{1+Z(\omega)}f_s^r(\boldsymbol{a}_s^r,\omega)\tag{3}$$

□

A. Choice of Utilities

Recall that, a slice represents a service that utilizes different resources. Hence, the utility of a slice cannot linearly increase by increasing the allocation of a single resource. This fact is captured by Leonteif utility functions given by:

$$\min\left\{\frac{\overline{x}_s^1}{w_s^1},\frac{\overline{x}_s^1}{w_s^1},\dots,\frac{\overline{x}_s^R}{w_s^R}\right\},$$

here w_s^r is the weight given by slice s for resource r. However, the use of such a utility could lead to the resources not getting cleared. To avoid this scenario, we use a weighted combination of the Leonteif utility and alpha fair utility.

$$U_s(\overline{x})=\min\left\{\frac{\overline{x}_s^1}{w_s^1},\frac{\overline{x}_s^1}{w_s^1},\dots,\frac{\overline{x}_s^R}{w_s^R}\right\}+\sum_{r\in\mathcal{R}}\frac{1}{1-\alpha}\left(\frac{\overline{x}_s^r}{w_s^r}\right)^{1-\alpha}$$

The first term ensures that the utility of a slice increases as all the resources are increased in their required proportion. The second term ensures an increase in the overall utility with the increase of a single resource also, but with a diminishing return property. This choice of utility ensures that the resources are used fully and also captures that utility increases linearly with the minimum weighted benefit.

B. Bidding Strategy for Slices

The total reward received by slice s is the utility it receives from the resource allocation mechanism minus the amount it has to pay the resource center. The reward received by slice s is given by

$$y_s(\boldsymbol{b}_s,\boldsymbol{b}_{-s})=U_s(\overline{\boldsymbol{x}}_s(\boldsymbol{b}))-\sum_{r\in\mathcal{R}}b_{s,r}\overline{x}_s^r(\boldsymbol{b}),\tag{4}$$

where $\overline{x}_s$ is the average benefit received by slice s given by the SMA mechanism (3). We assume that slices are rational in choosing their bids. Each slice will rationally choose a bid that maximizes its total reward, i.e.,

$$\begin{aligned}&\text{Maximize: } U_s(\overline{\boldsymbol{x}}_s(\boldsymbol{b}))-\sum_{r\in\mathcal{R}}b_{s,r}\overline{x}_s^r(\boldsymbol{b})\\&\text{such that: } \overline{x}_s^r(\boldsymbol{b})=\sum_{\omega\in\Omega}\mu(\omega)\frac{e^{\beta\sum_s b_{s,r}f_s^r(\boldsymbol{a}_s^r,\omega)}}{1+Z(\omega)}f_s^r(\boldsymbol{a}_s^r,\omega)\\&\qquad b_{s,r}\geq 0\end{aligned}\tag{5}$$

C. Slicing Game

We now have the slicing game $\mathcal{G}$, where the players are the set of slices $\mathcal{S}$, their actions are the bids $\boldsymbol{b}_s$ and their reward y_s is the given by (4); governed by SMA mechanism (2). We then have the following definition of Nash equilibrium for game $\mathcal{G}$.

Definition 1. *Nash Equilibrium: A set of bids $(\boldsymbol{b}_s,\ s\in\mathcal{S})$ is a Nash equilibrium of the game $\mathcal{G}$, if for each $s\in\mathcal{S}$ and any bid $\boldsymbol{b}_s'$, we have $y_s(b_s,b_{-s})\geq y_s(b_s',b_{-s})$, i.e., any unilateral deviation of a slice's action does not increase its reward.*

We are interested in the set of Nash equilibria that maximizes the social utility in (1). In the following theorem, we show the existence of a socially optimal Nash equilibrium and its uniqueness.

