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# Measuring natural source dependence

Cédric Gutierrez and Emmanuel Kemel\*

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## Abstract

The consequences of most economic decisions are uncertain: they are conditional on events with unknown probabilities that decision makers evaluate based on their beliefs. Beyond consequences and beliefs, decision makers' preferences for the source of uncertainty generating events can affect decisions, entailing a source-dependence of preferences. However, there is, to date, no direct way to measure and interpret source dependence for natural sources, preventing researchers from comparing source preference across decision makers or across natural sources. This paper presents a simple and general method to measure source dependence by introducing a source-dependence function that maps the subjective probabilities of events generated by two sources. The method allows to characterize source dependence from a limited number of observations and is compatible with commonly used, choice-based data. We implement the method on four datasets and show that it consistently captures clear, albeit heterogeneous, patterns of source dependence between natural sources.

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# 1 Introduction

The consequences of most economic decisions are uncertain: they are conditional on events with, usually, unknown probabilities. According to subjective expected utility (SEU, Savage 1954), rational decisions depend only on the decision maker’s beliefs about events, in the form of subjective probabilities, and the utility of the related consequences. However, the source generating events may also affect decisions: for equal beliefs, decision makers may prefer to bet on one source over another, a pattern called *source dependence*.<sup>1</sup> Answering a recent call to extend our understanding of “how ambiguity attitudes are related across different sources of uncertainty and across different persons” (Baillon 2018, p. 1852), this paper proposes and implements a method for quantifying source dependence, allowing for comparisons across sources and across individuals.

To illustrate source dependence, imagine two American investors, respectively experts in the telecommunications and food industries, who consider investing in the stocks of AT&T, British Telecom, and Coca-Cola. According to “home bias”—the tendency to favor domestic stocks—both investors may prefer AT&T over British Telecom (Lau et al. 2010). However, is the preference for the domestic stock weaker for the first investor due to her expertise in the telecommunications industry? Answering this question requires comparing the magnitude of source dependence *across individuals*. We may also wonder if the magnitude of source dependence varies *across sources* for the same individual. For example, the first investor may exhibit a preference for investing in AT&T rather than in Coca-Cola (two domestic stocks) because she feels more competent about the telecommunications industry (Tversky and Fox 1995). Is the magnitude of source dependence between AT&T and Coca-Cola (due to perceived competence) stronger than the one between AT&T and British Telecom (due to home bias)? The method that we propose allows us to answer these two questions.

In the literature, source dependence has long been illustrated by Ellsberg’s (1961) though experiments. Ellsberg conjectured that individuals would prefer to bet on an urn with known composition (e.g., containing 50 black balls and 50 red balls) rather than on an urn with unknown composition (e.g., with 100 black and red balls with unknown proportion) despite holding the same beliefs about the two urns. Following this insightful example, most empirical investigations of source dependence have focused on similar sources using urns with known and unknown compositions (for a review, see

Trautmann and van de Kuilen 2015). However, there are apparent limits to the insights of studying such artificial settings. The domain of uncertainty is rich (Li et al. 2017), and the diversity of natural sources may entail a broad diversity in attitudes that cannot be captured by analyzing urns with different compositions.<sup>2</sup> Therefore, it is essential to understand how people make decisions for natural sources (Camerer and Weber 1992, Li et al. 2017, Baillon et al. 2018).

Our characterization of source dependence builds on the source model developed by Chew and Sagi (2008) and Abdellaoui et al. (2011), in which probabilities are weighted by a function that is source specific: the *source function*. The conventional method for measuring source dependence using this model is to estimate and compare the parameters of the source functions of different natural sources (e.g., Abdellaoui et al. 2011). While this method is well adapted to analyzing attitudes towards a given source of uncertainty, it makes the interpretation of source dependence more difficult. To illustrate this, we return to the example of the two investors. Following standard practices, the researcher may estimate two-parameter (sensitivity to changes in likelihood and pessimism) source functions using, for instance, a Prelec specification. Let's imagine that the source functions of A (AT&T) and B (British Telecom) for the first investor have the same sensitivity parameter of 0.9 but different pessimism parameters of 1.4 and 1.8, respectively. For the second investor, the source functions of A and B have the same sensitivity parameter of 0.45 but different pessimism parameters of 0.7 and 0.9, respectively. The pessimism parameters indicate that both investors exhibit a preference for source A over source B, a pattern consistent with home bias. But, is this preference for source A stronger for one of the two investors? To answer this question, one may compare the difference in the two source functions' pessimism parameters. This difference is greater for the first investor (1.8 vs. 1.4) than for the second one (0.9 vs. 0.7), which seems to indicate that the first investor exhibits a stronger home bias. However, this interpretation would be erroneous, as we will see later.

Two elements make the analysis of source dependence by comparing source functions' parameters difficult. First, the parameters of non-linear functions, such as the Prelec specification, have non-linear effects on probabilities. Second, and more importantly, differences between source functions are expressed on a scale that does not have a physical unit, preventing direct comparisons of source dependence across sources and across individuals.<sup>3</sup> To overcome these limitations, we propose a new and direct approach based on estimating a transformation function, that we called a



*source-dependence function*.<sup>4</sup> This method allows for the straightforward measurement and interpretation of source dependence. In particular, it characterizes source dependence between natural sources of uncertainty independently from risk preferences (i.e., preference between situations with known probabilities) and ambiguity attitudes (i.e., preferences between situations with known versus unknown probabilities).

Our source-dependence function extends the approach proposed by Dimmock et al. (2016b) to capture ambiguity attitudes to the comparison of pairs of natural sources. Their method consists of measuring the matching probabilities for a series of ambiguous events, i.e., “the objective probability at which, for a given prize, a subject is indifferent between betting on the ambiguous event versus betting on the objective probability” (p. 1363). The function that maps probabilities of ambiguous events onto (objective) matching probabilities captures ambiguity attitudes toward the ambiguous source. This *ambiguity function* measures an *ambiguity premium* (expressed in probability units) that an ambiguity-averse decision maker is willing to forgo in order to bet on a risky rather than on an ambiguous event of equal belief. We propose a generalization of this approach to capture source dependence between two natural sources. Returning again to the example of an investor facing decisions involving the share prices of AT&T (source A) and British Telecom (source B), suppose that the investor is indifferent between betting (the same desirable consequence) on two events  $E_A$  and  $E_B$ , generated by sources  $A$  and  $B$  and with probability  $\mu_A$  and  $\mu_B$ . When  $\mu_A < \mu_B$  the decision maker exhibits a preference for source  $A$  over source  $B$  and is ready to accept a decrease in winning probability  $\mu_B - \mu_A$  in order to bet on the event generated by source  $A$  instead of the one generated by source  $B$  with probability  $\mu_B$ . Said differently, this investor would ask for an increase in winning probability of  $\mu_B - \mu_A$  in order to bet on source  $B$  rather than on source  $A$  with a winning event of probability  $\mu_A$ . Therefore, the difference between probabilities  $\mu_B$  and  $\mu_A$  can be interpreted as a *source premium* of source  $A$  over source  $B$ .<sup>5</sup> We call this mapping between the beliefs of events generated by two sources of uncertainty a *source-dependence function*  $\phi$ .

This function offers a cardinal and easily interpretable measure of source dependence. Importantly, and contrary to existing approaches that determine if (the parameters of) ambiguity attitudes toward two sources are different, the source-dependence function directly measures if, and by how much, a given source is preferred to another. To illustrate the advantage of this method, we plot in Figure 1 the source-dependence functions  $\phi_{AB}$  of the two investors from our il-

lustrative example. The two functions  $\phi_{AB}$  lie below the diagonal, indicating that, across the range of probabilities, both investors exhibit a preference for source  $A$  over source  $B$ : they are willing to accept a decrease in winning probability in order to bet on source  $A$  instead of source  $B$ . We also see that the source premium is systematically larger for the second investor than for the first one. For instance, the second investor is willing to sacrifice 0.2 in winning probabilities, instead of 0.1 for the first investor, in order to bet on an event of source  $A$  over an event of source  $B$  with winning probability 0.5. In other words, and contrary to the “intuitive” conclusion based on comparing the source functions’ parameters, our method directly shows that the second investor exhibits a stronger preference for source  $A$  over source  $B$  than the first investor. Our method also allows us to study within-subject source dependence. For instance, we could compare, for both investors, the magnitude of source dependence between AT&T and British Telecom (capturing home bias) and between AT&T and Coca-Cola (capturing perceived competence).

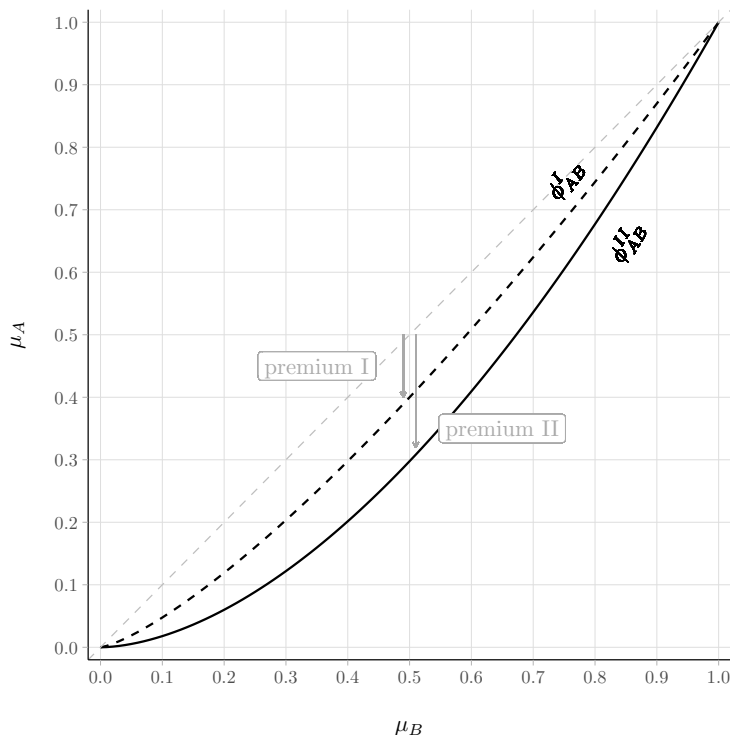


Figure 1: Illustration of the source-dependence functions  $\phi_{AB}$  for the two investors (I and II)

Our paper contributes to the growing literature on the heterogeneity of preferences. A siz-

able literature has investigated the heterogeneity of risk preferences (e.g., Von Gaudecker et al. 2011, l’Haridon and Vieider 2019) and, more recently, ambiguity attitudes across individuals (e.g., Dimmock et al. 2016a, Li 2017, Sutter et al. 2013). Such empirical investigations have generated interest in methods comparing ambiguity attitudes across decision makers (e.g., Wang 2019). Another stream of research has investigated differences of attitudes across sources (e.g., Li et al. 2017, Chew et al. 2012, Fox and Ülkümen 2011) and identified some factors explaining these differences (e.g., expertise, emotion, distance, epistemic vs. aleatory uncertainty). Our paper facilitates the study of the heterogeneity of source dependence by providing a method to measure and compare source dependence across individuals and sources.

We applied our method and estimated source-dependence functions on three datasets: one from a previously published paper (Abdellaoui et al. 2011) and two from original experiments. In all cases, one source was local and arguably more familiar to the subjects than the other. We chose datasets that represent the diversity of experimental designs (lab or online experiments, incentivized or hypothetical choices) and elicitation methods (certainty equivalents or matching probabilities) used in the empirical literature.

Our results show that source dependence can be easily estimated using a limited number of commonly used choice-based data. In particular, our method can be used not only with matching probabilities but also with certainty equivalents. For the latter, it does not require the measurement of utility or source (or probability-weighting) functions, thereby avoiding error propagation due to the measurement of these components. This means that our method could significantly reduce the number of choices required to measure source dependence compared to indirect methods.<sup>6</sup> Finally, we found that source-dependence functions offer an easy way to estimate and interpret heterogeneity in source dependence between individuals.

