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To cite this version:
Yi Wu, Erwan Fourn, Philippe Besnier, Cédric Quendo. Direct Synthesis of Multiband Bandpass Filters With Generalized Frequency Transformation Methods. IEEE Transactions on Microwave Theory and Techniques, 2021, 69 (8), pp.3820-3831. 10.1109/TMTT.2021.3086835. hal-03330274

HAL Id: hal-03330274
https://hal.science/hal-03330274
Submitted on 7 Sep 2021

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Direct Synthesis of Multiband Bandpass Filters With Generalized Frequency Transformation Methods

Yi Wu, Student Member, IEEE, Erwan Fourn, Member, IEEE, Philippe Besnier, Senior Member, IEEE, and Cédric Quendo, Senior Member, IEEE

Abstract—This article presents a new generalized direct synthesis method to design multiband bandpass filters (MBPFs). Based on the proposed frequency transformation technique, it can be used to synthesize an arbitrary number of bandpass filters starting from a low-pass prototype filter. This synthesis allows determining analytically all the resonant angular frequencies and slope parameters of all the bandpass filters from the low and high cut-off angular frequencies used as initial specifications whatever the number of bands. To validate our method, triple-, quad-, and quint-band third-order Chebyshev bandpass filters are designed and implemented in microstrip technologies. Very good agreements were achieved between simulation responses and measurements.

Index Terms—Frequency transformation methods, microstrip technology, multiband bandpass filter (MBPF), quad-band filter, quint-band filter, triple-band filter.

I. INTRODUCTION

FILTERS are key components in radio frequency transceivers in handing multiband operations. Their integration is challenging with regard to requirements. In new terminal generations providing several communication standards simultaneously, and especially in mobile terminals, embedded systems, or satellites, the footprint of the filtering parts needs to be reduced in order to meet mass and volume requirements of the whole system. Multiband bandpass filters (MBPFs) indeed then appear as an attractive solution to select multiple bands with a relatively compact circuit size. Thus, [1], [2] present some examples of MBPFs to simplify the design of communication system for satellite applications. Many researchers pay great attention on MBPFs leading to a large amount of circuit topologies implemented in different technologies such as microstrip [3]–[7], waveguide [8], or substrate integrated waveguide (SIW) [9]–[11]. More specifically, the synthesis of MBPFs, whatever the implementation technology, can be done through numerous methods. One of the most popular ones consists in deriving coupling matrix to achieve multiband responses [12]–[16]. Actually, this method is based on the optimization of the whole coupling matrix. High quality of initial values and good optimization algorithms (such as genetic algorithm or sequential quadratic programming) are needed to achieve satisfying results. Such methods may be considered inefficient in some cases because the convergence is not always guaranteed and the final topology is difficult to control [16], [17]. A second popular method to design MBPF is based on stepped-impedance resonators (SIRs) with two main concepts. The first one makes use of multisection SIRs with different resonant frequencies [18]–[24]. These resonators can have different shapes such as ring or stub-loaded shapes or consist of an assembling of quarter- or half-wavelength lines. For such filters, each resonator can resonate at the center frequencies of at least two bands. By coupling these resonators together, one can realize multibands responses. However, MBPFs design, especially beyond triple-bands, is still challenging because it is really difficult to simultaneously satisfy all the desired requirements for all bandpass, including resonant frequencies, bandwidths, and so on. A third method consists in using dual behavior resonator (DBR) filter [25], [26]. A DBR is based on the parallel association of N band-stop resonators, which can create N transmission zeros to form N − 1 bands. Based on this method, dual- and triple-band bandpass filters have been successfully synthesized and implemented. However, for this type of filter, it is hardly possible to achieve more bandwidth, because of the harmonic frequencies, and high-order filters because slope parameters are not taken into account into the synthesis.

Another MBPFs synthesis technique is based on frequency transformation methods. The first article was presented in [27] in the case of a dual-band filter. The main idea of this article is to map the lower and upper cut-off angular frequencies of each bandpass to the low-pass prototype ones (positive and negative). The dual-band resonator obtained using this transformation then consists in the association of a bandpass resonator and a band-stop one, the latter creating a transmission
The proposed frequency transformation function is a generalization that can be written as follows:

$$\Omega = T^{(N)}(\omega)$$

where $\Omega$ is the normalized angular frequency (associated with the normalized low-pass filter), $T^{(N)}(\omega)$ is the transformation function, $N$ is the number of bands of the MBPFs, and $\omega$ is the de-normalized angular frequency (associated with the MBPFs), $\omega_0$ and $b_0$ are the resonant angular frequency and the susceptance slope parameter of the wideband resonator, respectively, $\omega_k$ and $b_k$ ($k \in [1; N-1]$) are the ones of the stopband resonators.

The goal of the synthesis process is to link the $2N$ output parameters of the transformation function (the resonant angular frequencies $\omega_k$ and the slope parameters $b_k$), $k \in [1; N-1]$ to the input ones, i.e., the low and high cut-off angular frequencies of the $N$ bands ($\omega_{L_i}$ and $\omega_{H_i}$, $i \in [1; N]$) as defined in Fig. 1.

First, let us denote

$$U^{(N)}(\omega) = T^{(N)}(\omega) - 1.$$  \hspace{1cm} (2)

Substituting equation (1) for $T^{(N)}(\omega)$ in (2) leads to a new expression of $U^{(N)}(\omega)$ consisting in a ratio of two polynomials

$$U^{(N)}(\omega) = \frac{N(\omega)}{D(\omega)} = \frac{\omega^{2N} + \sum_{p=0}^{N-1} n_p \omega^p}{\sum_{q=0}^{2N-1} d_q \omega^q}$$  \hspace{1cm} (3)

where $d_q$ is equal to 0 when $q$ is even and $-n_q$ when $q$ is odd. A first expression of the $2N$ $n_p$ ($p \in [0; 2N-1]$) coefficients is then obtained as functions of the output parameters as detailed below.

