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Trading Signal, Functional Data Analysis, and Time Series Momentum[☆]

Sabri Boubaker^{a,b}, Zhenya Liu^{c,d,*}, Shanglin Lu^c, Yifan Zhang^c

^a*EM Normandie Business School, Métis Lab, France*

^b*International School, Vietnam National University, Hanoi, Vietnam*

^c*School of Finance, Renmin University of China, Beijing, China*

^d*CERGAM, Aix-Marseille University, Aix-en-Provence, France*

Abstract

Prior empirical results show that the time series momentum portfolio outperformed the buy-and-hold benchmark well from 1985 to 2009, but this profitable pattern unexpectedly vanishes after 2009. In this paper, we reconstruct the time series momentum portfolio by applying new trading rules derived from the functional data analysis approaches. Using a dataset that contains 24 commodities from January 2010 to December 2018, our daily-based strategy documents an improvement in the Sharpe ratio of 0.75 compared to 0.07 in terms of the original time series momentum portfolio. This finding offers an alternative strategy for trend-following investors in the commodity futures market.

Keywords: Asset Pricing, Futures Pricing, Time Series Momentum, Trading Signal, Functional Data Analysis

JEL: G12, G13, G15, G17

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*Corresponding author.

Email addresses: sboubaker@em-normandie.fr (Sabri Boubaker), zhenya.liu@ruc.edu.cn (Zhenya Liu), lushanglin@ruc.edu.cn (Shanglin Lu), zhangyifan@ruc.edu.cn (Yifan Zhang)

1. Introduction

Since the seminal work of Moskowitz et al. (2012), time series momentum (TSM, hereafter) effect has been documented as a universal phenomenon across international financial markets. It was later known as a widely implemented trend-following strategy in the CTA (commodity trading advisor) industry (Baltas & Kosowski, 2013, 2015). Based on the dataset of 58 futures contracts from January 1985 to December 2009, Moskowitz et al. (2012) proved that the TSM strategy could generate an impressive alpha, yielding more than 1% monthly excess return. However, adding the data sample from 2010 to 2018, we find that the previous significant performance of the TSM strategy can not be verified after 2009. In this paper, we explore possibilities of improving the performance of TSM strategy through implementing new rules of generating trading signals.

The theory of functional data analysis (FDA, hereafter) provides a data analyzing methodology when observations are curves or are generated from functional processes (Ramsay & Dalzell, 1991). It has been widely applied in many fields such as financial market modeling. Ramsay & Silverman (2005) and Horváth & Kokoszka (2012) provide comprehensive details on statistical tools and approaches involved in the theory. In this paper, we firstly apply the FDA approaches to obtain the functional patterns implied in discretely observed asset price sequences. Based on the extracted pattern, we then capture the cyclical and seasonal characteristics of the price trend over a given look-back window. Our proposed trading rules for reconstructing the TSM strategy rely on more information derived by the FDA approaches. We also compare the performance of various strategies under different settings.

The main finding of Moskowitz et al. (2012) is that the sign of cumulative return over the past twelve months predicts future price movement in the next month. However, this predictive power in terms of the TSM portfolio became unclear after 2009. This paper contributes to the literature with modeling price dynamics over TSM look-back window as Hilbert-space-valued random functions rather than scalar random variables, therefore identifying the cyclical and seasonal patterns of price trend involved in the TSM portfolio construction. From an academic perspective, our proposed portfolio constructing method provides insights into understanding the TSM strategy in the CTA industry. From the strategy evaluation perspective, our proposed strategy shows a significant improvement in the Sharpe ratio from 0.07 to 0.75 during the period 2010 to 2018 compared to the original TSM strategy.

The rest of this paper is organized as follows. Section 2 describes our dataset. Section 3 introduces the functional data analysis approach and the trading rules of the improved time series momentum strategy. Section 4 presents the empirical performance of the proposed strategies. Section 5 concludes this paper.

2. Data

Our dataset is constructed from Bloomberg. It consists of the price series of 24 commodities, which are among the most liquid futures contracts in the world. Following Bessembinder (1992), Moskowitz et al. (2012), and Asness et al. (2013), we collect daily settlement prices and adjust for rollovers so that the portfolios are always composed of the most active contracts, which are typically the nearest or next nearest-to-delivery contracts. Likewise, the excess returns over the risk-free rate are computed for all instruments. The sample covers more than fifty years from January 1966 to December 2018, with all 24 commodities became available after 1995. The summary

statistics of the excess returns of each instrument are presented in Table 1. Appendix A provides more details of each instrument.