Overall, we found clear evidence of source dependence in our experimental studies. We also observed that source dependence must be described by two dimensions, as the sign and the magnitude of the source premium depend on the probabilities level. In other words, subjects’ preferences for one source over another may vary with the level of likelihood. Finally, our analyses revealed very heterogeneous patterns of source dependence in our samples. On average, individuals in our datasets exhibited a preference for the “local” source. However, a substantial proportion of the subjects exhibited the opposite pattern of preferences. Accounting for such heterogeneity in source

dependence may be as important as accounting for heterogeneity in beliefs in explaining economic patterns (Bakshi et al. 2015, Croitoru and Lu 2014).

## 2 Definition and measurement of source dependence

We first introduce the theoretical framework that defines source dependence. Then, we present our measurement method and show that it is compatible with all the usual types of choice-based data generally used for assessing source dependence.

### 2.1 Source dependence defined

Expected utility (EU) is the benchmark model of rational choice for decisions under uncertainty (Savage 1954). Under this model, preferences are captured by two components: a utility function  $U$  and a probability distribution  $\mu$  over events. The value assigned to a binary prospect  $(x, E, y)$ , the object of choice studied in this paper, that yields  $x$  if event  $E$  occurs and  $y$  otherwise, with  $x \geq y \geq 0$ , is

$$\mu(E)U(x) + (1 - \mu(E))U(y). \tag{1}$$

We assume monetary outcomes and strictly increasing utility throughout. In the case of risk, objective probabilities are available, and the value of a (risky) prospect  $(x, p, y)$ , that gives  $x$  with probability  $p$  and  $y$  otherwise, is

$$pU(x) + (1 - p)U(y). \tag{2}$$

Despite its normative appeal, this model fails to capture two major psychological aspects of decision under uncertainty: probability weighting and (non-neutral) ambiguity attitudes. Probability weighting refers to the observation that decision makers do not treat probabilities linearly (Kahneman and Tversky 1979). Under risk, this bias can be accommodated by the introduction of a strictly increasing probability-weighting function  $w$  mapping  $[0, 1]$  to  $[0, 1]$  and by assuming that a prospect  $(x, p, y)$  is evaluated by

$$w(p)U(x) + (1 - w(p))U(y). \tag{3}$$

Non-neutral ambiguity attitudes, the other well-documented deviation from EU, refers to the observation that decision makers may exhibit a preference between known and unknown probability distributions over events; in other words, they behave as if they do not assign the same weight to known and unknown probabilities. In a famous illustration of this behavior, Ellsberg (1961) intuited that people would prefer to bet on an urn with known composition (i.e., risky) rather than on an urn with unknown composition (i.e., ambiguous), even if there were no reason to believe that one composition would be more favorable than the other. This behavior can be accommodated by the introduction of a specific weighting function  $w_a$  and by assuming that an ambiguous prospect  $(x, E, y)$  is evaluated by

$$w_a(\mu(E))U(x) + (1 - w_a(\mu(E)))U(y). \quad (4)$$

Under this model, ambiguity attitudes are captured by the difference between the weighting functions  $w_a$ , when probability distributions over events are unknown, and  $w$ , when probability distributions over events are known. This model allows us to account for ambiguity aversion while assuming the existence of a unique distribution of probabilities  $\mu$ . This probability is called a-neutral, as it corresponds to the willingness to bet that would be observed for an ambiguity-neutral decision maker. In this paper, unknown probabilities are a-neutral and are referred to as probabilities, for the sake of simplicity.<sup>7</sup>

Following Chew and Sagi’s (2008) work on probabilistic sophistication within a source of uncertainty, Abdellaoui et al. (2011) developed an approach assuming that the weighting function can be different for each source, calling this function a *source function*. Using the source function  $w_S$ , an ambiguous prospect  $(x, E, y)$  with event  $E$  generated by a source  $S$  is evaluated by

$$w_S(\mu(E))U(x) + (1 - w_S(\mu(E)))U(y) \quad (5)$$

Comparing  $w_S$  to  $w$  characterizes the ambiguity attitude toward a given source  $S$ . The difference between source functions  $w_A$  and  $w_B$  of two distinct sources  $A$  and  $B$  characterizes *source dependence*: i.e., the fact that ambiguity attitudes differ across sources.<sup>8</sup>

The vast majority of empirical studies on ambiguity attitudes have focussed on the unknown “Ellsberg” urn as a source of uncertainty (for a review, see Trautmann and van de Kuilen 2015).

This source offers the advantage that probability distributions  $\mu$  can be inferred from symmetry arguments and consequently do not need to be measured. Fewer studies have measured attitudes toward one or several natural sources of uncertainty. Most of these studies compare attitudes toward a given source to attitudes toward risk (i.e.,  $w_S$  versus  $w$ ), revealing ambiguity attitudes (van de Kuilen and Wakker 2011). In the present paper, we compare behavior towards natural sources  $A$  and  $B$  and, hence, assess source dependence.

## 2.2 The source-dependence function $\phi$

In this section, we introduce the source-dependence function  $\phi$  that allows a direct measure of source dependence of preferences between two natural sources. We then show that the source-dependence function  $\phi$  can be measured using either matching probabilities (MP), that assess attitudes toward a source on the scale of probabilities, or certainty equivalents (CE), that assess attitudes toward a source on the scale of outcomes.

### 2.2.1 A direct measure of source dependence using a $\phi$ transformation function

We consider two natural sources,  $A$  and  $B$ , and their functions  $w_A$  and  $w_B$ . We introduce the function  $\phi_{AB}$ , such that  $w_B = w_A \circ \phi_{AB}$  (i.e.,  $\phi_{AB} = w_A^{-1} \circ w_B$ ). The function  $\phi_{AB}$  is strictly increasing, satisfies  $\phi_{AB}(0) = 0$  and  $\phi_{AB}(1) = 1$ , and maps probabilities of events  $E_B$  generated by the source  $B$  to probabilities of events  $E_A$  generated by the source  $A$ , with events  $E_A$  and  $E_B$  being such that the decision maker is indifferent between betting (a same desirable consequence) on them. Deviations of  $\phi_{AB}$  from identity directly characterize source dependence:  $A$  is strictly preferred to  $B$  if  $\phi_{AB}(x) < x$ . In turn,  $x - \phi_{AB}(x)$  represents the *source premium* of source  $A$  over source  $B$ , i.e., the decrease in likelihood the decision maker is ready to accept in order to bet on source  $A$  instead of source  $B$ . Therefore, the transformation function  $\phi_{AB}$  offers a direct measure of source preference of  $A$  over  $B$ . Inversely, the source preference of  $B$  over  $A$  is captured by  $\phi_{BA} = \phi_{AB}^{-1}$ .

### 2.2.2 Measuring $\phi$ from matching probabilities

As introduced earlier, the method developed by Dimmock et al. (2016b) consists of fixing an outcome  $x > 0$  and measuring a series of matching probabilities  $M_S$  such that  $(x, M_S, 0) \sim (x, E_S, 0)$ , where  $E_S$  are events generated by  $S$  for which the a-neutral probabilities  $\mu(E_S) = \lambda_S$  are known.

The analysis then consists of eliciting an ambiguity function  $m_S$  that maps the probabilities  $\mu(E_S) = \lambda_S$  onto the matching probabilities  $M_S$ :

$$m_S(\lambda_S) = M_S \tag{6}$$

Under standard assumptions of monotonicity and continuity, the ambiguity function  $m_S$  is strictly increasing and satisfies  $m_S(0) = 0$  and  $m_S(1) = 1$ . According to Equation (5),  $m_S = w^{-1} \circ w_S$ . Differences in preferences between two sources  $A$  and  $B$ , i.e., source dependence, can therefore be obtained by comparing ambiguity functions  $m_A$  to  $m_B$  (e.g., Li et al. 2017). More specifically,

$$m_B = w^{-1} \circ w_B = w^{-1} \circ w_A \circ \phi_{AB} = m_A \circ \phi_{AB}$$

Hence,

$$\phi_{AB} = m_A^{-1} \circ m_B$$

The source-dependence function  $\phi_{AB}$  relies on a direct comparison of ambiguity functions  $m_A$  and  $m_B$ , with no need to measure the weighting function for risk  $w$  or the source functions  $w_A$  and  $w_B$ .

### 2.2.3 Measuring $\phi$ from certainty equivalents

Suppose that we fix an outcome  $x > 0$  and measure, for each source  $S$ , a series of certainty equivalents  $CE_S$  such that  $CE_S \sim (x, E_S, 0)$ , where  $E_S$  are events generated by  $S$ , for which the a-neutral probabilities  $\mu(E_S) = \lambda_S$  are known. The method then consists of eliciting a function  $c_S$  that maps these probabilities  $\mu(E_S) = \lambda_S$  to the normalized certainty equivalents  $CE_S$ :

$$c_S(\lambda_S) = \frac{CE_S}{x} \tag{7}$$

For parallelism with the ambiguity function, we refer to  $c_S$  as an *uncertainty function*. Under standard assumptions of monotonicity and continuity, the uncertainty function  $c_S$  is strictly increasing and satisfies  $c_S(0) = 0$  and  $c_S(1) = 1$ . According to Equation (5), and after rescaling the utility such that  $U(x) = 1$ ,  $c_S = \frac{U^{-1} \circ w_S}{x}$ . Assuming that utility is source independent

(an assumption generally made in applications of the source model and empirically supported by Abdellaoui et al. 2011), differences of uncertainty functions  $c_S$  across sources reveal differences of source functions. In particular, for two sources  $A$  and  $B$ , comparing  $c_A$  and  $c_B$  allows us to elicit the source-dependence function  $\phi_{AB}$ , as follows:

$$c_B = \frac{U^{-1} \circ w_B}{x} = \frac{U^{-1} \circ w_A \circ \phi_{AB}}{x} = c_A \circ \phi_{AB}$$

Hence,

$$\phi_{AB} = c_A^{-1} \circ c_B$$

It is therefore possible to measure  $\phi_{AB}$  from certainty equivalents, with no need to control for the utility function.<sup>9</sup>

#### 2.2.4 Comments on the source-dependence function $\phi$

Overall, the source-dependence function  $\phi_{AB}$  can be measured simply from either matching probabilities or certainty equivalents. It does not require measuring or controlling for the utility, the weighting function for risk or even the source functions. Source-dependence functions can therefore be measured from a smaller number of choices and avoid error propagation due to the measurement of utility and source (or risk) weighting functions.

The characterization of source dependence is independent of risk attitudes (related to  $u$  and  $w$ ) and ambiguity attitudes (related to the difference between  $w_A$  and  $w$  or between  $w_B$  and  $w$ ). Instead, it relates to the differences of attitudes across sources. A linear  $\phi$  does not necessarily mean that decision makers are risk neutral or ambiguity neutral for the two sources, only that they exhibit the same attitude for the two sources. Conversely, there may be source dependence even if decision makers are risk neutral or ambiguity neutral for one of the two sources. Therefore, the introduction of source dependence, as measured by our function  $\phi$ , enlarges the scope of analysis of attitudes toward natural sources of uncertainty beyond the concept of risk and ambiguity attitudes.

Eventually, when  $A$  is a risky source ( $R$ ), we have  $w_B = w \circ \phi_{RB}$  and  $\phi_{RB} = w^{-1} \circ w_B$ . In this case, the transformation function  $\phi_{RB}$  corresponds to the ambiguity function proposed by Dimmock et al. (2016b) for capturing ambiguity attitudes. To summarize, the function  $\phi$  generalizes Dimmock



et al.’s (2016b) approach in two ways: it extends the approach for capturing source dependence between natural sources, and it allows measurement using not only matching probabilities but also certainty equivalents.

### 2.3 Parametric specifications

As the source-dependence function can apply to both ambiguity and uncertainty functions, we define it as

$$\phi_{AB} = f_A^{-1} \circ f_B \tag{8}$$

with  $f_A$  and  $f_B$  being either ambiguity or uncertainty functions of sources  $A$  and  $B$ .