We now assume that the lower cut-off angular frequencies $\omega_{L_i}$ and the upper cut-off angular frequencies $\omega_{H_i}$ ($i \in [1; N]$) are mapped to $-1$ and $+1$ in the $\Omega$ normalized domain, respectively. As $T^{(N)}(\omega)$ is an odd function, we then have

$$T^{(N)}(-\omega_{L_i}) = T^{(N)}(\omega_{H_i}) = 1 \quad i \in [1; N].$$  \hspace{1cm} (4)

Therefore, the $N$ high cut-off angular frequencies and the opposite of the $N$ lower cut-off ones are the $2N$ roots of $U^{(N)}(\omega)$. A second expression of the $n_p$ coefficients is thus obtained as functions of the input parameters this time.

So, $2N$ equations (one for each $n_p$ coefficient) link the $N$ resonant angular frequencies, the $\omega_0$, and the $N$ slope parameters, $b_k$ ($k \in [0; N-1]$), to the $N$ low cut-off angular frequencies, $\omega_{L_i}$, and the $N$ high cut-off ones, $\omega_{H_i}$ ($i \in [1; N]$). As presented below, an analytical expression can then be obtained for each output parameter.

Section II-A–II-C present the general expressions of the $n_p$ coefficients ($p \in [0; 2N-1]$) according to the cut-off angular frequencies (see Section II-B), on the one hand, and according to the resonant angular frequencies and slope parameters (see Section II-C), on the other hand. To simplify the writing of these equations and make the understanding easier, we first introduce specific mathematical expressions in Section II-A.

A. Specific Mathematical Operators

Let $\mathcal{Z}^{(r \rightarrow s)}$ be a set of real numbers $z_i$ with $i \in [r; s]$ ($r \in \mathbb{N}$ and $s \in \mathbb{N}^*$)

$$\mathcal{Z}^{(r \rightarrow s)} = \{z_i\}_{i=r}^s.$$  \hspace{1cm} (5)

We then denote $\mathcal{Z}^{(r \rightarrow s)}_\{i \neq j\}$ the same set of real numbers $z_i$ with $i \neq j$ ($i; j \in [r; s]$)

$$\mathcal{Z}^{(r \rightarrow s)}_\{i \neq j\} = \{z_i\}_{i=r}^s \setminus \{z_j\} = \{z_r, z_{r+1}, \ldots, z_{j-1}, z_{j+1}, \ldots, z_s\}.$$  \hspace{1cm} (6)
Let \( \mathcal{P}_a(Z^{(r-s)}) \) be the sum of all the different products of \( a \) elements of \( Z^{(r-s)} \), \( \mathcal{P}_a(Z^{(r-s)}) \) is thus a sum of \( \binom{a+s-r+1}{a} \) terms, each of them being a product of \( a \) elements of \( Z^{(r-s)} \).

For instance

\[
\begin{align*}
\mathcal{P}_1(Z^{(1-3)}) &= z_1 + z_2 + z_3 \\
\mathcal{P}_2(Z^{(1-3)}) &= z_1 z_2 + z_1 z_3 + z_2 z_3 \\
\mathcal{P}_3(Z^{(1-3)}) &= z_1 z_2 z_3
\end{align*}
\]

and

\[
\mathcal{P}_2(Z^{(1-4)}) = z_1 z_2 + z_1 z_4 + z_2 z_4.
\]

Finally, we denote

\[ \mathcal{P}_0(Z^{(r-s)}) = 1. \]  \( (7) \)

### B. Expression of the \( n_p \) Coefficients as Functions of the Cut-Off Angular Frequencies

In this section, an expression of the \( 2N \) \( n_p \) coefficients used in (3) is obtained as functions of the frequency transformation input parameters, i.e., the cut-off angular frequencies of the \( N \) bands. We use here the fact that the \( N \) high cut-off angular frequencies \( \omega_{L_i} \) and the opposite of the low cut-off ones \( \omega_{L_i} \), with \( i \in [1; N] \), are the zeros of \( U^{(N)}(\omega) \). We thus obtain a first system of \( 2N \) equations (one for each cut-off angular frequency) with \( 2N \) unknowns \( (n_p, p \in [0; 2N - 1]) \).

According to the expressions presented in Section II-A, we denote \( \mathcal{L}^{(1-N)} = \{\omega_{L_i}\} \) and \( \mathcal{H}^{(1-N)} = \{\omega_{H_i}\}, i \in [1; N] \). Solving this first system leads to

\[
n_p = \sum_{r=0}^{p} (-1)^{N-r} \mathcal{P}_{N-r} \left( \mathcal{H}^{(1-N)} \right) \mathcal{P}_{N+r-p} \left( \mathcal{L}^{(1-N)} \right)
\]

for \( p \in [0; N-1] \)

\[
= \sum_{r=p}^{2N} (-1)^{2N-r} \mathcal{P}_{2N-r} \left( \mathcal{H}^{(1-N)} \right) \mathcal{P}_{r-p} \left( \mathcal{L}^{(1-N)} \right)
\]

for \( p \in [N; 2N-1] \).  \( (8) \)

### C. Expression of the \( n_p \) Coefficients as Functions of the Resonant Angular Frequencies and the Susceptance Slope Parameters

We now express the \( 2N \) \( n_p \) coefficients as functions of the frequency transformation output parameters, i.e., the resonant angular frequency, \( \omega_0 \), and the slope parameter, \( b_0 \), of the wide bandpass resonator, and the parameters \( \omega_k \) and \( b_k \) \( (k \in [1; N-1]) \). We recall that these parameters are the resonant angular frequencies and the slope parameters of the stopband resonators, respectively. This is based on the introduction of the expression of \( T^{(N)}(\omega) \) from (1) in (2), which allowed us to obtain the expression of \( U^{(N)}(\omega) \) given in (3). We thus obtain a second system of \( 2N \) equations giving each \( n_p \) coefficient as a function of the output parameters.

