

Qualitative Bipolar Decision Frameworks Viewed as Pessimistic/Optimistic Utilities

Florence Dupin de St-Cyr, **Romain Guillaume**

IRIT, Université de Toulouse, Toulouse, France

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Introduction

- Decision : crucial in everyday life
- Explainable for the decision maker himself and for others
- To persuade and commit people

⇒ Bipolar Layered Framework

- Qualitative and Argumentative setting
- Under incomplete knowledge
- Bipolar evaluation
- Visual aspect

Bipolar Layered Framework

- Bipolar Graph
 - Connection between possible knowledge and goals,
 - Explication of a decision choice
- Aim of this proposal:
 - Study the link between the uncertain knowledge aggregation made by the BLF and classical aggregation functions used in decision under uncertainty approaches
 - Study the link between polarity and pessimistic/optimistic

Savage's Omelet problem

Someone has just broken five good eggs into a bowl when an agent comes and volunteer to finish making the omelet. The sixth egg lies unbroken beside the bowl. For some reason it must either be used for the omelet or wasted altogether. The agent must decide what to do with this unbroken egg. The agent must decide among three acts only. Namely:

- bi: to break an egg to join the other five eggs,
- bb: to break an egg into a saucer for inspection,
- ta: to throw an egg away without inspection.

The eight situations with their realized goals

worlds	description			goals		
	good 6th egg e_6	shell on table st	new bowl nb	omelet size	squan- dered	extra- dishes
ω_1	0	0	0	5	0*	0
ω_2	0	0	1	5	0*	1
ω_3	0	1	0	0	5	0
ω_4	0	1	1	5	0*	1
ω_5	1	0	0	5	1	0
ω_6	1	0	1	5	1	1
ω_7	1	1	0	6	0	0
ω_8	1	1	1	6	0	1

*A bad egg that has been thrown away is not considered as squandered.

$$\omega_3 < \omega_6 < \omega_5 < \omega_2 = \omega_4 < \omega_1 < \omega_8 < \omega_7$$

The two situations with their realized goals after bb

After action bb ("break beside") it holds $st \wedge nb$

worlds	description			goals		
	good 6th egg e_6	shell on table st	new bowl nb	omelet size	squan- dered	extra- dishes
ω_1	0	0	0	5	0*	0
ω_2	0	0	1	5	0*	1
ω_3	0	1	0	0	5	0
ω_4	0	1	1	5	0*	1
ω_5	1	0	0	5	1	0
ω_6	1	0	1	5	1	1
ω_7	1	1	0	6	0	0
ω_8	1	1	1	6	0	1

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$$\omega_3 < \omega_6 < \omega_5 < \omega_2 = \omega_4 < \omega_1 < \omega_8 < \omega_7$$

The two situations with their realized goals after bi

After action bi ("break inside") it holds $st \wedge \neg nb$

worlds	description			goals		
	good 6th egg e_6	shell on table st	new bowl nb	omelet size	squan- dered	extra- dishes
ω_1	0	0	0	5	0*	0
ω_2	0	0	1	5	0*	1
ω_3	0	1	0	0	5	0
ω_4	0	1	1	5	0*	1
ω_5	1	0	0	5	1	0
ω_6	1	0	1	5	1	1
ω_7	1	1	0	6	0	0
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The two situations with their realized goals after ta

After action ta ("throw away") it holds $\neg st$ (the agent has not broken the 6th egg, it was thrown directly)

worlds	description			goals		
	good 6th egg e_6	shell on table st	new bowl nb	omelet size	squandered	extra-dishes
ω_1	0	0	0	5	0*	0
ω_2	0	0	1	5	0*	1
ω_3	0	1	0	0	5	0
ω_4	0	1	1	5	0*	1
ω_5	1	0	0	5	1	0
ω_6	1	0	1	5	1	1
ω_7	1	1	0	6	0	0
ω_8	1	1	1	6	0	1

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$$\omega_3 < \omega_6 < \omega_5 < \omega_2 = \omega_4 < \omega_1 < \omega_8 < \omega_7$$

Decision criteria under ignorance

Definition (pessimistic and optimistic criterion)

The best pessimistic decision value is $\max_{d \in \mathbf{D}} \min_{\omega \in \Omega^d} u(\omega)$

The best optimistic decision value is $\max_{d \in \mathbf{D}} \max_{\omega \in \Omega^d} u(\omega)$

Definition (R_*^e -criterion)

The best R_^e decision value is: $\max_{d \in \mathbf{D}} \begin{cases} \min_{\omega \in \Omega^d} u(\omega) & \text{if } \min_{\omega \in \Omega^d} u(\omega) < e \\ \max_{\omega \in \Omega^d} u(\omega) & \text{otherwise} \end{cases}$*

- The best pessimistic decision is *bb* (which leads to ω_4 at worst)

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- The best optimistic decision is *bi* (which leads to ω_7 at best)

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- The best pessimistic decision is *bb* (which leads to ω_4 at worst)
- The best optimistic decision is *bi* (which leads to ω_7 at best)
- If the threshold of optimism is that the decision is good as soon as no more than one egg is squandered: $e = u(s_1)$ then *bb* is R_*^e -preferred to *ta* and *bi*.