In our analyses, we used parametric specifications for the functions  $f$  and  $\phi$ . We considered two popular, non-linear, two-parameter specifications for the function  $f$  (see Table 1): the Goldstein Einhorn (1987, hereafter GE) and the Prelec (1998). Parametric specifications have been commonly used to model probability-weighting functions (Bruhin et al. 2010), ambiguity functions (Li et al. 2017) and even uncertainty functions (l’Haridon and Vieider 2019). In all these applications, the two parameters, relating respectively to elevation and curvature, have behavioral interpretations. The parameter capturing the global elevation of the function (denoted  $\delta$ ) is interpreted in terms of optimism and the one measuring the curvature of the function (denoted  $\gamma$ ) is interpreted in terms of sensitivity toward changes in probabilities.

These non-linear specifications usually offer a better goodness of fit than the neo-additive specification (for an illustration, see Li et al. 2017). However, there are limitations to their use. First, the interpretation of the parameters is different for each specification.<sup>10</sup> Second, due to their non-linearity, it is difficult to interpret the effect of a change in a parameter on the shape of the function. This issue is particularly salient when studying the effect of a parameter’s heterogeneity (as measured, for example, by its standard deviation) on the shape of the function.

In our method, any deviation from linearity of the transformation function  $\phi$  captures the source dependence of preferences. However, with the “standard” parameters, it is hard to interpret how the coefficients of the function  $\phi$  translate into deviations from linearity and consequently to estimate the magnitude of the source premium. As an illustration, imagine two individuals having the same

sensitivity parameter  $\gamma$  for the source-dependence function  $\phi$ , but different elevation parameters  $\delta = 0.5$  and  $\delta = 1.5$ , respectively. With this information, it is difficult to conclude for which individual the source premium is the largest, in absolute terms.

In order to alleviate this issue, we propose a reparametrization of the GE and the Prelec specifications using two parameters  $\beta = 1 - 2\phi(0.5)$  and  $\alpha = 1 - \frac{\partial\phi}{\partial\mu}(0.5)$ . We use  $\beta$  to denote the global elevation parameter which captures the overall elevation of the plot, and  $\alpha$  to denote the global sensitivity parameter, which governs curvature (e.g., inverse-S shape of the plot). Importantly, while simplifying the interpretation of the results, this reparametrization does not create any loss of generality.

The first parameter  $\beta$  captures the “global” preference for source  $A$  over source  $B$ . As we see in Figure 2, when  $\beta > 0$  (blue curves), the subject exhibits a preference for source  $A$  over source  $B$ , whereas when  $\beta < 0$  (red curves), the subject exhibits a preference for source  $B$  over source  $A$ . In addition, the value  $\beta/2$  represents the *source premium* of source  $A$  over source  $B$  in the middle of the probability interval, i.e., the decrease in likelihood the decision maker is willing to accept in order to bet on source  $A$  over source  $B$ . It takes value 0 when  $\phi(0.5) = 0.5$  (i.e., when there is no source premium).<sup>11</sup>

The second parameter  $\alpha$  relates to the slope (i.e., the derivative) of the function  $\phi$  for probability 0.5. It characterizes the sensitivity to changes in probabilities in the middle of the probability interval.  $\alpha$  takes value 0 when the slope is 1, and negative (positive) values in case of oversensitivity (insensitivity) to changes in probabilities. As we can see in Figure 2, the pattern of preference between the two sources, and therefore the magnitude and sign of the source premium, can depend on the level of likelihood. The sensitivity parameter  $\alpha$  captures the elasticity of the source premium to likelihood changes. When  $\alpha > 0$ , the premium of source  $A$  over source  $B$  increases (decreases) by  $\alpha$  percentage point of probability when the likelihood increases (decreases) by one percentage point of probability from probability 0.5. The reverse pattern is observed when  $\alpha < 0$ .

An interesting property is that these parameters can be directly computed from the original parameters of the two non-linear specifications considered in this paper (see Table 1 for the mapping between these parameters and the original ones). Importantly, while the parameters can be interpreted with reference to the value of the function or its derivative for probability 0.5, they are not estimated from the behavior of the function in the middle of the probability interval alone.

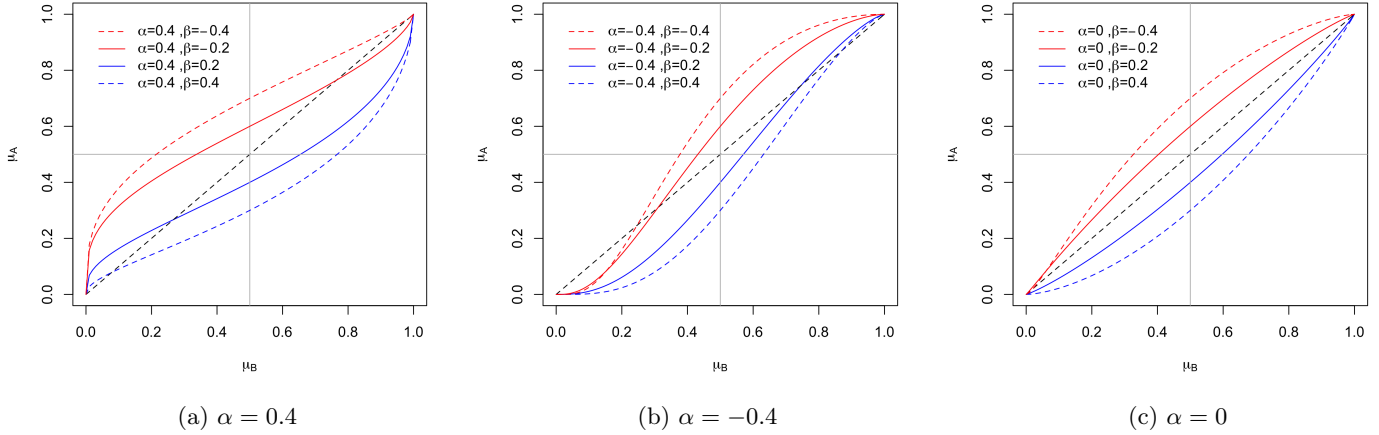


Figure 2: Illustration of the source-dependence function  $\phi$

Instead, they depend on the behavior of the function *over the whole interval*, like any other parametric specification. In this regard, the function estimated using our parameters is one-to-one related to the function estimated using the original parameters. However, the reparametrization allows for easier interpretation of the function parameters and their heterogeneity. In particular, the parameters have the same interpretation regardless of the chosen specification.<sup>12</sup>

		Prelec (1998)	Goldstein Einhorn(1987)
Expression		$exp(-\delta(-\log(p))^\gamma)$	$\frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$
Modified parameters	$\alpha$	$1 - 2\delta\gamma exp(-\delta(-\log(0.5))^\gamma)[- \log(0.5)]^{\gamma-1}$	$1 - \frac{\delta\gamma p^{\gamma-1}(1-p)^{\gamma-1}}{(\delta p^\gamma + (1-p)^\gamma)^2}$
	$\beta$	$1 - 2exp(-\delta(-\log(0.5))^\gamma)$	$1 - 2\frac{\delta}{\delta+1}$
Original parameters	$\gamma$	$\frac{\log(0.5)(1-\alpha)}{(1-\beta)\log(0.5(1-\beta))}$	$\frac{1-\alpha}{(1+\beta)(1-\beta)}$
	$\delta$	$\frac{-\log(0.5(1-\beta))}{[-\log(0.5)]^\gamma}$	$\frac{1-\beta}{1+\beta}$

Table 1: Specifications and their re-parametrization

### 3 Empirical implementation

#### 3.1 Data

We used three studies to test our method empirically, relying on one existing dataset (Study A) and designing two experiments (Studies B and C) such that we could scan several combinations of

approaches to evaluate prospects (CE vs. MP) and control for beliefs. Studying attitudes towards natural sources requires measuring or controlling beliefs that are not necessarily uniform. Because the methods disentangling beliefs from attitudes are not the paper’s main topic, we relegate their description to Appendix A.

In all of these studies, one source was local and arguably more familiar to the subjects than the other. We used this local source as the reference source. The studies used different experimental procedures (individual interviews in Studies A and B and an online experiment in Study C) and incentives (random incentives in Studies A and B and hypothetical choices in Study C). Table 2 summarizes the characteristics of these three datasets. We present the details of all three studies below and report the instructions of experiments B and C in Appendix E.

Study	N	Valuation method	Elicitation of beliefs	Sources
Study A	62	CE	EE	Temperature in Paris
				Temperature in a foreign city
Study B	95	MP	EE	Approval rating of French president E. Macron
				Approval rating of American president D. Trump
Study C	200	CE	BH	Temperature in Paris
				Temperature in Belgrade

*Note: EE stands for events exchangeability and BH for belief hedging.*

Table 2: Summary of the three datasets

### 3.1.1 Study A

For this study, we used the data on two natural sources  $S$  collected by Abdellaoui et al. (2011): the temperature in Paris ( $S = A$ ) and the temperature in a foreign city ( $S = B$ ). For each source, subjects’ beliefs were measured prior to eliciting their attitudes toward ambiguity.

*Measurement of beliefs:* Participants’ beliefs about the sources were elicited using the approach developed by Baillon (2008) based on exchangeable events (see Appendix A). For each source  $S$ , a sequential process was used to build a series of five events  $E_{k,S}$  with probabilities  $\mu_k \in (1/8, 1/4, 1/2, 3/4, 7/8)$ . Abdellaoui et al. (2011) provide more details about the procedure.

*Evaluation of prospects:* With these events (for which the researchers knew the a-neutral probability) at hand, the certainty equivalents  $CE_{k,S}$  of five prospects  $(1000, E_{k,S}, 0)$  were measured for each source. These CEs allowed us to assess the uncertainty function  $c_S$ , since  $c_S(\mu_{k,S}) = \frac{CE_{k,S}}{1000}$ .<sup>13</sup>

*Procedure:* 62 subjects participated in individual, computer-based interviews. Real incentives

were implemented for half of the subjects (real-incentive treatment) whereas the other half made hypothetical choices (hypothetical treatment).

### 3.1.2 Study B

In this study, we used a similar design as in Study A, but with different sources and a different valuation approach of ambiguous prospects (MPs instead of CEs). The two sources  $S$  were the approval ratings of the French president Emmanuel Macron (source  $S = A$ ) and US president Donald Trump (source  $S = B$ ).<sup>14</sup> These two variables take values between 0 and 100 percent and were revealed one month after the experiment.<sup>15</sup>

*Measurement of beliefs:* In a first task, we elicited a series of events  $E_{k,S} = [0, v_{k,S}]$  generated by  $S$  with a-neutral probabilities  $\mu(E_{k,S}) \in (1/8, 1/4, 1/2, 3/4, 7/8)$ , using events exchangeability as in Abdellaoui et al. (2011). Values  $v_{k,S}$  are percentages of approval ratings and were measured with a precision of one percentage point.

*Evaluation of prospects:* In a second task, we measured ambiguity attitudes using MPs. For each source, we measured the matching probabilities  $mp_{k,S}$  of prospects  $(100, E_{k,S}, 0)$  with a precision of 0.01. This allowed us to assess the ambiguity function  $m_S$  since  $m_S(\mu_{k,S}) = mp_{k,S}$ .

Both beliefs and attitudes rely on the measurement of indifference values, which we elicited with choice lists. We used a bisection procedure to complete these lists (see Abdellaoui et al. 2019). When a list was completed, the subjects reviewed all the choices from the list and were able to make changes, if necessary. Subjects then had to confirm the whole list in order for the software to move to the next choice list.

*Procedure:* We recruited 95 subjects to participate in a one-hour individual, computer-based interview for a compensation of €10. Upon arrival, subjects watched a 10-minute video describing the experiment. A survey including comprehension questions followed, in order to identify subjects who required additional clarifications from the research assistants. The experiment began with a few practice questions to familiarize subjects with the software. Subjects then completed the belief task and ambiguity task for one of the two sources before moving to the second source. For each source, the belief task always preceded the ambiguity task. The order of the questions in the ambiguity task was randomized.

Real incentives were implemented, and the related procedure was presented in the instructions.