Table 1: Descriptive Statistics of the Excess Returns of Each Commodity Future Contract.

	Start Date	Annualized Mean(%)	Annualized Volatility(%)	t-stat	Skewness	Kurtosis	Sharpe Ratio
Energy							
BRENT	Mar-95	4.06	32.60	0.608	-0.569	1.651	0.125
GASOIL	Mar-95	3.49	32.38	0.526	-0.370	1.098	0.108
WTI	May-83	-0.72	34.95	-0.124	-0.371	2.176	-0.021
GASOLINE	Jun-86	10.56	36.58	1.648	-0.124	2.870	0.289
HEATINGOIL	Sep-86	4.51	33.75	0.759	0.279	2.778	0.133
NATURALGAS	Jun-90	-18.53	55.17	-1.796	0.061	1.093	-0.336
Agriculture							
COTTON	May-84	-5.24	25.76	-1.198	0.039	0.898	-0.203
COFFEE	May-84	-13.15	35.26	-2.197	0.586	2.050	-0.373
COCOA	May-84	-10.49	28.31	-2.182	0.194	0.577	-0.371
SUGAR	May-84	-7.31	34.05	-1.264	0.222	1.446	-0.215
CORN	Jan-66	-8.93	24.30	-2.675	0.502	2.996	-0.367
SOYBEAN	Jan-66	-2.93	25.97	-0.821	0.508	4.674	-0.113
SOYMEAL	Jan-66	2.18	29.12	0.544	0.565	3.759	0.075
SOYOIL	Jan-66	-1.15	30.09	-0.278	0.895	4.838	-0.038
WHEAT	Jan-66	-9.52	26.39	-2.625	0.259	1.746	-0.361
Livestock							
LIVECATTLE	Apr-84	-2.13	14.24	-0.883	-0.583	2.530	-0.150
LEANHOGS	Jun-86	-8.65	25.46	-1.938	-0.399	0.919	-0.340
Metal							
ALUMINUM	Aug-87	-2.24	20.04	-0.626	-0.013	0.906	-0.112
COPPER	Jun-86	0.88	24.63	0.204	-0.481	5.169	0.036
NICKEL	Mar-87	1.78	36.69	0.273	0.972	5.940	0.048
ZINC	Mar-89	-2.38	24.03	-0.541	-0.444	3.014	-0.099
GOLD	Apr-80	-6.27	16.93	-2.306	-0.066	1.757	-0.370
SILVER	May-80	-8.99	29.24	-1.912	-0.118	1.358	-0.308
PLATINUM	Mar-84	-5.43	22.64	-1.416	-0.586	5.031	-0.240

Note: Returns are calculated by the changes in the logarithm of daily settlement prices from January 1966 to December 2018. The reported statistics include the annualized mean return, annualized standard deviation, t-statistic, skewness, kurtosis, and Sharpe ratio. For more details about our data sample, see Appendix A.

3. Methodology

3.1. Functional Data Analysis

The FDA theory has been demonstrated to provide sufficient statistical tools for exploring underlying features among discrete observations. Using functional smoothing method, we see a sequence of discrete observations as a curve, i.e., a functional observation. Let $p_{t,u}^s$, $t - k \leq u \leq t$ denote the prices of security s over latest k observable periods at time t , and the price is considered as an index compiled using the excess returns of corresponding security. Then, discrete prices

$p_{t,u}^s$, $t - k \leq u \leq t$ can be functionalized as a smooth curve in the form of $p_t^s(u)$, $0 \leq u \leq 1$. According to the theory, $p_t^s(u)$ is assumed to be a square-integrable function in $L^2[0, 1]$, of which the fourth moment is finite. Therefore, it can be approximated by a group of basis functions. In this case, the excess kurtosis and skewness of excess returns presented in Table 1 are satisfied in the functional modeling.