Here, we denote \( \mathcal{W}^{(0-N-1)} = \{\omega_0^2\} \) the set of all the resonant angular frequency squares and \( \mathcal{W}^{(1-N-1)} = \{\omega_0^2\} \) the set of the resonant angular frequency squares of the stopband resonators only. We distinguish three cases for \( p = 0, p \) odd, and \( p \) even, \( p \in [0; 2N - 1] \).

1. For \( p = 0 \), we have

\[
n_0 = (-1)^N \mathcal{P}_N \left( \mathcal{W}^{(0-N-1)} \right) = (-1)^N \prod_{r=0}^{N-1} \alpha_r^2. \]  \( (9) \)

2. For \( p \) odd, we denote \( p = 2q - 1, q \in [1; N] \). We then have

\[
n_p = n_{2q-1}
= (-1)^{N-q+1} \frac{\alpha_0}{b_0} \mathcal{P}_{N-q} \left( \mathcal{W}^{(1-N-1)} \right). \]  \( (10) \)

3. For \( p \) even, we denote \( p = 2q, q \in [1; N-1] \), and then we obtain

\[
n_p = n_{2q}
= (-1)^{N-q} \left\{ \mathcal{P}_{N-q} \left( \mathcal{W}^{(0-N-1)} \right) + \frac{\alpha_0}{b_0} \sum_{k=1}^{N-1} \frac{\alpha_k}{b_k} \mathcal{P}_{N-q-1} \left( \mathcal{W}^{(1-N-1)} \right) \right\}. \]  \( (11) \)

### D. Expression of the Resonance Angular Frequencies and Susceptance Slope Parameters

Equations (9) to (11) give an expression of the \( 2N \) \( n_p \) as functions of the output parameters of the frequency transformation technique, i.e., \( \omega_k \) and \( b_k, k \in [0; N-1] \). In a practical way, we have to reverse this equation system in order to express the output parameters as functions of the \( 2N \) \( n_p \) coefficients, these latter being calculated from the input parameters \( (\omega_{L_i} \) and \( \omega_{H_i} \), \( i \in [1; N] \)) using (8).

First, using the expressions of \( n_0, n_1, \) and \( n_{2N-1} \) and whatever the number of bands \( N \), we can easily demonstrate that the resonant frequency \( \omega_0 \) and the slope parameter \( b_0 \) of the wide bandpass filter can be written as

\[
\omega_0 = \sqrt{-\frac{n_0 n_{2N-1}}{n_1}}, \quad b_0 = \frac{\sqrt{n_0}}{n_1}. \]  \( (12) \)

\[
(13) \]

Using (10) for all the odd value of \( p \), we can also demonstrate (see Appendix A) that the resonant angular frequencies of the band-stop resonators \( (\omega_k, k \in [1; N-1]) \) are the \( N-1 \) positive solutions of (14)

\[
\sum_{r=0}^{N-1} n_{2r+1} \omega_0^{2r} = 0. \]  \( (14) \)

Once the resonant angular frequencies of the band-stop resonators are determined using (14), the remaining unknowns are the slope parameter coefficients \( b_k, k \in [1; N-1] \) in (11). This equation can also be rewritten as follows:

\[
\sum_{k=1}^{N-1} \left[ \frac{\alpha_k}{b_k} \mathcal{P}_{N-q-1} \left( \mathcal{W}^{(1-N-1)} \right) \right] = \mathcal{P}_{N-q} \left( \mathcal{W}^{(0-N-1)} \right) - (-1)^{N-q} n_{2q} \]  \( (15) \)
for \( q \in [1; N - 1] \). Using (15) for each value of \( q \), we obtain an \( N - 1 \) linear equation system with \((N - 1) \) unknowns (i.e., each \( b_k \) or more precisely \( 1/b_k \)). This system can be written as follows:

\[
X_{N-1,N-1} B_{N-1} = A_{N-1}
\]

(16)

where \( X_{N-1,N-1} \) is the \((N - 1 \times N - 1)\) matrix of \( x_{q,k} \) coefficients with

\[
x_{q,k} = a_k P_{N-q-1} \left( W_{N-q}^{1-N-1} \right).
\]

(17)

\( B_{N-1} \) is the vector of the \( N - 1 \) unknowns

\[
B_{N-1} = \left[ \frac{1}{b_1}, \frac{1}{b_2}, \ldots, \frac{1}{b_{N-1}} \right]^T
\]

(18)

and \( A_{N-1} \) is the vector of already known coefficients \( a_q^{(N)} \)

\[
A_{N-1} = [a_1, a_2, \ldots, a_{N-1}]^T
\]

(19)

with

\[
a_q = \frac{P_{N-q} \left( W_{N-q}^{(0-N-1)} \right) - (-1)^{N-q} n_{2q}}{n_{2N-1}}.
\]

(20)

The slope parameters are then determined by inverting the matrix \( X_{N-1,N-1} \)

\[
B_{N-1} = X_{N-1,N-1}^{-1} A_{N-1}.
\]

(21)

By solving the above matrices, the slope parameters can be written as

\[
b_1 = \frac{n_1 n_2^2 \omega_1}{n_1 n_2 n_3 - n_0 n_3^2 - n_1^2} \quad \text{for } N = 2
\]

(22)

\[
b_k = \frac{a_k \prod_{j=1, j \neq k}^{N-1} (\omega_k^2 - \omega_j^2)}{\sum_{j=1}^{N-1} Y_j^2 \omega_k^{2j-2}} \quad \text{for } N \geq 3
\]

(23)

where \( k \in [1; N - 1] \) and

\[
Y_j = \frac{n_{2j} - n_0 n_{2j+1}}{n_1 - n_{2N-1}}.
\]

(24)

One should note that this synthesis technique leads to a unique solution for a given specifications. So, there is theoretically only one transmission zero between two consecutive bands whatever the order of the designed filter.