Definition (BLF)

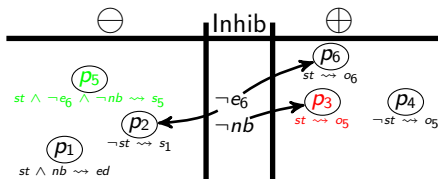
Given a utility function $u: LIT_G \rightarrow]-1, 1[$.

A BLF is a tuple $(G, \mathcal{P}, \mathcal{I}, l)$.

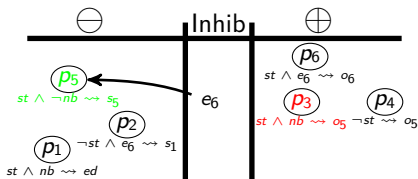
- $G \subseteq LIT_G$ is a set of literals goals s.t. $\forall g \in G, u(g) \neq 0$. G is separated into G^+ and G^- where $G^+ = \{g \in G | u(g) > 0\}$ and $G^- = \{g \in G | u(g) < 0\}$.
- \mathcal{P} is a set of decision principles (DPs). A decision principle is an expression $\varphi \rightsquigarrow g$ where $\varphi \in \mathcal{L}_F, g \in LIT_G$ which expresses that there is a causal link between φ and g of the form generally when φ holds g is realized
- \mathcal{I} is a set of inhibitors. An inhibitor is a pair $(\psi, \varphi \rightsquigarrow g)$ where $\psi \in \mathcal{L}_F$ and $\varphi \rightsquigarrow g \in \mathcal{P}$ which expresses that when ψ holds the causal link between φ and g is broken
- $l: G \rightarrow [0, 1]$ represents the level of importance of each goal of the BLF, it is defined by $\forall g \in G^+, l(g) = u(g)$ and $\forall g \in G^-, l(g) = -u(g)$.

Different BLFs for Savage's omelet

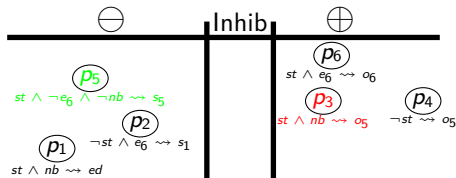
B1 given e_6 more plausible than $\neg e_6$



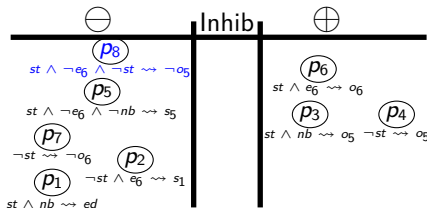
B2 given $\neg e_6$ more plausible than e_6



B3 given that e_6 as plausible as $\neg e_6$



B4: $\neg o_5$ worst neg. goal; e_6 as plaus. as $\neg e_6$



Reasoning and deciding with a BLF

Definition (realized goals)

Given a BLF $B = (G, \mathcal{P}, \mathcal{I}, I)$, a goal g in LIT_G is realized wrt B and $K \in \mathcal{L}_F$ if there is one DP p in \mathcal{P} s.t. $K \models p$ and p concludes g and p is not inhibited when knowing only K .

Decision	B1	B2	B3	B4
bi (causes $st \wedge \neg nb$)	$\{o_6\}$	$\{s_5\}$	\emptyset	\emptyset
bb (causes $st \wedge nb$)	$\{o_6, o_5, ed\}$	$\{o_5, ed\}$	$\{o_5, ed\}$	$\{o_5, ed\}$
ta (causes $\neg st$)	$\{s_1, o_5\}$	$\{o_5\}$	$\{o_5\}$	$\{\neg o_6, o_5\}$

Definition (utility, disutility of a decision)