Specifically, each subject had a 10% chance of being selected and having one of her choices played for real at the end of the experiment. During the instructions, subjects were informed that all the choices they would make during the experiment were eligible to be selected and played for real. The selected participants could gain up to €100 extra.

### 3.1.3 Study C

In this study, we measured beliefs and attitudes jointly using certainty equivalents and belief hedging (Baillon et al. 2017).

*Evaluation of prospects:* We considered two sources  $S$ : the temperature, in celsius degrees, in a local city, Paris, France (source  $S = A$ ) and a foreign city, Belgrade, Serbia (source  $S = B$ ). For each source  $S$ , we created an exhaustive partition of mutually exclusive events  $E_{1,S}, E_{2,S}, E_{3,S}$  and measured CEs for all prospects  $(20, E_{k,S}, 0)$ , where  $E_{k,S} \in \{E_{1,S}, E_{1,S}^c, E_{2,S}, E_{2,S}^c, E_{3,S}, E_{3,S}^c\}$ . The three events were  $E_{1,S} = (-\infty, 18]$ ,  $E_{2,S} = ]18, 22]$ , and  $E_{3,S} = ]22, +\infty)$ . We elicited CEs using the bisection with a precision of €1.

*Procedure:* We ran the experiment on a large pool of 200 subjects. It was conducted online, and choices were hypothetical.

## 3.2 Estimation Strategy

### 3.2.1 Errors specification and likelihood function

We used a unified statistical approach to measure source dependence between two sources  $s \subset \{A, B\}$  in the available datasets. In the three experiments, our measurement followed an equation of type

$$\begin{aligned} y_{i,k,s} &= f_i(\mu_{i,k,s}) \text{ if } s = A \\ &= f_i \circ \phi_i(\mu_{i,k,s}) \text{ if } s = B \end{aligned}$$

where  $y_{i,k,s}$  is the valuation (either a MP or a CE) by subject  $i$  of a prospect  $k$  involving event  $E_{i,k,s}$  with probability  $\mu_{i,k,s}$ ,  $f$  is either an ambiguity or uncertainty function, and  $\phi$  is a source-dependence function.

We assumed that subjects made decision errors, such that the measured indifference  $y_{i,k,s}^*$  followed  $y_{i,k,s}^* = y_{i,k,s} + \epsilon_{i,s}$  where  $\epsilon_{i,s} \sim N(0, \sigma_{i,s}^2)$ . Hence, we accounted for heteroscedasticity across sources and individuals. Indifferences were measured with a precision  $\eta$  such that the likelihood of each observations followed

$$\begin{aligned} \pi(y_{i,k,s} | \theta_i, \mu_{i,k,s}) &= p(y_{i,k,s}^* - \frac{\eta}{2} < y_{i,k,s} + \epsilon_{i,s} < y_{i,k,s}^* + \frac{\eta}{2}) \\ &= p(y_{i,k,s}^* - \frac{\eta}{2} - y_{i,k,s} < \epsilon_{i,s} < y_{i,k,s}^* + \frac{\eta}{2} - y_{i,k,s}) \\ &= \Psi\left(\frac{y_{i,k,s}^* - y_{i,k,s} + \frac{\eta}{2}}{\sigma_{i,s}}\right) - \Psi\left(\frac{y_{i,k,s}^* - y_{i,k,s} - \frac{\eta}{2}}{\sigma_{i,s}}\right) \end{aligned}$$

where  $\theta_i$  is the vector of function parameters  $a_i$  and  $b_i$  (the parameters of  $f_i$  for an arbitrarily chosen reference source),  $\alpha_i$  and  $\beta_i$  (the parameters of the source-dependence function  $\phi_i$ ), and  $\mu_{i,k,s}$  (the beliefs). The cumulative function of the normal distribution is denoted  $\Psi$ . With belief hedging (used in Study C), we estimated beliefs along with other parameters (see the details in Appendix C).

The likelihood for a given individual  $i$  is

$$l(\theta_i) = \prod_s \prod_k \pi(y_{i,k,s}, \theta_i, \mu_{i,k,s})$$

This likelihood specification aims to elicit the parameters of the function  $f$  that captures attitudes toward one of the two source (taken as the reference) and, more importantly, the parameters of the transformation function  $\phi$  that captures source dependence.

### 3.2.2 Accounting for preference heterogeneity

At the *aggregate level*, all the subjects were assumed to have the same preferences, i.e.,  $\theta_i$  did not depend on the index  $i$ . In particular, this meant that the preferences of all the subjects were the same for the reference source and revealed the same pattern of source dependence. Because this assumption may be unrealistic, we considered a random-coefficient model where source dependence (captured by parameters  $\alpha_i$  and  $\beta_i$ ) was randomly distributed across subjects. Parameters of ambiguity or uncertainty functions for the reference source were also assumed to be randomly

distributed.

We used  $\bar{\alpha}$  and  $\bar{\beta}$  ( $\sigma_\alpha$  and  $\sigma_\beta$ ) to denote the mean (standard deviation) of the insensitivity and elevation parameters of the source-dependence function. The random coefficient models were estimated using a Hierarchical Bayes (HB) simulation as these have recently been shown to be suitable for estimating risk models (Murphy and ten Brincke 2017). To do so, we used the RSGHB R package, with priors corresponding to linear uncertainty or ambiguity function with virtually no heterogeneity. Such priors correspond to rational representative agent models (ambiguity-neutral or uncertainty-neutral attitudes) with no between-subject heterogeneity and no source dependence. Our choice of priors was therefore based on rational-choice models, and played “against” our results, which revealed non-linear and heterogeneous functions with heterogeneous source effects.

## 4 Results

This section presents the results of the empirical implementation of our econometric set-up for the three studies. For each study, we report the econometrics estimations of the means and standard deviations of the parameters of the source-dependence function  $\Phi_{AB}$  (see Table 3). The descriptive statistics of studies B and C are provided in Appendix B.<sup>16</sup> We focus on the results obtained with the Prelec specification as it is compatible with the parametric approaches used for modeling both uncertainty and ambiguity functions.<sup>17</sup> The results obtained with the Goldstein Einhorn specification were similar (see Appendix D).

	Study A		Study A (only real incentives)		Study B		Study C	
$\bar{\alpha}$	0.000	[-0.054; 0.055]	-0.205	[-0.286; -0.096]	0.353	[0.251; 0.449]	0.051	[0.012; 0.091]
$\bar{\beta}$	0.028	[-0.071; 0.129]	0.104	[0.030; 0.176]	0.277	[0.171; 0.377]	0.059	[0.032; 0.085]
$\sigma_\alpha$	0.163	[0.125; 0.212]	0.181	[0.115; 0.279]	0.319	[0.255; 0.396]	0.229	[0.200; 0.264]
$\sigma_\beta$	0.335	[0.268; 0.416]	0.137	[0.086; 0.198]	0.360	[0.294; 0.436]	0.125	[0.101; 0.164]
LL	-1817.354		-982.404		-3313.293		-3449.682	

Note: 95% credible intervals between brackets.

Table 3: Summary of HB estimations - Studies A, B and C

### 4.1 Study A

Study A compared attitudes toward temperature in a local city (Paris, France) and temperature



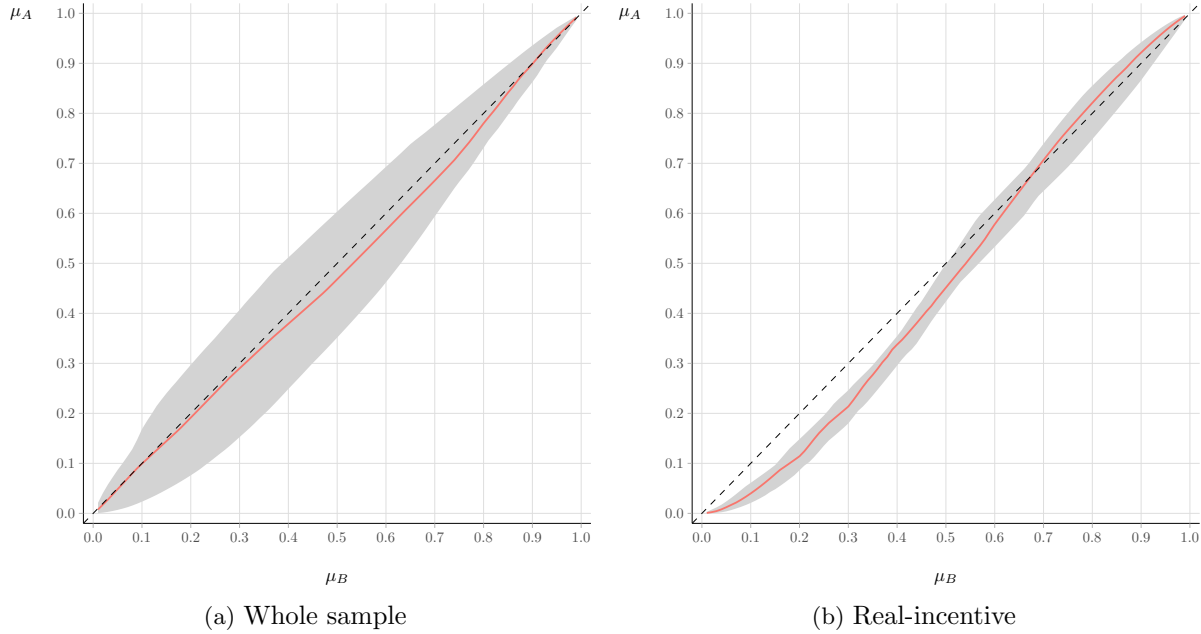


Figure 3: Source-dependence function (median and IQR) for Study A

in a foreign city, which was different for each subject. Figure 3 displays the quartile behavior (median and interquartile range) of the estimated source-dependence function  $\phi_{AB}$ .

Looking at the parameters of  $\phi_{AB}$ , we observed no average source dependence, as the 95% credible interval (hereafter CI) of both  $\bar{\alpha}$  and  $\bar{\beta}$  included 0. Following Abdellaoui et al. (2011), we further performed analyses focusing on the group with real incentives. In this subsample, we found evidence of source dependence. The parameter  $\bar{\beta}$  was positive (95% CI = [0.030; 0.176]), indicating that, on average, subjects exhibited a preference for the local source over the foreign source. The re-parametrization offers an easy interpretation of this parameter. The source premium in the middle of the likelihood interval was equal to 0.05 (i.e.,  $\bar{\beta}/2$ ). In other words, the average subject was willing to give up a 0.05 winning probability to bet on an event generated by the local source rather than on an event of probability 0.5 generated by the foreign source.

The average insensitivity parameter  $\bar{\alpha}$  for the group with real incentives was negative (95% CI = [-0.286; -0.096]), suggesting that the premium paid to avoid betting on the foreign source decreased with the level of likelihood: subjects exhibited a preference for the local source for low and medium levels of likelihood, but this pattern was reversed for higher levels of likelihood. This result is consistent with Abdellaoui et al. (2011), who found a preference for betting on the temperature

in Paris over the temperature in a foreign city for  $p < 0.5$  with the real-incentive subsample.

We also observed a considerable heterogeneity in source dependence. Looking at the whole sample, a large portion of the subjects exhibited a strong preference for one source over the other. Indeed, the standard deviation of the elevation parameter ( $\sigma_\beta = 0.33$ ) suggested that  $\beta$  was greater than 0.3, in absolute value, for one-third of the subjects.<sup>18</sup> In other words, one-third of the subjects behaved as if they inflated or deflated a 0.5 winning event probability by at least 0.15, depending on their source preference. We also found heterogeneity in the sensitivity dimension of source dependence, indicating that the rate at which the source premium increased/decreased with changes in likelihood differed between subjects.

Overall, Study A confirmed the source dependence of preferences. In the real-incentive subsample, subjects exhibited a preference for the local source over the foreign source. Besides, our analyses showed that the magnitude, and direction, of this source premium could depend on the level of likelihood. This study also revealed considerable heterogeneity in source dependence and provided evidence of pronounced source dependence for a sizable part of the sample. Interestingly, despite considerable source dependence at the individual level in the whole sample (i.e., when pooling incentivized and non-incentivized groups), the effects cancelled out at the aggregate level, resulting in no average source dependence. Therefore, this study showed that an apparent absence of average source dependence may hide important effects, though in opposite directions, at the individual level.

