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Suppression of the Kapitza instability in confined falling liquid films

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Suppression of the Kapitza instability in confined falling liquid films

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We revisit the linear stability of a falling liquid film flowing through an inclined narrow channel in interaction with a gas phase. We focus on a particular region of parameter space, small inclination and very strong confinement, where we have found the gas to strongly stabilize the film, up to the point of fully suppressing the long-wave interfacial instability attributed to Kapitza (Kapitza, *Zh. Eksp. Teor. Fiz.* **18**, 1948). The stabilization occurs both when the gas is merely subject to an aerostatic pressure difference, *i.e.* when the pressure difference balances the weight of the gas column, and when it flows counter-currently. In the latter case, the degree of stabilization increases with the gas velocity. Our investigation is based on a numerical solution of the Orr-Sommerfeld temporal linear stability problem as well as stability experiments that clearly confirm the observed effect. We quantify the degree of stabilization by comparing the linear stability threshold with its passive-gas limit, and perform a parametric study, varying the relative confinement, the Reynolds number, the inclination angle and the Kapitza number. For example, we find a 30% reduction of the cut-off wavenumber of the instability for a water film in contact with air, flowing through a channel inclined at 3 degrees and of height 2.8 times the film thickness. We also identify the critical conditions for the full suppression of the instability in terms of the governing parameters. The stabilization is caused by the strong confinement of the gas, which produces perturbations of the adverse interfacial tangential shear stress that are shifted by half a wavelength with respect to the wavy film surface. This tends to reduce flow rate variations within the film, thus attenuating the inertia-based driving mechanism of the Kapitza instability.

Key words:

1. Introduction

We consider a liquid film falling along the bottom wall of an inclined narrow channel, under the action of gravity and in interaction with a laminar gas flow (see figure 1). We distinguish two scenarios for the gas flow: (i) either the gas is subject to an aerostatic pressure gradient, *i.e.* a pressure difference which balances the weight of the gas column; (ii) or the gas flows counter-currently to the liquid film at an imposed flow rate. Examples of the velocity profiles corresponding to these two scenarios are depicted in figure 2. We revisit the linear stability of this flow by numerically solving the Orr-Sommerfeld temporal linear stability problem and by performing stability experiments. We are

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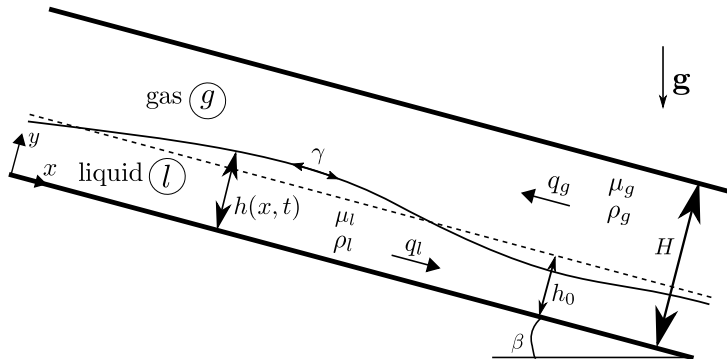


FIGURE 1. Sketch of the considered problem: a liquid film falling down an inclined wall whilst interacting with a strongly-confined gas phase. The gas is either subject to an aerostatic pressure difference or flows counter-currently at an imposed flow rate. The flow rates q_l and q_g and the heights h , h_0 and H are dimensionless.

39 particularly interested in how the stability of the falling film is affected by the gas phase
 40 in the case of strong confinement. Among the susceptible instability modes of the film
 41 (Boomkamp & Miesen 1996), we focus solely on the convective long-wave interfacial mode
 42 (Trifonov 2017) known as Kapitza instability (Kapitza 1948). For example, short-wave
 43 modes occurring in the liquid (Floryan *et al.* 1987) and/or gas (Schmidt *et al.* 2016) do
 44 not play a role in our study.

45 We start by recalling some theoretical results for the case of a liquid film falling
 46 in a *passive* atmosphere. Benjamin (1957) and Yih (1963) solved the associated Orr-
 47 Sommerfeld temporal linear stability problem and proved that a vertical liquid film
 48 is always unstable to long-wave surface disturbances. Further, they determined the
 49 threshold of this long-wave instability in terms of the critical Reynolds number $Re_{cr} =$
 50 $5/6 \cot \beta$, which depends on the inclination angle β . The instability mechanism is due to
 51 inertia, while the role of the longitudinal component of gravity is to advect and steepen
 52 the wave, whereas surface tension stabilizes the film. The way inertia destabilizes the film
 53 has been explained either via the shift between the vorticity and the displacement of the
 54 interface (Kelly *et al.* 1989; Smith 1990; Kalliadasis *et al.* 2012), following the argument
 55 introduced by Hinch (1984) for two-layer Couette flows, or via the inertia-related lag of
 56 flow-rate perturbations with respect to film-thickness perturbations (Dietze 2016).

57 Investigations of the effect of an *active* outer phase on the stability of the film started
 58 with works dedicated to a slightly different configuration. Yih (1967) investigated the
 59 pressure-driven channel flow of two co-current fluid layers and showed that a viscosity
 60 contrast between layers of equal density and thickness causes a long-wave interfacial
 61 instability. This work was extended by Yiantsios & Higgins (1988), who accounted for
 62 a density contrast and a non-unity thickness ratio, and included gravity as well as
 63 surface tension. More recently, spatio-temporal linear stability analysis was applied to
 64 this problem (Valluri *et al.* 2010; Ó Náraigh *et al.* 2013).

65 By additionally accounting for an inclination of the channel, Tilley *et al.* (1994) were
 66 able to investigate (among other scenarios) the stability of a falling water film in contact
 67 with air, which is the configuration we are interested in. In fact, their temporal Orr-
 68 Sommerfeld linear stability problem is exactly the same as the one we solve here (see §2).
 69 The authors argued that confining the gas phase by an upper wall is likely to stabilize
 70 the falling film through an adverse shear stress at the liquid/gas interface. However, their
 71 calculations for the water-air system led them to conclude that this effect is negligible.

72 Indeed, the minimum cut-off wavenumber upon varying the total flow rate changed
73 only slightly with the channel height. In our current study, we have come to a different
74 conclusion. We find that the cut-off wavenumber can be reduced up to the point of fully
75 suppressing the Kapitza instability by diminishing the channel height at a fixed liquid
76 flow rate. This holds both when the gas is subject to an aerostatic pressure difference
77 and when it flows counter-currently.

78 A number of studies have investigated the effect of a turbulent counter-current gas flow
79 on the stability of a falling liquid film. By applying a spatio-temporal analysis, Vellingiri
80 *et al.* (2015) found a convective/absolute/upward-convective transition of the instability
81 when increasing the interfacial shear stress exerted by the gas (conditions studied in our
82 current manuscript are far removed from this transition). The authors also found that
83 the cut-off wavenumber of the convective instability may either diminish, increase, or
84 display a non-monotonic behaviour with increasing gas shear stress. Trifonov (2017) made
85 similar observations (also for turbulent gas flow) based on extensive temporal stability
86 calculations, where the inclination angle, confinement, and liquid Reynolds number were
87 additionally varied. For the strong confinement levels on which we mainly focus in our
88 current study, we have observed only a monotonic decrease of the cut-off wavenumber
89 with increasing gas shear stress. However, we have checked that we recover the same
90 qualitative behaviour as Trifonov (2017) and Vellingiri *et al.* (2015) when the confinement
91 level is decreased, notwithstanding that the gas flow in our case is laminar and not
92 turbulent.

93 We proceed by discussing several related experimental works that have focused on
94 the linear stability of falling liquid films. Krantz & Goren (1971) studied liquid films
95 flowing down a strongly inclined plane at low Reynolds numbers. They imposed inlet
96 disturbances of controlled amplitude and frequency, and measured wave celerities as well
97 as spatial growth rates. Later, Pierson & Whitaker (1977) and Alekseenko *et al.* (1985)
98 studied liquid films flowing down the outside of a vertical cylinder. They measured
99 the wavelength, wave celerity and spatial growth rate for the fastest growing waves
100 over a large range of Reynolds numbers, showing reasonable agreement with the theory
101 developed in previous studies (Pierson & Whitaker 1977; Whitaker 1964).

102 Liu *et al.* (1993) studied liquid films falling down a weakly inclined plane at moderate
103 Reynolds numbers. By imposing a small-amplitude perturbation of controlled frequency
104 on the liquid flow rate, the authors were able to measure the critical Reynolds number
105 in terms of the inclination angle, the cut-off wavenumber as a function of the Reynolds
106 number, and the dispersion curves of the spatial growth rate and wave celerity. Their
107 experimental data agreed with the neutral stability curve of Anshus & Goren (1966).

108 Alekseenko *et al.* (2009) investigated the additional effect of an active gas phase by
109 measuring the spatial growth rate of small-amplitude surface waves excited on a liquid
110 film falling down the inner surface of a vertical tube in interaction with a co- or counter-
111 current turbulent gas flow. In particular, the authors found that increasing the counter-
112 current gas flow reduces the cut-off wavenumber of the Kapitza instability, while the
113 maximal growth rate is increased. Thus, shorter waves are stabilized while longer waves
114 are amplified. Nonlinearly, this manifests itself in the attenuation of the precursory
115 capillary ripples typically forming on wavy falling liquid films (Trifonov 2010). This
116 has been also detected experimentally by Kofman *et al.* (2017). We observe a different
117 behaviour in our current study. For the strong confinement levels considered, we find that
118 the linear growth rate is decreased at all unstable wavenumbers when the counter-current
119 gas velocity is increased. However, in the case of a weaker confinement, we recover the
120 same qualitative behaviour as Alekseenko *et al.* (2009) for the laminar gas flow conditions
121 studied here (see section 4.3).

Our manuscript is structured as follows. In §2, we write the Orr-Sommerfeld temporal linear stability eigenvalue problem governing the stability of the falling film; the set-up and measurement methodology for the linear stability experiments are presented in §3; in §4 we present the results of our stability analysis focusing on the stabilizing effect of confinement in terms of the liquid Reynolds number, the inclination angle, the Kapitza number, and the velocity of a counter-current gas flow; the mechanism responsible for the stabilization induced by the confinement is presented in §5; finally, concluding remarks are presented in §6.

2. Governing equations and linear stability problem

For the configuration depicted in figure 1, the governing equations in dimensionless form read as (subscripts l and g are phase indicators):

$$\nabla \cdot \mathbf{u}_l = 0, \quad \nabla \cdot \mathbf{u}_g = 0, \quad (2.1a)$$

$$D_t \mathbf{u}_l = -\Pi_\rho \nabla p_l + \mathbf{B} + \Delta \mathbf{u}_l, \quad (2.1b)$$

$$D_t \mathbf{u}_g = -\nabla p_g + \mathbf{B} + \Pi_\mu \Pi_\rho^{-1} \Delta \mathbf{u}_g, \quad (2.1c)$$

where $\Pi_\rho = \rho_g/\rho_l$ and $\Pi_\mu = \mu_g/\mu_l$ designate the density and viscosity ratios. Here \mathbf{u} is the velocity vector of components (u, v) along (x, y) , p the pressure, Δ the Laplace operator, D_t the material derivative, and $\mathbf{B} = (\sin \beta, -\cos \beta)$ the inclination vector with the inclination angle β . For non-dimensionalization, we have employed reference scales obtained from balancing viscous drag and gravity, i.e. $\mathcal{L} = \nu_l^{2/3} g^{-1/3}$, $\mathcal{U} = (\nu_l g)^{1/3}$ and $\mathcal{T} = \mathcal{L}/\mathcal{U}$ for length, velocity, and time, and $\rho_g \mathcal{U}^2$ for pressure. The reader is warned that the full gravitational acceleration g rather than its streamwise projection $g_x = g \sin \beta$ is used here, in contrast to Ruyer-Quil & Manneville (2000). This way, the scales are independent of the control parameters. As a result of our scaling, the Reynolds and Froude numbers do not appear explicitly in (2.1). We thus introduce the following definition for the Reynolds number in the liquid and gas:

$$Re_l = \frac{\tilde{q}_l}{\nu_l}, \quad Re_g = \frac{\tilde{q}_g}{\nu_g}, \quad (2.2)$$

where \tilde{q}_l and \tilde{q}_g are the dimensional liquid and gas flow rates per unit width (the tilde will denote dimensional quantities throughout the manuscript). The boundary conditions at the walls read:

$$\mathbf{u}(0) = 0, \quad \mathbf{u}(H) = 0. \quad (2.3)$$

The kinematic and dynamic coupling conditions at the interface $y = h$ are:

$$u_l = u_g, \quad (2.4a)$$

$$(\mathbf{S}_l \cdot \mathbf{n}) \cdot \mathbf{t} = \Pi_\mu (\mathbf{S}_g \cdot \mathbf{n}) \cdot \mathbf{t}, \quad (2.4b)$$

$$(\mathbf{S}_l \cdot \mathbf{n}) \cdot \mathbf{n} + \kappa We = \Pi_\rho (\mathbf{S}_g \cdot \mathbf{n}) \cdot \mathbf{n}, \quad (2.4c)$$

where $We = \gamma(\rho_l \mathcal{L} \mathcal{U}^2)^{-1}$ is the Weber number with surface tension γ . Note that, due to the chosen scaling, our Weber number is identical to the Kapitza number $Ka = \gamma(\rho_l g^{1/3} \nu_l^{4/3})^{-1}$ which prevails in the literature on falling films. In (2.4b) and (2.4c), $\mathbf{S} = \mathbf{T} - p\mathbf{I}$ is the stress tensor, \mathbf{T} the viscous stress tensor and \mathbf{I} the identity matrix. The normal and tangential vectors to the interface are defined as:

$$\mathbf{n} = \frac{[-\partial_x h, 1]}{\sqrt{1 + (\partial_x h)^2}}, \quad \mathbf{t} = \frac{[1, \partial_x h]}{\sqrt{1 + (\partial_x h)^2}}, \quad (2.5)$$

153 whereas $\kappa = \nabla \cdot \mathbf{n} = -\partial_{xx}h[1+(\partial_x h)^2]^{-3/2}$ is the curvature of the interface. The kinematic
154 condition at the interface reads:

$$\partial_t h + u_l \partial_x h = v_l. \quad (2.6)$$

155 In our analysis, we will vary the relative confinement of the film, which is defined as:

$$\eta = \frac{H}{h_0}, \quad (2.7)$$

156 where h_0 designates the film thickness of the primary flow and H the gap height. It
157 constitutes one of the control parameters of the problem, together with the inclination
158 angle β and the flow rates q_l and q_g .

