



**HAL**  
open science

**Paolo Serafini: Mathematics to the Rescue of  
Democracy. What does Voting Mean and How can it be  
Improved? XIII, 135 pp., Springer Nature Switzerland  
AG 2020.**

Eric Kamwa

► **To cite this version:**

Eric Kamwa. Paolo Serafini: Mathematics to the Rescue of Democracy. What does Voting Mean and How can it be Improved? XIII, 135 pp., Springer Nature Switzerland AG 2020.. Journal of Economics, 2021, 10.1007/s00712-021-00757-5 . hal-03322709

**HAL Id: hal-03322709**

**<https://hal.science/hal-03322709>**

Submitted on 21 Aug 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

**Paolo Serafini: Mathematics to the Rescue of Democracy. What does Voting Mean and How can it be Improved?**  
**XIII, 135 pp., Springer Nature Switzerland AG 2020, €26.36**

**Eric Kamwa**

Received: date / Accepted: date

The book under review here was published well after Paolo Serafini's retirement in 2016; it seems to trace all that he learned from his familiarization with the voting theory and the interest aroused in him by the debates that marked the elections in Italy, his country of birth. Introduced late to voting theory, Paolo Serafini is a mathematician whose research topics have mainly been optimization problems. His late familiarization with voting theory explains well the fact that of the vast list of his scientific publications, very few are devoted to voting theory; these are mostly about *Biproportional Representation* which we will talk about in later.

This 135-page book is divided into 15 chapters easy to read and that can mostly be read separately. It should be noted that a fine spirit of synthesis was used to present in each chapter a good number of concepts without lengthening the size of the chapters which are on average eight pages. Chapter 1 sets the context for the content of the book and the objective of the book as assumed by the author: to present in a simple way how electoral systems work and to point out some of their virtues and defects. Paolo Serafini is careful to point out that, as the title of the book mentions, even if mathematics is the basis of any voting system, the book presents the concepts in a way that can be understood by non-mathematicians. Chapter 2, which is complementary to the previous one, allows the author to present in a relaxed way the main purposes of voting in a democratic society. This chapter details in substance, the contents of the other chapters of the book which can be grouped in three great blocks treating rather close topics: 1) the formation of a government by electoral methods, 2) apportionment (proportional representation), and 3) voting power. It is therefore a good idea to present the content of the book based on this block-by-block structuring.

The first block, which includes Chapters 3 to 8, deals with the fundamentals of voting theory. Chapter 3 presents the different methods and purposes of aggregating individual preferences and then gives a quick overview of the genesis of the main families of collective aggregation rules. The aggregation rule proposed by Le Marquis de Condorcet as well as the associated majority criteria are introduced by Chapter 4. The majority cycles which are the main limits of the Condorcet's rule, allows the author to introduce some of the voting rules allowing to get out of this deadlock. In Chapter 5, the author takes as a starting point the Borda rule and the Range Voting to introduce the family of scoring rules and confront them with the majority criteria. Chapter 6 presents the runoff rules. This book does not derogate from the tradition and Chapter 7 presents the two major (im)possibility theorems which are at the heart of voting theory: Arrow's theorem supports the idea according to which there is no indisputable social choice process that makes it possible to express a coherent hierarchy of preferences for a community from the aggregation of

the individual preferences expressed by each of the members of this same community; according to the Gibbard-Satterthwaite theorem, if a voting rule has at least three possible outcomes and if it is non-manipulable, then it is dictatorial. Two paradoxes affecting voting rules are discussed, namely the abstention paradox and the monotony paradox. Of the wide range of voting paradoxes that exist, the author does not justify why he restricts his analysis to these paradoxes. Are the other paradoxes of lesser importance? The book does not answer this question. The author, like many others in the literature, seems to have some admiration for the majority judgment a voting rule developed by [Balinski and Laraki \(2007\)](#). This is evident throughout the book; moreover, Chapter 8, which is devoted to the presentation of this rule, is the longest in the book and extends over twenty-two pages. In this chapter, although the author reviews some virtues and weaknesses of the majority judgment, this presentation seems to ignore recent debate on this rule<sup>1</sup> which might have allowed the author to either reaffirm his admiration for this rule or to nuance his position.

