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Robust Fuzzy Model Predictive Control with time delays for Nonlinear Systems

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Abstract

In this work, a robust fuzzy model predictive control (RFMPC) is designed for uncertain time-delay systems under input constraints with external disturbances. The proposed controller is based on an augmented state space model that includes the state variables and output tracking error variable of system. As a result, the proposed control based on augmented state space model can guarantee the convergence and tracking performance of system. To ensure the stability of the closed loop constrained time-delays system, in the form of linear matrix inequalities (LMIs), the Lyapunov–Krasovskii (L-K) theory is employed to derive sufficient stability conditions where the L-K theory has the ability to reflect the system's original state space and its advantages in controller synthesis and computation. As a result, less conservative stable conditions in terms of LMIs are given and used to ensure the asymptotically robust stability of closed-loop constrained system, and the controller construction procedure is shifted into a minimization problem subjected to certain constraints in terms of LMI. The robustness properties of the proposed RFMPC approach are compared with the results of a standard predictive control MPC applied to control a nonlinear system with parameter uncertainties and time-delays.

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1. Introduction

In many industrial processes, time delays, also known as lags or as dead-times, are caused by the process's instrumentation, actuation and/or control technology. Time delays can also be considered as a consequence of

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certain system' limitations in data transmission, information processing time, and feedback delays in the case of feedback control. The time delays have a significant impact on the stability of the industrial systems, which may lead to the instability of the systems, and in many cases the degradation in the performances of the systems.

Generally, the presence of delays in any system may increase the difficulties of controlling the system since these delays can affect the state input or/and output. Generally, delays in systems are modelled as constant or time varying delays, known or unknown delays, while in many cases these delays are considered deterministic or stochastic delays depending on the targeted systems [1, 2, 3]. As one of the most famous sources of delays in control system [4-7] is the computation times required to implement one of the advanced process control algorithms such as Model Based Control (MPC) and its recent variants (Adaptive MPC, Distributed and Economic MPC, Cooperative MPC...). Usually, the time needed for computation to perform one controlling step varies from one control algorithm to another which may generate possible delays within the system itself and arise as feedback delays in the control loops.

Generally, the MPC approach for nonlinear systems can be formulated as a LMI problem in which an exploration of a common positive definite matrix is performed in order to satisfy the LMIs constraints for all local models of the nonlinear plant [8-10]. Unfortunately, this may lead to several difficulties to construct the MPC controller since the solution of this problem may not be feasible and the existence of such matrix might not possible in many cases, especially for highly nonlinear systems. For the last few decades, fuzzy techniques have been successfully used in many engineering areas and widely applied to control and model nonlinear systems. Generally, fuzzy techniques, such as Takagi–Sugeno (T–S) inference system, are very useful to approximate the nonlinearities of real systems. The fuzzy inference systems can also be used to transform ordinary linear MPC algorithm into a nonlinear control approach via T–S fuzzy model [11-13]. This can be done using a T–S fuzzy model, in which the complex dynamic model of the nonlinear system is composed of a set of local linear subsystems by the use of fuzzy rules [14, 15]. The MPC technique has been extensively used to control linear systems, and fused many times with T-S fuzzy to form a nonlinear MPC controller. The MPC controller for constrained linear systems to track piecewise-constant references is implemented and investigated in [16]. A Robust model predictive control (RMPC) has been designed for an uncertain system under bounded control signals in [17]. In [18, 19], a finite-horizon MPC problem investigated for max-plus-linear systems and a control solution was derived for this problem.

Furthermore, the fuzzy inference techniques were successfully used to address the optimization problem in MPC algorithms for nonlinear process systems [20-23]. The investigations of fuzzy MPC methods for nonlinear systems can also be seen in [24–27]. In [28] a sufficient stability conditions of the fuzzy MPC controller is derived in which the stability analysis and the control procedure are reduced to LMI problems. Similar work has been done in [29] where the authors were more focused on solving the problems of time delays stability and stabilization fuzzy model predictive control of nonlinear systems with time delays. However, in the above-mentioned works on fuzzy MPC for time-delay nonlinear systems, the disturbances have not been fully considered despite that disturbance exists extensively in many practical applications.