159 We perform a temporal stability analysis following the works of Yih (1967), Yiantsios
160 & Higgins (1988) and Tilley *et al.* (1994). We start by writing the base flow velocity
161 profiles:

$$U_l = K_l \left(\frac{y^2}{2} - h_0 y \right) + \Pi_\rho T_t y, \quad (2.8a)$$

$$U_g = K_g \frac{h_0^2}{2} \left(\eta - \frac{y}{h_0} \right) \left(2 - \eta - \frac{y}{h_0} \right) - \frac{\Pi_\rho}{\Pi_\mu} T_t h_0 \left(\eta - \frac{y}{h_0} \right). \quad (2.8b)$$

162 The constants K_l and K_g are defined as:

$$K_l = \Pi_\rho \partial_x P - \sin \beta (1 - \Pi_\rho M), \quad (2.9a)$$

$$K_g = \Pi_\rho \Pi_\mu^{-1} [\partial_x P - \sin \beta (1 - M)], \quad (2.9b)$$

163 where $\partial_x P$ is the driving pressure gradient and $M = \Delta p / \Delta p^a$ relates the driving pressure
164 difference Δp to the aerostatic pressure difference $\Delta p^a = \Lambda \sin \beta$, Λ designating the
165 (dimensionless) wavelength. The coefficient M allows to distinguish between the two
166 studied scenarios for the gas phase (see figure 2): (i) an aerostatic pressure difference,
167 where $M = 1$; (ii) a counter-current gas flow, where $M > 1$.

168 The interfacial tangential shear stress appearing in (2.8) is scaled with $\rho_g \mathcal{U}^2$ and reads:

$$T_t = \frac{h_0}{2\Pi_\rho} \left[K_l - K_g \left(\eta - 1 \right)^2 \right] \left[1 + \frac{1}{\Pi_\mu} \left(\eta - 1 \right) \right]^{-1}, \quad (2.10)$$

169 while the pressure profiles read:

$$P_l = P|_{h_0}(x=0) + x \partial_x P + \Pi_\rho^{-1} \cos \beta (h_0 - y), \quad (2.11a)$$

$$P_g = P|_{h_0}(x=0) + x \partial_x P + \cos \beta (h_0 - y), \quad (2.11b)$$

170 where $P|_{h_0}(x=0)$ is an arbitrary reference pressure at $y = h_0$ and $x = 0$.

171 We linearly perturb the primary flow as follows:

$$u = U + u^*, \quad v = v^*, \quad p = P + p^*, \quad (2.12)$$

172 where upper case letters refer to the base flow, while the star symbol denotes a small-
173 amplitude perturbation. We introduce the stream-function perturbations ϕ^* in the liquid
174 phase and ψ^* in the gas phase, satisfying:

$$u_l^* = \partial_y \phi^*, \quad v_l^* = -\partial_x \phi^*, \quad (2.13a)$$

$$u_g^* = \partial_y \psi^*, \quad v_g^* = -\partial_x \psi^*, \quad (2.13b)$$

175 and seek solutions of the form:

$$\{\phi^*, p_l^*\} = \{\hat{\phi}(y), \hat{p}_l(y)\} \exp[i\alpha(x - ct)], \quad (2.14a)$$

$$\{\psi^*, p_g^*\} = \{\hat{\psi}(y), \hat{p}_g(y)\} \exp[i\alpha(x - ct)], \quad (2.14b)$$

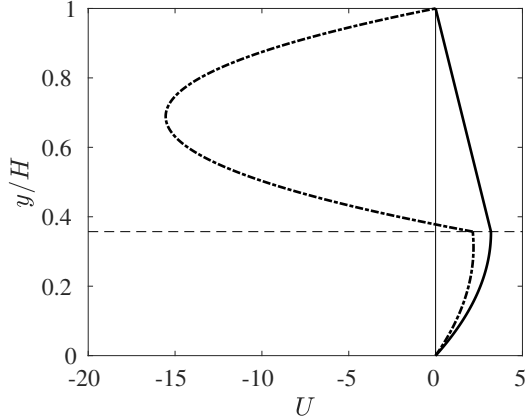


FIGURE 2. Base-state velocity profiles for an air-water flow: $\beta = 3^\circ$, $\eta = 2.8$ ($\tilde{H} = 1.5$ mm). The flat dashed line represents the interface, the solid line the aerostatic case ($M = 1$ in (2.9)) with $Re_l = 23.6$, whereas the dashed-dotted line the counter-current case ($M = 100$ in (2.9)) with $Re_l = 17.1$ and $Re_g = -12.8$.

176 where $\alpha \in \mathbb{R}$ is the wavenumber and $c = c_r + ic_i$ the complex wave celerity, whereas the hat
 177 symbol denotes the amplitudes of the corresponding perturbations. Substituting (2.14)
 178 into (2.1) linearized around the base flow, and eliminating the pressure in (2.1b) by
 179 substituting (2.1c), we obtain the well-known Orr-Sommerfeld equations:

$$\hat{\phi}'''' - 2\alpha^2 \hat{\phi}'' + \alpha^4 \hat{\phi} = i\alpha[(U_l - c)(\hat{\phi}'' - \alpha^2 \hat{\phi}) - U_l'' \hat{\phi}], \quad (2.15a)$$

$$\hat{\psi}'''' - 2\alpha^2 \hat{\psi}'' + \alpha^4 \hat{\psi} = i\alpha[(U_g - c)(\hat{\psi}'' - \alpha^2 \hat{\psi}) - U_g'' \hat{\psi}], \quad (2.15b)$$

180 subject to the boundary conditions:

$$\hat{\phi}(0) = 0, \quad \hat{\phi}'(0) = 0, \quad (2.16)$$

$$\hat{\psi}(H) = 0, \quad \hat{\psi}'(H) = 0, \quad (2.17)$$

181 where the prime symbol denotes differentiation with respect to y . Linearization of the
 182 coupling conditions (2.4) must account for the perturbation of the variable fields as well
 183 as a perturbation of the position at which the condition is applied, i.e. at the interface.
 184 Consider the perturbed interfacial value $\{f\}|_h$ of the variable $f(x, y, t)$ with base state
 185 $F(y)$. This can be decomposed into:

$$\{f\}|_h = \{F\}|_{h_0} + \{f^*\}|_{h_0} + \{F'\}|_{h_0} h^*, \quad (2.18)$$

186 where $f^*(x, y, t) = \hat{f}(y) \exp[i\alpha(x - ct)]$ stems from the perturbation of the variable field
 187 and $\{F'\}|_{h_0} h^*$ is the contribution from changing the position of the film surface within
 188 this field.

189 Applying (2.18) to the coupling conditions (2.4) leads to:

$$\hat{\phi}' + U_l' \hat{\phi} \underline{c}^{-1} = \hat{\psi}' + U_g' \hat{\phi} \underline{c}^{-1}, \quad (2.19a)$$

$$\hat{\phi}'' + U_l'' \hat{\phi} \underline{c}^{-1} + \alpha^2 \hat{\phi} = \Pi_\mu [\hat{\psi}'' + U_g'' \hat{\phi} \underline{c}^{-1} + \alpha^2 \hat{\psi}], \quad (2.19b)$$

$$2\alpha^2 \hat{\phi}' - i\alpha \Pi_\rho [\hat{p}_l + P_l' \hat{\phi} \underline{c}^{-1}] + We i\alpha^3 \hat{\phi} \underline{c}^{-1} = 2\alpha^2 \Pi_\mu \hat{\psi}' - i\alpha \Pi_\rho [\hat{p}_g + P_g' \hat{\phi} \underline{c}^{-1}] \quad (2.19c)$$

190 where all quantities are evaluated at $y = h_0$ and where we have introduced $\underline{c} = c - U|_{h_0}$

191 and used the relation:

$$h - h_0 = h^* = \hat{\phi} \underline{c}^{-1} \exp[i\alpha(x - ct)], \quad (2.20)$$

192 which follows directly from (2.6) together with (2.18). The pressure perturbation ampli-
193 tudes \hat{p}_l and \hat{p}_g in (2.19c), evaluated at $y = h_0$, can be recovered directly from (2.1):

$$\hat{p}_l = \Pi_\rho^{-1} [\underline{c} \hat{\phi}' + U_l' \hat{\phi}] + (i\alpha \Pi_\rho)^{-1} (\hat{\phi}''' - \alpha^2 \hat{\phi}'), \quad (2.21a)$$

$$\hat{p}_g = [\underline{c} \hat{\psi}' + U_g' \hat{\psi}] + \Pi_\mu (i\alpha \Pi_\rho)^{-1} (\hat{\psi}''' - \alpha^2 \hat{\psi}'). \quad (2.21b)$$

194 The Orr-Sommerfeld problem (2.15)-(2.21) is identical to the problem studied by Tilley
195 *et al.* (1994). We solve it by means of continuation using a code based on the continuation
196 software AUTO-07p (Doedel 2008). This code was previously used in Dietze & Ruyer-
197 Quil (2013). The continuation is started from the asymptotic limit $\alpha = 0$, for which c_r
198 and $c_i = 0$ are known analytically, and continued by varying one of the control parameters
199 while calculating c_r and c_i . We have validated our numerical procedure by comparing
200 stability results with those of Brevdo *et al.* (1999) for falling films in a passive atmosphere,
201 those of Yiantsios & Higgins (1989) in the asymptotic limit of long waves, and those
202 of Yiantsios & Higgins (1988) and Tilley *et al.* (1994) for a pressure-driven horizontal
203 liquid-gas flow. These validations are presented in the appendix A.

204 We point out that the considered gas flow rates are far from the convective-absolute
205 transition studied by Vellingiri *et al.* (2015), Schmidt *et al.* (2016), and Lavalle *et al.*
206 (2017), thus no spatio-temporal analysis is required here. For additional security, we
207 have validated our stability calculations with our own experiments, which are described
208 in the next section 3. Due to the convective nature of the instability, we measure a
209 spatial growth rate in our experiments, whereas our stability calculations yield temporal
210 growth rates (in a reference frame moving at the wave celerity). Both approaches yield
211 the same stability threshold and most of our comparisons between experiments and linear
212 stability analysis concern this stability threshold. However, in one instance (figure 9b),
213 we compare experimental and numerical growth rate dispersion curves and for this we
214 apply the so-called Gaster transformation (Gaster 1962) that relates temporal and spatial
215 growth rates:

$$\alpha_r c_i = -\zeta (c_r + \alpha_r \partial_{\alpha_r} c_r), \quad (2.22)$$

216 where ζ is the spatial growth rate and $\alpha_r c_i$ the temporal one. This transformation is
217 valid when the spatial and temporal growth rates are small. This is generally the case
218 for falling liquid films, as shown by Brevdo *et al.* (1999), who compared spatial growth
219 rates obtained directly from a spatio-temporal analysis with values obtained by applying
220 the Gaster transformation to temporal stability results, and showed that the error is
221 smaller than 0.1%. In the single instance where we have applied the Gaster transformation
222 (figure 9b), flow conditions are comparable to those in Brevdo *et al.* (1999).

223 3. Experiments

224 Our experimental apparatus, sketched in figure 3, has been employed in previous
225 studies (Kofman 2014; Kofman *et al.* 2017) and has been modified here to impose a
226 much stronger confinement of the gas phase above the falling liquid film (figure 4). The
227 liquid film flows down an inclined glass plate (150 cm long, 27 cm wide, 5 mm thick)
228 fixed in a framework mounted on rubber feet to reduce the influence of environmental
229 vibrations. The inclination angle β can be changed in the range $0^\circ - 20^\circ$ and is measured
230 using an inclinometer with a precision of 0.05° .

231 A gear pump conveys the liquid from a collection tank located at the end of the plate to

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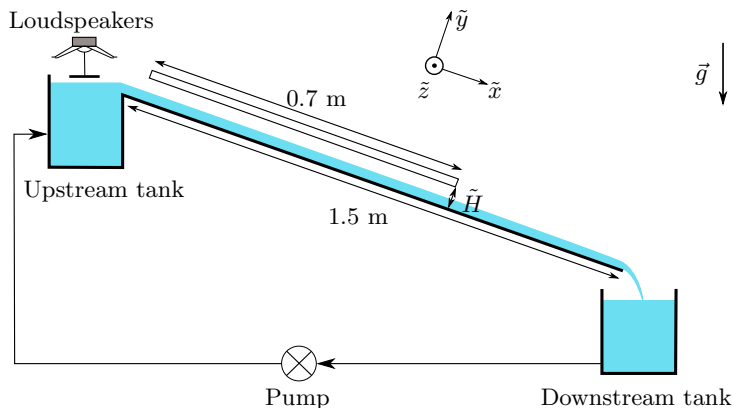


FIGURE 3. Sketch of the experimental set-up.

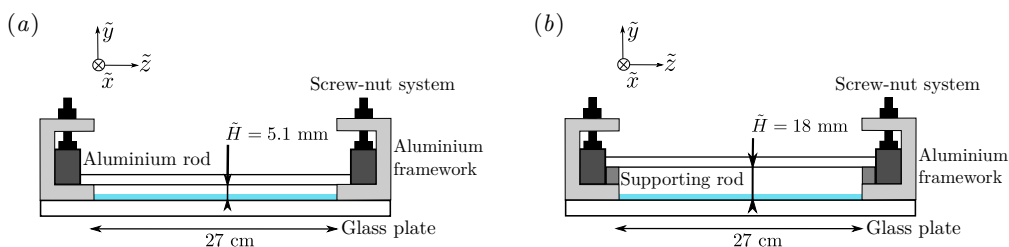


FIGURE 4. Sketch of the channel cross-section: (a) confined and (b) “unconfined” configuration.

232 an inlet tank, from which the liquid overflows onto the glass plate. The inlet tank is filled
 233 with several layers of glass beads in order to homogenize the entering flow. A temporal
 234 periodic forcing of the film is introduced at the inlet to trigger sinusoidal surface waves
 235 of prescribed frequency and amplitude. This is achieved by a vibrating aluminum plate,
 236 which is fixed to the membrane of two loudspeakers and generates harmonic vibrations
 237 above the liquid surface over the whole width of the film (Kofman *et al.* 2017).