III. IMPLEMENTATION WITH LC ELEMENTS

Starting from a classical ladder structure composed of series inductances and shunt capacitances, a low-pass prototype filter can be modified to use only parallel capacitances separated by \( J \) admittance inverters as shown in [32]. Using the generalized frequency transformation technique presented in Section II, the \( j^{th} \) capacitance, \( C_{LP}^{(j)} \), of such a low-pass prototype can be transformed into a multiband bandpass resonator as shown in Fig. 2. The multiband bandpass resonator then obtained consists in the parallel association of one bandpass resonator \((L_0^{(i)}, C_0^{(i)})\) and \( N - 1 \) band-stop ones \((L_k^{(i)}, C_k^{(i)}), k \in [1; N - 1] \) whose values are given by the following equations:

\[
L_0^{(i)} = \frac{1}{b_0 C_{LP}^{(j)} a_0}
\]

(25)
values for band-stop and bandpass resonators with a view to the implementation in microstrip technology.

IV. SYNTHESIS AND EXPERIMENTAL VALIDATION OF THREE TEST MBPFs

For validating the frequency transformation method and the implementation proposed in the previous sections, three examples of MBPFs have been synthesized and experimentally validated with 3, 4, and 5 bands, respectively. In the first case (tri-band filter), all the output parameter expressions obtained from (12) to (21) are presented directly here as an example. In the other two cases, for the sake of clarity, these expressions are presented in Appendix (see Sections B and C).

The three test filters are of order 3 with a Chebyshev approximation and 20-dB bandpass return loss (RL). There are implemented in a low-cost microstrip technology using a RO4003C Rogers substrate (dielectric constant: \(\varepsilon_r \)). The fabrication of the circuits was made by laser engraving using a LPKF Protolaser U4.

A. Triple-Band Bandpass Filters

To synthesize a triple-band bandpass filter using the frequency transformation method proposed here, one needs to take \(N = 3\) in (12) to (21). The resonance angular frequencies and slope parameters are then given by

\[
\omega_0 = \frac{-n_3n_5}{n_1} \quad \text{(33)}
\]

\[
\omega_1 = \sqrt{-n_3 - \frac{n_3^2 - 4n_5n_1}{2n_5}} \quad \text{(34)}
\]

The filter was then implemented in microstrip technology with the substrate characteristics given in the introduction of this section. Fig. 6 presents the layout of the fabricated MBPRs with parallel LC resonators.

![Fig. 4. MBPRs with parallel LC resonators.](image)

![TABLE I](image)

**Component Values of the Ideal LC Tri-Band Third-Order Bandpass Filter**

<table>
<thead>
<tr>
<th>Resonators ((i \in [1:3]))</th>
<th>(L_i^{(i)} = 1.567) nH</th>
<th>(C_i^{(i)} = 1.689) nH</th>
<th>(L_i^{(i)} = 1.380) nH</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_i^{(i)} = 2.384) pF</td>
<td>(C_i^{(i)} = 3.124) pF</td>
<td>(C_i^{(i)} = 2.321) pF</td>
<td></td>
</tr>
<tr>
<td>(J_1^{(i)} = 0.0072) S</td>
<td>(J_1^{(i)} = 0.0071) S</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(J_01 = J_{34} = 0.012) S</td>
<td>(J_{12} = J_{23} = 0.0067) S</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

with

\[
b_0 = \sqrt{-n_0} \sqrt{n_1n_5} \quad \text{(36)}
\]

\[
b_1 = \frac{\omega_0(\omega_0^2 - \omega_1^2)}{Y_1 + Y_2\omega_1^2} \quad \text{(37)}
\]

\[
b_2 = \frac{\omega_0(\omega_0^2 - \omega_2^2)}{Y_1 + Y_2\omega_2^2} \quad \text{(38)}
\]

where the \(n_p\) \((p \in [0; 5])\) are determined using (8) according to the specifications of the designed triple-band bandpass filter given in terms of low and high cut-off angular frequencies. The latter are then transposed in terms of resonant angular frequencies and slope parameters. As an example, the arbitrary following specifications are proposed.

1) Bandpass 1: 2.00–2.10 GHz (BW: 100 MHz).
2) Bandpass 2: 2.45–2.65 GHz (BW: 200 MHz).
3) Bandpass 3: 2.95–3.20 GHz (BW: 250 MHz).

The resulting resonant frequencies and slope parameters are \(f_0 = \omega_0/2\pi = 2.604\) GHz, \(f_1 = \omega_1/2\pi = 2.191\) GHz, \(f_2 = \omega_2/2\pi = 2.812\) GHz, \(b_0 = 4.735\), \(b_1 = 4.277\), and \(b_2 = 5.818\). A tri-band third-order bandpass filter is first implemented in an ideal LC structure (see Fig. 3), each resonator following the model of Fig. 4. All the resonator components and J-inverters, calculated from (25) to (32), are summarized in Table I. Note that the three tri-band resonators are identical in order to simplify the microstrip implementation in the next step. This is possible by fixing the three \(C_{jLP}^{(i)}\) components at the same value using an admittance scale factor when introducing the J-inverters in the low-pass prototype [32]. Fig. 5 shows the frequency response of this filter. The three simulated bands are in perfect agreement with the specifications.
This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

Fig. 5. Ideal frequency response of the specified triple-band bandpass filter.

Fig. 6. Layout of the fabricated tri-band bandpass filter.

Fig. 7. Photograph of the three fabricated filters: tri-band (top), quad-band (bottom left), and quint-band (bottom right).