In this paper, a robust model predictive control for time-delay T-S fuzzy model is designed to control nonlinear systems with time-delays state and external disturbances with input constraints. The main focus of this paper is to provide the stability conditions for the MPC control which is based on T-S prediction. First, the T-S fuzzy systems are employed to approximate nonlinear uncertain discrete-time systems with state delays and external disturbances. Next, an augmented system is proposed and used to design the MPC control law, which guarantees the convergence as well as a good tracking performance of system. Finally, a sufficient condition based on LMI is provided to address the robust fuzzy MPC control synthesis of T-S fuzzy systems with state delays and external disturbances, while the LMI optimization will be solved online at each time step to ensure a good tracking performance of system. Based on Lyapunov–Krasovskii approach, the synthesized controller has a guaranteed asymptotical stability.

The rest of this paper is structured as follows: In next section, the uncertain non-linear systems with delays based on a T-S fuzzy inference system and proposed augmented system are introduced. Next, the robust FMPC approach based on linear matrix inequalities (LMIs) is presented in Section 3. The performance of the proposed robust fuzzy controller is illustrated through an example in Section 4. Finally, section 5 concludes the paper.

2. Problem formulation

2.1. System Description

In this section, a discrete-time state space form with state time delay and disturbances represented by a T–S fuzzy dynamic model, is considered for this study where the i th rule is described as follows:

Plan rule j : If $z_1(k)$ is F_{i1} , and $z_2(k)$ is F_{i2} and And $z_b(k)$ is F_{ib} then

$$\begin{cases} x(k+1) = \tilde{A}_j x(k) + \tilde{A}_{dj} x(k-d) + \tilde{B}_j u(k) + w(k) \\ y(k) = \tilde{C}_j x(k) \end{cases} \quad (1)$$

where $z_1(k), z_2(k), \dots, z_b(k)$ are the premise variable, and $F_{ih}, j = 1, 2, \dots, r, h = 1, 2, \dots, b$ are fuzzy sets, $x(k) \in R^n, u(k) \in R^m, y(k) \in R^q$ and $w(k) \in R^w$ are the state variable, control input, measurable output and external disturbance, respectively. Also, $d > 0$ is the time-invarying input delay. Assuming that $\tilde{A}_j = A_j + \Delta A_j, \tilde{A}_{dj} = A_{dj} + \Delta A_{dj}, \tilde{B}_j = B_j + \Delta B_j, \tilde{C}_j = C_j + \Delta C_j$. Where $\Delta A_i, \Delta B_i, \Delta A_{dj}$ and ΔC_i are uncertain perturbations, while the matrices A_i, A_{dj}, B_i and C_i are constant matrices corresponding to appropriate dimensions. r represents the number of IF–THEN rules. For the sake of notational convenience, let $\vartheta := 1, 2, \dots, r$.

Given a pair $(x(k), u(k))$, and by applying the fuzzy inference rules, the final output of the fuzzy system is described as follows:

$$\begin{cases} x(k+1) = \sum_{j=1}^r h_j(z(k)) \tilde{A}_j x(k) + \tilde{A}_{dj} x(k-d) + \tilde{B}_j u(k) + w(k) \\ y(k) = \sum_{j=1}^r h_j(z(k)) (C_j + \delta C_j) x(k) \end{cases} \quad (2)$$

Let $F_{ib}(z(k))$ denote the membership function of $z(k)$ in F_{ib} . Then $\mu_i(z(k)) = \prod_{h=1}^b F_{ih}(h_h(k))$, and $h_j(z(k)) = \mu_j(z(k)) / \sum_{i=1}^r \mu_i(z(k))$. Considering $h_i(z(k)) \geq 0, i = 1, 2, \dots, r, \sum_{i=1}^r \mu_i(z(k)) > 0$. Therefore, the value of $h_j(z(k)) \geq 0, j = 1, 2, \dots, r$, and $\sum_{j=1}^r h_j(z(k)) = 1$.