238 At a position 5 cm from the inlet, the falling film enters a region where the gas phase
 239 is confined by an upper 5 mm thick glass plate of length 70 cm and width 27 cm (see
 240 figure 3). In our study, the gap height \tilde{H} of the channel formed between the two glass
 241 plates was set to two different values: (i) $\tilde{H} = 5.1 \text{ mm} \pm 0.1 \text{ mm}$, which we call the
 242 confined case; (ii) $\tilde{H} = 18 \text{ mm}$, which we loosely refer to as the “unconfined” case. The
 243 shift from one case to the other is achieved by raising the top glass plate upon supporting
 244 rods (see figure 4). For the confined configuration, the uniformity of the gap height was
 245 verified both in the streamwise and the transverse directions.

246 Water is used as working liquid. The temperature of the liquid is measured in the inlet
 247 and downstream tanks, and the surface tension is regularly monitored by measuring a
 248 water sample in a pending drop tensiometer. The liquid flow rate is fixed large enough
 249 that the film is unstable but small enough that the film surface without inlet excitation
 250 is virtually flat in the measurement section (i.e. surface waves are undetectable). The
 251 experimental control parameters and the range of variation of the physical parameters
 252 are specified in tables 1 and 2.

253 To experimentally characterize the interfacial instability of the falling film, a one-
 254 point measurement of the film thickness time trace is performed using the CCI (Confocal
 255 Chromatic Imaging) technique (Cohen-Sabban *et al.* 2001). The sensor model used in
 256 our experiments (STIL CCS-PRIMA CL4) allows us to detect two interface locations

Control parameter	Notation	Domain of variation
Inclination angle	β	1.69° and 3.05°
Reynolds number	Re_l	17.7 – 41.6
Forcing frequency	\tilde{f}	1.8 – 8.4 Hz

TABLE 1. Range of variation of the control parameters for the experiments.

Physical property	Notation	Range of variation
Density	ρ_l	997 – 999 kg/m ³
Kinematic viscosity	ν_l	8.9 – 11.1 × 10 ⁻⁷ m ² /s
Surface tension	γ	71.1 – 71.7 mN/m

TABLE 2. Physical properties of water at ambient temperature and their range of variation during our experiments.

257 simultaneously (the glass-water and the water-air interface) with a precision of 250 nm
 258 and an acquisition frequency up to 2 kHz. The CCI probe is mounted on a linear
 259 translation stage in order to enable measurements along the streamwise axis of the
 260 channel. The film thickness is measured at mid-width of the channel and at different
 261 longitudinal locations from the inlet. Measurements in the transverse direction were also
 262 carried out to check the uniformity of the film thickness, at sufficient distance from the
 263 lateral boundaries.

264 Figure 5 displays the streamwise evolution of the time-averaged film thickness \tilde{h}_m along
 265 the central axis of the channel, in the unconfined configuration. Two wave regimes are
 266 represented: (i) a virtually flat film, observed without inlet excitation (empty circles),
 267 and (ii) periodically-excited waves, observed with inlet forcing (filled triangles).

268 The actual flow rate of the falling liquid film is deduced from a film thickness mea-
 269 surement without inlet excitation, i.e. when the film surface is virtually flat. Indeed,
 270 for a fully-developed waveless film, the film thickness h_0 and the Reynolds number Re_l
 271 are linked through the base flow velocity profile (2.8) subject to an aerostatic pressure
 272 difference:

$$Re_l = -K_l \frac{h_0^3}{3} + T_t \frac{h_0^2}{2}, \quad (3.1)$$

273 where K_l is defined in (2.9a) (using $M = 1$ for the aerostatic scenario) and the interfacial
 274 tangential shear stress T_t in (2.10). By measuring the film thickness without inlet
 275 excitation in the developed region of the film (where \tilde{h}_m in figure 5 no longer changes),
 276 we obtain Re_l from (3.1) with $h_0 = h_m$.

277 The same waveless signal also serves a second purpose. It is compared to the film thickness
 278 measured *with* inlet excitation to determine an upper limit for the excitation amplitude
 279 below which the excited surface waves can be considered linear, i.e. governed by the
 280 linearized Navier-Stokes equations (2.15). In that case, the film thickness perturbation
 281 is sinusoidal in time (2.20) and thus the time-average of the perturbed film thickness
 282 $h_0 + h^*$ corresponds to the thickness of the waveless film h_0 . The excitation amplitude
 283 in the experiments was limited to values where this condition was satisfied in order to
 284 allow comparisons with the linear stability analysis.

285 Consequently, the waves excited in our experiments were sinusoidal (see also inset in
 286 figure 6a), and their amplitude \tilde{A} , defined here as the peak-to-peak value of the time

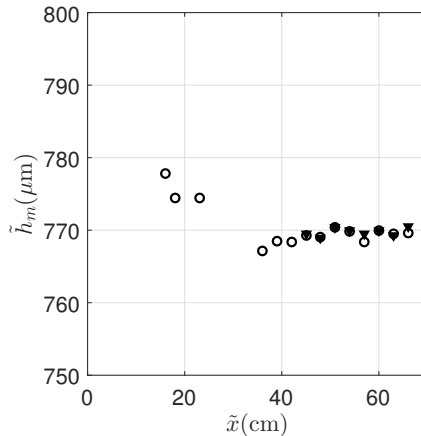


FIGURE 5. Streamwise evolution of the time-averaged film thickness \tilde{h}_m along the central axis of the channel, with inlet excitation ($\tilde{f} = 4.2$ Hz, filled triangles) and without inlet excitation (empty circles). Parameters: $\beta = 1.69^\circ$, $Re_l = 41.5$, unconfined configuration $\tilde{H} = 18$ mm.

287 signal, can be obtained in a statistically robust manner by performing a discrete Fourier
 288 transform of the film thickness time trace measured with the CCI system.

289 The frequency resolution $\Delta f = f_p/N_p$ of the discrete Fourier analysis is limited by the
 290 acquisition frequency f_p and the number of sampling points N_p . In all our measurements,
 291 a minimum of 5 series of at least 80 periods of the forcing mode was acquired. Thereby,
 292 f_p and N_p were chosen such that $\Delta f \leq 0.025$ Hz. An example of the CCI signal and
 293 the corresponding modulus of the discrete Fourier transform are shown in panels 6a and
 294 6b. Here the forcing frequency is 3 Hz ($\beta = 1.69^\circ$, $Re_l = 34.8$). The predominant peak
 295 appearing in figure 6b indicates that the signal is mono-periodic with a frequency equal
 296 to the forcing frequency, as expected. The amplitude of this mode is directly obtained
 297 from the peak height, provided that the signal contains an integer number of periods of
 298 the relevant mode (Bergé *et al.* 1988). Under this condition, the amplitude is four times
 299 the height of the peak (here $\tilde{A} = 4 \times 4.4 = 17.6 \mu\text{m}$).

300 In panels 6c and 6d we have additionally represented a CCI signal and the corresponding
 301 spectrum obtained without inlet forcing for the same angle and Reynolds number as in
 302 panels 6a and 6b. The spectrum exhibits peaks of very small amplitude (the scale has
 303 been divided by 10 compared to panel 6b) that are the signature of the intrinsic noise of
 304 the set-up. The bump around 3 Hz and the low-frequency peaks are intrinsic to the set-up
 305 as they are present in all our signals. The peak observed at 4.2 Hz is most-probably due
 306 to the pump, the frequency of which increases with the Reynolds number, i.e. with the
 307 flow rate delivered by the pump. The noise level is low enough that the quality of the
 308 signal measured with inlet forcing is not deteriorated.

309 To investigate the linear stability of the film, we measure the spatial evolution of the
 310 wave amplitude along the channel. This allows us to obtain: (i) the spatial growth rate
 311 of the instability as a function of the forcing frequency for a given inclination angle and
 312 Reynolds number, i.e. the dispersion relation; (ii) the cut-off frequency of the instability,
 313 \tilde{f}_c , as a function of the Reynolds number for a given angle β , i.e. the neutral stability
 314 curve $\tilde{f}_c(Re_l)$.

315 The spatial growth rate for a given frequency at a fixed (supercritical) Reynolds number
 316 is determined as follows. In the linear regime, the wave amplitude grows exponentially as
 317 a function of \tilde{x} , $\tilde{A}(\tilde{x}) \propto \exp(\tilde{\zeta}\tilde{x})$ where $\tilde{\zeta}$ is the spatial growth rate and \tilde{x} is the streamwise
 318 location of the measurement point. The spatial growth rate $\tilde{\zeta}$ is obtained by performing

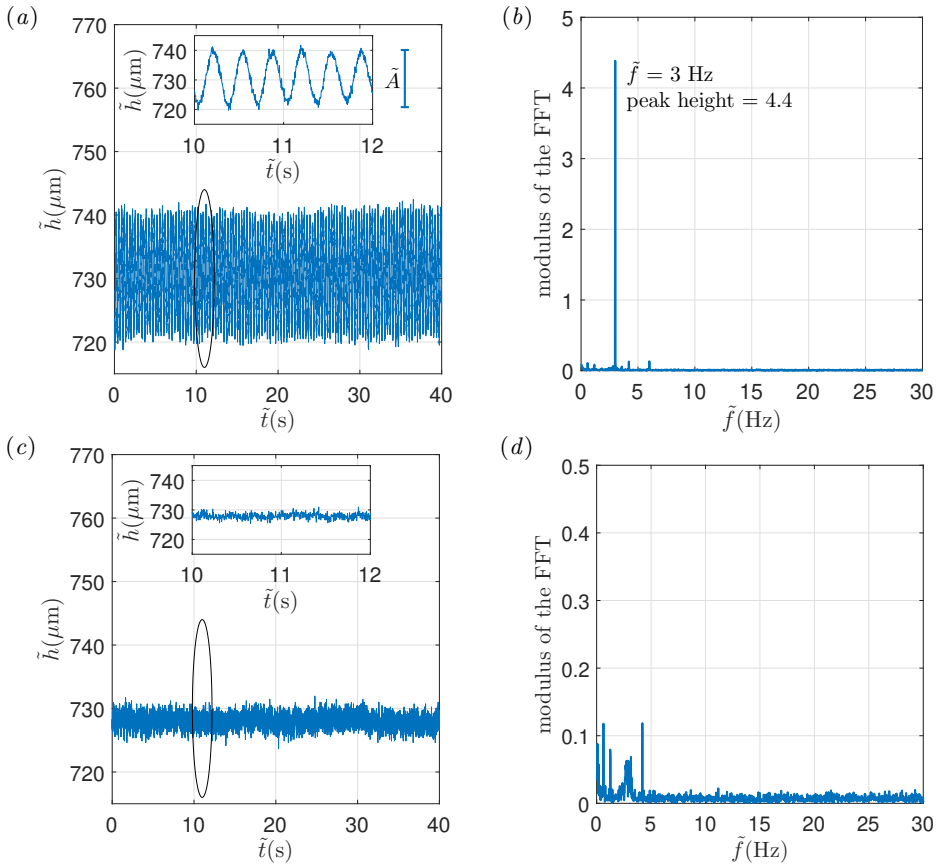


FIGURE 6. Examples of measurement signals with and without inlet forcing: $\beta = 1.69^\circ$, $Re_l = 34.8$, unconfined configuration $\tilde{H} = 18$ mm. (a) Time trace of the film thickness measured by the CCI system at a fixed longitudinal location $\tilde{x} = 63$ cm and enlarged inset. Forcing frequency: $\tilde{f} = 3$ Hz. (b) Corresponding modulus of the discrete Fourier transform. (c) Time evolution of the film thickness without inlet forcing. (d) Corresponding modulus of the discrete Fourier transform (the scale is divided by 10 compared to panel 6b).

319 a linear regression on the logarithm of $\tilde{A}(\tilde{x})$. The precision of $\tilde{\zeta}$, i.e. $\Delta\tilde{\zeta}$, depends on the
 320 standard deviation of the residual of the regression, σ_{res} , and on the length $\Delta\tilde{x}$ over which
 321 the measurements are conducted: $\Delta\tilde{\zeta} = 2\sigma_{res}\Delta\tilde{x}^{-1}$. These experiments require extremely
 322 precise amplitude measurements at several positions along the channel, especially when
 323 the growth rate is weak. Figure 7a shows an example of the streamwise spatial evolution
 324 of the wave amplitude. Here, $\tilde{\zeta}$ is obtained by a fitting from $\tilde{x} = 10$ to 30 cm. We
 325 observe that for $\tilde{x} > 30$ cm the spatial evolution of the wave amplitude is no longer
 326 exponential. The region of exponential growth varies depending on the experimental
 327 parameters (inclination angle, Reynolds number, forcing frequency). Accordingly, the
 328 amplitude of the inlet excitation is optimized for each experimental run so that the wave
 329 amplitude is large enough to be detected over the length of the channel while remaining
 330 in the linear regime.