## 4.2 Study B

Study B measured attitudes toward approval ratings of the local (French, the reference source) president and a foreign (US) president. The study used matching probabilities with beliefs measured independently using the events-exchangeability method.

We captured an average source-dependence effect for the elevation parameter ( $\bar{\beta} = 0.277$ , 95% CI [0.171; 0.377]); overall, there was a preference for the approval rating of the local president over that of the foreign president. Despite the fact that this study used a different method to evaluate prospects than Study A (matching probabilities instead of certainty equivalents), the parameters of the function  $\phi_{AB}$  can be interpreted in the same way as in Study A. The source premium of source A over source B at probability 0.5 was consequential: 0.14 (as  $\bar{\beta} = 0.277$ ). On average,

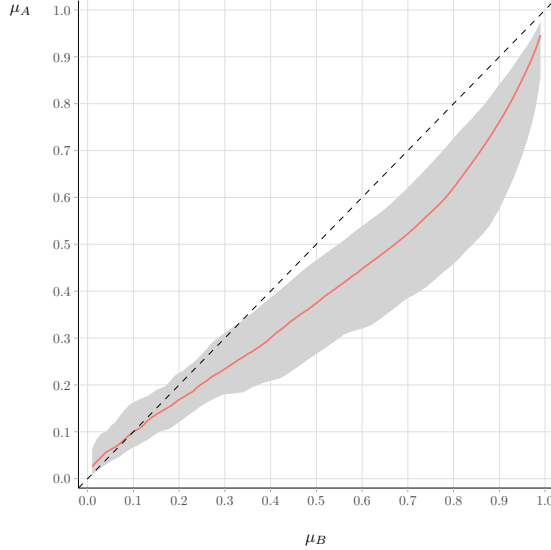


Figure 4: Source-dependence function (median and IQR) for Study B

subjects were ready to give up a 0.14 winning probability in order to bet on the local source rather than on a 0.5-probability event generated by the foreign source. In addition, we reported a positive average sensitivity parameter ( $\bar{\alpha} = 0.353$ , 95% CI [0.251; 0.449]). Therefore, the magnitude of the source premium was not constant but instead increased with an increase in the likelihood. While the average source premium (for betting on the local rather than the foreign source) was 0.14 at probability 0.5, it increased (decreased) by 0.35 percentile points (i.e.,  $0.01 \times 0.35$ ) for every 1 percentile point increase (decrease) in the probability of events generated by the foreign source. As a consequence, subjects exhibited a preference for the foreign source over the local one for very low levels of likelihood, but the pattern was reversed for medium to high levels of likelihood.

Regarding the heterogeneity in the sample, we observed large between-subjects differences for the source-dependence function  $\phi_{AB}$  (see Figure 4). On average, subjects exhibited a preference for the local source, and this preference was very strong ( $\beta > 0.5$ ) for about 25% of the subjects. In contrast, around 20% of the subjects exhibited a preference for the foreign source ( $\beta < 0$ ). There was also a high level of heterogeneity in terms of the sensitivity parameter  $\alpha$ . For instance, the parameter  $\alpha$  was greater than 0.5 for about 30% of the subjects, indicating a strong likelihood dependence of the source premium.

### 4.3 Study C

Study C used certainty equivalents to jointly measure beliefs and attitudes toward local (Paris, France, the reference source) versus foreign (Belgrade, Serbia) temperatures.

Our estimations captured a significant mean effect for both elevation and insensitivity parameters of the source-dependence function  $\phi_{AB}$ . The average subject exhibited a preference for the local source over the foreign source ( $\bar{\beta} = 0.059$ , 95% CI = [0.032; 0.085]). On average, subjects were ready to give up a 0.03 winning probability (as  $\bar{\beta} = 0.059$ ) in order to bet on the local source rather than on a 0.5-probability event generated by the foreign source. The average insensitivity parameter indicated that source preferences were likelihood dependent ( $\bar{\alpha} = 0.05$ , 95% CI = [0.012; 0.091]). Consequently, the source premium of source A over source B slowly increased (decreased) when the likelihood level increased (decreased). As consequence, for very low levels of likelihood of event generated by the foreign source, the source premium was negative and preferences were reversed: the average subject then preferred to bet on the foreign rather than on the local source.

We also observed sizable heterogeneity in terms of source dependence for the parameter  $\beta$ . For instance, while the average subject exhibited a preference for the local over the foreign source, around 30% of the subjects exhibited the opposite pattern. Heterogeneity was even larger for the sensitivity parameter  $\alpha$ . Around 40% of the subjects exhibited a pattern opposite to the average behavior, i.e., they preferred the local source for low levels of likelihood and the foreign source for higher levels of likelihood.

Overall, study C revealed patterns similar to study B, despite using different sources, measurement method (CE vs. MP), and experimental procedure (online vs. lab experiment). Regarding source dependence, preference for the local source prevailed. However, this was not a universal pattern. A portion of the sample exhibited the opposite behavior. Besides, the pattern of preference between local and foreign sources depended on the level of likelihood: the magnitude and sign of the source premium was likelihood dependent.

Contrasting the results of this study with the ones of Study A reinforces the importance of taking into account heterogeneity in attitudes. While the magnitude of the source dependence was, on average, larger in Study C than in Study A, subjects were more likely to exhibit extreme preferences for one source over the other in Study A. Unlike Study C, which used the same foreign

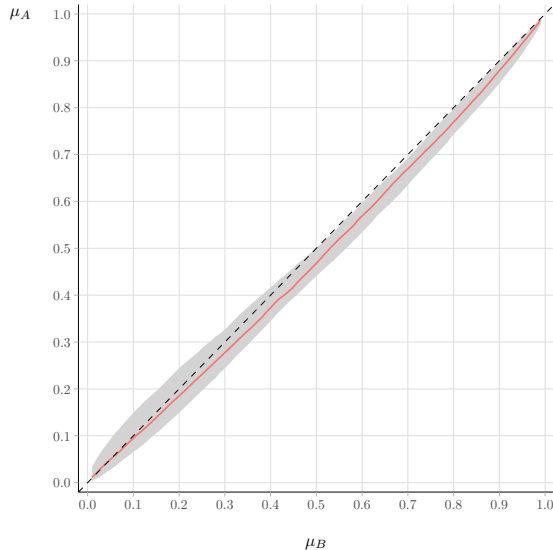


Figure 5: Source-dependence function (median and IQR) for Study C

source for all subjects, study A used different foreign sources for different subjects. This difference in design may explain why we observed more heterogeneity in source dependence in Study A than in Study C.

## 5 Discussion

### 5.1 Methodological contribution: a simple and general method to measure source dependence

The paper’s first contribution is to offer a simple and general method to measure source dependence by introducing a source-dependence function. This function  $\phi$  maps probabilities of events generated by two sources and allows for a direct and easily interpretable measure of source dependence. Any deviation of the function  $\phi$  from linearity indicates the presence of source dependence. In addition, the distance of  $\phi$  from the identity function measures a source premium, i.e., the decrease in winning probability a decision maker is ready to accept to bet on one source rather than another. This premium is measured on a cardinal scale (probabilities) and is therefore easily interpretable and comparable across studies. Finally, the source-dependence function  $\phi$  is also suitable to study the likelihood dependence of preferences between sources for a given decision maker or between decision makers for a given source.

We have shown that this source-dependence function can be measured using either matching probabilities or certainty equivalents. Using matching probabilities is an efficient approach as it does not require the measurement of the utility (Dimmock et al. 2016b). However, it can be challenging to implement when subjects are not familiar with probabilities (Bouchouicha et al. 2017). The alternative approach, using certainty equivalents, is cognitively easier for the decision maker but requires measuring the utility function (e.g., Abdellaoui et al. 2011), which considerably increases the complexity and the duration of the experiment. Our approach, implemented with uncertainty functions, offers a simple way to analyze source dependence by measuring attitudes on the outcome scale (using certainty equivalents, for example) but with no need to measure the utility function or weighting functions. As an illustration, we applied this approach to the dataset from Abdellaoui et al. (2011). We measured source dependence using only a subset of the choice tasks, as our approach did not require the use of the certainty equivalents dedicated to the measurement of the utility function. Overall, the results that we obtained, in terms of source dependence, were similar to those obtained by Abdellaoui et al. (2011).

Our measurement method is also general since it is compatible with the two most common approaches for controlling for beliefs: explicitly measuring beliefs using the events-exchangeability method (Abdellaoui et al. 2011) or controlling them using the belief-hedging method (Baillon et al. 2018). In particular, our method generalizes the belief-hedging approach proposed by Baillon et al. (2018) in two ways. First, while the belief-hedging approach requires eliciting matching probabilities, our method can be deployed using either matching probabilities or certainty equivalents. Second, it is compatible not only with the neo-additive function but also with other non-linear functions.

Another methodological contribution of this paper is to unify the use of some popular two-parameter functions. We propose a re-parametrization of the two most popular specifications (i.e., Prelec and Goldstein-Einhorn) that offers two easily interpretable and linear parameters. This re-parametrization facilitates the comparison of the results across specifications and the interpretation of the heterogeneity in parameters. We estimated these parameters using Hierarchical Bayesian analyses and observed that the results obtained were consistent across specifications (see Appendix D).

## 5.2 Empirical results: source dependence and its heterogeneity

We found evidence of source dependence in the three experimental studies. Notably, this effect was sometimes masked when looking at the mean parameters but unveiled when considering the heterogeneity in the data. For instance, in Study A, even though many subjects exhibited strong preferences for one source over the other, we did not find any average source-dependence effect.

Studies B and C, as well as Study A when focusing on the subjects in the real-incentive condition, captured an average source-dependence effect. In other words, subjects exhibited different attitudes toward “local” sources than similar “foreign” sources: they generally preferred the local source to the foreign one, as captured by the source-dependence function’s elevation parameter.

We also observed that the source premium was not constant but instead often varied depending on the likelihood of events. The sensitivity parameter of the source dependence function captured this likelihood-dependence of source premiums. Although, on average, subjects preferred the local source for likely events, the opposite pattern was sometimes observed for unlikely events. A one-parameter (e.g., power) source-dependence function would not be able to capture this pattern. Instead, different specifications are needed to capture the several inflection points of the source-dependence function, as intuited by Fox and Tversky (1995). In the same way that Li et al. (2017) showed that two-parameter specifications are needed to capture attitudes toward natural sources of uncertainty, we showed that these functions are also needed to capture the complexity of source dependence.

Prior studies measuring source dependence have mostly focused on mean or median differences between sources. These differences are computed either directly from aggregate or random coefficient estimations (e.g., Baillon et al. 2017, Li et al. 2017) or individual-level estimations (e.g., Abdellaoui et al. 2011). Studying modal (i.e., mean or median) source dependence allows us to capture a modal direction of the effect, but it fails to capture heterogeneity in source dependence. Taking heterogeneity into account is essential because it allows us to refine our understanding of specific economic mechanisms and can lead to different predictions from the ones generated with a representative agent. For instance, heterogeneity in beliefs has been shown to explain some patterns in asset pricing (Croitoru and Lu 2014), the volatility of outcomes (Bakshi et al. 2015, Li 2013), speculative bubbles (Scheinkman and Xiong 2003), and political extremism (Glaeser et al. 2005).

Similarly, heterogeneity in risk attitudes has been used theoretically to study phenomena as varied as insurance demand (Cutler et al. 2008), asset pricing (Dumas 1989, Longstaff and Wang 2012), and occupational choice (Kihlstrom and Laffont 1979).

Influential empirical papers have revealed the heterogeneity of risk preferences (Bruhin et al. 2010, Von Gaudecker et al. 2011, Falk et al. 2018). Scholars have also started to investigate the heterogeneity of ambiguity attitudes *within* a source, i.e., across subjects, using Ellsberg urns as a source of uncertainty (e.g., Dimmock et al. 2016b), showing, for example, that ambiguity aversion is not universal (Trautmann and van de Kuilen 2015). More recently, Baillon et al. (2017) and Abdellaoui et al. (2021) have used mixed modeling techniques to capture heterogeneity in attitudes toward natural sources (i.e., within source). Our paper contributes to this literature by showing evidence of heterogeneous patterns not only in terms of ambiguity/uncertainty attitudes but also in source dependence.