Consequently, the entire fuzzy system can be described as:

$$\begin{cases} x(k+1) = A_v x(k) + \tilde{A}_{dv} x(k-d) + B_v u(k) + w(k) \\ y(k) = C_v x(k) \end{cases} \quad (3)$$

where

$$\tilde{A}_v = \sum_{j=1}^r h_j(z(k)) \tilde{A}_j, \quad \tilde{A}_{dv} = \sum_{j=1}^r h_j(z(k)) \tilde{A}_{dj}, \quad \tilde{B}_v = \sum_{j=1}^r h_j(z(k)) \tilde{B}_j, \quad \tilde{C}_v = \sum_{j=1}^r h_j(z(k)) \tilde{C}_j,$$

Lemma 1 (Schur Complement). [30] Given any real matrices X, Y and Z with $X = X^T$ and $Z > 0$. Then we have

$$X - YZ^{-1}Y^T < 0$$

If and only if

$$\begin{bmatrix} X & Y^T \\ Y & Z \end{bmatrix} < 0$$

Assumption 1. actuator fault $f_a(k)$, sensors faults $f_s(k)$ and the external disturbance $\omega(k)$ are bounded, and there exists a positive scalar β such that $\|\omega(k)\| < \beta$.

Remark 1. Assumption 1 can be used to ensure that the increments of the disturbances as well as the faults between

two sampling time instants are bounded.

In the next section, the control strategy is presented where constraints, such as control effort and state variables, are considered in obtaining the control law.

2.2. Augmented state space model

Let $y_r(k)$ denote the desired set-point and $e(k)$ the tracking error, defined as

$$e(k) = y(k) - y_r(k)$$

Then, the error dynamics at time $n+1$ is formulated as:

$$e(k+1) = y(k+1) - y_r(k+1) \quad (4)$$

For a constant set-point $y_r(k+1) = y_r(k)$

Using Eqs. (3) and (4), the following $e(k+1)$ can be formulated as:

$$e(k+1) = e(k) + \tilde{C}\tilde{A}_d x(k-d) + \tilde{C}\tilde{B}u(k) + \tilde{C}w(k) \quad (5)$$

As a result, the augmented system can be expressed as the following discrete-time system with state delay:

$$\begin{cases} \bar{x}(k+1) = \mathcal{A}\bar{x}(k) + \mathcal{A}_d\bar{x}(k-d) + \mathcal{B}u(k) + \mathcal{D}w(k) \\ y(k) = \mathcal{C}\bar{x}(k) \end{cases} \quad (6)$$

Where $\bar{x} = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}$, $\bar{x}(k-d) = \begin{bmatrix} x(k-d) \\ e(k-d) \end{bmatrix}$

$$\mathcal{A} = \begin{bmatrix} \tilde{A} & 0 \\ 0 & I \end{bmatrix}, \mathcal{A}_d = \begin{bmatrix} \tilde{A}_d & 0 \\ \tilde{C}\tilde{A}_d & 0 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} \tilde{B} \\ \tilde{C}\tilde{B} \end{bmatrix}, \mathcal{D} = \begin{bmatrix} I \\ \tilde{C} \end{bmatrix}, \mathcal{C} = [\tilde{C} \quad 0]$$

Hence, through the above analysis, the fuzzy control for the extended state-space model in (6) is obtained as follows:

$$u(k+i/k) = \sum_{l=1}^r K_l \bar{x}(k+i/k) \quad (7)$$

$$K_z = \sum_{l=1}^r h_l(z(k)) K_l$$

where $K_l (l = 1, 2, \dots, r)$ are the local control gains.

Then, the following augmented closed-loop fuzzy system can be obtained as:

$$\bar{x}(k+1) = (\mathcal{A}_z + \mathcal{B}_z K_z) \bar{x}(k) + \mathcal{A}_{dz} \bar{x}(k-d) + \mathcal{D}w(k) \quad (8)$$

The augmented model that includes both state variables and the output tracking error variable of the model is given in Eq. (8). Based on the features of the augmented model, the synthesized controller will not only guarantee a better convergence and tracking performance but also provides more degrees of freedom compared to other standard MPC control approach.

3. Robust Model Predictive controller with constraints

In this section, the basic concepts of the MPC technique are presented in details. At each time step, the main objective of an MPC problem is to calculate the control move $u(k+i/k)$ by minimizing the predefined performance function described below:

$$\min_{u(k+i/k), i \geq 0} \max J_{\infty}(k) = \sum_{j=0}^{\infty} \|\bar{x}(k+i/k)\|_Q^2 + \|u(k+i/k)\|_R^2 - \alpha^2 \|\bar{w}(k+i/k)\|^2 \quad (9)$$

Furthermore, the constraint on control input is considered as follows:

$$|u(k+i/k)| \leq u_{s,max}, i \geq 0, s = 1, \dots, m \quad (10)$$

where $Q > 0$ and $R > 0$, are the corresponding weighting matrices.