331 The neutral stability curve $\tilde{f}_c(Re_l)$ is determined as follows. The wave amplitude is
 332 measured at two locations along the channel (amplitude \tilde{A}_1 at $\tilde{x}_1 = 54$ cm and amplitude
 333 \tilde{A}_2 at $\tilde{x}_2 = 63$ cm), within the exponential growth region. Fixing β and the forcing
 334 frequency \tilde{f} , we compare the two amplitudes for several Reynolds numbers to find the

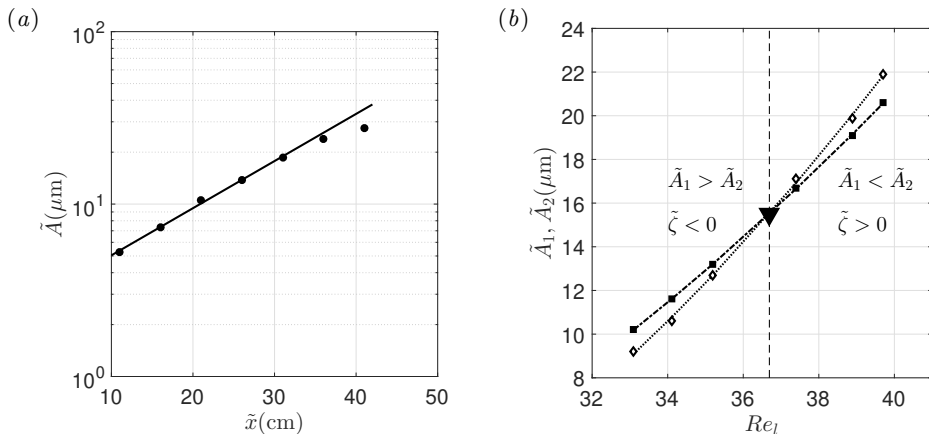


FIGURE 7. Determining the growth rate and stability threshold from experiments. (a) Streamwise evolution of the wave amplitude along the channel ($\tilde{f} = 2.6$ Hz, $\beta = 3.05^\circ$, $Re_l = 34.9$, unconfined configuration $\tilde{H} = 18$ mm). Experimental points (symbols) are fitted using an exponential curve (solid line) in the range $\tilde{x} = 10 - 30$ cm to obtain the spatial growth rate: $\tilde{\zeta} = 6.3$ (1/m) \pm 0.4 (1/m). (b) Determination of the critical Reynolds number from the change in wave amplitude at two streamwise locations \tilde{x}_1 and \tilde{x}_2 when varying Re_l ($\tilde{f} = 4.2$ Hz, $\beta = 1.69^\circ$, unconfined configuration $\tilde{H} = 18$ mm). Filled squares: amplitude \tilde{A}_1 at $\tilde{x}_1 = 54$ cm, empty diamonds: amplitude \tilde{A}_2 at $\tilde{x}_2 = 63$ cm. The downward triangle marks the intersection, yielding the critical Reynolds number.

critical Reynolds number at which the spatial growth rate $\tilde{\zeta}$ changes sign. This threshold is determined by fitting curves to the $\tilde{A}_i(Re_l)$ data for the two locations and locating their intersection, as illustrated in figure 7b. The two curves in figure 7b represents the wave amplitudes \tilde{A}_1 and \tilde{A}_2 measured at two different locations \tilde{x}_1 and $\tilde{x}_2 > \tilde{x}_1$ as a function of Re_l . At the value of Re_l where the two curves intersect, i.e. where $\tilde{A}_1 = \tilde{A}_2$, the spatial growth rate is zero and thus $Re_l = Re_{cr}$. To the left of the intersection, the downstream amplitude is smaller than the upstream one ($\tilde{A}_1 > \tilde{A}_2$) and thus the film is stable. To the right of the intersection, the downstream amplitude is larger than the upstream one ($\tilde{A}_1 < \tilde{A}_2$) and thus the film is unstable. This measurement of the critical Reynolds number is performed for several forcing frequencies, thus obtaining different points on the neutral stability curve $\tilde{f}_c(Re_l)$. Subsequently, in order to cross-validate these points, the Reynolds number is fixed at the critical value corresponding to a given forcing frequency, and frequencies slightly above and below this value are scanned to verify that the film indeed changes from stable to unstable.

4. Results

In this section, we present our experimental and numerical linear stability results for three existing fluid combinations: (i) water in contact with air, which we mainly focus on; (ii) aqueous solution of dymethylsulfoxide (DMSO) in contact with air; (iii) aqueous solution of glycerin in contact with air. The fluid properties and the range of variation of the control parameters used for the numerical stability calculations are summarized in tables 3 and 4.

Fluid system	ρ_l (kg/m ³)	ρ_g (kg/m ³)	ν_l (m ² /s)	ν_g (m ² /s)	γ (mN/m)
Water-air	1000.0	1.185	1.00×10^{-6}	15.58×10^{-6}	76.9
DMSO(83.11%)-air	1098.3	1.185	2.85×10^{-6}	15.58×10^{-6}	48.4
Glycerin(50%)-air	1130.0	1.185	5.02×10^{-6}	15.58×10^{-6}	69.0

TABLE 3. Physical properties of the fluid systems considered in the numerical stability calculations. The working liquids are: water, an aqueous solution of dymethylsulfoxide (DMSO) at 83.11% by weight, and an aqueous solution of glycerin at 50% by weight. The outer phase is air in all cases.

Control parameter	Notation	Range of variation
Inclination angle	β	$1.65^\circ - 20^\circ$
Relative confinement	η	$1.4 - 25.5$
Liquid Reynolds number	Re_l	$17 - 70$
Gas Reynolds number	Re_g	$-250 - 4.8$
Kapitza number	Ka	$331.8 - 3592$

TABLE 4. Range of variation of the control parameters for the numerical stability calculations.

4.1. Unconfined gas phase

We start by validating our temporal linear stability calculations with our experiments in the “unconfined” configuration of figure 4b ($\tilde{H} = 18$ mm). We do this comparison for two types of curves: the neutral stability curve $\tilde{f}_c(Re_l)$ and the dispersion curve of the spatial growth rate $\tilde{\zeta}(\tilde{f})$. In the latter case, we use the Gaster transformation (2.22) to transform the temporal growth rate from our numerical calculations into a spatial growth rate.

Figure 8 shows the neutral stability curve $\tilde{f}_c(Re_l)$ for an inclination angle of $\beta = 1.69^\circ$. Therein, different types of symbols, which represent measurements, indicate the stability of the investigated point: downward-pointing triangles for stable, upward-pointing triangles for unstable, and circles for neutral conditions. The solid line corresponds to the stability calculation. We point out that the precision of the inclination angle measurement using the inclinometer was not sufficient to obtain such good agreement between the two data sets. Instead, β , which is the same for all points on the neutral stability curve, was adjusted in the stability calculation until the numerical curve best matched the experimental one.

For the case of figure 8, where the inclinometer measurement yielded $\beta = 1.70^\circ$ with a precision of 0.05° , we computed numerical curves for inclination angles ranging from 1.67° to 1.73° with a 0.01° increment. The numerical curve fitting best with the experimental data yielded an inclination angle of $\beta = 1.69^\circ$. When varying β , not only the numerical curve changes but also the experimental points, because the flow rate is obtained from equation (3.1), where K_l depends on β . To demonstrate the sensitivity of the comparison in figure 8 w.r.t. to the inclination angle β , we have included two additional figures in appendix B, showing the comparison for $\beta = 1.68^\circ$ and $\beta = 1.70^\circ$.

Figure 9 compares numerical and experimental data for a larger inclination angle. Panel 9a represents the neutral stability curve for $\beta = 3^\circ$, and panel 9b the dispersion curve of the spatial growth rate for $\beta = 3.08^\circ$ and $Re_l = 31$. As experimental data were not acquired on the same day, the precise value of β is slightly different for the two

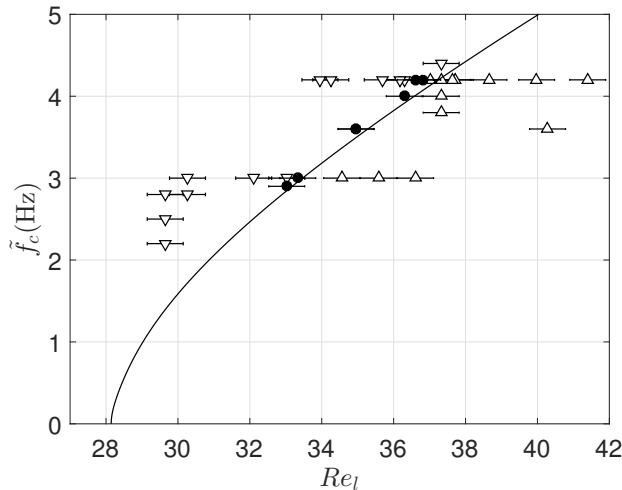


FIGURE 8. Comparison of numerical (line) and experimental (symbols) stability results at $\beta = 1.69^\circ$ in the “unconfined” configuration (figure 4b): neutral stability curve. Symbols indicate the stability of the investigated point: downward-pointing triangles for stable, upward-pointing triangles for unstable, and circles for neutral conditions.

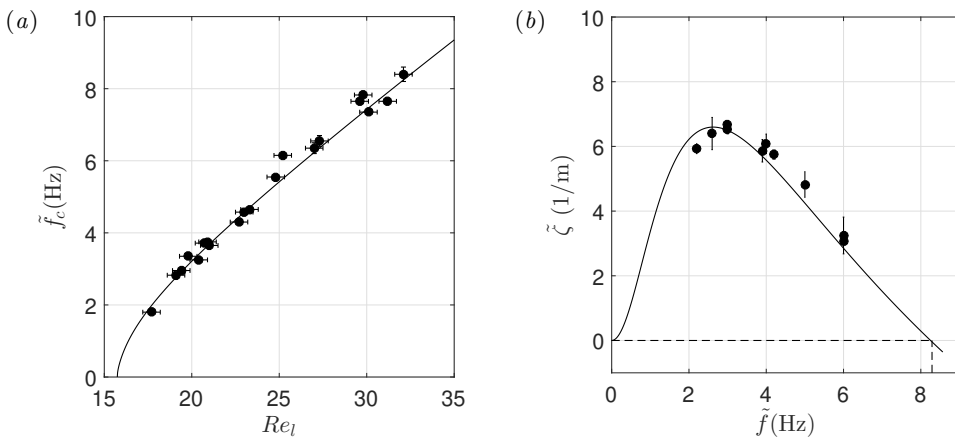


FIGURE 9. Comparison of numerical (solid line) and experimental (filled circles) stability results: unconfined configuration. (a) Neutral stability curve for $\beta = 3^\circ$; (b) dispersion curve of the spatial growth rate for $\beta = 3.08^\circ$ and $Re_l = 31$. The precise value of β is slightly different for the two panels as experimental data were obtained from experimental runs on different days.

384 panels. Agreement between the experimental and numerical dispersion curves confirms
 385 that the Gaster transformation is applicable here (see also Brevdo *et al.* (1999)).

386 4.2. Confined gas phase: aerostatic pressure difference

387 We now study how the stability of the falling liquid film changes when the gas phase
 388 is significantly confined. To quantify the degree of confinement, we use the ratio η (2.7)
 389 relating the gap height H and the thickness of the primary film flow h_0 . To quantify the
 390 effect of η on film stability, we will compare our stability results with the limiting case
 391 of a passive gas ($\Pi_\mu, \Pi_\rho \rightarrow 0$), where the confinement plays no role. This limit will be
 392 denoted with the superscript ∞ .

393 We first study the situation where the gas is subject to an aerostatic pressure gradient.

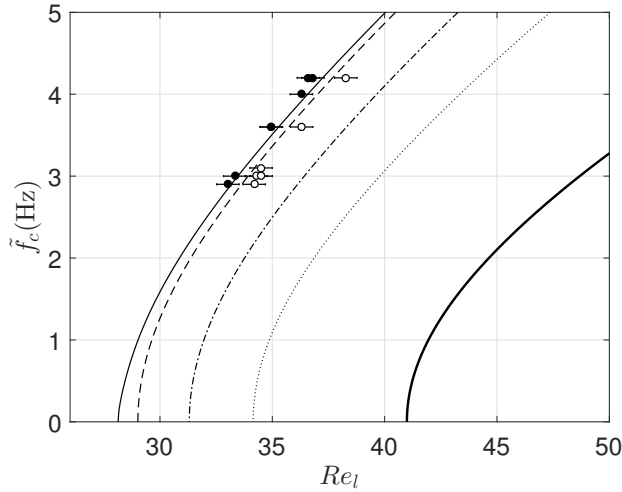


FIGURE 10. Neutral stability curves of a water film ($Ka = 3592$) inclined at $\beta = 1.69^\circ$ in contact with aerostatic air at different confinement levels $\eta_{cr} = \eta(\alpha_c = 0)$. Filled symbols: “unconfined” experiment with $\eta_{cr} = 25.5$ (panel 4b, $\tilde{H} = 18$ mm); open symbols: confined experiment with $\eta_{cr} = 7.54$ (panel 4a, $\tilde{H} = 5.1$ mm). Stability calculations are represented with lines. Thin solid: “unconfined” ($\eta_{cr} = 25.5$, $\tilde{H} = 18$ mm); dashed line: confined at $\tilde{H} = 5.1$ mm ($\eta_{cr} = 7.54$); dash-dotted line: $\tilde{H} = 2.5$ mm ($\eta_{cr} = 3.60$); dotted line: $\tilde{H} = 2$ mm ($\eta_{cr} = 2.80$); thick solid line: $\tilde{H} = 1.7$ mm ($\eta_{cr} = 2.23$).

394 In that case, the gas moves only as a reaction to the film’s motion. We start by
 395 validating our temporal stability calculations with our own experiments in the confined
 396 configuration $\tilde{H} = 5.1$ mm (panel 4a) and the “unconfined” configuration $\tilde{H} = 18$ mm
 397 (panel 4b) at an inclination angle $\beta = 1.69^\circ$, using water as working liquid.

398 The two leftmost curves in figure 10 represent the calculated neutral stability curves
 399 $\tilde{f}_c(Re_l)$ for these two configurations, while symbols represent the corresponding exper-
 400 imental data. Both data sets evidence a clear stabilization of the falling film due to
 401 increased confinement of the gas phase (dashed curve, open circles), and their agreement
 402 is gratifying.

403 The dimensional gap height \tilde{H} along the curves in figure 10 is constant, meaning that
 404 the relative confinement η changes, as \tilde{h}_0 changes with Re_l . Nonetheless, a representative
 405 value for η can be given for each curve by evaluating it at the critical Reynolds number
 406 Re_{cr} (where the curve intersects the x -axis), i.e. $\eta_{cr} = \eta(\alpha_c = 0)$, α_c being the cut-off
 407 wavenumber. The quantity η_{cr} is quite large for the two leftmost curves in figure 10
 408 ($\eta_{cr} > 7$), and this explains why the observed confinement-induced stabilization is
 409 rather weak (experimental wave amplitudes are reported in figure 24 of appendix C).
 410 However, when decreasing η_{cr} , the confinement-induced stabilization becomes much
 411 stronger, as evidenced by the three additional curves in figure 10 obtained from our
 412 stability calculations. For example, at $\eta_{cr} = 2.23$ (thick solid line), the critical Reynolds
 413 number number Re_{cr} , below which the film is always stable, is increased by over 50%
 414 compared to the “unconfined” configuration (thin solid line).