Table II

<table>
<thead>
<tr>
<th>Resonators</th>
<th>$L_0$ = 17.50</th>
<th>$L_1$ = 18.80</th>
<th>$L_2$ = 14.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_0$ = 0.92</td>
<td>$W_1$ = 3.36</td>
<td>$W_2$ = 3.50</td>
<td></td>
</tr>
<tr>
<td>$L_{J1}$ = 18.70</td>
<td>$L_{J2}$ = 16.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W_{J1}$ = 0.15</td>
<td>$W_{J2}$ = 0.12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inverters</th>
<th>$L_{01}$ = 17.70</th>
<th>$L_{12}$ = 18.34</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{01}$ = 0.51</td>
<td>$W_{12}$ = 0.11</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 8 presents the EM-simulated and measured frequency responses of the fabricated tri-band bandpass filter. The correlation between both responses is very good over the entire frequency band. The measured RL are 12.3, 14.9, and 14.1 dB from the first to the last bandpass and the measured insertion loss (IL) are 1.98, 2.17, and 2.01 dB, respectively. The isolation between the three bandpass are better than 58.1 and 57.6 dB.

B. Quad-Band Bandpass Filters

In the same way as for the tri-band case, the synthesis of a quad-band bandpass filter using this frequency transformation
method starts with the determination of the expression of the resonant angular frequencies and the associated slope parameters as functions of the low and high cut-off frequencies. To do so, one takes \( N = 4 \) in (12) to (21). The resulting equations are given in Appendix B for a sake of clarity.

Arbitrary specifications are also proposed here as an example to design, fabricate, and measure a third-order quad-band bandpass filter. Note that a first example of quad-band bandpass filter was already presented in [33] with four equal absolute bandwidths. The equations presented in [33] and here are slightly different in their writing (especially the \( n_p \) coefficients) but completely equivalent. This is only because the generalization to any number of bands was not established at the time [33] was written. The specifications proposed in this article show different bandwidths for each band are as follows:

1) Bandpass 1: 2.00–2.08 GHz (BW: 80 MHz).
2) Bandpass 2: 2.30–2.40 GHz (BW: 100 MHz).
3) Bandpass 3: 2.65–2.80 GHz (BW: 150 MHz).
4) Bandpass 4: 3.04–3.20 GHz (BW: 160 MHz).

The resulting resonant frequencies and slope parameters are

\[
\begin{align*}
    f_0 &= \frac{\omega_0}{2\pi} = 2.573 \text{ GHz}, \\
    f_1 &= \frac{\omega_1}{2\pi} = 2.146 \text{ GHz}, \\
    f_2 &= \frac{\omega_2}{2\pi} = 2.501 \text{ GHz}, \\
    f_3 &= \frac{\omega_3}{2\pi} = 2.951 \text{ GHz}, \\
    b_0 &= 5.251, \\
    b_1 &= 6.381, \\
    b_2 &= 5.697, \quad \text{and} \quad b_3 = 8.066.
\end{align*}
\]

An ideal LC structure implementation is then made as in the tri-band case and all the resonators are again fixed identical by taking all the \( C_{iP} \) equal. All the resonator component values and J-inverter ones are summarized in Table III. As shown in Fig. 9, the four obtained bands are, as previously, in perfect agreement with the specifications.

Fig. 10 presents the layout of the fabricated quad-band bandpass filter and a picture is also shown in Fig. 7. Again, three stars separated by J-inverters are obtained, each star including six branches this time: one bandpass element, three stopband ones, and the previous and next J-inverters. All lengths and widths of the resonators and the inverters are summarized in Table IV. The quad-band filter is 97.7 × 68.9 mm² without taking into account the two 50 Ω access lines.

Fig. 11 presents the EM-simulated and measured frequency responses of the fabricated quad-band bandpass filter. As in the tri-band case, both responses are in near perfect agreement. The measured RL are 12.1, 14.2, 14.2, and 14.3 dB from the first to the fourth bandpass and the measured IL are 2.90, 2.94, 2.95, and 2.65 dB, respectively. Very good isolation are here again achieved between all the bandpass.

### TABLE III

Component Values of the Ideal LC Quad-Band Third-Order Bandpass Filter

<table>
<thead>
<tr>
<th>Resonators ( i \in {1;3} )</th>
<th>( L_0 )</th>
<th>( L_1 )</th>
<th>( L_2 )</th>
<th>( L_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.586 nH ( \times 10^3 )</td>
<td>1.902 nH ( \times 10^3 )</td>
<td>1.603 nH ( \times 10^3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.412 pF ( \times 10^6 )</td>
<td>2.892 pF ( \times 10^6 )</td>
<td>2.355 pF ( \times 10^6 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C_{iP} )</td>
<td>1.315 nH ( \times 10^{-6} )</td>
<td>2.211 pF ( \times 10^6 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( J_1 )</td>
<td>0.0078 S</td>
<td>0.0074 S</td>
<td>0.0067 S</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE IV

Microstrip Quad-Band Third-Order Bandpass Filter: Length and Width Dimensions (in mm)

<table>
<thead>
<tr>
<th>Resonators</th>
<th>( L_0 )</th>
<th>( L_1 )</th>
<th>( L_2 )</th>
<th>( L_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.70</td>
<td>19.01</td>
<td>16.2</td>
<td>15.30</td>
<td></td>
</tr>
<tr>
<td>1.30</td>
<td>3.80</td>
<td>3.30</td>
<td>3.51</td>
<td></td>
</tr>
<tr>
<td>( W_0 )</td>
<td>( W_1 )</td>
<td>( W_2 )</td>
<td>( W_3 )</td>
<td></td>
</tr>
<tr>
<td>21.30</td>
<td>18.40</td>
<td>15.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.11</td>
<td>0.12</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inverters</th>
<th>( W_0 )</th>
<th>( W_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.23</td>
<td>19.85</td>
<td></td>
</tr>
<tr>
<td>0.51</td>
<td>0.11</td>
<td></td>
</tr>
</tbody>
</table>

C. Quint-Band Bandpass Filters

In the same way as for the two previous cases, one can derive the equations of the resonant angular frequencies and the slope parameters for the quint-band bandpass filters as functions of the low and high cut-off angular frequencies by taking \( N = 5 \) in (12) to (21). The resulting equations are given in Appendix C for the sake of clarity.