Equations (9) and (10) are constrained as min-max optimization problem that corresponds to a worst-case infinite-horizon MPC problem with a quadratic objective function.

Next, a Lyapunov-Krasovskii function, which is defined in Eq. (11), is used to obtain the stability conditions:

$$V(\bar{x}(k/k)) = \bar{x}^T(k/k) S \bar{x}(k/k) + \sum_{n=1}^d \bar{x}^T(k-n/k) P \bar{x}(k-n) \quad (11)$$

Where $S^T > 0, P^T > 0$ and

$$V(\bar{x}(k+i)) = \bar{x}^T(k+i) S \bar{x}(k+i) + \sum_{i=1}^d \bar{x}^T(k+i-n) P \bar{x}(k+i-n)$$

For any $i \geq 0$, suppose $V(\bar{x}(k/k))$ satisfies the following stability constraint:

$$V(\bar{x}(k+i+1/k)) - V(\bar{x}(k+i/k)) \leq -[\|\bar{x}(k+i/k)\|_Q^2 + \|u(k+i/k)\|_R^2] + \alpha^2 \|\bar{w}(k+i/k)\|^2 \quad (12)$$

As assumed earlier, the summation should be up to ∞ , ie., $i \rightarrow \infty, x(\infty) = 0$. Summing from $i = 0$ to ∞ yields the following:

$$J(k) \leq V(\bar{x}(k/k)) \quad (13)$$

By defining $V(\bar{x}(k/k)) \leq \gamma$ and from Eq.(12), we can see that an upper bound of the fitness function $J(k)$ is obtained as:

$$J(k) \leq \gamma.$$

By summarizing the above analysis, the following theorem can be obtained:

Theorem1: For the augmented time delay system in Eq. (8) and at the sampling time k , the state feedback control Eq. (7) minimize the worst case MPC objective function with input constraints Eq. (10), if there exists a symmetric positive definite matrices $M_v > 0, X_v > 0, G$ and Y_v and a positive scalar γ by solving the following convex optimization problem:

$$\min_{\gamma, G_v, Y_v, M_v} \gamma \quad (14)$$

Subject to

$$\begin{bmatrix} -1 & * & * & * & * \\ \bar{x}(k) & -M_v & * & * & * \\ \bar{x}(k-1) & 0 & -X_v & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{x}(k-d) & 0 & 0 & \dots & -X_v \end{bmatrix} \leq 0 \quad (15)$$

$$\begin{bmatrix} -U & Y_v \\ Y_v^T & -G - G^T + M_v \end{bmatrix} \leq 0 \quad j, l \in v$$

$$\begin{bmatrix} -\Pi & * & * & * & * & * & * \\ R^{1/2}Y_v & -\gamma I & * & * & * & * & * \\ Q^{1/2}G_v & 0 & -\gamma I & * & * & * & * \\ G_v & 0 & 0 & -X_v & * & * & * \\ 0 & 0 & 0 & 0 & -\alpha^2 I & * & * \\ \mathcal{A}_z G_v + \mathcal{B}_z Y_v & 0 & \mathcal{D}G_v & 0 & 0 & -M_v & * \\ 0 & 0 & 0 & 0 & 0 & X_v \mathcal{A}_{dz} & -X_v \end{bmatrix} < 0 \tag{16}$$

where $\Pi = G_v + G_v^T - M_v$

Proof: At sampling time k , a Lyapunov-Krasovskii function is defined as:

$$V(\bar{x}(k/k)) = \bar{x}^T(k/k) S \bar{x}(k/k) + \sum_{n=1}^d \bar{x}^T(k-n/k) P \bar{x}(k-n) \tag{17}$$

Let us suppose $V(\bar{x}(k/k))$ satisfies the following robust stability condition at each sampling instant

$$V(\bar{x}(k+i+1/k)) - V(\bar{x}(k+i/k)) \leq -[\|\bar{x}(k+i/k)\|_Q^2 + \|u(k+i/k)\|_R^2] + \alpha^2 \|\bar{w}(k+i/k)\|^2 \tag{18}$$