Accounting for heterogeneity in source dependence may be as important as accounting for heterogeneity in beliefs or risk attitudes. Indeed, “the domain of nonprobabilized uncertainties is rich just like the domain of non-monetary commodities, with many kinds of informational and emotional configurations” (Li et al. 2017, p. 1). Attitudes toward a given source may be explained by different dimensions such as the perceived degree of competence or familiarity with the source or the type of uncertainty (i.e., epistemic or aleatory). The perception of these dimensions may vary broadly from one individual to another. As an illustration, the high level of heterogeneity in Study A may be due to the fact the foreign cities varied across subjects (see Abdellaoui et al. 2011). Different cities may generate different valence, memories, levels of expertise, or forecast difficulties. Future research could explore the effects of source characteristics on individuals’ attitudes and how these characteristics interact among themselves and with individuals’ characteristics.

## 6 Conclusion

This paper introduces a source-dependence function that directly captures source dependence, i.e., differences of attitudes across natural sources of uncertainty. We show that the function can be measured from a limited number of choices and is compatible with commonly used approaches for valuing prospects and controlling beliefs. The empirical implementation on three experimental



datasets shows that source dependence (1) is robust to different sources and modeling assumptions but (2) highly heterogeneous and (3) can be studied using two intuitive parameters of elevation and insensitivity.

## Endnotes

<sup>1</sup>We define a source of uncertainty as a family of events generated by a similar mechanism of uncertainty (Tversky and Fox 1995, Abdellaoui et al. 2011).

<sup>2</sup>We follow the convention in the literature and use the term “natural” to qualify sources that are distinct from more “artificial” sources of uncertainty based on, for instance, urns with different (known or unknown) compositions.

<sup>3</sup>If decision makers have different utility functions, the comparison of the difference between their respective source functions may be distorted or even reversed when expressed on the outcome scale (i.e., in terms of differences in willingness to pay). Similarly, if decision makers treat known probabilities differently, the comparison of the difference between their respective source functions may be distorted or reversed when expressed on the scale of known probabilities (i.e., in terms of difference of willingness to bet).

<sup>4</sup>Although transformation functions have not yet been used to capture differences of attitude between sources, they have been used throughout decision theory to capture comparisons (Kreps and Porteus 1978, Klibanoff et al. 2005).

<sup>5</sup>The opposite situation can occur: If  $\mu_A > \mu_B$ , the decision maker exhibits a preference for source  $B$  over source  $A$  and would require an increase in winning probability  $\mu_A - \mu_B$  in order to bet on the event generated by source  $A$  instead of the one generated by source  $B$ . The source premium of source  $B$  over source  $A$  is defined as the difference between  $\mu_A$  and  $\mu_B$ .

<sup>6</sup>In the empirical section, we show that, using the dataset from Abdellaoui et al. (2011), we obtained similar results with a lower number of choices.

<sup>7</sup>Following Dimmock et al. (2016b), we use the notation  $a$ -neutral probabilities instead of subjective probabilities.  $A$ -neutral probabilities “can be interpreted as the beliefs of the ambiguity neutral twin of the decision maker” (Baillon et al. 2019).

<sup>8</sup>Several authors have proposed considering risk as a specific source of uncertainty. Under this convention, ambiguity aversion ( $w_a \neq w$ ) is a specific case of source dependence: a preference for sources with known probabilities over sources with unknown probabilities.

<sup>9</sup>In this paper, we do not interpret the uncertainty functions on their own. We instead use them as a measurement tool for capturing the source-dependence functions.

<sup>10</sup>As an illustration, Li et al. (2017, p. 10) have noted that “in Prelec’s family, the insensitivity parameter  $[\gamma]$  overlaps partly with the aversion parameter  $[\delta]$ , also capturing some aversion.”

<sup>11</sup>We note that the Prelec and the GE specifications measure the global elevation for different probability levels ( $p = 1/e$  for the Prelec specification and  $p = 0.5$  for the GE). We propose expressing the global elevation and

sensitivity at probability 0.5, which is a natural benchmark for assessing the global shape characteristics.

<sup>12</sup>We note that this reparametrization can also be employed for modeling other functions for which the Prelec or GE specifications are suitable. This is the case, for example, of probability-weighting functions, source functions, ambiguity functions, or even uncertainty functions.

<sup>13</sup>In the present paper, we focus on this relationship, even though Abdellaoui et al. (2011) employed a different approach. They used additional CEs to elicit the utility function and “correct” the function  $c_s$  for the utility curvature in order to assess the source function  $w_S$ .

<sup>14</sup>We use the following two information sources for Donald Trump and Emmanuel Macron’s approval ratings: <https://elections.huffingtonpost.com> and <http://www.tns-sofres.com>.

<sup>15</sup>In the experiment, we used two periods of time (one month and nine months after the experiment). In this paper, we report only the results obtained for the approval rating one month after the experiment.

<sup>16</sup>For the descriptive statistics of Study A, see the original paper: Abdellaoui et al. (2011).

<sup>17</sup>In the case of an uncertainty function  $c_S = u^{-1} \circ w_S$ , if  $u$  follows a power specification (i.e.,  $u(x) = x^\alpha$ ) and the source function  $w_S$  follows a Prelec specification with parameters  $\delta'$  and  $\gamma'$  then  $c$  also follows a Prelec with parameter  $\delta = \delta'/\alpha$  and  $\gamma = \gamma'$ . In the case of ambiguity function  $m_S = w^{-1} \circ w_S$ , if  $w$  and  $w_S$  both follow a Prelec, then  $m_S$  also follows a Prelec.

<sup>18</sup>This is because the mean value  $\bar{\beta}$  was almost equal to 0.

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## Appendix A: Methods for controlling for beliefs

Several methods have been developed in the literature to control for beliefs, independently of the approach employed for evaluating ambiguous prospects.

- Considering symmetric sources for which beliefs can be inferred without measurement.

In specific cases, the symmetry argument allows us to assume that beliefs about the likelihood of events of a given source are uniform. For instance, in the case of the two-color Ellsberg urn, subjects have no reason to believe that one color is more likely than the other; it can therefore be assumed that subjects assign the same probability to both colors.<sup>19</sup> Sources for which beliefs can be assumed to be uniform, based on symmetry arguments, are called *symmetric sources*. Urns à la Ellsberg have this convenient property of symmetry but are artificial. In order to extend the analysis of source preferences to natural sources of uncertainty, several authors have considered natural symmetric sources. For instance, Chew et al. (2012) considered the trailing digit (odd or even) of a stock's closing price. More recently, Li et al. (2017) designed cases of natural sources featuring the symmetry argument. However, more often than not, natural sources do not feature symmetry arguments, and consequently, beliefs cannot be assumed to be uniform and must be explicitly measured.

- Measuring beliefs from stated probabilities.

Pioneering studies on source effects controlled for beliefs by explicitly asking subjects to state their beliefs about a series of events generated by a given source (e.g., Fox and Tversky 1995). A limitation of this approach is that these measures are usually neither choice based nor incentivized, and modern experimental economists are therefore reluctant to use them.

- Measuring beliefs from events exchangeability.

Scoring rules are popular choice-based methods for measuring beliefs. Still, they generally rely on the assumption of risk and ambiguity neutrality, which makes them inconsistent for the analysis of source preferences (for an illustration of biases induced by the use of scoring rules, see Armantier and Treich 2013).

A choice-based method for measuring beliefs without restrictive assumptions on risk or ambiguity attitudes has been proposed by Baillon (2008). The method uses the concept of exchangeability

of events to build a series of events  $E_k$  with a known a-neutral probability  $\lambda_k$ . Two events  $E_1$  and  $E_2$  are exchangeable if  $(x, E_1, y) \sim (x, E_2, y)$  which implies that  $\mu(E_1) = \mu(E_2)$ .

The method starts by splitting the universal event  $U$  into two exchangeable events  $E_1$  and  $E_2$ , such that  $\mu(E_1) = \mu(E_2) = 1/2$ . The procedure then proceeds iteratively by splitting  $E_1$  and  $E_2$  into exchangeable events. The researchers then repeat the procedure until a given level of precision in beliefs is attained. This method has been applied by Abdellaoui et al. (2011) for several sources, and a non-chained version of the method has been developed and implemented by Abdellaoui et al. (2021).

- Measuring beliefs jointly with attitudes: Belief hedging.

Another approach consists of neutralizing the role of beliefs by using complementary events. For example, Fox and Tversky (1995) used complementary events to show that Stanford students had a preference for betting on the temperature in San Francisco (source  $S_1$ ) rather than Istanbul (source  $S_2$ ). Concretely, they considered a series of events  $E_{k,S}$  (and their complementary  $E_{k,S}^c$ ) defined as temperature intervals. They measured, the certainty equivalents  $CE_{E_{k,S}} \sim (x, E_{k,S}, 0)$  and  $CE'_{E_{k,S}} \sim (x, E_{k,S}^c, 0)$  for each source  $S$ . Observing that  $CE_{S_1} + CE'_{S_1} > CE_{S_2} + CE'_{S_2}$  indicates a preference for source  $S_1$  over source  $S_2$  that cannot be explained by beliefs. Regarding uncertainty functions, the difference  $1 - (CE_S/x + CE'_S/x)$  captures the deviation of the uncertainty function  $c_S$  from the identity function.<sup>20</sup> A difference between  $CE_{S_1} + CE'_{S_1}$  and  $CE_{S_2} + CE'_{S_2}$  captures a difference in the (curvatures of the) uncertainty functions of source  $S_1$  and source  $S_2$ , i.e., a source dependence.

The idea that complementary events allow us to measure attitudes by hedging the role of beliefs has been refined and used by Baillon et al. (2017). The authors measured certainty equivalents for a series of events and their complementary events to estimate the components of uncertainty functions (utility source functions). More recently, Baillon et al. (2018) proposed using complementary events to estimate neo-additive ambiguity functions  $m_S$  while canceling out the role of beliefs.



## Appendix B: Descriptive statistics of the original experiments

### Study B

In Table A.1, we report the approval ratings  $v_{k,S}$ , such that the events  $E_{k,S} = [0, v_{k,S}]$  have a-neutral probabilities  $\mu(E_{k,S})$  of 0.125, 0.25, 0.5, 0.75, and 0.875, respectively. In Table A.2, we report the matching probabilities  $mp_{k,S}$  of events  $E_{k,S}$ .

$\mu(E_{k,S})$	US president's approval rating				French president's approval rating			
	Mean	Median	Std. Dev.	IQR	Mean	Median	Std. Dev.	IQR
0.125	29.90	24.50	21.76	[14.50; 39.50]	37.95	36.50	21.21	[24.50; 49.50]
0.25	34.44	25.50	23.51	[18.50; 46.25]	43.68	40.50	23.47	[25.75; 54.25]
0.50	47.38	43.50	22.62	[31.00; 61.25]	58.21	52.50	21.42	[45.75; 75.50]
0.75	57.78	54.50	21.75	[42.75; 75.50]	66.73	69.50	19.31	[50.50; 78.25]
0.875	63.02	62.50	22.08	[49.50; 80.25]	71.70	75.50	19.34	[54.50; 87.50]

Table A.1: Descriptive statistics of the beliefs (EE) data for Study B

$\mu(E_{k,S})$	US president's approval rating				French president's approval rating			
	Mean	Median	Std. Dev.	IQR	Mean	Median	Std. Dev.	IQR
0.125	25.51	25.00	12.01	[19.50; 32.50]	41.64	42.00	12.41	[33.50; 50.50]
0.25	28.88	29.50	11.88	[21.50; 35.50]	45.50	46.00	12.26	[36.50; 53.50]
0.50	33.49	34.00	12.32	[24.50; 39.50]	50.65	50.50	12.68	[42.50; 57.50]
0.75	38.91	38.50	12.54	[30.50; 45.50]	56.05	55.50	12.49	[47.50; 64.50]
0.875	42.62	41.50	13.89	[32.50; 51.50]	59.49	58.50	12.97	[50.50; 67.50]

Note: The matching probabilities are expressed in percentages.