The arbitrary specifications proposed here to design an example of quint-band bandpass filter are as follows:
They leads to the following resonant frequencies and slope parameters:

\[ f_0 = \frac{\omega_0}{2\pi} = 2.548 \text{ GHz}, \quad f_1 = \frac{\omega_1}{2\pi} = 2.178 \text{ GHz}, \quad f_2 = \frac{\omega_2}{2\pi} = 2.480 \text{ GHz}, \quad f_3 = \frac{\omega_3}{2\pi} = 2.797 \text{ GHz}, \quad f_4 = \frac{\omega_4}{2\pi} = 3.137 \text{ GHz}, \quad b_0 = 5.421, \]
\[ b_1 = 6.075, \quad b_2 = 5.735, \quad b_3 = 8.323, \quad \text{and} \quad b_4 = 10.050. \]

An ideal LC quint-band bandpass filter is also implemented to validate the method. The resonator component and J-inverter values are summarized in Table V and the frequency responses shown in Fig. 12. Once again, all the bands are in perfect agreement with the specifications.

Fig. 13 presents the layout of the fabricated quint-band bandpass filter and a picture is also shown in Fig. 7. Three stars separated by J-inverters are obtained, each star including seven branches with an additional stopband one compared to the quad-band case. All lengths and widths of the resonators and the inverters are summarized in Table VI. The quint-band filter is 91.5 x 77.3 mm² without taking into account the two 50 Ω access lines.

Table V

<table>
<thead>
<tr>
<th>Component Values of the Ideal LC Quint-Band Third-Order Bandpass Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Resonators (i ∈ [1; 3])</strong></td>
</tr>
<tr>
<td>( L_0^{(i)} = 1.602 \text{ nH} )</td>
</tr>
<tr>
<td>( L_1^{(i)} = 1.874 \text{ nH} )</td>
</tr>
<tr>
<td>( L_2^{(i)} = 1.645 \text{ nH} )</td>
</tr>
<tr>
<td>( L_3^{(i)} = 1.388 \text{ nH} )</td>
</tr>
<tr>
<td>( L_4^{(i)} = 1.241 \text{ nH} )</td>
</tr>
<tr>
<td>( C_0^{(i)} = 2.436 \text{ pF} )</td>
</tr>
<tr>
<td>( C_1^{(i)} = 2.850 \text{ pF} )</td>
</tr>
<tr>
<td>( C_2^{(i)} = 2.502 \text{ pF} )</td>
</tr>
<tr>
<td><strong>J-inverters ((J_{i+1}))</strong></td>
</tr>
<tr>
<td>( J_0^{(i)} = 0.0070 \text{ S} )</td>
</tr>
<tr>
<td>( J_1^{(i)} = 0.0072 \text{ S} )</td>
</tr>
<tr>
<td>( J_2^{(i)} = 0.0069 \text{ S} )</td>
</tr>
<tr>
<td><strong>Inverters ((J_{i+1}))</strong></td>
</tr>
<tr>
<td>( J_{01} = J_{34} = 0.013 \text{ S} )</td>
</tr>
<tr>
<td>( J_{12} = J_{23} = 0.0069 \text{ S} )</td>
</tr>
</tbody>
</table>

1) Bandpass 1: 2.00–2.10 GHz (BW: 100 MHz).
2) Bandpass 2: 2.30–2.38 GHz (BW: 80 MHz).
3) Bandpass 3: 2.60–2.69 GHz (BW: 90 MHz).
4) Bandpass 4: 2.88–2.98 GHz (BW: 100 MHz).
5) Bandpass 5: 3.20–3.30 GHz (BW: 100 MHz).

Fig. 14 presents the simulated and measured frequency responses of the fabricated quint-band bandpass filter. Here again, simulations and measurements are in a good correlation, except a slight frequency shift in the higher transmission zeros. This can be explained by the low-cost technology process, which implies in particular that the vias were made by hand.
The measured RL are 24.8, 17.5, 13.9, 10.9, and 10.8 dB from the first to the fifth band and the measured IL are 3.01, 4.95, 4.98, 4.75, and 3.45 dB, respectively. Very good isolation are again achieved between all the bandpass.

V. DISCUSSION

Using our proposed transform function, we always assume that lower cut-off angular frequencies \( \omega_L \) and the upper cut-off angular frequencies \( \omega_H \) \((i \in [1; N])\) are mapped to \(-1\) and \(+1\) in the \( \Omega \) normalized symmetrical domain, respectively. This features that only transfer functions that are symmetric in the low-pass domain can be used in this method. Chebyshev, Butterworth, and elliptic filters have all symmetrical structures where the symmetry is defined as \( g_0 = g_{n+1}, g_1 = g_n, g_2 = g_{n-1}, \) etc., in the low-pass domain. There are all therefore compatible with our proposed synthesis method. For Gaussian low-pass prototype (flat-group-delay), filters with order \( n = 1 \) are identical to the first-order Butterworth low-pass prototype and can also be synthesized with this method. However, Gaussian filters with order \( n \geq 2 \) are structurally asymmetrical as discussed in [22]. It is therefore impossible to synthesize multiband filter with this method. Nevertheless, flat group-delay can also be obtained with symmetrical structures using cross-coupling as presented in [34] and [35].