Following previous financial applications of the FDA, we employ the *B-Spline* basis system to transform the discrete data into smooth functions (De Boor, 1978; Cao et al., 2020). The core of functional smoothing is to minimize the sum of squared errors between a linear combination of basis functions and target observations. As a result of dealing with the trade-off between fitting capacity and function smoothness, the roughness penalty is used to generate better results. Moreover, the fourth-order derivative is chosen in the penalty term, considering that we apply the second derivative of the smoothed functions in the latter analysis.¹

3.2. Trading Rules

This paper improves the TSM strategy performance in terms of its trading signals generated from the asset past performance over k look-back periods. As proposed in Moskowitz et al. (2012), the long or short trading signal is determined by the sign of cumulative return over past k periods:

$$\text{sign}_{s,t}^{\text{TSM}} = \begin{cases} +1, & r_{t-k,t}^s > 0, \\ -1, & r_{t-k,t}^s < 0, \end{cases}$$

where $r_{t-k,t}^s$ indicates the cumulative return of security s from $t - k$ to t .

Consequently, the portfolio return of TSM strategy with 12-months look-back period and 1-month holding period across S_t securities from time t to $t + 1$ is given by:

$$r_{t,t+1}^{\text{TSM}} = \frac{1}{S_t} \sum_{s=1}^{S_t} \text{sign}_{s,t}^{\text{TSM}} \left(r_{t-12,t}^s \right) \frac{\sigma_{\text{target}}}{\sigma_t^s} r_{t,t+1}^s, \quad (1)$$

where S_t is the number of securities available at time t . We suggest interpreting the economic intuition behind the success of TSM formula, also known as trend-following strategy, through its three decomposed components: i) trading signal, identifying the price trend by comparing the latest price to its counterpart 12 months ago; ii) constant risk parity, scaling the time-varying volatility of each security by a consistent target volatility;² iii) equally weighted portfolio, distributing an equal amount of funds to each security within the portfolio following the “naive 1/N rule”.³

¹For more details about functional smoothing methodology, we refer to Ramsay & Silverman (2005) and Horváth & Kokoszka (2012).

²The annual target volatility is chosen to be 40% due to its similarity to the risk of an average individual stock, as stated in Moskowitz et al. (2012). We keep the same 40% target volatility to compare our proposed methods with them. Likewise, the ex-ante annualized variance σ_t^2 for each security is calculated as follows:

$$\sigma_t^2 = 261 \sum_{i=0}^{\infty} (1 - \delta) \delta^i (r_{t-1-i} - \bar{r}_t)^2,$$

where δ is chosen to satisfy $\sum_{i=0}^{\infty} (1 - \delta) \delta^i = 60$ days, and \bar{r}_t is the exponentially weighted average return calculated similarly.

³Note that we only focus on exploring the possibilities of changing the trading signal component in this study.

Assuming the asset price follows a continuous-time diffusion process with a stochastic trend, He et al. (2018) suggest more complicated buy/sell decisions if the goal is to maximize the expected future wealth. Importantly, the FDA method enables us to capture more features implied in price dynamics within the look-back window $[t - k, t]$, not merely concentrating on the sign of the partial sum of past returns. Although the partial sum is helpful for filtering noises in asset returns, considering its arithmetic sign as an upward or downward price trend indicator is insufficient. Its deficiency comes in the sense that the aggregated value fails to capture the informative signals of stochastic price trends, particularly in the fragile post-crisis recovery period of the global economy.

We suggest new rules for generating trading signals using additional information derived from the smoothed curve of past price dynamics. On the one hand, the smoothed curve keeps the long-memory pattern of the path-dependent asset price process while filtering out noises. On the other hand, the first-order and second-order derivatives of the smoothed function capture cyclical and seasonal characteristics of the price movements. As an analogy of velocity and acceleration in Physics, the sign consistency of both derivatives identifies a segment of a strong price trend, whereas the inconsistency indicates a segment of a trend reversal. We expect to see improvement in portfolio returns considering that more information is involved in this case when identifying the ongoing price trend, and refer to the new strategy briefly as ITSM.