Table A.2: Descriptive statistics of the attitude data (matching probabilities) for Study B

## Study C

The three exclusive events, constituting an exhaustive partition are  $E_1 = (-\infty, 18]$  ,  $E_2 = ]18, 22]$  and  $E_3 = ]22, \infty)$ .

Source	Event	Mean	Median	Std. Dev.	IQR
Paris	$E_1$	10.38	9.50	4.96	[6.50; 13.50]
	$E_3^c$	13.44	14.50	4.66	[9.50; 17.50]
	$E_2$	8.05	8.50	4.56	[4.50; 10.50]
	$E_1^c$	10.07	9.50	5.17	[6.50; 14.50]
	$E_3$	6.73	6.50	4.53	[3.50; 9.50]
	$E_2^c$	12.44	13.50	4.31	[9.50; 15.50]
Belgrade	$E_1$	10.19	9.50	4.92	[6.50; 13.50]
	$E_3^c$	12.38	12.50	4.64	[9.50; 16.50]
	$E_2$	7.30	7.50	4.30	[4.50; 9.50]
	$E_1^c$	9.40	9.50	5.08	[5.50; 13.50]
	$E_3$	7.12	7.50	4.54	[3.50; 9.50]
	$E_2^c$	12.32	12.50	4.53	[9.50; 15.50]

Table A.3: Descriptive statistics on beliefs and attitude data (CEs) for Study C

## Appendix C: Estimations of beliefs and attitudes for study C

For study C, we estimated probabilities (beliefs) jointly with the parameters of uncertainty functions. For each source (A and B), we considered a three-event partition  $\{E_1, E_2, E_3\}$ , with beliefs characterized by two values  $\mu_1 = \mu(E_1)$  and  $\mu_3 = \mu(E_3)$  (given that  $\mu(E_2) = 1 - \mu(E_1) - \mu(E_3)$ ). In order to ensure that  $0 < \mu(E_1) + \mu(E_3) < 1$ , we used a multinomial logit transformation  $\mu_1 = \frac{\exp(\nu_1)}{1 + \exp(\nu_1) + \exp(\nu_3)}$  and  $\mu_3 = \frac{\exp(\nu_3)}{1 + \exp(\nu_1) + \exp(\nu_3)}$ .

The likelihood of event  $E_2$  was the reference, and  $\nu_1$  and  $\nu_3$  measured the log odd-ratios with reference to this likelihood. Concretely, if  $\nu_1 = 0$  then  $\mu_1 = \mu_2$ , and if  $\nu_1 > 0$  ( $\nu_1 < 0$ ) then  $\mu_1 > \mu_2$  ( $\mu_1 < \mu_2$ ). The same applied to  $\nu_3$  and  $\mu_3$ . For random-coefficient estimations that account for heterogeneity in beliefs (and attitudes),  $\nu_1$  and  $\nu_3$  were considered as (possibly correlated) random variables that vary across subjects.

	Prelec		GE	
$\nu_{1, \bar{S}=A}$	0.487	[0.199; 0.780]	0.524	[0.236; 0.817]
$\nu_{3, \bar{S}=A}$	-0.466	[-0.709; -0.236]	-0.366	[-0.521; -0.200]
$\nu_{1, \bar{S}=B}$	0.719	[0.497; 0.935]	0.456	[0.187; 0.726]
$\nu_{3, \bar{S}=B}$	0.091	[-0.128; 0.307]	-0.037	[-0.173; 0.110]
$\bar{a}$	0.570	[0.532; 0.607]	0.567	[0.518; 0.614]
$\bar{b}$	-0.007	[-0.050; 0.036]	-0.007	[-0.050; 0.036]
$\bar{\alpha}$	0.051	[0.012; 0.091]	0.059	[-0.007; 0.119]
$\bar{\beta}$	0.059	[0.032; 0.085]	0.060	[0.039; 0.079]
$\sigma_{\nu_{1, S=A}}$	1.638	[1.381; 1.929]	1.537	[1.268; 1.846]
$\sigma_{\nu_{3, S=A}}$	1.110	[0.902; 1.349]	0.878	[0.754; 1.022]
$\sigma_{\nu_{1, S=B}}$	1.293	[1.127; 1.489]	1.504	[1.287; 1.752]
$\sigma_{\nu_{3, S=B}}$	1.028	[0.832; 1.249]	0.724	[0.613; 0.852]
$\sigma_a$	0.207	[0.180; 0.238]	0.209	[0.161; 0.257]
$\sigma_b$	0.285	[0.255; 0.319]	0.281	[0.249; 0.319]
$\sigma_\alpha$	0.229	[0.200; 0.264]	0.283	[0.245; 0.326]
$\sigma_\beta$	0.125	[0.101; 0.164]	0.071	[0.046; 0.099]
LL	-3449.682		-3468.245	

Note: 95% credible intervals between brackets.  $a$  and  $b$  are the likelihood insensitivity and uncertainty aversion parameters of the reference source.

Table A.4: HB estimations on study C

## Appendix D: Estimations with other specifications

	Study A		Study A (only real incentives)		Study B		Study C	
$\bar{a}$	0.281	[0.183; 0.375]	0.314	[0.166; 0.453]	0.544	[0.484; 0.601]	0.570	[0.532; 0.607]
$\bar{b}$	0.149	[0.032; 0.266]	0.193	[0.043; 0.341]	-0.105	[-0.185; -0.025]	-0.007	[-0.050; 0.036]
$\bar{\alpha}$	0.000	[-0.054; 0.055]	-0.205	[-0.286; -0.096]	0.353	[0.251; 0.449]	0.051	[0.012; 0.091]
$\bar{\beta}$	0.028	[-0.071; 0.129]	0.104	[0.030; 0.176]	0.277	[0.171; 0.377]	0.059	[0.032; 0.085]
$\sigma_a$	0.345	[0.281; 0.425]	0.362	[0.273; 0.479]	0.250	[0.207; 0.300]	0.207	[0.180; 0.238]
$\sigma_b$	0.440	[0.368; 0.529]	0.394	[0.306; 0.509]	0.371	[0.320; 0.431]	0.285	[0.255; 0.319]
$\sigma_\alpha$	0.163	[0.125; 0.212]	0.181	[0.115; 0.279]	0.319	[0.255; 0.396]	0.229	[0.200; 0.264]
$\sigma_\beta$	0.335	[0.268; 0.416]	0.137	[0.086; 0.198]	0.360	[0.294; 0.436]	0.125	[0.101; 0.164]
LL	-1817.354		-982.404		-3313.293		-3449.682	

Note: 95% credible intervals between brackets.

$a$  and  $b$  are the likelihood insensitivity and uncertainty (ambiguity) aversion parameters of the reference source.

Table A.5: Summary of HB estimations with Prelec specification - Studies A, B and C

	Study A		Study A incentives only (only real incentives)		Study B		Study C	
$\bar{a}$	0.310	[0.210; 0.405]	0.338	[0.189; 0.483]	0.572	[0.520; 0.622]	0.567	[0.518; 0.614]
$\bar{b}$	0.164	[0.047; 0.280]	0.172	[0.016; 0.326]	-0.119	[-0.200; -0.039]	-0.007	[-0.050; 0.036]
$\bar{\alpha}$	0.020	[-0.039; 0.085]	-0.268	[-0.397; -0.145]	0.456	[0.365; 0.545]	0.059	[-0.007; 0.119]
$\bar{\beta}$	-0.005	[-0.104; 0.093]	0.143	[0.081; 0.202]	0.303	[0.191; 0.408]	0.060	[0.039; 0.079]
$\sigma_a$	0.345	[0.278; 0.428]	0.379	[0.292; 0.494]	0.212	[0.176; 0.254]	0.209	[0.161; 0.257]
$\sigma_b$	0.440	[0.368; 0.529]	0.408	[0.319; 0.526]	0.372	[0.321; 0.433]	0.281	[0.249; 0.319]
$\sigma_\alpha$	0.183	[0.144; 0.231]	0.297	[0.198; 0.411]	0.312	[0.254; 0.381]	0.283	[0.245; 0.326]
$\sigma_\beta$	0.333	[0.270; 0.410]	0.106	[0.066; 0.159]	0.363	[0.292; 0.446]	0.071	[0.046; 0.099]
LL	-1815.071		-953.680		-3297.273		-3468.245	

Note: 95% credible intervals between brackets.

$a$  and  $b$  are the likelihood insensitivity and uncertainty (ambiguity) aversion parameters of the reference source.

Table A.6: HB estimations with GE specification - Studies A, B and C

# Appendix E: Instructions of the experiments

## E1 - Instructions for experiment B (translated in English)

### General information

- The study will last approximately 1 hour
- You will receive 10€ for your participation
- You will also have the possibility of earning an extra monetary amount
- There are no right or wrong answers
- We are trying to measure your preferences in situations of uncertainty

1

### Course of the experiment

1. Instructions
2. Training
3. Experiment

Please don't hesitate to ask us questions anytime. The experimenter will remain by your side throughout the experience.

2

### Example of choices

### First example

What would you prefer between:

- Option at the top: win 100€ if Angela Merkel's approval rating in two months is lower than 60% and nothing otherwise.
- Option at the bottom: a draw, played in 2 months, that offers a 50% chance of winning 100€ and a 50% chance of winning nothing.

4

## First example: illustration

We are not interested in your political preferences.

We use the approval ratings of politicians as a source of uncertainty.

5

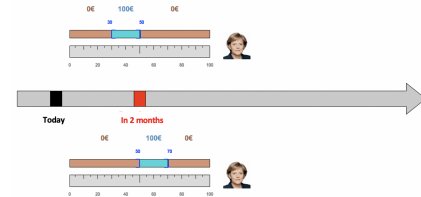
## Second example

Example of a choice in which both options involve the same variable:

Do you prefer:

- Option at the top: an option that offers 100€ if Angela Merkel's approval rating in two months is between 30% and 50%.
- Option at the bottom: an option that offers 100€ if Angela Merkel's approval rating in two months is between 50% and 70%.

Here, you should choose the interval in which you believe Angela Merkel's approval rating is the most likely to fall.



6

## The different sources of uncertainty

The previous examples involved Angela Merkel's approval rating in two months and the color of a ball drawn from a bag, also in two months.

In this experiment, we will consider 4 sources on which you will be able to bet, and each source will be considered at two periods of time.

7

## The different sources of uncertainty

The four sources of uncertainty are:

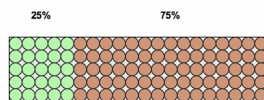
- The color of a ball drawn from a bag with a known composition
- The color of a ball drawn from a bag with an unknown composition
- The approval rating of Donald Trump (US president), measured by a monthly survey, official source: <http://elections.huffingtonpost.com/pollster/donald-trump-favorable-rating>
- The approval rating of Emmanuel Macron (French president), measured by a monthly survey, official source: <http://www.tns-sofres.com>

8

## The bag with a known composition: known probabilities

The bag with a known composition contains 100 balls that are either green or brown.

- You know the number of green and brown balls.
- You know the probability of drawing a green ball from the bag.



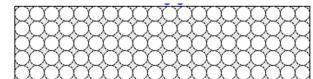
9

## The bag with an unknown composition: unknown probabilities

The bag with unknown composition contains 100 balls of 8 different colors: blue, purple, red, orange, yellow, green, grey and black

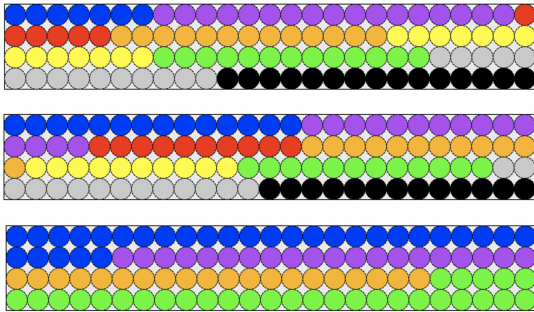
You do not know the exact number of each color. There can be:

- 0,1... even 100 blue balls,
- 0,1... even 100 purple balls,
- 0,1... even 100 red balls,
- 0,1... even 100 orange balls,
- 0,1... even 100 yellow balls,
- 0,1... even 100 green balls,
- 0,1... even 100 grey balls,
- 0,1... even 100 black balls.