415 This means that the confinement can fully stabilize the Kapitza instability at a given
 416 Reynolds number. We now investigate this suppression of the instability in further detail
 417 based on our stability calculations. In figure 11a, we have plotted the temporal growth
 418 rate for a water film at $Re_l = 23.9$ and $\beta = 3^\circ$ as a function of the wavenumber α for
 419 different values of η (panel 11b represents the corresponding wave celerity c_r). We see

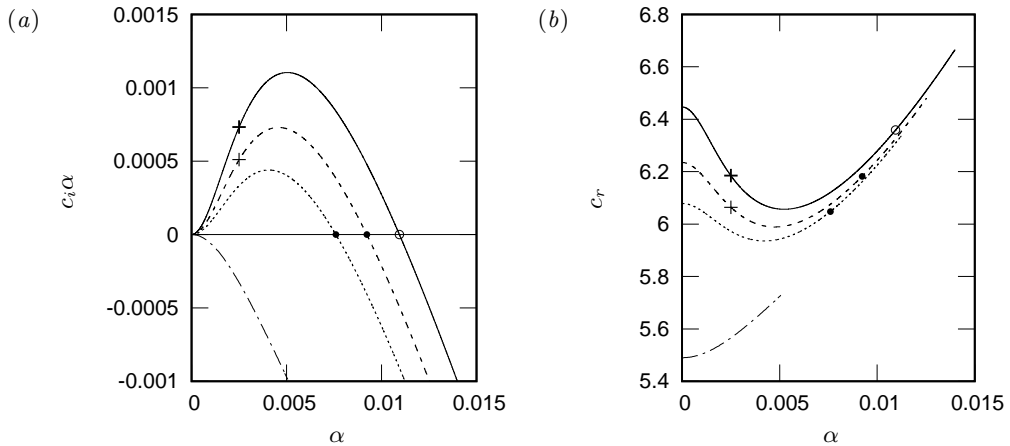


FIGURE 11. Stability calculations for a water film ($Ka = 3592$) at $Re_l = 23.9$ and $\beta = 3^\circ$ in contact with aerostatic air. Effect of confinement on (a) temporal growth rate and (b) wave celerity. Solid line: passive-gas limit; dashed line: $\eta = 3.6$; dotted line: $\eta = 2.8$; dash-dotted line: $\eta = 2$. The + symbol marks data points that will be discussed in §5. The filled and open circles correspond to the cut-off wavenumber α_c for the confined and passive-gas configurations.

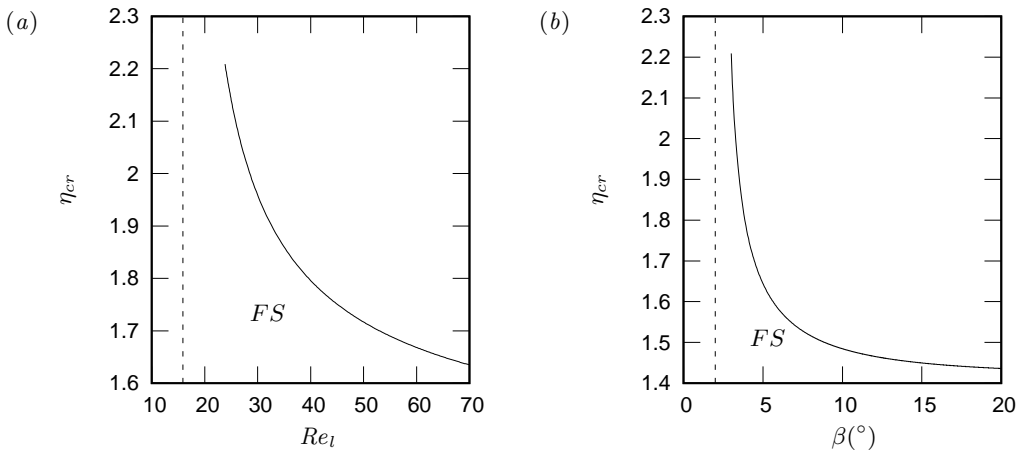


FIGURE 12. Numerical calculation of the critical confinement $\eta_{cr} = \eta(\alpha_c = 0)$ below which a water film ($Ka = 3592$) in contact with aerostatic air is fully stabilized (FS). (a) Effect of the liquid Reynolds number Re_l at $\beta = 3^\circ$. The vertical dashed line marks the critical Reynolds number in the passive-gas limit $Re_{cr}^\infty = 5/6 \cot \beta = 15.9$; (b) effect of the inclination angle β at $Re_l = 23.9$. The vertical dashed line marks the critical angle in the passive-gas limit $\beta_{cr}^\infty = \arctan(5/6/Re_l) = 2^\circ$.

420 that the unstable range, i.e. where the wavenumber α is smaller than the cut-off value
 421 α_c (intersection of the curves with $c_i \alpha = 0$) decreases with η and entirely disappears at
 422 $\eta = 2$. At this point, the Kapitza instability has been fully suppressed.

423 The critical relative confinement η_{cr} for the suppression of the instability is plotted in
 424 figure 12 against the liquid Reynolds number Re_l (panel 12a) and the inclination angle
 425 β (panel 12b), respectively. We see that it is easier to suppress the falling film instability
 426 the smaller Re_l and β .

427 No such curve can be obtained for the remaining control parameter, the Kapitza
 428 number Ka , as the onset of the long-wave instability is unaffected by surface tension.

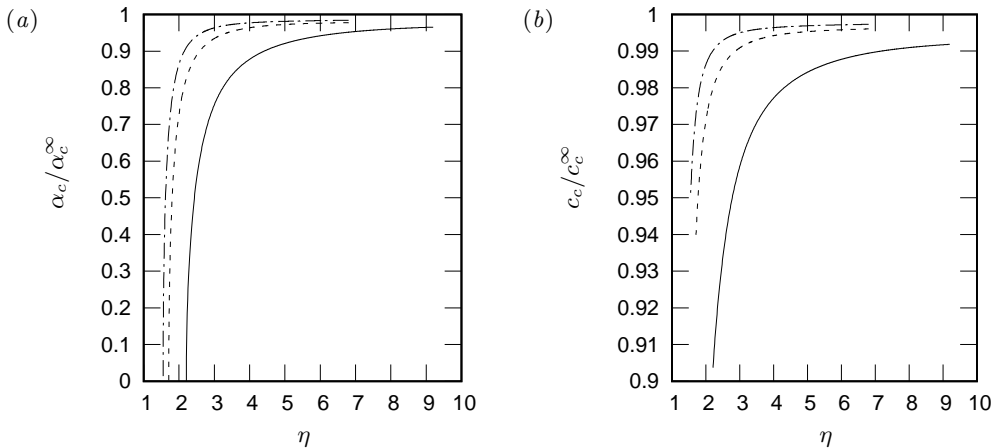


FIGURE 13. Stabilization due to confinement for different working liquids at $Re_l = 23.9$ and $\beta = 3^\circ$ in contact with aerostatic air. (a) Cut-off wavenumber and (b) cut-off wave celerity compared to their passive-gas limits α_c^∞ and c_c^∞ (open circles in panels 11a and 11b). Solid line: water ($Ka = 3592$); dashed line: aqueous solution of dymethylsulfoxide at 83.11% by weight ($Ka = 509.5$); dash-dotted line: aqueous solution of glycerin at 50% by weight ($Ka = 331.8$).

429 Instead, we have plotted in figure 13 the cut-off wavenumber (panel 13a) and the
 430 corresponding wave celerity (panel 13b) as a function of η for the three studied liquids,
 431 which display a substantial variation of Ka . In both panels, the plotted quantity is related
 432 to its value in the limit of a passive gas. We see that the stabilizing effect of confinement
 433 is felt earlier for water ($Ka = 3592$) compared to aqueous solutions of dymethylsulfoxide
 434 ($Ka = 509.5$) and glycerol ($Ka = 331.8$). Panel 13b shows that the confinement-induced
 435 stabilization of the liquid film is accompanied by a decrease of the wave celerity.

4.3. Confined gas phase: counter-current flow

437 We now investigate how confinement affects the stability of the falling liquid film
 438 when it is sheared by a counter-current gas flow. For this, we apply an adverse pressure
 439 difference $\Delta p > 0$, which is larger than the aerostatic pressure difference $\Delta p^a = \Lambda \sin \beta$.
 440 The strength of the gas flow is quantified by the pressure difference ratio $M = \Delta p / \Delta p^a$.

441 Figure 14 represents stability calculations for a falling water film at $\beta = 3^\circ$ and $Re_l =$
 442 23.9 . The relative confinement is fixed to $\eta = 3.6$ and the adverse pressure difference
 443 is increased. Panel 14a shows that increasing the counter-current gas flow increasingly
 444 stabilizes the film until the point of fully suppressing the Kapitza instability (dash-dotted-
 445 dotted line). Panel 14b shows that this stabilization is accompanied by a decrease in wave
 446 celerity.

447 The level of stabilization strongly depends on the inclination angle β , as shown in
 448 panels 15a and 15b, which represent contours of the cut-off wavenumber (for $Re_l = 23.9$)
 449 and critical Reynolds number related to their aerostatic limit as a function of β and
 450 M . At low inclination angles and relatively high adverse pressure gradients, the cut-off
 451 wavenumber is reduced by up to 70% with respect to the aerostatic case and the critical
 452 Reynolds number is increased by up to 40%.

453 The dispersion curves in figure 14 exhibit a number of features that seem to contradict
 454 the stability results of Alekseenko *et al.* (2009), Vellingiri *et al.* (2015), and Trifonov
 455 (2017), which were obtained for a weakly-confined turbulent gas flow ($\eta = 24 - 260$
 456 in Trifonov (2017)). Firstly, increasing the gas flow, monotonically reduces our unstable
 457 range of wavenumbers, whereas Vellingiri *et al.* (2015) and Trifonov (2017) found that the

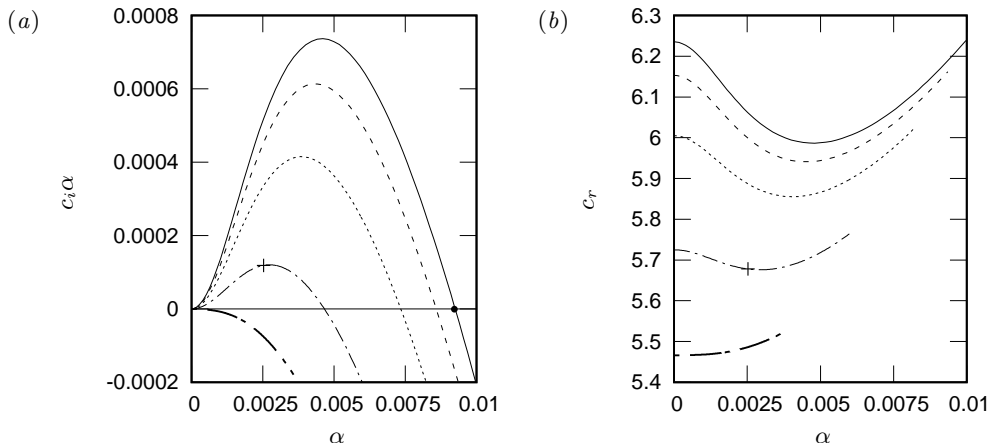


FIGURE 14. Influence of an increasingly strong counter-current air flow on the stability of a falling water film ($Ka = 3592$) at $\beta = 3^\circ$, $Re_l = 23.9$ and $\eta = 3.6$. (a) Temporal growth rate; (b) wave celerity. Solid line: $M = 1$ ($Re_g = 3.0$); dashed line: $M = 10$ ($Re_g = -0.94$); dotted line: $M = 25$ ($Re_g = -7.2$); dash-dotted line: $M = 50$ ($Re_g = -17.1$); thick dash-dotted line: $M = 70$ ($Re_g = -24.4$). The + symbol marks data points that will be discussed in §5. The filled circle in (a) marks the cut-off wavenumber α_c^a for the aerostatic configuration.

458 cut-off wavenumber can first decrease and then increase with the gas velocity magnitude.
 459 Secondly, our growth rate is reduced at all wavenumbers, whereas Alekseenko *et al.* (2009)
 460 found that increasing the gas velocity magnitude may reduce the cut-off wavenumber but
 461 increase the maximal growth rate at the same time. In that case, long waves are amplified
 462 and short waves are stabilized.

463 It turns out that these differences are not due to the laminar gas flow conditions, but
 464 to the strong confinement levels studied here. Indeed, for weaker confinement levels, we
 465 recover the same qualitative behaviour as Alekseenko *et al.* (2009), Vellingiri *et al.* (2015),
 466 and Trifonov (2017), notwithstanding that the gas flow is laminar in our case. This is
 467 shown in figures 16, where we consider a significantly weaker confinement ($\eta \geq 7$). Figure
 468 16a shows that the cut-off wavenumber (filled circles) now varies non-monotonically
 469 with increasing gas flow. Interestingly, this curve displays an additional local maximum in
 470 addition to the local minimum reported by Vellingiri *et al.* (2015) and Trifonov (2017).
 471 In the same graph, we have also plotted the maximally-amplified wavenumber (filled
 472 triangles), which varies in the same manner. Panel 16b shows the corresponding curve
 473 for the maximum growth rate, which displays only a local minimum.

474 In panel 16c, we have plotted growth rate dispersion curves for three representative
 475 points from panel 16a. In contrast to the dispersion curves in panel 14a (where $\eta = 3.6$),
 476 we see that increasing the gas flow can now stabilize short waves while amplifying long
 477 waves (compare solid and dot-dashed lines), in accordance with Alekseenko *et al.* (2009).
 478 For an even weaker confinement level ($\eta = 9$), we find that an increased gas flow can
 479 *destabilize* the film at all wavenumbers, as shown in figure 17a. Panel 17b, which plots
 480 different characteristic points of the growth rate dispersion curve as a function of η , allows
 481 to identify the critical confinement at which this transition occurs ($\eta = 8$ for $Re_l = 17$
 482 and $\beta = 3^\circ$). Just before the transition ($\eta = 7.5 - 8$), the cut-off wavenumber is reduced
 483 w.r.t. the aerostatic limit while the maximum growth rate is increased, similarly to what
 484 was observed by Alekseenko *et al.* (2009).