Theorem 1. *Let $\boldsymbol{b}=(b_s,\ s\in\mathcal{S})>0$ be the bids chosen by the slices. Then, there exists a unique Nash equilibrium $\boldsymbol{b}_{NE}$ for the slicing game $\mathcal{G}$. Also, at this unique Nash equilibrium the network utility is close to the socially optimal utility, i.e.,*

$$\sum_s U_s(\overline{\boldsymbol{x}}_s(\boldsymbol{b}_{NE}))\geq\sum_s U_s(\overline{\boldsymbol{x}}^*)-\frac{\sum_r\log(|\mathcal{A}^r|)}{\beta}\tag{6}$$

Since $\mathcal{A}^r$ is finite for a large β, the sum utility of the slices is close to the optimal utility.

Proof. Consider the optimization problem,

$$\begin{aligned}\text{Maximize: }&\sum_{s\in\mathcal{S}}U_s\left(\hat{\boldsymbol{x}}_{\boldsymbol{s}}\right)\\&-\frac{1}{\beta}\sum_{\omega\in\Omega}\mu(\omega)\sum_{r\in\mathcal{R}}\sum_{A\in\mathcal{A}^r}p(A,\omega)\log p(A,\omega)\\&-\frac{1}{\beta}\sum_{\omega\in\Omega}\mu(\omega)\sum_{r\in\mathcal{R}}p(O_r,\omega)log(p(O_r,\omega))\\ \text{such that: }&\hat{x}_s^r\leq\sum_{\omega\in\Omega}\mu(\omega)\sum_{A\in\mathcal{A}^r}p(A,\omega)f_s^r(\boldsymbol{a}_s,\omega),\\&p(A,\omega)\geq 0,\ \sum_{A\in\mathcal{A}^r}p(A,\omega)=1-p(O_r,\omega)\end{aligned}\tag{7}$$

Consider the partial Lagrangian with parameters $b_{s,r}$

$$\begin{aligned} L(\hat{\boldsymbol{x}}, p, \boldsymbol{b}) = &\sum_{s\in\mathcal{S}} \left(U_s\left(\hat{\boldsymbol{x}}_{\boldsymbol{s}}\right) - \sum_{r\in\mathcal{R}} b_{s,r}\hat{x}_s^r \right) \\ &+ \sum_{s}\sum_{r\in\mathcal{R}} b_{s,r} \sum_{\omega\in\Omega}\mu(\omega) \sum_{A\in\mathcal{A}^r} p(A,\omega) f_s^r(\boldsymbol{a}_s,\omega) \\ &- \frac{1}{\beta}\sum_{\omega\in\Omega}\mu(\omega)\sum_{r\in\mathcal{R}}\sum_{A\in\mathcal{A}^r} p(A,\omega)\log p(A,\omega) \end{aligned}$$

The dual of the above problem is given by,

$$\begin{aligned} d(\boldsymbol{b}) = &\sup_{\hat{\boldsymbol{x}},p} L(\hat{\boldsymbol{x}}, p, \boldsymbol{b}) \\ &\text{s.t. } p(A,\omega)\geq 0, \quad \sum_{A\in\mathcal{A}^r} p(A,\omega) = 1 \end{aligned}$$

We observe that the maximization over p in the above is the SMA mechanism. Substituting the SMA allocation from Lemma 1,

$$\begin{aligned} d(\boldsymbol{b}) = &\sup_{\hat{\boldsymbol{x}}} \sum_s U_s(\hat{\boldsymbol{x}}_s) - \sum_{r\in\mathcal{R}} b_{s,r}\hat{x}_s^r \\ &+ \sum_{\omega\in\Omega}\sum_{r\in\mathcal{R}} \log\left(1 + \sum_{A\in\mathcal{A}} e^{\beta\sum_s b_{s,r} f_s^r(\boldsymbol{a}_s^r)}\right) \end{aligned}$$

It can be shown that the last log-sum-exp term is strictly convex. Also, the first two terms are also convex in $\boldsymbol{b}$, since it is a point wise supremum of affine functions. Hence, the dual is strictly convex and has a unique minimum.