Then, and from Eq. (13), the cost function in Eq. (9) is bounded as follows:

$$J(k) \leq \bar{x}^T(k/k) S \bar{x}(k/k) + \sum_{n=1}^d \bar{x}^T(k-n/k) P \bar{x}(k-n) \tag{19}$$

Since $V(\bar{x}(k/k)) \leq \gamma$, the performance index $J(k)$ has an upper bound γ . Then:

$$\bar{x}^T(k/k) S \bar{x}(k/k) + \sum_{n=1}^d \bar{x}^T(k-n/k) P \bar{x}(k-n) \leq \gamma \tag{20}$$

Thus, the lowest upper bound can be obtained from an optimization problem. The inequality (20) may be written as follows:

$$\frac{1}{\gamma} \bar{x}^T(k/k) S \bar{x}(k/k) + \frac{1}{\gamma} \sum_{n=1}^d \bar{x}^T(k-n/k) P \bar{x}(k-n) \leq I \tag{21}$$

As a result, the inequality in Eq. (15) can be easily be obtained using Schur’s complement in **lemma 1**.

According to system in Eq. (8), the condition in Eq. (16) is guaranteed only if the inequality below in Eq. (22) is satisfied:

$$[\bar{x}^T(k+i+1/k) S_v \bar{x}(k+i+1/k)] - [\bar{x}^T(k+i/k) S_v \bar{x}(k+i/k)] + [\bar{x}^T(k+i/k) P_v \bar{x}(k+i/k)] - [\bar{x}^T(k+i-d/k) P_v \bar{x}(k+i-d/k)] < -[(\bar{x}^T(k+i/k) Q \bar{x}(k+i/k)) + (u^T(k+i/k) R u(k+i/k))] + \alpha^2 \bar{w}^T(k+i/k) \bar{w}(k+i/k) \tag{22}$$

$$[\bar{x}^T(k+i/k) \bar{x}^T(k+i-d/k)] \begin{bmatrix} (\mathcal{A}_z + \mathcal{B}_z K_z)^T \\ \mathcal{A}_{dz}^T \\ \mathcal{D} \end{bmatrix} S_v [\mathcal{A}_z + \mathcal{B}_z K_z \quad \mathcal{A}_{dz} \quad \mathcal{D}] \begin{bmatrix} \bar{x}(k+i/k) \\ \bar{x}(k+i-d/k) \end{bmatrix} - [\bar{x}^T(k+i/k) \bar{x}^T(k+i-d/k)] \begin{bmatrix} -S_v + P_v + Q + K^T R K & * & * \\ 0 & -P_v & * \\ 0 & 0 & -\alpha^2 I \end{bmatrix} \begin{bmatrix} \bar{x}(k+i/k) \\ \bar{x}(k+i-d/k) \end{bmatrix} < 0 \tag{23}$$

By applying Schur’s complement, the inequality in Eq. (23) can be transformed into:

$$\begin{bmatrix} -S_v + P_v + Q + K^T R K & * & * & * \\ 0 & -P_v & * & * \\ 0 & 0 & -\alpha^2 I & * \\ \mathcal{A}_z + \mathcal{B}_z K_z & \mathcal{A}_{dz} & \mathcal{D} & -S_v^{-1} \end{bmatrix} < 0 \tag{24}$$

Now, we define $M_v = \gamma S_v^{-1}$ and $X_v = \gamma P_v^{-1}$ and by applying the Schur's complement again, we obtain the following:

$$\begin{bmatrix} -\gamma M_v^{-1} & * & * & * & * & * & * \\ R^{1/2} K_v & -\gamma I & * & * & * & * & * \\ Q^{1/2} & 0 & -\gamma I & * & * & * & * \\ I & 0 & 0 & -\gamma X_v & * & * & * \\ 0 & 0 & 0 & 0 & -\gamma X_v^{-1} & * & * \\ 0 & 0 & 0 & 0 & 0 & -\alpha^2 I / \gamma & * \\ \mathcal{A}_z + \mathcal{B}_z K_v & \mathcal{A}_{dz} & \mathcal{D} & 0 & 0 & 0 & -\gamma M_v \end{bmatrix} < 0 \tag{25}$$

where $(G - M_v)^T M_v^{-1} (G - M_v) \geq 0 \Rightarrow G^T M_v^{-1} G \geq G + G^T - M_v$, and pre-multiplying and post-multiplying the above inequality by $\text{diag} \left\{ \frac{1}{\gamma^{1/2}} G, I, I, \frac{1}{\gamma^{1/2}} I, \frac{1}{\gamma^{1/2}} I, \frac{1}{\gamma^{1/2}} X_v \right\}$, and substituting $K_v = -Y_v G^{-1}$, inequality (25) will eventually leads to the condition in Eq. (16).

