

Fourier-Transform Acousto-Optic Imaging with Off-axis Holographic Detection: supplemental document

A tagging function was introduced in our main manuscript as a multiplicative spatial function which plays a role in mapping the origin of tagged photon in the ultrasonic propagation plane. This function was the result of constructive interference between the tagged photons and the tailored optical reference beam during the camera integration time. By performing a linear combination of four frames acquired for different input phase of the structured acoustic wave, the complex Fourier component of the image to reconstruct is fetched, as demonstrated here.

1. DERIVATION

According to the definition given in the main body of this article, the complex correlation function $M_{m,n}^\phi$ exhibits the selection of the tagged photons operated between the two modulations during the exposure time of the camera τ_e . Here we fully develop its analytical derivation.

$$M_{m,n}^\phi(x, z) = \frac{1}{\tau_e} \int_{\tau_e} dt \sin^2 \left(\pi \frac{n}{T_0} t \right) A(x, z, t). \quad (S1)$$

As presented in the article, the pressure field in propagation across the plane (x, z) has the expression:

$$A(x, z, t) = P_0 h_{m,n}^\phi(t - c_s^{-1} z, x). \quad (S2)$$

with c_s the celerity of sound in the sample. By injecting Eq S2 in Eq S1 we have:

$$M_{m,n}^\phi(x, z) = \frac{P_0}{\tau_e} \int_{\tau_e} dt \sin^2 \left(\pi \frac{n}{T_0} t \right) h_{m,n}^\phi \left(t - c_s^{-1} z, x \right). \quad (S3)$$

To follow the derivation, we need to exhibit the analytical expression of $h_{m,n}^\phi$:

$$h_{m,n}^\phi(t, x) = \sum_k \Pi_n \left(t - \frac{m v_x^0}{n v_z^0} \frac{x}{c_s} - \frac{k T_0}{n} + \phi \right) \quad \Pi_n(t) = \begin{cases} 1 & \text{if } 0 \leq t < T_0/2n \\ 0 & \text{otherwise} \end{cases} \quad (S4)$$

With this definition of Π_n , $h_{m,n}^\phi$ is a binary T_0/n -periodic function. We draw it on [Figure S1](#).

Experimentally we can choose the integration time of the camera independently from the temporal length of the sequence. In order to have the same total insonified time along the measure, we choose commensurable quantities:

$$\tau_e = N T_0 \quad v_x^0 = L^{-1} \quad v_z^0 = (c_s T_0)^{-1} \quad (S5)$$

with N an integer. We chose $N = 5$, $T_0 = 20 \mu\text{s}$ and $\tau_e = 100 \mu\text{s}$, such as:

$$\forall (m, n) \quad \forall x \in [0, L] \quad \int_{\tau_e} h_{m,n}^\phi(t, x) dt = \frac{N T_0}{2}. \quad (S6)$$

We use the trigonometric identity:

$$\sin^2 \left(\pi \frac{n}{T_0} t \right) = \frac{1}{2} \left[1 - \cos \left(2\pi \frac{n}{T_0} t \right) \right] \quad (S7)$$

to develop the Eq S3:

$$\begin{aligned} M_{m,n}^\phi(x, z) &= \frac{P_0}{2 N T_0} \int_{N T_0} dt \left[1 - \cos \left(2\pi \frac{n}{T_0} t \right) \right] h_{m,n}^\phi \left(t - c_s^{-1} z, x \right) \\ &= \frac{P_0}{2} \left[\frac{1}{2} - \frac{1}{N T_0} \int_{N T_0} dt \cos \left(2\pi \frac{n}{T_0} t \right) h_{m,n}^\phi \left(t - c_s^{-1} z, x \right) \right] \end{aligned} \quad (S8)$$