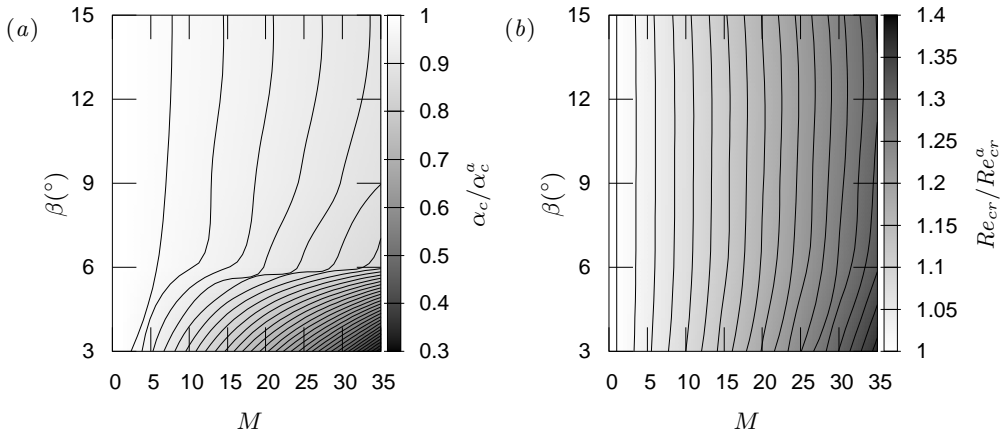


FIGURE 15. Effect of an increasingly strong counter-current air flow ($M = \Delta p / \Delta p^a$) on the stability of a falling water film ($Ka = 3592$) at $\beta = 3^\circ$ and $\eta = 2.8$. (a) Deviation of the cut-off wavenumber α_c from its aerostatic limit α_c^a at $Re_l = 23.9$; (b) deviation of the critical Reynolds number Re_{cr} from its aerostatic limit Re_{cr}^a . Darker regions correspond to a greater stabilization.

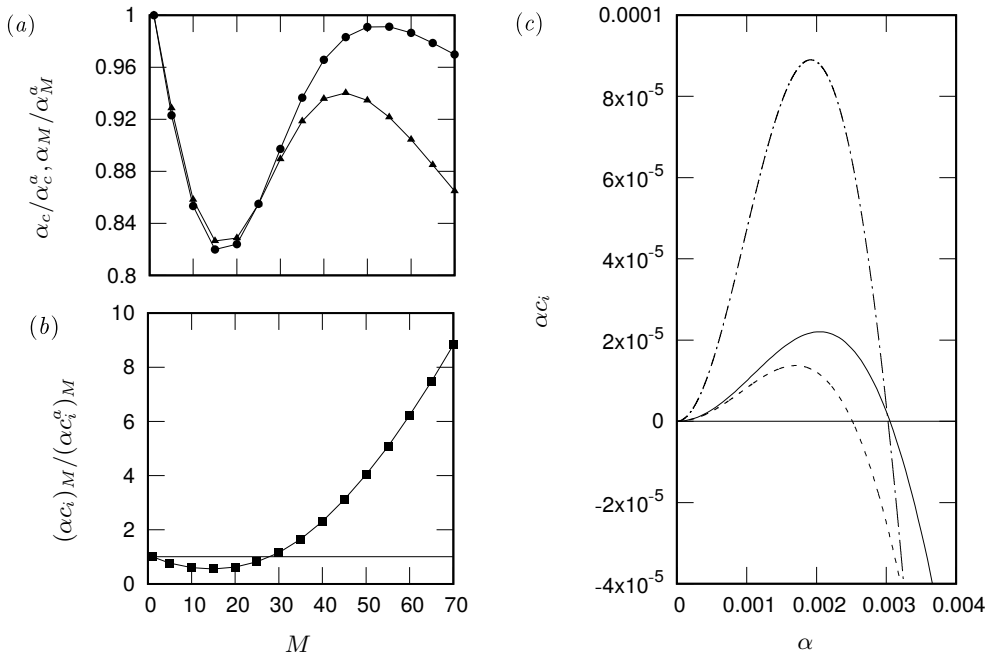


FIGURE 16. Effect of an increasingly strong counter-current air flow ($M = \Delta p / \Delta p^a$) on the stability of a falling water film ($Ka = 3592$) at $\beta = 3^\circ$, $\eta = 7$ and $Re_l = 17$. (a) Deviation of the cut-off wavenumber α_c (filled circles) and most unstable wavenumber α_M (filled triangles) from their respective aerostatic limits (superscript a); (b) deviation of the maximum growth rate $(\alpha_c)_M$ from the corresponding aerostatic limit $(\alpha_c^a)_M$. The horizontal solid line denotes $(\alpha_c)_M = (\alpha_c^a)_M$; (c) temporal growth rate. Solid line: $M = 1$ ($Re_g = 5.0$); dashed line: $M = 20$ ($Re_g = -67.5$); dash-dotted line: $M = 50$ ($Re_g = -175$).

485 5. Explanation of the stabilization mechanism

486 To identify the mechanism responsible for the confinement-induced stabilization of the
 487 Kapitza instability, we focus on three representative cases for which linear stability data

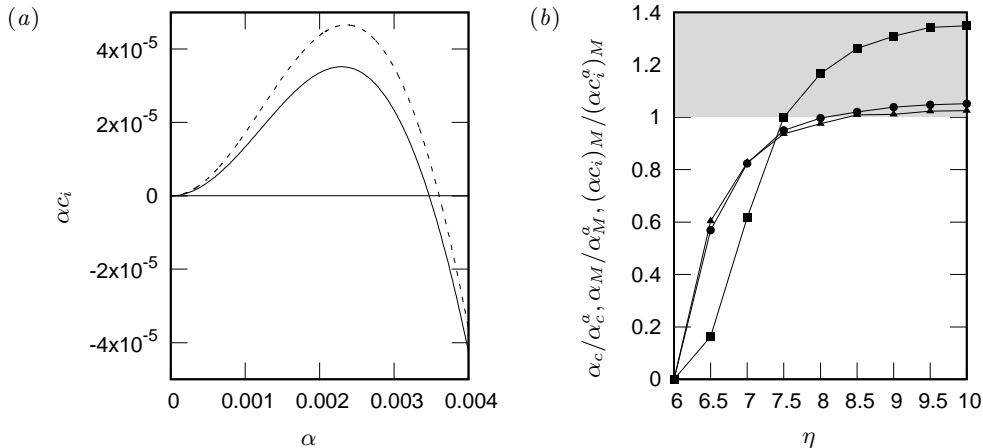


FIGURE 17. Effect of the relative confinement η on the stability of a falling water film ($Ka = 3592$) at $\beta = 3^\circ$, $Re_l = 17$ and $Re_g = -67.5$. (a) temporal growth rate. Solid line: $M = 1$ and $\eta = 9$; dashed line: $Re_g = -67.5$ ($M = 9.1$) and $\eta = 9$; (b) deviation of the cut-off wavenumber α_c (filled circles), most unstable wavenumber α_M (filled triangles) and maximum growth rate $(\alpha c_i)_M$ (filled squares) from their respective aerostatic limits. The grey area marks the unstable region compared to the aerostatic scenario.

488 were reported in §4 (the two points marked by plus symbols in figure 11 and the single
 489 point also marked by a plus symbol in figure 14). In all three cases, we have $Re_l = 23.9$
 490 and $\beta = 3^\circ$, whereas the conditions for the gas differ as follows: (i) aerostatic gas with
 491 weak relative confinement $\eta = 12.4$; (ii) aerostatic gas with strong relative confinement
 492 $\eta = 3.6$; (iii) counter-current gas with $Re_g = -17.1$ and strong relative confinement
 493 $\eta = 3.6$.

494 For each case, we evaluate the linear perturbation of different interfacial quantities
 495 based on a representative unstable mode fixed by the wavenumber $\alpha = 0.0025$ (situated
 496 on the ascending branch of the growth rate curves in figures 11 and 14). These pertur-
 497 bations can all be expressed in terms of the streamfunction perturbation amplitudes $\hat{\phi}$
 498 and $\hat{\psi}$ (2.14), which are the eigenfunctions of the Orr-Sommerfeld problem (2.15). For
 499 example, the film thickness perturbation h^* (2.20) can be written as:

$$h^* = \hat{h} \exp[i\alpha(x - ct) + i\Theta_h], \quad (5.1)$$

$$\hat{h} = \hat{\phi} \sqrt{\text{Re}\{1/\underline{c}\}^2 + \text{Im}\{1/\underline{c}\}^2}, \quad \Theta_h = \arctan[\text{Im}\{1/\underline{c}\} / \text{Re}\{1/\underline{c}\}],$$

500 where Θ_h designates the phase shift with respect to the stream-function perturbation
 501 (2.14a). Further, we are interested in the perturbed adverse tangential shear stress τ_t
 502 exerted by the gas on the film surface:

$$\tau_t = T_t + \tau_t^*, \quad (5.2)$$

503 where $T_t < 0$ is the tangential interfacial shear stress of the primary flow and the
 504 shear stress perturbation τ_t^* subsumes the two contributions associated with interfacial

505 quantities defined by the last two terms in (2.18):

$$\begin{aligned} \tau_t^* &= \hat{\tau}_t \exp[i\alpha(x - ct) + i\Theta_\tau], \\ \hat{\tau}_t &= \sqrt{\operatorname{Re}\{Z_\tau\}^2 + \operatorname{Im}\{Z_\tau\}^2}, \quad \Theta_\tau = \arctan[\operatorname{Im}\{Z_\tau\}/\operatorname{Re}\{Z_\tau\}], \end{aligned} \quad (5.3)$$

$$Z_\tau = \Pi_\rho^{-1}[\hat{\phi}'' + U_l'' \hat{\phi} \underline{c}^{-1} + \alpha^2 \hat{\phi}],$$

506 where the purely real amplitude $\hat{\tau}_t$ of the perturbation and its phase shift Θ_τ follow from
507 the complex amplitude Z_τ . Similarly, and with the help of (2.21b), we obtain for the
508 perturbed interfacial gas pressure p_g^* :

$$\begin{aligned} p_g^* &= \hat{p}_g \exp[i\alpha(x - ct) + i\Theta_p], \\ \hat{p}_g &= \sqrt{\operatorname{Re}\{Z_p\}^2 + \operatorname{Im}\{Z_p\}^2}, \quad \Theta_p = \arctan[\operatorname{Im}\{Z_p\}/\operatorname{Re}\{Z_p\}], \end{aligned} \quad (5.4)$$

$$Z_p = \underbrace{\underline{c}\hat{\psi}' + U_g'\hat{\psi}}_{Z_p^\rho} + \underbrace{\Pi_\mu(i\alpha\Pi_\rho)^{-1}(\hat{\psi}''' - \alpha^2\hat{\psi}')}_{Z_p^\mu} - \underbrace{\cos\beta\hat{\phi}\underline{c}^{-1}}_{Z_p^\beta},$$

509 where the components Z_p^ρ and Z_p^μ of the complex amplitude Z_p , through (2.21b), can
510 be traced directly to the inertial and viscous contributions in the streamwise momentum
511 equation, whereas Z_p^β comes from the last term in (2.18).

512 Figures 18b and 18c represent the perturbation profiles of the shear stress (5.3) and
513 pressure (5.4) for the three studied cases, allowing to discern their phase shifts $\Theta_\tau - \Theta_h$
514 and $\Theta_p - \Theta_h$ with respect to the film thickness perturbation profile (panel 18a). These
515 profiles have been scaled differently for the three cases, in order to facilitate a comparison
516 within the same graph. The respective perturbation amplitudes $\hat{\tau}_t$ and \hat{p}_g and that of
517 the gas-side normal viscous stress $\hat{\tau}_n$ are given in table 5. In addition, table 6 compares
518 the weight of the inertial and viscous contributions to the pressure perturbation's phase
519 shift Θ_p via the arguments $\operatorname{Im}\{Z_p^\rho\}/\operatorname{Re}\{Z_p\}$ and $\operatorname{Im}\{Z_p^\mu\}/\operatorname{Re}\{Z_p\}$.

520 From these data, we may conclude: (i) the perturbation of the normal viscous stress is
521 negligible compared to that of the pressure; (ii) the perturbation amplitudes $\hat{\tau}_t$ and \hat{p}_g
522 increase both with confinement and gas velocity; (iii) for strong confinement, the phase
523 shift with respect to the film thickness perturbation is fixed to $\Theta_\tau - \Theta_h \approx \pi$ for the shear
524 stress perturbation and $\Theta_p - \Theta_h \approx \pi/2$ for the pressure perturbation, and this does not
525 change significantly when the gas velocity is increased; (iv) for strong confinement, the
526 phase shift Θ_p is entirely governed by viscous effects and inertia plays no role.

527 The $\Theta_p - \Theta_h \approx \pi/2$ phase shift of the pressure perturbation means that its second
528 derivative $\partial_{xx}p_g^*$ vanishes at the film thickness extrema. As a result, its effect on stability
529 is neutral, i.e. it neither reinforces nor attenuates the film thickness perturbation. This
530 phase shift follows directly from the dominance of viscous effects in the gas flow,
531 i.e. $\operatorname{Im}\{Z_p^\mu\} \gg \operatorname{Im}\{Z_p^\rho\}$. Indeed, in this inertialess limit, the pressure gradient is
532 proportional to one over the third power of the local gas layer thickness (locally developed
533 Poiseuille-Couette flow). Conversely, in the case of inertia-dominated flow, the pressure
534 would be proportional to the square of the local gas velocity (following the Bernoulli
535 equation) and thus its minimum would arise at the maximum of the film thickness
536 perturbation, giving an anti-cyclic ($\Theta_p - \Theta_h = \pi$) phase shift. This would have a
537 destabilizing effect. However, for our case of strongly-confined gas flow, $\Theta_p - \Theta_h \approx \pi/2$
538 and the pressure perturbation plays no role in determining the state of stability. This
539 is in agreement with Trifonov (2017), who, in his stability analysis, did not find any

η	\tilde{H} (mm)	M	$\hat{\tau}_t/\hat{\tau}_t^{(\eta=12.4)}$	$\hat{p}_g/\hat{p}_g^{(\eta=12.4)}$	$\hat{\tau}_n/\hat{p}_g$
12.4	6.43	1	1.0	1.0	0.09227
3.6	1.87	1	5.30231	17.2737	0.00531
3.6	1.98	50	12.5435	61.1488	0.04583

TABLE 5. Ratio between the perturbation amplitudes (computed through (2.18)) of the stresses exerted by an air flow on a water film inclined at $\beta = 3^\circ$ with $Re_l = 23.9$: tangential shear stress $\hat{\tau}_t$, static pressure \hat{p}_g and normal viscous stress $\hat{\tau}_n$. Wavenumber: $\alpha = 0.0025$.