When we move on to the second block of the book (Chapters 9 to 13), we find ourselves at the heart of the issues that fascinate Paolo Serafini and on which most of his publications in voting theory are based. The main methods of apportionment/seats allocation are presented in Chapter 9 as well as the related paradoxes; for the assimilation of the concepts, the reader is accompanied by intuitive examples both theoretical and from real life (case of the European Parliament). This is complemented in Chapter 10 by an analysis of the historical evolution of seat allocation in the U.S. Congress and the debate over which method is less problematic. Like other books and works on the subject, this book does not make a clear-cut decision in favor of a certain method. It is up to the reader to draw his or her own conclusions. In Chapter 11, the question of how to choose the elected candidates on a list when there are several seats to be filled (in a constituency) is raised. With the help of an example, the author reviews some of the problems or paradoxical situations to which the various popular vote rules can lead, such as the Condorcet Majority Rule, the Borda Rule, Single Non-Transferable Vote and Single Transferable Vote. So how can we avoid these drawbacks? The author seems to argue that "Since the indication of several choices requires some effort on the part of the voter, we might as well ask for some extra effort to express the judgements required by Majority judgement" (p.92). Once again, Paolo Serafini shows his admiration for Majority Judgement. Chapter 12 deals with the Biproportional Representation, whose main objective is to match as closely as possible the total number of votes (or percentage of votes) collected by the parties and the seats allocated to them without eliminating the existing electoral districts. It is known that the ambition of the Biproportional Representation is to correct the anomalies often created by the proportional distribution systems, such as those that occurred in the Italian elections of 1996, 2006, 2008 and 2013, as Paolo shows. Being a mathematician, the author stresses that the problem of Biproportional Representation is above all a mathematical problem and should be left to mathematicians. I think he is in a good position to say this and I totally agree. The author uses examples (and in particular the Italian elections) to make the reader understand this without however breaking out the mathematical artillery. At the end of this chapter, it would not be unreasonable to ask the question why the Biproportional Representation System is only rarely used throughout the world, even though it was built on a logic of fairness. Based on historical facts, Chapter 13 returns to the problem of electoral districting, which remains unresolved today in several countries such as the United States, Italy and Germany. Electoral redistricting is often used as a tool of electoral manipulation for the purpose of profit; this is known as *gerrymandering*. The author presents and discusses the scope of some of the main criteria that have been suggested in the literature as needing to be met in order to ensure fair political districting. The author does not fail to mention the various technical and computational solutions that have been proposed. The complexity of the implementation of the proposed solutions and the impossibility of simultaneously satisfying all the criteria mean that today the problem of electoral districting continues to be a preoccupation of all political actors.

---

<sup>1</sup> See among others, [Balinski \(2019\)](#), [Laslier \(2019\)](#).

---

The third and last block of the book holds in Chapter 14 which deals with the measurement of voting power. Roughly speaking, the voting power of an individual is his or her probability to be decisive or to change the outcome of a vote. After a very brief enumeration of the main and historical voting power indices, the author focuses only on the Penrose index. This choice seems to be motivated by what we can read at the very beginning of page 114, "... Penrose index (or Penrose-Banzhaf, as it is also called) is the right concept for measuring voting power". Using a simple example, the author shows how the Penrose index is calculated and how this calculation can become more complex when the number of agents (players) is large. We also note an interesting application of the calculation of the Penrose index to the case of the 2020 US presidential elections. This makes it possible to realize that if it is the popular vote that determines the election of the president, the principle "one man, one vote" would not be met because the voters of certain states would have more power than those of other states. It is true that the author has chosen to focus only on the Penrose index. As this index is not the only one that exists and is used in the literature and in practice, this chapter alone cannot be representative of the work on indices of voting power.

The last chapter (Chap. 15), which seems to serve as a conclusion, offers the author a space where he questions and expresses his vision of things. The author paints a rather bleak, even chaotic picture of the future of democracy and voting if nothing is done to modify, adapt, evolve and improve the current framework in which voters and political parties make their decisions. This requires a better reliability of institutions by means of the fight against corruption, better information (or education) of the voters, as well as more ethics and social justice. As the author notes so well, democracy, in order to function properly, needs information, knowledge and awareness.

At the end of the reading, I must admit that the title of the book is a bit disconcerting. Positively disconcerting as I note in the following. Knowing that this book was written by a mathematician, I was expecting a deep "mathematical theory of democracy". But no! This book does not go into excessive mathematical formalization that would put off readers with a phobia of mathematics. Paolo Serafini has the merit of presenting in a synthetic and pedagogical way the main aspects of the voting theory. This book would be well suited for undergraduate and graduate students who are new to voting theory; it would also be suitable for any layman who is curious to understand electoral processes and systems. Let's get down to the content. Although the book is very enjoyable to read, the only regret I have is that the literature developed around the election of multiple winners (committee selection) is very little mentioned. Yet this literature has grown tremendously in the last ten years. It is true that one cannot unfortunately put all the theory of voting in a single book. The present book adds its stone to the series of books on voting theory while distinguishing itself by its pedagogy and simplicity.

## References

- Balinski M. (2019) Réponse à des critiques du jugement majoritaire. *Revue économique* 70 (4): 589-610.
- Balinski M. and R. Laraki (2007) A theory of measuring, electing and ranking. *Proceedings of the National Academy of Sciences*, 104(21): 8720-8725.
- Laslier JF. (2019) The strange "Majority Judgment". *Revue économique*, 70(4): 569-588.