Furthermore, to deal with the problem of conservatism, the following theorem represents a good compromise between conservatism and complexity.

Theorem2. [31], If there exists symmetric positive-definite matrices $G = \{\Phi_{ii}\}_{i \in v}$, general matrices $\{\Phi_{ij} = \Phi_{ij}^T\}_{i \neq j, i, j \in v}$, $\{Y_i\}_{i \in v}$, $\{M_i\}_{i \in v}$ and $\{X_i\}_{i \in v}$ such that the following LMIs are verified:

$$\min_{\gamma, G, \Phi, M, Y, X, \gamma} \tag{26a}$$

Subject to

$$\begin{bmatrix} -1 & * & * & * & * \\ \bar{x}(k) & -M_i & * & * & * \\ \bar{x}(k-1) & 0 & -X_s & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{x}(k-\tau) & 0 & 0 & \dots & -X_s \end{bmatrix} \leq 0 \quad i, s \in v$$

$$\begin{bmatrix} -U & Y_l \\ Y_l^T & -G - G^T + M_j \end{bmatrix} \leq 0 \quad j, l \in v \tag{26b}$$

$$Y_{ii} > \Phi_{ii}, \quad i \in v \tag{27}$$

$$Y_{ij} + Y_{ij}^T > \Phi_{ij} + \Phi_{ij}^T, \quad i \neq j, \quad j \in v \tag{28}$$

$$\Phi_l = \begin{bmatrix} 2\Phi_{11} & \dots & \Phi_{1r} \\ \vdots & \ddots & \vdots \\ \Phi_{r1} & \dots & 2\Phi_{rr} \end{bmatrix} > 0, \quad l \in v \tag{29}$$

where

$$Y_{ij} = \begin{bmatrix} -\Pi & * & * & * & * & * & * \\ R^{1/2}Y_l & -\gamma I & * & * & * & * & * \\ Q^{1/2}G & 0 & -\gamma I & * & * & * & * \\ G & 0 & 0 & -X_t & * & * & * \\ 0 & 0 & 0 & 0 & -\alpha^2 I & * & * \\ \mathcal{A}_j G + \mathcal{B}_j Y_l & 0 & \mathcal{D}G_l & 0 & 0 & -M_k & * \\ 0 & 0 & 0 & 0 & 0 & X_v \mathcal{A}_{dj} & -X_c \end{bmatrix}$$

with $i, j, l, t, c \in v$ and $\Pi = G + G^T - M_i$.

The constraint Eq. (10) is satisfied if there is an existence of a symmetric matrix U such that:

$$\begin{bmatrix} -U & Y_l \\ Y_l^T & -G - G^T + M_j \end{bmatrix} \leq 0 \quad j, l \in v \quad (30)$$

With $U_r \leq u_{s,max}$ $s = 1, \dots, m$, and U_r is a diagonal element of the matrix U .

As a conclusion, the closed-loop fuzzy discrete system will be asymptotically stabilized by the proposed control law Eq. (7) with $K_l = -Y_l G^{-1}$, while the positive constant γ will be an upper bound of the proposed cost function Eq. (9) at time instant k .

Proof: the proof can be achieved by a similar approach as in the proof of **theorem1**.

4. Simulation results

In this section, a nonlinear system with time-delay state is considered for investigating the proposed controller:

$$\begin{cases} \dot{x}_1 = x_2 + 0.5x_{1t-d} \\ \dot{x}_2 = -0.1x_1 - x_2^3 + u \\ y = x_1 \\ x_1 \in [-2, 2], x_2 \in [-1, 1] \end{cases}$$

Using the sector nonlinearity approach, the continuous T-S fuzzy model of the above system is obtained. Then, the discrete T-S fuzzy model is obtained by applying an Euler discretization with sampling time $T = 1s$:

If z is M_1 then $\tilde{A}_1 x(k) + \tilde{A}_{d1} x(k-d) + \tilde{B}_1 u(k) + w(k)$

$y(k) = \tilde{C}_1 x(k)$.