η	\tilde{H} (mm)	M	$\text{Im}\{Z_p^\rho\}/\text{Re}\{Z_p\}$	$\text{Im}\{Z_p^\mu\}/\text{Re}\{Z_p\}$	$\text{Im}\{Z_p^\beta\}/\text{Re}\{Z_p\}$
12.4	6.43	1	0.08325	0.99695	0.05462
3.6	1.87	1	0.04240	14.7428	0.02596
3.6	1.98	50	0.00324	7.68910	0.00021

TABLE 6. Different contribution to the pressure perturbation's phase shift according to (5.4). Flow conditions as in table 5.

540 [unstable mode related to the Kelvin-Helmholtz instability for flow conditions similar to](#)
541 [those studied here.](#)

542 Thus, it is the perturbation of the interfacial tangential shear stress that is responsible
543 for the confinement-related stabilization of the falling liquid film, as suggested by Tilley
544 *et al.* (1994). In particular, this results from the $\Theta_\tau - \Theta_h \approx \pi$ phase shift of the shear stress
545 perturbation with respect to the film thickness perturbation h^* , which remains virtually
546 unchanged over a wide range of confinement levels and gas velocities (see panels 19a and
547 19b). Due to this, the adverse tangential shear stress τ_t (5.2) is more negative at the wave
548 hump and less negative at the wave trough. This is principally caused by the change in
549 cross section for the gas flow, which is smaller above the wave hump and greater above
550 the wave trough.

551 So, how does the profile of the tangential stress perturbation τ_t^* in figure 18b stabilize
552 the film? First, we recall that the driving mechanism of the Kapitza instability is inertia,
553 which causes the liquid flow rate $q_l(x, t)$ to lag behind its inertialess target value:

$$q(x, t) = -K_l \frac{h^3}{3} + \tau_t \frac{h^2}{2}, \quad (5.5)$$

554 where K_l is defined in (2.9a). Just upstream of the wave hump, where q decreases in time
555 as the wave passes by, the actual flow rate q_l is slightly too high, while it is slightly too
556 low just downstream of the hump, where q increases in time. The resulting discrepancy
557 in flow into and out of the hump causes it to grow. The effect is stronger the more the
558 actual flow rate q_l lags behind its target value q , and that depends on how steeply q
559 changes within the wavy film.

560 According to (5.5), the effect of gravity (through the h^3 term in (5.5)) tends to increase
561 variations in q between the wave hump and wave trough as a result of the change in film
562 thickness h . The adverse tangential interfacial stress τ_t , through its perturbation τ_t^*
563 (figure 18b), counter-acts this effect. Indeed, τ_t^* acts to homogenize the flow rate within
564 the film, slowing the flow in the wave hump to a greater extent than underneath the
565 wave trough. This is a direct result of the phase lag observed between the film thickness
566 and the shear stress perturbation (figure 18), and the effect increases with the relevance
567 of the second term in (5.5).

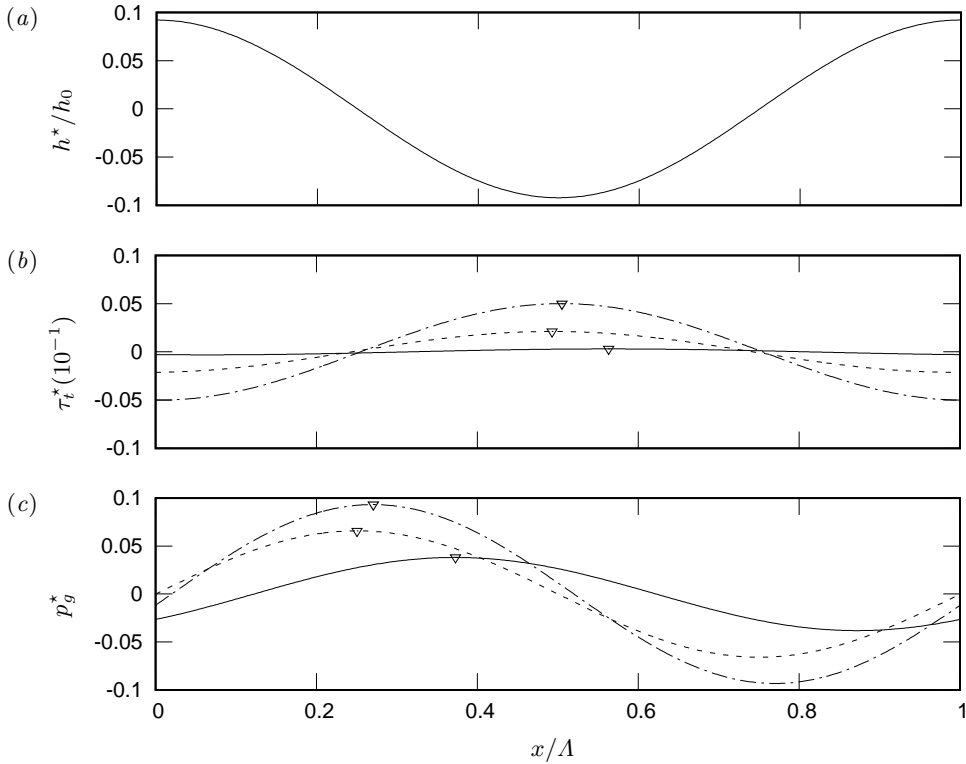


FIGURE 18. Water film at $\beta = 3^\circ$: linear perturbation of (a) film thickness (5.1), (b) tangential stress (5.3) and (c) **interfacial gas pressure** (5.4) associated with a particular eigenmode, i.e. $\alpha = 0.0025$ (wavelength $\Lambda = 2\pi/\alpha$). Solid line: $\eta = 12.4$ aerostatic ($M = 1$); dashed line: $\eta = 3.6$ aerostatic ($M = 1$); dash-dotted line: $\eta = 3.6$ counter-current gas ($M = 50$). Triangles in panels (b) and (c) mark the location of the maximum. Pressure perturbations are scaled with different coefficients to facilitate the representation: 60 for $\eta = 12.4$; 600 for $\eta = 3.6$ aerostatic; 1500 for $\eta = 3.6$ with $M = 50$.

568 We quantify this by considering the linear perturbation amplitude of the inertialess
 569 flow rate (5.5) and by evaluating the contributions due to gravity \hat{q}^g (actually, this term
 570 also subsumes the effect of pressure) and shear stress \hat{q}^τ :

$$\hat{q} = \underbrace{-K_l h_0^2 \hat{h}}_{\hat{q}^g} + \underbrace{\frac{1}{2} h_0^2 \hat{\tau}_t + h_0 T_t \hat{h}}_{\hat{q}^\tau}. \quad (5.6)$$

571 Panels (c) and (d) of figure 19 show that the ratio $|\hat{q}^\tau/\hat{q}|$ increases with confinement
 572 and gas velocity, which is in accordance with the increase in stabilization observed in
 573 figures 11 and 14.

574 The tangential shear stress also produces an indirect stabilizing effect. It is known that
 575 an adverse interfacial shear stress diminishes the celerity c of surface waves traveling on
 576 falling films (Aleksenko *et al.* 2009; Samanta 2014). This can be explicitly shown in the
 577 limit of infinitely-long waves ($\alpha \rightarrow 0$):

$$c|_{\alpha \rightarrow 0} = -K_l h_0^2 + T_t h_0. \quad (5.7)$$

578 The wave celerity c determines how fast the flow rate within the film would change in
 579 the inertialess limit as a wave passes by and reducing it weakens the inertial instability

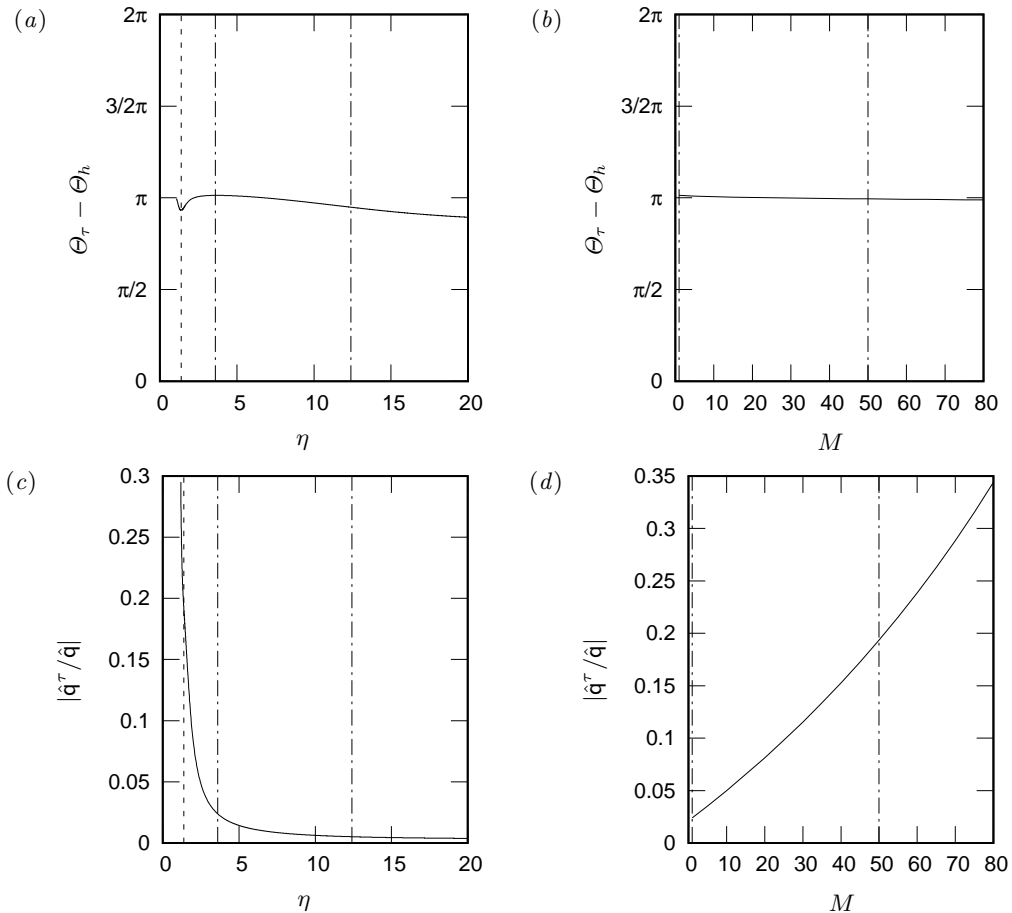


FIGURE 19. Water film at $\beta = 3^\circ$: phase lag $\Theta_\tau - \Theta_h$ of the perturbations of the film thickness and interfacial shear stress. (a) Variation with η with aerostatic gas ($M = 1$); (b) variation with M at $\tilde{H} = 1.87$ mm ($\eta = 3.3 - 3.6$). Contribution of the interfacial tangential shear stress to the inertialess flow rate (5.6): (c) variation with η under aerostatic gas ($M = 1$); (d) variation with M at $\tilde{H} = 1.87$ mm ($\eta = 3.3 - 3.6$). The vertical dash-dotted lines mark the three cases represented in figure 18; the vertical dashed line in (a) and (c) marks $c_r = U|_{h_0}$.

580 mechanism. In our stability results (figures 11b and 14b), we have observed c to diminish
 581 with increasing confinement and gas velocity, coinciding with an increased stabilization
 582 of the film.

583 6. Conclusion

584 Using linear stability calculations and experiments, we have revisited the stability of a
 585 falling liquid film flowing down an incline in interaction with a strongly-confined laminar
 586 gas flow and subject to the Kapitza instability. The gas is either subject to an aerostatic
 587 pressure gradient or flows counter-currently.

588 Tilley *et al.* (1994) suggested that confining the gas phase should stabilize the Kapitza
 589 instability, but concluded that this effect is small for water films in contact with air in
 590 the studied parameter range. For the same fluid combination, we have found a parameter
 591 range where the confinement-related stabilization is very strong, up to the point of

592 suppressing the Kapitza instability altogether. We have shown that the stabilizing effect
 593 of confinement is stronger when the inclination is small ($\beta = 1^\circ - 8^\circ$) and/or the Reynolds
 594 number is small, and we have determined the critical confinement necessary to fully
 595 stabilize the film as a function of these two parameters (figure 12). For example, a water
 596 film of 0.5 mm thickness flowing down a wall inclined at 3° can be fully stabilized by
 597 placing a confining plate at a distance of 1 mm from the incline. Water films, which have a
 598 high Kapitza number, are found to feel earlier the confinement-induced stabilization than
 599 other working fluids with lower Kapitza number, such as aqueous solutions of glycerin
 600 or dymethylsulfoxide. The suppression of the Kapitza instability is observed both when
 601 the gas is subject to an aerostatic pressure gradient and when it flows counter-currently.

602 In the latter case, at the strong confinement levels we have focused on, the stabilization
 603 is monotonically intensified with increasing gas flow velocity and occurs over the entire
 604 range of unstable wavenumbers. This is different from observations by Alekseenko *et al.*
 605 (2009), Vellingiri *et al.* (2015), and Trifonov (2017), who found a non-monotonic effect
 606 of the gas velocity and an only partial destabilization of the wave spectrum for weakly-
 607 confined turbulent gas flows. We have recovered these results also for the laminar gas
 608 flow conditions considered here, when reducing the confinement level. In this context, we
 609 have identified a confinement threshold above which the gas *destabilizes* the film at all
 610 wavenumbers (see figure 17).