More specifically, we maintain a strong long (short) trend signal only when the signs of both the first-order and second-order derivatives are positive (negative). The inconsistent cases are suggested to represent a relative weak trend signal. Therefore, we define the trading rules of a conservative strategy as:

$$\text{sign}_{s,t}^{\text{ITSM}_c}(u) = \begin{cases} +1, & Dp_t^s(u) \geq 0 \text{ and } D^2p_t^s(u) \geq 0, \\ 0, & Dp_t^s(u) \times D^2p_t^s(u) < 0, \\ -1, & Dp_t^s(u) < 0 \text{ and } D^2p_t^s(u) < 0, \end{cases} \quad (2)$$

and an aggressive strategy as:

$$\text{sign}_{s,t}^{\text{ITSM}_a}(u) = \begin{cases} +1, & Dp_t^s(u) \geq 0 \text{ and } D^2p_t^s(u) \geq 0, \\ +0.5, & Dp_t^s(u) \geq 0 \text{ and } D^2p_t^s(u) < 0, \\ -0.5, & Dp_t^s(u) < 0 \text{ and } D^2p_t^s(u) \geq 0, \\ -1, & Dp_t^s(u) < 0 \text{ and } D^2p_t^s(u) < 0, \end{cases} \quad (3)$$

where $0 \leq u \leq 1$, and $D(\cdot)$ and $D^2(\cdot)$ are the first and second derivative operators. The conservative strategy is designed to clear out position when the trend signal becomes weak, while the aggressive strategy still keeps a half position.

In the empirical studies, we consider testing the performance of ITSM strategies on both monthly and daily rebalancing basis. For the monthly rebalancing strategy, we collect the price index series over the past 12 months and smooth it into function $p_t^s(u)$, $0 \leq u \leq 1$ at the end of year t . Then we apply the trading signals generated from Eq. (2) and (3) for the following 12 months from t to $t + 12$. As for the daily rebalancing strategy, we similarly functionalize the daily price indices in each of the past 12 months into 12 smoothed curves at the end of month t . By averaging these 12 curves, we see the trading signals generated from the mean function with Eq. (2) and (3) as the prediction of price trend pattern over the trading days in the next holding month t to $t + 1$.

4. Strategy Performance

4.1. Time Series Momentum

Following Eq. (1), we calculate the TSM return for all commodities futures contracts with available data from January 1985 to December 2018. Table 2 reports some evaluation statistics of the strategy over the whole sample and two sub-periods. As shown in the last row of Table 2, the well-documented TSM effect in Moskowitz et al. (2012) vanishes after 2009, with a slump in the Sharpe ratio from 0.95 to 0.07.

Table 2: Performance of the Original Time Series Momentum Strategy.

	1985-2009			2010-2018			1985-2018		
	t-stat	SR	D-SR	t-stat	SR	D-SR	t-stat	SR	D-SR
Energy									
BRENT	2.46	0.66	0.72	1.21	0.40	0.46	2.65	0.55	0.62
GASOIL	2.34	0.63	0.73	1.70	0.57	0.55	2.89	0.60	0.65
WTI	2.16	0.43	0.49	0.24	0.08	0.10	1.99	0.34	0.40
GASOLINE	2.23	0.47	0.55	0.63	0.21	0.22	2.23	0.40	0.45
HEATINGOIL	2.39	0.51	0.61	0.75	0.25	0.24	2.44	0.44	0.49
NATURALGAS	1.47	0.34	0.45	1.20	0.40	0.48	1.87	0.36	0.46
Agriculture									
COTTON	1.87	0.38	0.41	0.08	0.03	0.03	1.61	0.28	0.32
COFFEE	0.49	0.10	0.11	0.49	0.16	0.13	0.66	0.11	0.11
COCOA	0.72	0.15	0.16	-2.56	-0.85	-0.91	-0.78	-0.13	-0.14
SUGAR	0.17	0.03	0.04	0.21	0.07	0.07	0.26	0.04	0.05
CORN	1.02	0.20	0.25	0.89	0.29	0.31	1.32	0.23	0.27
SOYBEAN	0.48	0.10	0.10	-1.64	-0.55	-0.52	-0.54	-0.09	-0.09
SOYMEAL	0.10	0.02	0.02	-1.74	-0.58	-0.51	-0.93	-0.16	-0.16
SOYOIL	0.70	0.14	0.14	1.05	0.35	0.40	1.11	0.19	0.20
WHEAT	1.62	0.32	0.41	-0.57	-0.19	-0.19	1.06	0.18	0.21
Livestock									
LIVECATTLE	2.33	0.47	0.53	-0.90	-0.30	-0.33	1.60	0.27	0.31
LEANHOGS	1.59	0.33	0.47	-0.10	-0.03	-0.04	1.28	0.23	0.31
Metal									
ALUMINUM	0.86	0.19	0.18	0.93	0.31	0.38	1.22	0.22	0.23
COPPER	2.55	0.54	0.69	0.37	0.12	0.12	2.41	0.43	0.51
NICKEL	2.38	0.51	0.59	-1.27	-0.42	-0.48	1.40	0.25	0.29
ZINC	1.93	0.43	0.46	-0.73	-0.24	-0.27	1.26	0.24	0.26
GOLD	2.01	0.40	0.40	0.97	0.32	0.40	2.23	0.38	0.40
SILVER	0.98	0.20	0.20	0.50	0.17	0.18	1.09	0.19	0.19
PLATINUM	0.92	0.18	0.17	-0.07	-0.02	-0.03	0.77	0.13	0.13
ALL	4.73	0.95	1.08	0.21	0.07	0.06	4.02	0.69	0.72