To show that there is unique Nash equilibrium, we show that the game $\mathcal{G}$ is an ordinal potential game. Consider the potential function $\Psi(\boldsymbol{b}) = -d(\boldsymbol{b})$. We need to show that,

$$\text{sign}\left(\frac{\partial}{\partial b_{s,r}}\Psi(\boldsymbol{b})\right) = \text{sign}\left(\frac{\partial}{\partial b_{s,r}} y(\boldsymbol{b})\right) \tag{8}$$

Let us denote the benefit received by slice s as

$$\overline{x}_s^r = \sum_{\omega\in\Omega}\mu(\omega)\frac{e^{\beta\sum_s b_{s,r} f_s^r(\boldsymbol{a}_s^r,\omega)}}{1+Z(\omega)} f_s^r(\boldsymbol{a}_s^r,\omega)$$

Then the subgradient of the potential function Ψ is given by,

$$\nabla_{\boldsymbol{b}_s}\Psi(\boldsymbol{b}) = \overline{\boldsymbol{x}}_s - (\nabla_{\overline{\boldsymbol{x}}_s} U_s)^{-1}(\boldsymbol{b}_s)$$

Also, the gradient of the reward is given by,

$$\nabla_{\boldsymbol{b}_s} y(\boldsymbol{b}_s, \boldsymbol{b}_{-s}) = \left(\nabla_{\boldsymbol{b}_s}\overline{\boldsymbol{x}}(\boldsymbol{b})\right)^T\left(\nabla_{\overline{\boldsymbol{x}}} U_s(\overline{\boldsymbol{x}}) - \boldsymbol{b}_s\right)$$

It can be noted that all coordinates of $\nabla_{\boldsymbol{b}_s}\overline{\boldsymbol{x}}(\boldsymbol{b})$ is non-negative. Hence, (8) follows.

Also, since the potential function Ψ has a unique maximum and noting that entropy is bounded by $\log(|\mathcal{A}^r|)$, the unique non-trivial Nash equilibrium satisfies (6). $\square$

IV. Distributed Algorithm for Socially Optimal Equilibrium

We now present a dual sub-gradient algorithm for learning the socially optimal Nash equilibrium. We assume that the state $\omega(t)\in\Omega$ of the network at time t is known to the resource center prior to allocation of resources. Each slice will choose an initial bid b_0. For the chosen bid, slice s calculates the average benefit $\hat{\boldsymbol{x}}_s$, that maximizes its total reward i.e.,

$$\hat{\boldsymbol{x}}_s(t) = \arg\max_{\boldsymbol{\beta}\in[0,B]^R} U_s(\boldsymbol{\beta}) - \sum_{r\in\mathcal{R}} b_{s,r}(t)\beta_r.$$

With the assumption of fixed bids $\{b_{s,r}(t),\ s\in\mathcal{S}\}$, at time t, resource center r chooses the allocation $A^r(t+1)$ with a probability distribution given by

$$p(A^r,\omega(t+1);\boldsymbol{b}(t)) = \frac{e^{\beta\sum_s b_{s,r}(t) f_s^r(\boldsymbol{a}_s^r,\omega(t+1))}}{1+Z(\omega(t+1))}$$

Let $\tilde{x}_s^r(t)$ be the benefit attained by slice s from resource center r at time t. Finally, the bids are updated by the following dual subgradient algorithm

$$b_{s,r}(t+1) = \left[b_{s,r}(t) - \frac{1}{t}\left(\hat{x}_s^r(t) - \tilde{x}_s^r(t)\right)\right]^+ \tag{9}$$

Here, $[x]^+ = x$ if $x>0$ and $[x]^+ = 0$ otherwise.

Intuitively, the algorithm is explained as follows: If the attained benefit is more than the maximum benefit for that bid, it is useful to reduce the bid, thereby increasing the total reward. If, on the other hand, the attained benefit is less than the maximum benefit for that bid, then the slice increases its bid so that it gets a better reward.