If z is M_2 then $\tilde{A}_2 x(k) + \tilde{A}_{d2} x(k-d) + \tilde{B}_2 u(k) + w(k)$

$y(k) = \tilde{C}_2 x(k)$.

where $z = -x_2^2, M_1 = (z+1)/2, M_2 = 1 - M_1$.

$$A_1 = \begin{bmatrix} 0.9504 & 0.9834 \\ -0.09834 & 0.9504 \end{bmatrix}, A_2 = \begin{bmatrix} 0.9635 & 0.6218 \\ -0.06218 & 0.3417 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.4958 \\ 0.9834 \end{bmatrix}, B_2 = \begin{bmatrix} 0.365 \\ 0.6218 \end{bmatrix}, c_1 = c_2 = [1 \ 0], A_{d1} = \begin{bmatrix} 0.19 & 0.11 \\ -0.3 & 0.15 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.21 & 0.51 \\ -0.01 & 0.35 \end{bmatrix}.$$

To control the proposed nonlinear system, the MPC parameters are set as follows: The weight matrices of the cost function in Eq. (16) are given by $Q=I$ and $R=0.5I$. The input control constraint is defined as $\|u\| \leq 0.2$, and the delayed state is set as $x(k-2)$. Furthermore, the initial values and external disturbance are set as follows:

$$x(-2) = [0, 0], x(-1) = [0, 0], x(0) = [0, 0], w(k) = 0.5\sin(0.2k).$$

According to theorem 1, an *LMI* tool-box is used to solve the optimization problem (25-29), the control law and state curves are shown in Fig. 1-3.

In this section, simulation study is performed to examine the feasibility and the performance of the proposed method, while the proposed nonlinear system with constraints and time-delay state, provided earlier, is controlled by the proposed RFMPC and a regular MPC. The simulation results of both controllers are compared in order to verify if the proposed controller have the ability to resolve the problems of constraints, disturbances, and time-delay, and lead the nonlinear system towards an optimal performance. Although, the cost function Eq. (47) is considered to design the RMPC controller, while the transient responses (i.e., overshoot, settling time...) may considerably be improved in comparison with the regular MPC method.

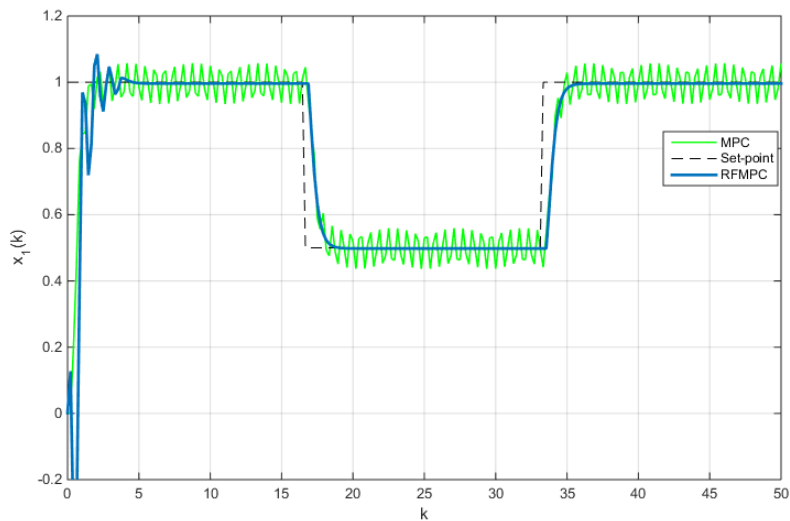


Fig. 1. Comparison between the Set point tracking state $x_1(t)$ for the proposed method RFMPC and standard MPC controller.