611 We have identified the mechanism responsible for the instability suppression. As
 612 suggested by Tilley *et al.* (1994), viscous stresses in the gas play a decisive role. In
 613 particular, it is the tangential viscous stress exerted by the gas on the film surface that
 614 is decisive. The key of the stabilization mechanism is that there is a phase shift of almost
 615 exactly half a wavelength between the film thickness perturbation and the associated
 616 shear stress perturbation. Consequently, the adverse shear stress is greater at a wave
 617 hump than at a wave trough. This tends to homogenize the local-equilibrium flow rate
 618 within the liquid film, thus counter-acting the effect of gravity, which acts to force larger
 619 flow rates where the film is thick (wave hump) and smaller ones where the film is thin
 620 (wave trough). As a result, the inertia-related lag between the actual local flow rate and
 621 its equilibrium value is reduced and this weakens the inertia-driven mechanism of the
 622 Kapitza instability.

623 The stabilization mechanism increases whenever the contribution of the interfacial
 624 shear stress in the equilibrium flow rate becomes stronger with respect to that of gravity
 625 (and pressure). This is the case when the inclination is weak, the film is thin (Re_l is
 626 small) and the counter-current gas flow is strong. It is under those conditions that our
 627 stability calculations and experiments display the strongest stabilization.

628 We point out that confining the gas phase also contributes to the observed stabilization
 629 through an indirect effect, i.e. by reducing the wave celerity. Reducing the wave speed
 630 reduces inertia-induced flow rate imbalances within the liquid film, which again weakens
 631 the Kapitza instability.

632 Acknowledgements

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 634 01 of the French Agence Nationale de la Recherche.

635 Appendix A. Numerical validation of the stability code

636 To validate our two-phase linear stability code numerically, we have compared its
 637 predictions to data from the literature for three examples of interfacial instability.

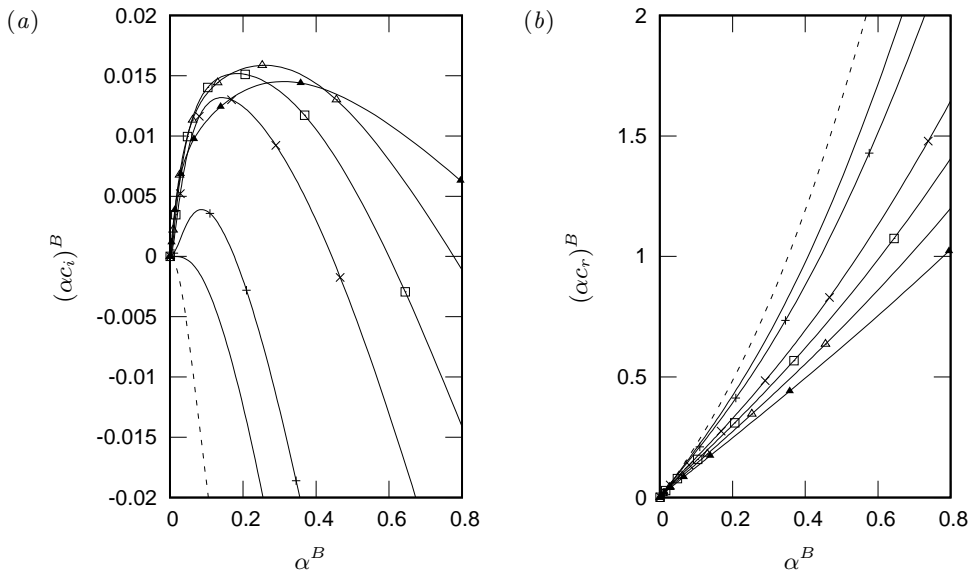


FIGURE 20. Comparison of (a) temporal growth rate and (b) angular wave frequency between our work and Brevdo *et al.* (1999) (figure 2 there). Parameter values: $\beta = 4.6^\circ$, $\nu_l = 5.02 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$, $\gamma = 69 \times 10^{-3} \text{ N m}^{-1}$, $\rho_l = 1130 \text{ kg m}^{-3}$, $Ka = \gamma(\rho_l g^{1/3} \nu_l^{4/3})^{-1} = 331.85$. Values of the Reynolds number $Re^B = 3/2 Re_l$: 10 (dashed line), $Re_{cr}^B = 5/4 \cot \beta$ (line), 20 (plus), 40 (cross), 60 (empty squares), 100 (empty triangles), 200 (filled triangles). Note that α^B , $(\alpha c_r)^B$ and $(\alpha c_i)^B$ are scaled as in Brevdo *et al.* (1999), i.e. using the Nusselt film thickness and the free-surface velocity as length and velocity scales.

638 Figure 20 compares our stability calculations with landmark calculations of Brevdo
 639 *et al.* (1999) for the case of an inclined liquid film falling in a passive atmosphere. We
 640 point out that the conditions studied by Brevdo *et al.* (1999) concern a liquid film flowing
 641 in a passive atmosphere ($\Pi_\mu = \Pi_\rho = 0$), whereas in the present work we have considered
 642 an active phase.

643 Figure 21 presents the neutral stability diagram for the long-wave Yih instability
 644 (horizontal pressure-driven two-layer flow with $\Pi_\rho = 1$) as obtained by Yiantsios &
 645 Higgins (1989), in comparison with our calculations (circles).

646 Finally, table 7 compares our stability data to those of Yiantsios & Higgins (1988) and
 647 Tilley *et al.* (1994) for a pressure-driven two-layer flow through a horizontal channel, in
 648 the absence of gravity. In addition to the numerical punctual values of table 7, we also
 649 present in figure 22 the real and imaginary part of the wave celerity as a function of the
 650 wavenumber for the case at $Q^T = 1.6$ and $We^T = 0.2$ (forth and seventh row of table 7).

651

652 Appendix B. Sensitivity of the comparison in figure 8 w.r.t. the 653 inclination angle β

654 To remedy the insufficient resolution of the inclinometer, the inclination angle β of
 655 the experimental setup was obtained by fine-tuning (within the precision range $\pm 0.05^\circ$
 656 of the inclinometer) the value of β in the numerical calculation of the neutral stability
 657 curve for the unconfined case, until this matched with the corresponding experimental
 658 curve. Figure 8 shows the best match of these curves for one set of conditions, obtained
 659 for an inclination angle of $\beta = 1.69^\circ$.

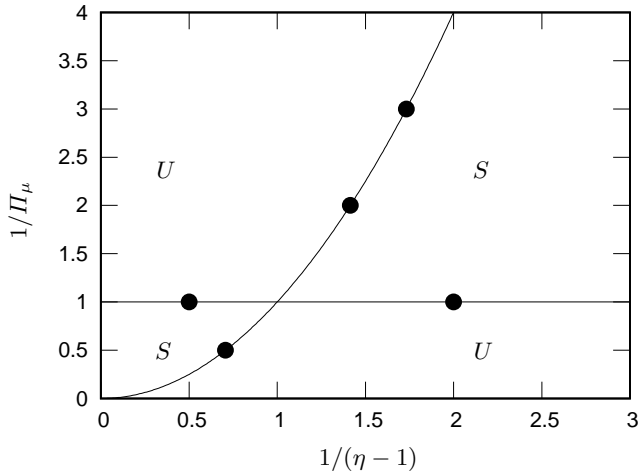


FIGURE 21. Neutral stability diagram for a horizontal pressure-driven two-layer flow with $\Pi_\rho = 1$: comparison of our numerical results (circles) with figure 2b of Yiantsios & Higgins (1989) (lines). Stable regions are denoted by S , unstable regions by U .

α^T	Q^T	We^T	$(c_r, c_i)^T$ (YH)	$(c_r, c_i)^T$ (TDB)	$(c_r, c_i)H\mathcal{L}^{-1}$ (this work)
20	0.32	0.016	(0.399984 , -0.0066148)	(0.399986 , -0.0066151)	(0.399984 , -0.0066151)
20	0.32	0.008	(0.399992 , -0.0032796)	(0.399992 , -0.0032796)	(0.399991 , -0.0032798)
20	1.6	0.4	(1.99814 , -0.166982)	(1.99815 , -0.166982)	(1.99814 , -0.166982)
20	1.6	0.2	(1.99912 , -0.082368)	(1.99912 , -0.082369)	(1.99912 , -0.082369)
40	0.32	0.008	(0.3999976, -0.003325)	(0.3999977, -0.003325)	(0.3999974, -0.003325)
40	0.32	0.004	(0.3999988, -0.001658)	(0.3999989, -0.001658)	(0.3999985, -0.001658)
40	1.6	0.2	(1.99971 , -0.083328)	(1.99971 , -0.083329)	(1.99971 , -0.083328)
40	1.6	0.1	(1.999858 , -0.04150)	(1.999859 , -0.04150)	(1.999857 , -0.04150)

TABLE 7. Real and imaginary part of the wave celerity c : $\Pi_\mu = 1/5$, $\Pi_\rho = 1$ and $\eta = 2$. Comparison between our work and results of Yiantsios & Higgins (1988) (table I there) and Tilley *et al.* (1994) (table I there). Scales used in Tilley *et al.* (1994) are related to ours as: $\alpha^T = \alpha H\mathcal{L}^{-1}$, $(c_r, c_i)^T = (c_r, c_i)H\mathcal{L}^{-1}$, $Q^T = Re_l + Re_g$, $We^T = WeH\mathcal{L}^{-1}$.

660 To demonstrate the sensitivity of this comparison w.r.t. the inclination angle, figure 23
 661 shows this comparison for two slightly different values of β , i.e. $\beta = 1.68^\circ$ (panel *a*) and
 662 $\beta = 1.70^\circ$ (panel *b*).

663 Appendix C. Measured streamwise evolution of the wave-amplitude

664 Figure 24 shows the measured streamwise evolution of the wave amplitude along the
 665 channel for the “unconfined” (figure 4*b*) and the confined (figure 4*a*) configurations at
 666 $Re_l = 38.7$, for the same inlet forcing amplitude and frequency. The film is unstable in
 667 both cases. The decrease of the wave amplitude induced by strengthening the confinement
 668 (changing $\eta = 22$ to $\eta = 6.65$) is approximately 7%.

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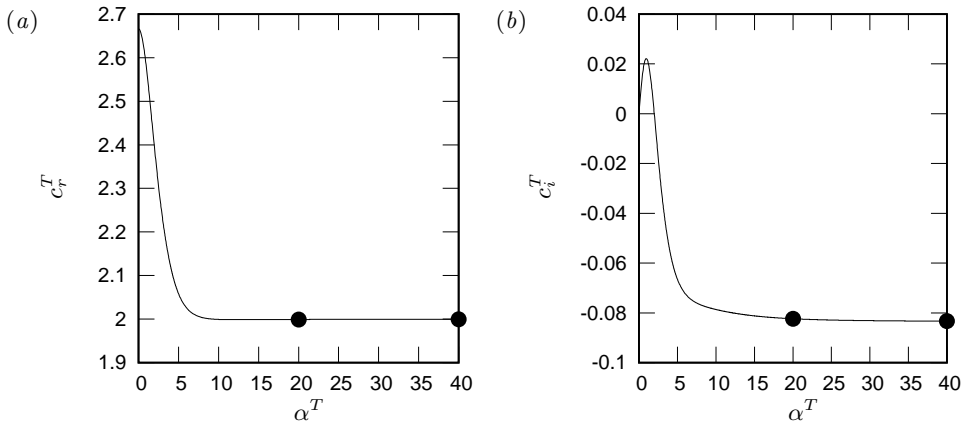


FIGURE 22. (a) Real and (b) imaginary wave celerity c as a function of the wavenumber: $\Pi_\mu = 1/5$, $\Pi_\rho = 1$, $\eta = 2$, $Q^T = 1.6$ and $We^T = 0.2$ (same parameters as the fourth and seventh row of table 7, whose numerical values corresponding to our calculations are marked with filled circles). Scales used in Tilley *et al.* (1994) are related to ours as: $\alpha^T = \alpha H \mathcal{L}^{-1}$, $(c_r, c_i)^T = (c_r, c_i) H \mathcal{L}^{-1}$, $Q^T = Re_l + Re_g$, $We^T = We H \mathcal{L}^{-1}$.

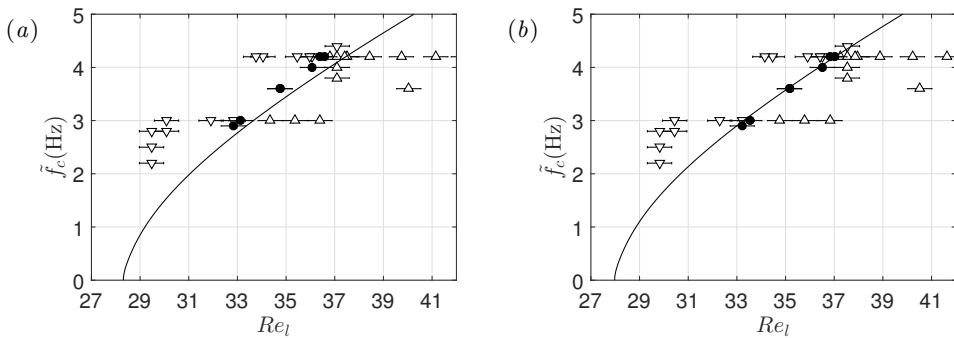


FIGURE 23. Sensitivity of the comparison in figure 8 w.r.t. the assumed inclination angle β : (a) $\beta = 1.68^\circ$; (b) $\beta = 1.70^\circ$.

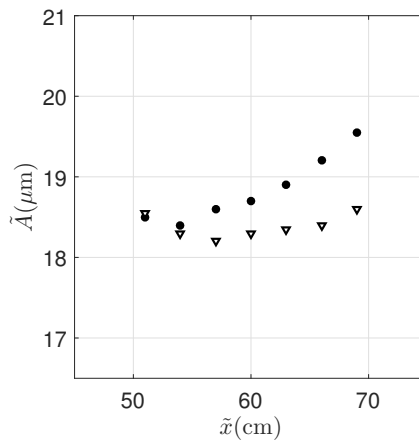


FIGURE 24. Experimental streamwise evolution of the wave amplitude for a water film at $Re_l = 38.7$ and $\beta = 1.69^\circ$ with aerostatic air. Open triangles: confined ($\eta = 6.65$); filled circles: “unconfined” ($\eta = 22$). Forcing frequency $\tilde{f} = 3.6$ Hz.

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