Algorithm 1 : Learning Efficient Nash Equilibrium

Initialize:

For all $s\in\mathcal{S}$ and initialize bid $b_{s,r} = b_0$

Resource Allocation Mechanism at time t:

Choose $A^r(t+1)$ with probability

$$p(A^r,\omega(t+1);\boldsymbol{b}(t)) = \frac{e^{\beta\sum_s b_{s,r}(t) f_s^r(\boldsymbol{a}_s^r,\omega(t+1))}}{1+Z(\omega(t+1))}$$

Update $\tilde{\boldsymbol{x}}_s$ and $\hat{\boldsymbol{x}}_s$ at time t

$$\begin{aligned} \tilde{x}_s^r(t) &= f_s^r(a^r(t+1),\omega(t+1)) \\ \hat{\boldsymbol{x}}_s(t) &= \arg\max_{\boldsymbol{\beta}\in[0,B]^R} U_s(\boldsymbol{\beta}) - \sum_{r\in\mathcal{R}} b_{s,r}(t)\beta^r \end{aligned}$$

Bid update for slice s at time t:

$$b_{s,r}(t+1) = \left[b_{s,r}(t) + \frac{1}{t}\left(\hat{x}_s^r(t) - \tilde{x}_s^r(t)\right)\right]^+$$

Next we have the theorem that shows that Algorithm 1 converges to the near-efficient Nash equilibrium.

Theorem 2. *Let the bids in Algorithm 1 be such that $b_{s,r}(t) < b_{\max}$, for all s, r and t. Then, Algorithm 1 converges to the optimal bids $\boldsymbol{b}^*$.*

Proof. Let $\boldsymbol{\Delta}(t) = \boldsymbol{b}(t) - \boldsymbol{b}^*$ and $\Delta_{s,r}(t) = b_{s,r}(t) - b^*_{s,r}$.

$$\begin{aligned}
\|\boldsymbol{\Delta}(t+1)\|^2 &= \sum_{s\in\mathcal{S}}\sum_{r\in\mathcal{R}} \Delta^2_{s,r}(t+1) \\
&\overset{(a)}{\leq} \sum_{s\in\mathcal{S}}\sum_{r\in\mathcal{R}} \left(b_{s,r}(t) - \frac{1}{t}(\hat{x}^r_s(t) - \tilde{x}^r_s(t)) - b^*_{s,r}\right)^2 \\
&\overset{(b)}{\leq} \sum_{s\in\mathcal{S}}\sum_{r\in\mathcal{R}} \Big(\Delta^2_{s,r}(t) - 2\frac{1}{t}\Delta_{s,r}(t)\left(\hat{x}^r_s(t) - \tilde{x}^r_s(t)\right) \\
&\quad + \frac{B^2}{t^2}\Big). \qquad (10)
\end{aligned}$$

Here (a) follows due to the non-expansiveness of projection operator $[\cdot]^+$ and (b) follows since $\tilde{x}^r_s(t),\ \hat{x}^r_s(t) < B$.

Let us define,

$$\overline{x}^r_s(t) = \sum_{\omega\in\Omega} \mu(\omega) \frac{e^{\beta \sum_s b_{s,r}(t) f^r_s(\boldsymbol{a}^r_s,\omega)}}{1 + Z(\omega)} f^r_s(\boldsymbol{a}^r_s, \omega).$$

Also, the subgradient of the dual function is,

$$\nabla_{\boldsymbol{b}_s(t)} d(\boldsymbol{b}(t)) = \hat{\boldsymbol{x}}_s(t) - \overline{\boldsymbol{x}}_s(t).$$

Then by gradient inequality for convex functions, we have

$$\big(\hat{\boldsymbol{x}}_s(t) - \overline{\boldsymbol{x}}_s(t)\big)^T(\boldsymbol{b}(t) - \boldsymbol{b}^*) \geq d(\boldsymbol{b}(t)) - d(\boldsymbol{b}^*)$$

Substituting the above in (10), we get

$$\begin{aligned}
\|\boldsymbol{\Delta}(t+1)\|^2 \leq &\|\boldsymbol{\Delta}(t)\|^2 - \frac{2}{t}\big(d(\boldsymbol{b}(t)) - d(\boldsymbol{b}^*)\big) \\
&+ \frac{B^2RS}{t^2} - \frac{2}{t}\boldsymbol{\Delta}(t)^T\big(\overline{x}^r_s(t) - \tilde{x}^r_s(t)\big) \quad (11)
\end{aligned}$$