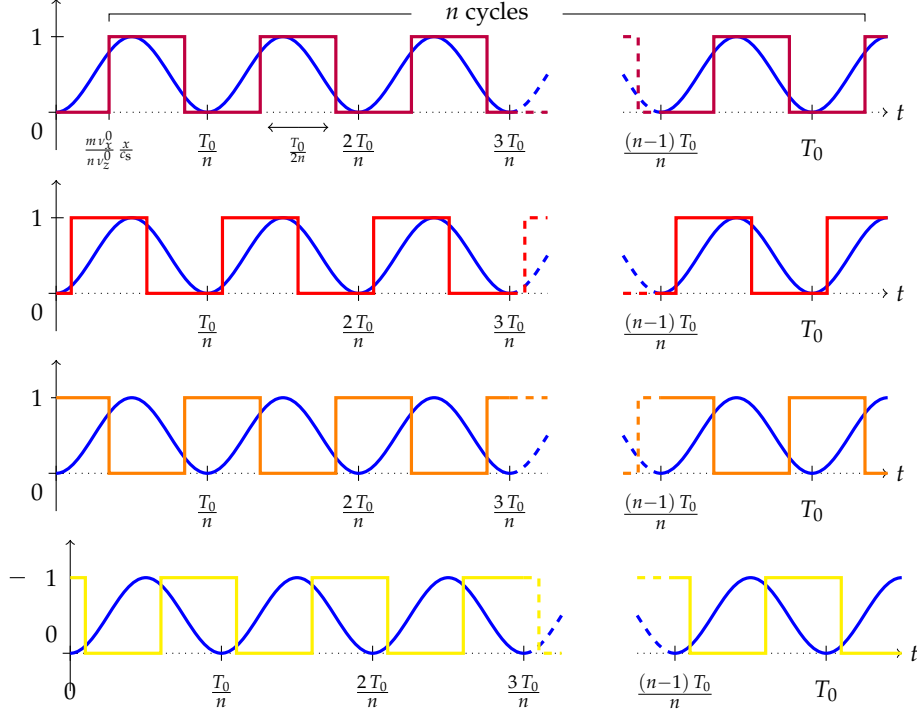


Fig. S1. $\sin^2(\pi n t/T_0)$ in blue and $h_{m,n}^\phi(t,x)$ for an arbitrarily chosen position x and for $\phi \in \{0, \pi/2, \pi, 3\pi/2\}$ in respectively $\{\blacksquare, \blacksquare, \blacksquare, \blacksquare\}$.

As $h_{m,n}^\phi$ is a binary T_0/n -periodic function, we can split the integral into nN small narrower integrals (See Fig.S1 for a graphical example):

$$\begin{aligned}
M_{m,n}^\phi(x,z) &= \frac{P_0}{2} \left[\frac{1}{2} - \frac{n}{T_0} \int_{\frac{m v_x^0}{n v_z^0} \frac{x}{c_s} + \frac{z}{c_s} - \phi + \frac{T_0}{2n}}^{\frac{m v_x^0}{n v_z^0} \frac{x}{c_s} + \frac{z}{c_s} - \phi} dt \cos\left(2\pi \frac{n}{T_0} t\right) \right] \\
&= \frac{P_0}{2} \left(\frac{1}{2} - \frac{1}{2\pi} \left\{ \sin\left[2\pi\left(m v_x^0 x + n v_z^0 z\right) - \phi + \pi\right] \right. \right. \\
&\quad \left. \left. - \sin\left[2\pi\left(m v_x^0 x + n v_z^0 z\right) - \phi\right] \right\} \right) \\
&= \frac{P_0}{2} \left[\frac{1}{2} + \frac{1}{\pi} \sin(2\pi(m v_x^0 x + n v_z^0 z) - \phi) \right]
\end{aligned} \tag{S9}$$

We define $I(x,z)$ is the acousto-optic image within the medium, $I(x,z)$ is a real function. \bar{I} is its 2 dimensional Fourier transform.