Let $d \in \mathcal{L}_F$ be a decision and $B = (G, \mathcal{P}, \mathcal{I}, I)$ a BLF, the BLF Pareto-evaluation of d is a pair $(disut_B(d), ut_B(d))$ s.t.:

$$disut_B(d) = \max_{g \in G^{\ominus}} (I(g)) | g \text{ is realized wrt } B \text{ and } d)$$

$$ut_B(d) = \max_{g \in G^{\oplus}} (I(g)) | g \text{ is realized wrt } B \text{ and } d)$$

Decision rules

Definition (Pareto dominance)

Given a BLF $B = (G, \mathcal{P}, \mathcal{I}, I)$, d_1 and d_2 in \mathcal{L}_F ,
 $d_1 \succeq_{\text{Pareto}} d_2$ if $ut_B(d_1) \geq ut_B(d_2)$ and $disut_B(d_1) \leq disut_B(d_2)$.

Definition (bi-poss preference)

Given a BLF $B = (G, \mathcal{P}, \mathcal{I}, I)$, d_1 and d_2 in \mathcal{L}_F ,
 $d_1 \succeq d_2$ if $\max(ut_B(d_1), disut_B(d_2)) \geq \max(ut_B(d_2), disut_B(d_1))$

- B1: $bi \succ_{\text{Pareto}} bb$ and $bi \succ_{\text{Pareto}} ta$
- B1: $bi \sim_{\text{bi-poss}} bb$ and $bi \succ_{\text{bi-poss}} ta$ since the disutility of the extra-dish is neglected in front of the utility of achieving o_6 .
- B2: the best decision for Pareto-dominance is ta .
- B4: bb is the Bi-poss most preferred decision.

Fuzzy BLF

In order to compare BLF with classical decision criteria under uncertainty. We first introduce the notion of fuzzy BLF

Definition (Fuzzy BLF)

A BLF $B = (G, \mathcal{P}, \mathcal{I}, I)$ is a fuzzy BLF if

$$\begin{cases} \forall g, g' \in G^{\oplus}, & \text{if } I(g) \geq I(g') \text{ then } g \models g' \text{ and} \\ \forall g, g' \in G^{\ominus}, & \text{if } I(g) \geq I(g') \text{ then } g \models g' \end{cases}$$

Remark

Any BLF can be transformed into a Fuzzy BLF by transforming each goal in a lower level to the disjunction of the goals of the higher levels, the two BLFs are equivalent wrt decision making when the original BLF has each positive (resp. negative) goal on a distinct level (as it is the case in our examples).

Bipolarity and optimism of the decision maker

Proposition

Given a fuzzy-BLF B and a decision $d \in \mathcal{L}_F$, it holds that^a

$$ut_B(d) = \min_{\omega \in \Omega^d} ut(\omega) \quad disut_B(d) = \min_{\omega \in \Omega^d} disut(\omega)$$

^aWith $\min_{\omega \in \emptyset} ut(\omega) = 0$ and $\min_{\omega \in \emptyset} disut(\omega) = 0$.

Proposition

Considering a fuzzy-BLF $B = (G, \mathcal{P}, \mathcal{I}, I)$ based on u ,

- *if $G^\ominus = \emptyset$ (i.e., $G = G^\oplus$) then B -Pareto and B -Biposs pref. orderings are same as maxmin pessimistic criterion wrt ut_B .*
- *if $G^\oplus = \emptyset$ (i.e., $G = G^\ominus$), then B -Pareto and B -Biposs pref. ordering are same as maxmax optimistic criterion wrt $(1 - disut_B)$.*
- *if $\forall g \in G^\oplus$ then $\neg g \in G^\ominus$ and vice versa with $I(g) = I(\neg g)$. then d_1 is B -Pareto-pref. to d_2 iff d_1 is minmax pref. to d_2*

R_*^e -BLF

Definition

A R_*^e -BLF is a fuzzy-BLF such that

- there is a universal positive goal g^u and a DP: $\top \rightsquigarrow g^u$ with $l(g^u) > \max_{g \in G^\ominus} l(g)$,
- for all g in $G^\oplus \setminus \{g^u\}$, $\neg g \in G^\ominus$ and vice-versa,
- $\max_{g \in G^\oplus} l(g) < e$,
- and if $l(g) > l(g')$ then $l(\neg g) > l(\neg g')$.

Proposition

An optimal solution using Pareto pref. rules on a R_*^e -BLF is optimal for R_*^e preference.

Conclusion

- We have shown that the extension to a bipolar scale is important to help to differentiate the ways to deal with the uncertainty associated with positive or negative goal,
- We have provided a formal characterization of the fact that BLF is a generalization of maxmin and maxmax criterion
- and we have situated it wrt to R_*^e -criterion.