Notes: †The time series momentum strategy returns are computed following Eq. (1). For comparison with Moskowitz et al. (2012), we divide the whole sample into two subsamples: 1985 to 2009 and 2010 to 2018.

‡SR indicates the gross Sharpe ratio of portfolio returns, and D-SR indicates the downside-risk Sharpe ratio introduced by Ziemba (2005).

4.2. Improved Time Series Momentum

As we explained earlier, the FDA theory provides sufficient statistical tools for investigating the past price movements as a curve, not just a scalar variable. We evaluate the strategy performance

of applying proposed trading rules instead of the original TSM trading signals in this subsection.

Fig. 1 describes the cumulative returns of the daily and monthly rebalancing conservative ITSM strategies: $ITSM_c^d$ and $ITSM_c^m$, and aggressive ITSM strategies: $ITSM_a^d$ and $ITSM_a^m$ from 2010 to 2018. The buy-and-hold (BAH) and TSM strategies are also plotted as comparable counterparts. We further present the strategy performance evaluating statistics, including Sharpe ratio (SR) and maximum drawdown (MDD) for each strategy in Table 3.

As illustrated in Table 3, the TSM strategy significantly outperformed the BAH benchmark from 1985 to 2009. The Sharpe ratio was 0.95 relative to 0.12 in this period. Nevertheless, an almost zero profit TSM investment was demonstrated after 2009 in Fig. 1 suffering long and deep equity drawdowns, even though it was still better than the BAH benchmark. Meanwhile, we see from the subgraphs of Fig. 1 that both the conservative and aggressive ITSM strategies exhibited better profitability relative to the BAH and TSM strategies from 2010 to 2018. Despite no holding positions when the trend signals are weak, it turned out that the conservative ITSM strategies still capture the price trend patterns. The aggressive trading signals worked slightly better than the conservative ones at the daily frequency, achieving a higher terminal wealth in December 2018.

Figure 1: Equity Curves of the TSM and ITSM investments.