As an example, we present a multiband elliptic filter with quint-band response. We consider a fourth-order filter with an elliptic approximation (23-dB RL with normalized TZs at \( \pm \frac{2.2}{j} \)). The specifications for this case are as follows:

1) Bandpass 1: 3.00–3.10 GHz (BW: 100 MHz).
2) Bandpass 2: 3.30–3.40 GHz (BW: 100 MHz).
3) Bandpass 3: 3.60–3.70 GHz (BW: 100 MHz).
4) Bandpass 4: 3.90–4.00 GHz (BW: 100 MHz).
5) Bandpass 5: 4.20–4.30 GHz (BW: 100 MHz).

The resonating frequencies and slope parameters can be easily obtained by the generalized formulas in Section II. Each sub-band is a bandwidth-scaled and frequency-translated copy of the original low-pass prototype (i.e., each bandpass reaches 23-dB in-band RL and owns two TZs on both sides, as shown in Fig. 15). This example also shows that it is possible to introduce several TZs between each bandpass with this method using elliptic filters.

VI. CONCLUSION

In this article, a new generalized synthesis method for designing MBPF has been proposed. This method is relatively simple and leads to a completely analytical equation system. It uses slope parameters offering flexibility to implement the designed filters with noncoupled structures, using them directly, or with coupled ones calculating the coupling coefficients from these parameters [36]–[38]. Note that from six bands and beyond a numerical resolution of (14) will be necessary because of the degree of this equation.

Three third-order MBPFs with a different number of bands (3, 4, and 5, respectively) were implemented in microstrip technologies to illustrate the efficiency of the synthesis method. A star-like structure has been chosen to realize each multiband bandpass resonator of the filters. This enabled to keep the filters compact and the implementation simple. Very good agreement has been achieved between theoretical responses and measurements. This implementation solution is in fact mainly limited by the maximum bandwidth that can be obtained with the bandpass elements. Increasing the number of bands would lead to narrow bandwidth for each band and sharp rejection between them. Such a rejection could be difficult to realize because it implies that the impedance of J-inverters separating the bandpass part to the stopband ones in each multiband resonator will increase potentially reaching technological limits. Nevertheless, such an implementation is just an example, which has the interests to be low cost and easy to do, and any other topology or technology allowing wider bandwidths can be used.

APPENDIX A

EXPRESSION GOVERNING THE RESONANT ANGULAR FREQUENCIES OF THE BAND-STOP RESONATORS

As presented in Section II, the resonant angular frequencies of the stopband parts of an \( N \)-band bandpass resonator (MBPR) are the positive solutions of (14).

\[\varphi_i = \frac{\pi}{2N} \left(2 - \frac{1}{N} \right), \quad \varphi_{i+1} = \frac{\pi}{2N} \left(2 + \frac{1}{N} \right), \quad i = 0, 1, \ldots, N-1.\]
In the case of an $N$-band bandpass filters, the coefficients $n_p$ of the $U^{(N)}(\omega)$ function [see (3)] are given by (10) in the case of an odd value of $p$ ($p = 2q - 1$, $q \in \{1; N\}$).

For $q = N$ and so $p = 2N - 1$, this equation leads to
\[ n_{2N-1} = -\frac{\alpha_0}{b_0} \]  
(41)
knowing that $P_0(\mathcal{W}^{(1-N-1)}) = 1$ as defined in Section II-A. So, (10) becomes
\[ n_p = n_{2q-1} = (-1)^{N-q}n_{2N-1}P_{N-q}(\mathcal{W}^{(1-N-1)}) \]  
(42)
Applying (42) with $q = N - 1$ ($p = 2N - 3$) leads to
\[ n_{2N-3} = -n_{2N-1}P_1(\mathcal{W}^{(1-N-1)}) \]  
(43)
Considering any angular frequency of the stopband parts $\omega_k$, $k \in \{1; N-1\}$, this expression can be written as follows:
\[ n_{2N-3} = -n_{2N-1}[P_1(\mathcal{W}^{(1-N-1)}) + \omega_k^2] \]  
(44)
which implies that
\[ n_{2N-1}P_1(\mathcal{W}^{(1-N-1)}) = \left[-n_{2N-3} + n_{2N-1}\omega_k^2\right]. \]  
(45)
In the same way, for $q = N-2$ ($p = 2N - 5$), (42) becomes
\[ n_{2N-3} = n_{2N-1}P_2(\mathcal{W}^{(1-N-1)}) \]  
(46)
where
\[ n_{2N-5} = n_{2N-1}P_2(\mathcal{W}^{(1-N-1)\_k\_l}) \]  
(47)
Continuing thus for the other values of $q$ from $N - 1$ to $1$, one can show that
\[ n_{2N-1}P_{N-q}(\mathcal{W}^{(1-N-1)}) = \sum_{r=0}^{N-q} n_{2(q+r)-2} \alpha_k^{2r}. \]  
(48)
Let’s focus now on the case $q = 1$ ($p = 1$). First, using (42), we have
\[ n_1 = (-1)^{N-1}n_{2N-1}P_{1-N}(\mathcal{W}^{(1-N-1)}) \]  
(49)
Using (48), this expression becomes
\[ n_1 = (-1)^{N-1}\omega_k^2 \frac{1}{(-1)^{N-2}} \sum_{r=0}^{N-2} n_{2r+3} \alpha_k^{2r} \]  
\[ = -\sum_{r=1}^{N-1} n_{2r+1} \alpha_k^{2r}. \]  
(50)
Therefore, we obtain the equation governing the resonant angular frequencies of the band-stop resonators
\[ \sum_{r=0}^{N-1} n_{2r+1} \alpha_k^{2r} = 0. \]  
(51)
This equation has $2(N-1)$ solutions, one half being positive and the other half negative. Of course, in this case, only the positive solutions are retained.