10

### The bag with an unknown composition: unknown probabilities - Illustrations



11

### Approval rating of a personality

In this experiment, we will also consider two "natural" sources of uncertainty:

The approval rating of:

the US president,  
Donald Trump



the French president,  
Emmanuel Macron



Here, we are not interested in your political preferences.

We consider these variables as sources of uncertainty:

- that nobody can predict with certainty
- On which everybody can have an opinion.

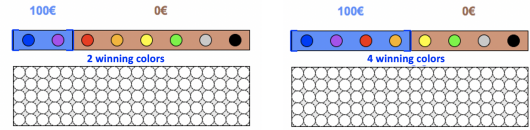
13

### The software

### The bag with an unknown composition: unknown probabilities

Consider the following situations:

- You win 100€ if the ball drawn from the bag is of one of the winning color.



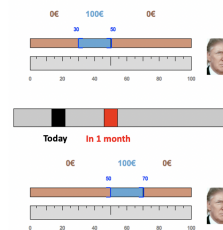
Note that the higher the number of winning colors, the higher your chances of winning.

What matters is the number of winning colors. In practice, you will be able to choose the winning colors.

12

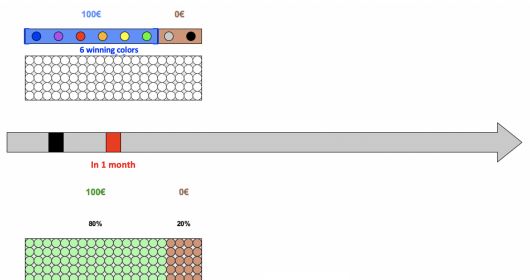
### Examples

- Each of these sources will be considered at different periods of time.
- When you choose between two sources, they will always refer to the same period.
- In all cases, in case of monetary gain, you will receive the payment for any monetary gain 12 months from now.

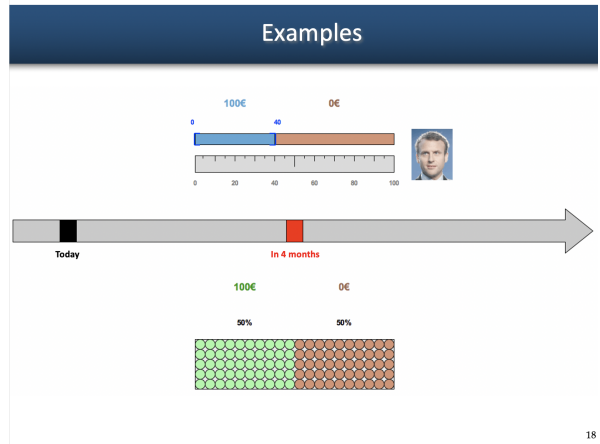
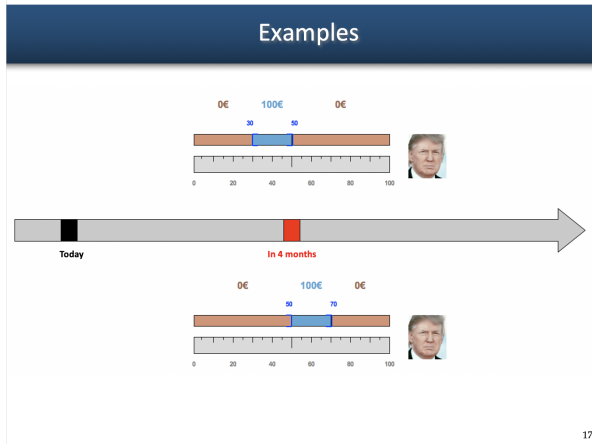


14

### Examples



16



### General principles

For each choice, it is important to look at:

- What sources are concerned
- The dates

Note that the uncertainty about the two options is always resolved at the same time and, in case of monetary gain, you will receive payment the same date for both options: in 12 months. Your choice should therefore focus on the event on which you prefer to bet.

19

### Choice lists

You will have to make a large number of choices.

To facilitate the choices, we have grouped them in lists. Within a choice list.

- The sources and their dates are fixed.
- Only the winning event varies.

20

### Filling in a choice list

In order to speed up the filling of a list, we will implement the following procedure. The procedure begins with a given choice, for example: bet on the color of a ball drawn from a bag containing 50 green balls and 50 brown balls, or bet on the popularity rating being greater than 60.

- If you prefer to bet on the color of a ball drawn from the bag being green, it means that you would also prefer this option if the bag had more than 50 green balls. There is therefore no need to ask you these questions; they are automatically completed by the computer. The procedure will ask you questions for probabilities between 0 and 50.
- Conversely, if you prefer to bet on the approval rating, it means that you would also prefer this option if the bag contained less than 50 green balls. There is therefore no need to ask you these questions; they are automatically completed by the computer. The procedure will ask you questions for probabilities between 50 and 100.

21

### Real incentives



## Real incentives

In order to encourage sincere answers, we will use a real incentive system.

Every subject will have a 10% chance of having one of his/her choices randomly selected and played for real.

- Before the experiment, we created a list of envelopes. 1 out of 10 envelopes contains a winning ticket.
- You will receive a random envelope: you therefore have a 10% chance of getting a winning ticket.



23

## Real incentives

At the end of the session, you will open the envelope and discover if you have been selected to have one of your choices played for real.

If this is the case, a computer program will randomly choose one of the choices that you have made during the session.

We will look at your answer to this question to determine your gains.

Note that:

- You will not be able to modify your answer.
- It is therefore in your interest to answer each question sincerely in order not to regret your choices at the moment of the random draw.

24

## Real incentives

- We will organize a Skype call at the time indicated in the question.
- For instance, if you answer the questionnaire on September 1st and the question refers to an event occurring in one month, we would organize a Skype call on October 1st.
- Depending on the question randomly selected and on the answer to this question, your gain will depend on:



25

## Real incentives



The gain depends on the **color of the ball drawn from a bag with a known composition**:

- We will fill up a bag with the number of green balls and brown balls as indicated in the selected question.
- We will randomly draw a ball.
- The color of the ball will determine your gain.



The gain depends on the **color of the ball drawn from a bag with an unknown composition**:

- We will fill up the bag with 100 balls
- You will know that the balls can be of 8 different colors but you won't know the exact composition of the bag
- You will choose a certain number of winning colors depending on the number indicated in the question.
- We will randomly draw a ball.
- The color of the ball will determine your gain.

26

## Real incentives

The gain depends on **Donald Trump's approval rating**:



- We will visit the website:
  - <http://elections.huffingtonpost.com/pollster/donald-trump-favorable-rating>.
- We will use the percentage of favorable opinions as a measure of Donald Trump's approval rating.
- The gain will depend on your answer and the approval rating.



The gain depends on **Emmanuel Macron's approval rating**:

- We will visit the website:
  - <http://www.tns-sofres.com/cotes-de-popularites>.
- We will use the percentage of French people who trust Emmanuel Macron as a measure of approval rating.
- The gain will depend on your answer and the approval rating.

27

## Payment of the gains

In any case, all the gains will be **paid 12 months after the date of the session**:

- Regardless of the period indicated in the selected question
- Regardless of the source used to determine your gain

28

## E2 - Instructions of experiment C

Experiment C was run online, and started with a video presenting the instructions. The video is available upon request. In what follows, we report screenshots from the video (translated) and a

translation of the verbatim that came with each part of the video.

## Choices under uncertainty

Thank you for your participation.

This experiment lasts about fifteen minutes, including this video.

The experiment consists of a series of questions where you must indicate your preference between a certain gain and a gain that depends on a situation of uncertainty.

The situation of uncertainty is related to the temperature in a given city on **May 15, 2019 at noon**.

More precisely, the uncertainty carries on:

- the temperature on May 15 at noon in **Paris** (France),
- or the temperature on May 15 at noon in **Belgrade** (Serbia).

Figure A.1: Instructions of Study C: slide 1

Translation of the script:

*Thank you for your participation. This experiment lasts about fifteen minutes, including this video. The experiment consists of a series of questions where you must indicate your preference between a certain gain and a gain that depends on a situation of uncertainty. The situation of uncertainty is related to the temperature in a given city on May 15 at noon. More precisely, the uncertainty carries on:*

- *the temperature on May 15 at noon in Paris (France)*
- *or the temperature on May 15 at noon in Belgrade (Serbia).*

# Example of a question

Indicate your preference between option A or option B.

**Option A** **Option B**

Win €0 or €20 depending on the temperature on May 15 at noon in Paris.

Win

€10

€20

€0

below or equal to 22°C

strictly higher than 22°C

I prefer option A

I prefer option B

Figure A.2: Instructions of Study C: slide 2

Translation of the script:

*Here is an example of a question. You must indicate your preference between option A, on the left, and option B, on the right. Option A gives 10 euros for sure. Option B gives 20 euros only if the temperature on May 15 at noon in Paris is below or equal to 22 degrees. If the temperature is strictly higher than 22 degrees, you do not win anything.*

# Another example of a question

Indicate your preference between option A or option B.

<b>Option A</b>	<b>Option B</b>
	Win €0 or €20 depending on the temperature on May 15 at noon in <b>Belgrade</b> .
	
Win	€0                      €20
€15	
	below or equal to 22°C                      strictly higher than 22°C
<input type="button" value="I prefer option A"/>	<input type="button" value="I prefer option B"/>

Figure A.3: Instructions of Study C: slide 3

Translation of the script:


*Here is another example of a question. You must indicate your preference between option A on the left and option B on the right. Option A gives you 15 euros for sure. Option B gives you 20 euros only if the temperature on May 15 at noon in Belgrade is strictly higher than 22 degrees. If the temperature is below or equal to 22 degrees, you do not win anything.*

# Another example of a question

Indicate your preference between option A or option B.

**Option A**

**Option B**  
Win €0 or €20  
depending on  
the temperature on May 15 at noon in **Belgrade**.



Win

€10

€20      €0      €20

below or equal to 18°C      strictly higher than 18°C and below or equal to 22°C      strictly higher than 22°C

Figure A.4: Instructions of Study C: slide 4

Translation of the script:

*Here is a final example of a question. Option A gives 10 euros for sure. Option B gives 20 euros only if the temperature on May 15 at noon in Belgrade is below or equal to 18 degrees, or strictly higher than 22 degrees. If the temperature is between 18 degrees and 22 degrees, you do not win anything.*

# Series of questions

Questions are independent of one another. You are asked to answer as if each question were a unique choice.

Questions are grouped in series. Within a series,

- option B, which is uncertain, does not change,
- but the gain offered for sure by option A varies.

A message will indicate when you move from one series to another. This means that option B will change. In this case, it will be important to look at the new option B to answer the questions.

Figure A.5: Instructions of Study C: slide 5

Translation of the script:

*Questions are independent of one another. You are asked to answer as if each question were a unique choice. Questions are grouped in series. Within a series, option B, which is uncertain does not change, but the gain offered for sure by option A varies. A message will indicate when you move from one series to another. This means that option B will change. In this case, it will be important to look at the new option B to answer the questions.*

# Concluding remarks

All the choices are hypothetical; you are asked to answer as if you had to make this type of choice.

There is no right or wrong answer. We only want to observe your preferences for this type of choice.

For our study, it is important that you answer these questions seriously. We count on you to answer the questions carefully.

Figure A.6: Instructions of Study C: slide 6

Translation of the script:

*Concluding remarks. All the choices are hypothetical; you are asked to answer as if you had to make this type of choice.*

*There is no right or wrong answer. We only want to observe your preferences for this type of choice. For our study, it is important that you answer these questions seriously. We count on you to answer the question carefully.*