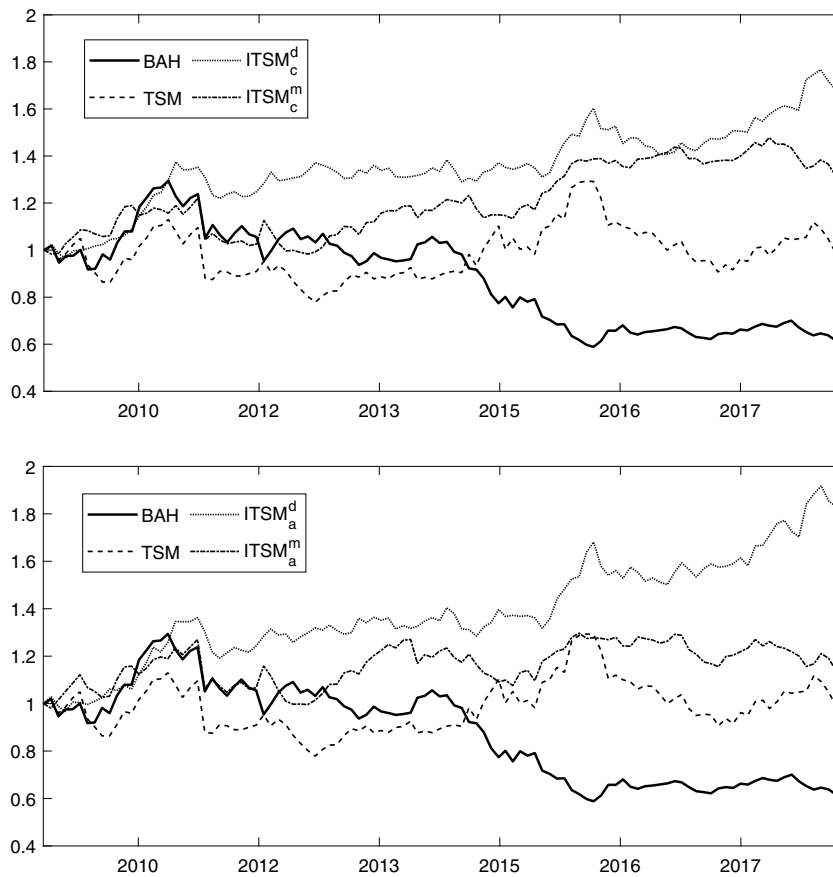


Table 3: Performance Evaluating Statistics of the Improved Time Series Momentum Strategies.

		1985-2009			2010-2018		
		SR	MDD(%)	t-stat	SR	MDD(%)	t-stat
All Commodities	BAH	0.12	53.25	0.61	-0.38	54.71	-1.14
	TSM	0.95	24.08	4.73	0.07	31.07	0.21
	ITSM _c ^d	0.42	21.02	2.12	0.73	12.32	2.19
	ITSM _c ^m	0.89	10.35	4.14	0.33	15.57	0.89
	ITSM _a ^d	0.46	21.17	2.32	0.75	12.54	2.25
	ITSM _a ^m	0.63	15.39	3.17	0.20	17.01	0.55
Energy Sector	BAH	0.24	76.19	1.20	-0.35	74.33	-1.06
	TSM	0.55	66.37	2.73	0.49	38.29	1.46
	ITSM _c ^d	-0.01	70.93	-0.05	0.92	25.75	2.74
	ITSM _c ^m	0.27	67.30	1.34	0.62	17.31	1.85
	ITSM _a ^d	-0.06	77.81	-0.32	0.98	29.37	2.92
	ITSM _a ^m	0.23	68.86	1.15	0.50	26.40	1.50
Agriculture Sector	BAH	-0.15	73.12	-0.76	-0.29	60.24	-0.86
	TSM	0.36	57.31	1.78	-0.34	58.30	-1.02
	ITSM _c ^d	0.42	41.88	2.11	0.46	27.16	1.36
	ITSM _c ^m	0.65	21.09	3.26	-0.26	34.82	-0.78
	ITSM _a ^d	0.42	39.34	2.10	0.45	27.91	1.36
	ITSM _a ^m	0.54	20.13	2.69	-0.28	39.95	-0.83
Livestock Sector	BAH	-0.12	83.03	-0.59	-0.24	50.68	-0.71
	TSM	0.55	76.54	2.72	-0.22	64.58	-0.65
	ITSM _c ^d	0.39	48.27	1.93	-0.06	68.39	-0.17
	ITSM _c ^m	0.40	41.46	1.96	0.17	16.68	0.51
	ITSM _a ^d	0.29	55.24	1.38	0.53	46.96	1.58
	ITSM _a ^m	0.22	51.56	1.08	-0.04	26.47	-0.12
Metal Sector	BAH	0.16	-64.43	0.82	-0.18	54.43	-0.53
	TSM	0.60	44.71	3.01	0.07	47.60	0.20
	ITSM _c ^d	0.13	45.75	0.65	0.01	28.62	0.02
	ITSM _c ^m	0.31	18.61	1.56	0.35	15.32	1.05
	ITSM _a ^d	0.10	52.15	0.51	0.13	17.00	0.38
	ITSM _a ^m	0.25	21.59	1.24	0.25	22.02	0.74

Notes: †BAH is short for the buy-and-hold portfolio, which is equal-weighted and long-only. TSM is the 12-1 time series momentum strategy in line with Eq. (1). ITSM_c^d, ITSM_c^m, ITSM_a^d, and ITSM_a^m are the conservative and aggressive improved time series momentum strategies with daily and monthly rebalancing.