Summing the above from $t = 0$ to T,

$$\begin{aligned}
\|\boldsymbol{\Delta}(T+1)\|^2 \leq \|\boldsymbol{\Delta}(0)\|^2 - \sum_{t=0}^{T}\Big(&\frac{2}{t}\big(d(\boldsymbol{b}(t)) - d(\boldsymbol{b}^*)\big) + \frac{B^2RS}{t^2} \\
&- \frac{2}{t}\boldsymbol{\Delta}(t)^T\big(\overline{x}^r_s(t) - \tilde{x}^r_s(t)\big)\Big)
\end{aligned}$$

Rearranging, and taking expectation, we have,

$$\sum_{t=0}^{T}\frac{2}{t}E\big(d(\boldsymbol{b}(t)) - d(\boldsymbol{b}^*)\big) \leq E\big(\|\boldsymbol{\Delta}(0)\|^2\big) + \sum_{t=0}^{T}\frac{B^2RS}{t^2}$$

This implies, $\liminf_{T\to\infty}\sum_{t=0}^{T}\frac{2}{t}E\big(d(\boldsymbol{b}(t)) - d(\boldsymbol{b}^*)\big) \leq \infty$. By Kronecker's Lemma [12], we have

$$\liminf_{t\to\infty} E\big(d(\boldsymbol{b}(t))\big) - d(\boldsymbol{b}^*) = 0$$

By Fatou's lemma and since $d(\boldsymbol{b}(t)) \geq d(\boldsymbol{b}^*)$, we have *w.p.* 1

$$\liminf_{t\to\infty}\big(d(\boldsymbol{b}(t))\big) - d(\boldsymbol{b}^*) = 0$$

Since, $d(\cdot)$ is strictly convex and b^* is unique, we have *w.p.* 1

$$\liminf_{t\to\infty}\|\boldsymbol{b}(t) - \boldsymbol{b}^*\| = 0 \qquad (12)$$

From (11), we have

$$\|\boldsymbol{\Delta}(t+1)\|^2 \leq \|\boldsymbol{\Delta}(t)\|^2 + \frac{B^2RS}{t^2} - \frac{2}{t}\boldsymbol{\Delta}(t)^T\big(\overline{\boldsymbol{x}}(t) - \tilde{\boldsymbol{x}}(t)\big)$$

Let $c_t = (2/t)\boldsymbol{\Delta}(t)^T\big(\overline{\boldsymbol{x}} - \tilde{\boldsymbol{x}}(t)\big)$. Since $b(t)$ is bounded and $\sum_{t=0}^{T} c_t$ is an $\mathcal{F}_T$ martingale, by martingale convergence theorem, we have *w.p.* 1, $\lim_{T\to\infty}\sum_{t=0}^{T} c_t < \infty$. From the above and Lemma 2.3 in [13] $\|\boldsymbol{\Delta}(t)\|$ converges *w.p.* 1. Together with (12) we have the result. □

V. Conclusion

In this paper we present a mechanism for resource allocation for network slices. A slice is considered as a service that requires different resources; each resource being owned by an independent entity. We argue that to make efficient use of the available resources, at least a subset of the resources needs to be allocated dynamically. Also, at the time scale in which these resources are allocated, we do not restrict the complexity of the resource allocation mechanism. Under this setup, we propose an allocation mechanism called the soft-max allocation mechanism. We argue that the soft-max allocation mechanism translates to a game among the slices. Further, we show that there exists a unique Nash equilibrium for the game and that the Nash equilibrium is also socially optimal. Finally, we present a distributed stochastic gradient algorithm that converge to this unique Nash equilibrium. In future, to obtain a complete view, we intend to add a static component for the resource allocation among slices and discuss how these resources need to be allocated to the end users.

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