$$\begin{aligned}
s_{m,n}^\phi &\propto \int_{x,z} dx dz I(x,z) |M(x,z)|^2 \\
&\propto \int_{x,z} dx dz I(x,z) \left\{ \frac{1}{2} + \frac{1}{\pi} \sin\left[2\pi\left(m v_x^0 x + n v_z^0 z\right) - \phi\right] \right\}^2 \\
&\propto \int_{x,z} dx dz I(x,z) \left\{ \frac{1}{4} + \frac{1}{2\pi^2} + \frac{1}{\pi} \sin\left[2\pi\left(m v_x^0 x + n v_z^0 z\right) - \phi\right] \right. \\
&\quad \left. - \frac{1}{2\pi^2} \cos\left[4\pi\left(m v_x^0 x + n v_z^0 z\right) - 2\phi\right] \right\} \\
&\propto \left(\frac{1}{4} + \frac{1}{2\pi^2} \right) \int_{x,z} dx dz I(x,z) + \frac{1}{2\pi i} \left[-\bar{I}\left(m v_x^0, n v_z^0\right) e^{i\phi} + \bar{I}\left(-m v_x^0, -n v_z^0\right) e^{-i\phi} \right] \\
&\quad - \frac{1}{4\pi^2} \left[\bar{I}\left(2m v_x^0, 2n v_z^0\right) e^{2i\phi} + \bar{I}\left(-2m v_x^0, -2n v_z^0\right) e^{-2i\phi} \right]
\end{aligned} \tag{S10}$$

As I is a real function:

$$\tilde{I}(m v_x^0, n v_z^0) = \tilde{I}^*(-m v_x^0, -n v_z^0) \quad (\text{S11})$$

and

$$\tilde{I}(2m v_x^0, 2n v_z^0) = \tilde{I}^*(-2m v_x^0, -2n v_z^0) \quad (\text{S12})$$

so

$$s_{m,n}^\phi \propto \left(\frac{1}{4} + \frac{1}{2\pi^2} \right) \int_{x,z} dx dz I(x,z) + \frac{1}{2\pi i} \left[-\tilde{I}(m v_x^0, n v_z^0) e^{i\phi} + \tilde{I}^*(m v_x^0, n v_z^0) e^{-i\phi} \right] - \frac{1}{4\pi^2} \left[\tilde{I}(2m v_x^0, 2n v_z^0) e^{2i\phi} + \tilde{I}^*(2m v_x^0, 2n v_z^0) e^{-2i\phi} \right] \quad (\text{S13})$$

Measurements for various phase shifts, $\phi \in \{0, \pi/2, \pi, 3\pi/2\}$, give access to the real and the imaginary parts:

$$\begin{aligned} s_{m,n}^0 &\propto \left(\frac{1}{4} + \frac{1}{2\pi^2} \right) \int_{x,z} I(x,z) - \frac{1}{\pi} \Im \left[\tilde{I}(m v_x^0, n v_z^0) \right] - \frac{1}{2\pi^2} \Re \left[\tilde{I}(2m v_x^0, 2n v_z^0) \right] \\ s_{m,n}^{\pi/2} &\propto \left(\frac{1}{4} + \frac{1}{2\pi^2} \right) \int_{x,z} I(x,z) - \frac{1}{\pi} \Re \left[\tilde{I}(m v_x^0, n v_z^0) \right] + \frac{1}{2\pi^2} \Re \left[\tilde{I}(2m v_x^0, 2n v_z^0) \right] \\ s_{m,n}^\pi &\propto \left(\frac{1}{4} + \frac{1}{2\pi^2} \right) \int_{x,z} I(x,z) + \frac{1}{\pi} \Im \left[\tilde{I}(m v_x^0, n v_z^0) \right] - \frac{1}{2\pi^2} \Re \left[\tilde{I}(2m v_x^0, 2n v_z^0) \right] \\ s_{m,n}^{3\pi/2} &\propto \left(\frac{1}{4} + \frac{1}{2\pi^2} \right) \int_{x,z} I(x,z) + \frac{1}{\pi} \Re \left[\tilde{I}(m v_x^0, n v_z^0) \right] + \frac{1}{2\pi^2} \Re \left[\tilde{I}(2m v_x^0, 2n v_z^0) \right] \end{aligned} \quad (\text{S14})$$

$$s_{m,n}^{3\pi/2} - s_{m,n}^{\pi/2} + i(s_{m,n}^\pi - s_{m,n}^0) \propto \tilde{I}(m v_x^0, n v_z^0) \quad (\text{S15})$$