‡For the convenience of comparing with Moskowitz et al. (2012), we present the strategy performance on two divided subsamples: 1985 to 2009 and 2010 to 2018.

§SR indicates the gross Sharpe ratio of portfolio returns, and MDD represents the maximum drawdown of investment strategy.

Moreover, we report the evaluation statistics according to different commodity classes over two subsamples in Table 3. Generally, all ITSM strategies reported higher SR and smaller MDD from 2010 to 2018, not only in the class of all commodities, but also in four commodity classes of Energy, Agriculture, Livestock, and Metal. Furthermore, the daily rebalanced strategies performed much better than the monthly rebalanced strategies in all commodity classes except for the Metal Sector, since more detailed information of price dynamics is taken into consideration on a daily basis. Additionally, we note that the daily rebalanced conservative strategy $ITSM_c^d$ and aggressive strategy $ITSM_a^d$ obtained a Sharpe ratio of 0.75 and 0.73 from 2010 to 2018. Both of them turned out to be a significant improvement relative to 0.07 that of the TSM strategy in the same testing sample.

4.3. Performance in Crisis Periods

Since our data sample covers several financial crisis periods, we are also interested in assessing whether the proposed ITSM strategies are resistant to these financial crises. We consider three crisis periods, namely, the Asian Financial Crisis from 1997 to 1999, the Global Financial Crisis from 2007 to 2009, and the European Debt Crisis from 2010 to 2012. Table 4 reports the Sharpe ratio and maximum drawdown statistics for different ITSM strategies in terms of the class of all commodities.

MDD statistics show substantial differences between the proposed ITSM strategies and the TSM strategy in each crisis period. All ITSM strategies present significantly smaller drawdowns compared to the TSM strategy. Except for the Asian Financial Crisis period, the ITSM strategies also document a higher Sharpe ratio than TSM in periods of the Global Financial Crisis and the European Debt Crisis. We note that a smaller drawdown in the crisis period is of higher practical meaningfulness for market participants, particularly fund managers.

Table 4: Performance of ITSM Strategies during the Crisis Periods.

Strategy	Asian Financial Crisis (1997-1999)			Global Financial Crisis (2007-2009)			European Debt Crisis (2010-2012)		
	SR	MDD(%)	t-stat	SR	MDD(%)	t-stat	SR	MDD(%)	t-stat
BAH	-0.79	40.13	-1.35	-0.13	53.25	-0.23	0.11	26.28	0.20
TSM	1.03	13.48	1.75	0.18	24.08	0.31	-0.40	31.07	-0.68
$ITSM_c^d$	0.47	7.97	0.81	0.59	6.98	1.00	1.16	11.31	1.98
$ITSM_c^m$	0.98	4.73	1.68	0.31	9.95	0.53	0.02	15.57	0.03
$ITSM_a^d$	0.43	9.21	0.74	0.09	13.07	0.15	0.88	12.54	1.49
$ITSM_a^m$	0.57	6.76	0.98	0.22	15.39	0.38	0.10	17.01	0.17

Notes: †BAH is short for the buy-and-hold portfolio, which is equal-weighted and long-only. TSM is the 12-1 time series momentum strategy in line with Eq. (1). $ITSM_c^d$, $ITSM_c^m$, $ITSM_a^d$, and $ITSM_a^m$ are the conservative and aggressive improved time series momentum strategies with daily and monthly rebalancing.

‡SR indicates the gross Sharpe ratio of portfolio returns, and MDD represents the maximum drawdown of investment strategy.

5. Conclusion

The construction of a time series momentum portfolio, usually characterized as the trend-following strategy, can be decomposed into three components: trading signal, constant risk parity, and equally weighted portfolio. Based upon this decomposition, we have focused on improving the profitability of the time series momentum portfolio in terms of trading signals over a sample period from January 2010 to December 2018, posterior to that of Moskowitz et al. (2012). The dataset containing the most liquid contracts of 24 commodities futures from Bloomberg enabled us to investigate the patterns of commodities price movements over a given look-back window.