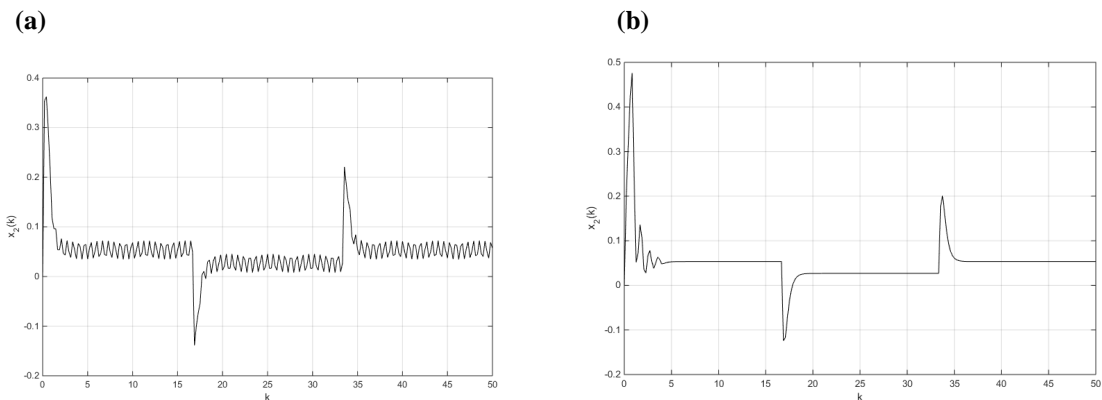


Fig.2 . Comparison between the set point tracking state $x_2(t)$ for proposed method RFMPC, and the Standard MPC controller ((a) MPC, (b) RFMPC)

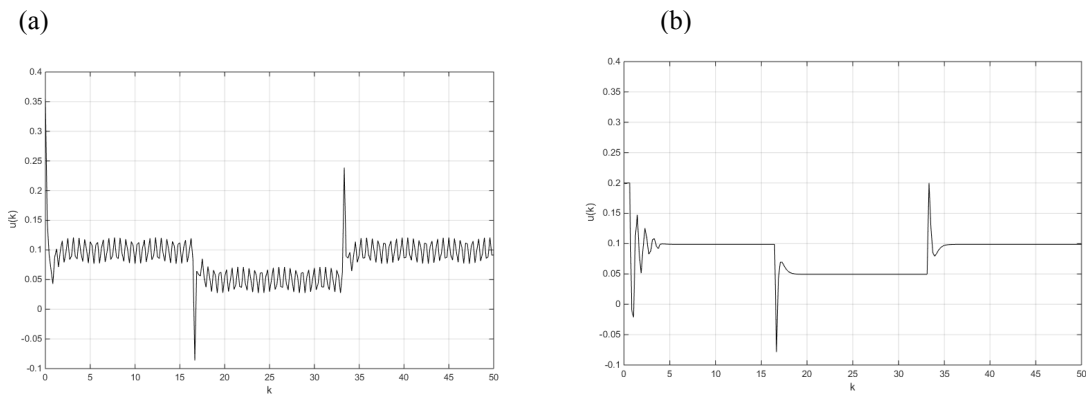


Fig. 3. Comparison between the proposed method RFMPC, and the Standard MPC controller ((a) MPC, (b) RFMPC)

Based on the given initial conditions, the tracking performances for both methods were performed and presented in Fig. 1, 2 and 3. Both Fig. 1 and 2 clearly show that, and by using the proposed approach, the states $x_1(k)$, $x_2(k)$ have approached their set point quickly (less than 4 time units) with a rejection of disturbances, while the input constraints were satisfied over the entire horizon. On the other hand, the standard MPC controller could not converge the states to their set points in the presence of disturbances.

As expected, the response input control obtained by the proposed method have the ability to control the nonlinear systems in a way that the control signal force applied to the system is reduced to the minimum.

5. Conclusion

In this paper, a robust fuzzy model predictive control for constrained uncertain systems affected by external disturbances and time-state delay is proposed to control nonlinear systems. To achieve a control law, the state space model, used by the proposed method, has been described by an augmented state space model in which both the state variables and tracking error are included. Based on this extended state model, the main concepts related to the proposed RFMPC were described in details and the proper control law has been constructed in a way that guarantees a good convergence as well as tracking performance for the system. Also, the Lyapunov-Krasovskii approach was used to develop the sufficient conditions ensuring the stability of the targeted system. Finally, a state feedback controller is constructed based on LMIs conditions, which is obtained by minimizing the worst-case objective function. As a results, sufficient conditions for the existence of the RFMPC controller are then obtained. Also, it has been shown that the receding horizon implementation of the obtained solutions guarantees the closed-loop fuzzy system to be asymptotically stable. To demonstrate the advantages of the proposed solutions, a simulation study has been conducted and the results show that the proposed method has overcome the performance of the standard MPC.

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