**APPENDIX B**

**Resonant Angular Frequencies and Slope Parameters for a Quad-Band Bandpass Filter**

\[ \omega_0 = \sqrt{-\frac{n_0n_7}{n_1}} \]  
(52)
\[ \omega_1 = \sqrt{-\frac{n_5}{3n_7}} + \sqrt{\omega_1} + \sqrt{\omega_2} \]  
(53)
\[ \omega_2 = \sqrt{-\frac{n_5}{3n_7}} + \Delta_3\sqrt{\omega_1} + \Delta_4\sqrt{\omega_2} \]  
(54)
\[ \omega_3 = \sqrt{-\frac{n_5}{3n_7}} + \Delta_4\sqrt{\omega_1} + \Delta_3\sqrt{\omega_2} \]  
(55)
\[ b_0 = \sqrt{-\frac{n_0}{n_1n_7}} \]  
(56)
\[ b_1 = \omega_1(\omega_1^2 - \omega_2^2)(\omega_1^2 - \omega_2^2) \]  
(57)
\[ b_2 = \omega_2(\omega_1^2 - \omega_2^2)(\omega_1^2 - \omega_2^2) \]  
(58)
\[ b_3 = \omega_3(\omega_1^2 - \omega_2^2)(\omega_1^2 - \omega_2^2) \]  
(59)
\[ \Omega_1 = \Delta_1 + \sqrt{\Delta_2^2 + \Delta_3^2} \]  
(60)
\[ \Omega_2 = \Delta_1 - \sqrt{\Delta_2^2 + \Delta_3^2} \]  
(61)
\[ \Delta_1 = \frac{n_5n_5}{3n_7^2} - \frac{n_1^2}{2n_7} \]  
(62)
\[ \Delta_2 = \frac{n_3}{3n_7} - \frac{n_2^2}{9n_7} \]  
(63)
\[ \Delta_3 = \frac{1 + j\sqrt{3}}{2} \]  
(64)
\[ \Delta_4 = \frac{1 - j\sqrt{3}}{2} \]  
(65)
\[ Y_1 = -\frac{n_2}{n_1} - \frac{n_1}{n_7} \]  
(66)
\[ Y_2 = -\frac{n_4}{n_1} - \frac{n_1}{n_7} \]  
(67)
\[ Y_3 = -\frac{n_6}{n_1} - \frac{n_1}{n_7} \]  
(68)

**APPENDIX C**

**Resonant Angular Frequencies and Slope Parameters for a Quad-Band Bandpass Filter**

\[ \omega_0 = \sqrt{-\frac{n_0n_9}{n_1}} \]  
(69)
\[ \omega_1 = \sqrt{-\frac{n_7}{4n_9}} - \frac{1}{2}\left(\sqrt{R_0} + \sqrt{R_1}\right) \]  
(70)
\[ \omega_2 = \sqrt{-\frac{n_7}{4n_9}} - \frac{1}{2}\left(\sqrt{R_0} - \sqrt{R_1}\right) \]  
(71)
\[
\omega_3 = \sqrt{-\frac{n_7}{4n_9} + \frac{1}{2} \left(\sqrt{R_0 - R_1}\right)} \\
\omega_4 = \sqrt{-\frac{n_7}{4n_9} + \frac{1}{2} \left(\sqrt{R_0 + R_1}\right)}
\]

where

\[
R_0 = \frac{n_7^2}{2n_9} - \frac{2n_5}{3n_9} + R_3 + n_5^2 - 3n_3n_7 + 12n_1n_9 \\
+ \frac{1}{(3n_9)^2} R_3^2 \\
R_2 = \frac{n_7^2}{2n_9} - \frac{2n_5}{3n_9} - R_3 - n_5^2 - 3n_3n_7 + 12n_1n_9 \\
- \frac{1}{(3n_9)^2} R_3^2 \\
R_3 = \frac{3n_9 \sqrt{2} \left( R_4 + \sqrt{(R_4^2 - 4R_5^2)}\right)}{R_5} \\
R_4 = 2n_7^2 - 9n_3n_7n_9 + 27n_3^2n_9 + 27n_5^2n_1 - 72n_1n_9n_9 \\
+ 4n_9n_5n_9 - n_5^2 - 8n_3n_9^2 \\
R_1 = R_2 - \frac{4n_9n_5n_9 - n_5^2 - 8n_3n_9^2}{4n_9 \sqrt{R_0}} \\
R_5 = n_3^2 - 3n_1n_7 + 12n_1n_9 \\
b_0 = \sqrt{\frac{n_1}{n_9 n_5}} \\
b_1 = \frac{a_1 (a_1^2 - \omega_2^2)}{Y_1 \omega_1^2 + Y_2 \omega_1^2 + Y_3 \omega_1^2 + Y_1} \\
b_2 = \frac{a_2 (a_2^2 - \omega_2^2)}{Y_2 \omega_2^2 + Y_3 \omega_2^2 + Y_4 \omega_2^2 + Y_1} \\
b_3 = \frac{a_3 (a_3^2 - \omega_2^2)}{Y_3 \omega_3^2 + Y_4 \omega_3^2 + Y_5 \omega_3^2 + Y_1} \\
b_4 = \frac{a_4 (a_4^2 - \omega_2^2)}{Y_4 \omega_4^2 + Y_5 \omega_4^2 + Y_6 \omega_4^2 + Y_1} \\
Y_1 = \frac{n_2 - \frac{n_5}{n_9 n_5}}{n_1} \\
Y_2 = \frac{n_9}{n_9 n_5} - \frac{n_3}{n_9} \\
Y_3 = \frac{n_6}{n_1 n_9} + \frac{n_7}{n_9} \\
Y_4 = \frac{n_8}{n_9} - \frac{n_7}{n_9}
\]

Acknowledgment

The authors would like to thank Christophe Guitton (University of Rennes 1, Rennes, France) and Gilles Picoult (INSA Rennes, Rennes) for the fabrication of the printed circuit board.

References


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This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.