With the implementation of functional data analysis approaches, we propose new trading rules for identifying the price trend patterns and how the improved time series momentum strategies are constructed. Our trading signals are defined by the signs of the first-order and second-order derivatives of the smoothed functional observations of past price dynamics. As an analogy of velocity and acceleration in Physics, these signals capture the cyclical characteristics of time series momentum effect to some extent. Finally, two differently rebalanced strategies were investigated during the period from 2010 to 2018. All suggested strategies outperform the original TSM strategy in the sense of Sharpe ratio and maximum drawdown of portfolio returns. These findings are consistent for the class of all commodities and four commodities sectors of Energy, Agriculture, Livestock, and Metal. We suggest further research on exploring more financial markets, such as equity index futures, bond futures, currency forwards, and crypto currencies.

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Appendix A. Commodity Futures Dataset

We provide additional details about the commodities futures contracts involved in this paper. Our dataset from Bloomberg consists of 24 commodities: six of them are in the energy sector (Brent Crude, Gas Oil, WTI Crude, RBOB Gasoline, Heating Oil, Natural Gas), nine of them are in the agriculture sector (Cotton, Coffee, Cocoa, Sugar, Corn, Soybeans, Soy Meal, Soy Oil, Wheat), two of them are in the livestock sector (Live Cattle, Lean Hogs), and seven of them are in the metal sector (Aluminum, Copper, Nickel, Zinc, Gold, Silver, Platinum). For all commodities futures contracts, the exchanges at which they are traded, and tickers from Bloomberg are given in the following table.

Table A1: Details of Commodities Futures Contracts.

Commodity Name	Exchange	Contract Ticker
Energy		
BRENT	Intercontinental Exchange (ICE)	COx Comdty
GASOIL	Intercontinental Exchange (ICE)	QSx Comdty
WTI	New York Mercantile Exchange (NYMEX)	CLx Comdty
GASOLINE	New York Mercantile Exchange (NYMEX)	XBWx Comdty
HEATINGOIL	New York Mercantile Exchange (NYMEX)	HOx Comdty
NATURALGAS	New York Mercantile Exchange (NYMEX)	NGx Comdty
Agriculture		
COTTON	Intercontinental Exchange (ICE)	CTx Comdty
COFFEE	Intercontinental Exchange (ICE)	KCx Comdty
COCOA	Intercontinental Exchange (ICE)	CCx Comdty
SUGAR	Intercontinental Exchange (ICE)	SBx Comdty
CORN	Chicago Board of Trade (CBoT)	C x Comdty
SOYBEAN	Chicago Board of Trade (CBoT)	S x Comdty
SOYMEAL	Chicago Board of Trade (CBoT)	SMx Comdty
SOYOIL	Chicago Board of Trade (CBoT)	BOx Comdty
WHEAT	Chicago Board of Trade (CBoT)	W x Comdty
Livestock		
LIVECATTLE	Chicago Mercantile Exchange (CME)	LCx Comdty
LEANHOGS	Chicago Mercantile Exchange (CME)	LHx Comdty
Metal		
ALUMINUM	London Metal Exchange (LME)	LMAHDS03
COPPER	London Metal Exchange (LME)	LMCADS03
NICKEL	London Metal Exchange (LME)	LMNIDS03
ZINC	London Metal Exchange (LME)	LMZSDS03
GOLD	New York Commodities Exchange (COMEX)	GCx Comdty
SILVER	New York Commodities Exchange (COMEX)	SIx Comdty
PLATINUM	Tokyo Commodity Exchange (TOCOM)	JAx Comdty

Note: The first and second nearest future contract can be obtained from Bloomberg by substituting 1 and 2 with 'x' in the contract ticker. For example, 'CO1 Comdty' and 'CO2 Comdty' are the tickers of the first and second nearest active contract of brent crude oil.

Declaration of Competing Interest

None.

Highlights:

1. Empirical results present that the well-documented time series momentum effect on the commodity futures market vanishes after 2009.
2. The past asset price dynamics are employed to generate new trading signals, other than the sign of past asset returns.
3. The discrete sample observations of the asset prices are functionalized to curves using the functional smoothing method.
4. The first-order and second-order derivatives of the functionalized curves are designed to generate new trading signals for the return formula of time series momentum.
5. Compared with the original time series momentum, the empirical application of the newly generated trading signals shows a significant improvement in the Sharpe ratio from 0.07 to 0.75 after 2009.