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Study of and work on mathematics teacher talk in classroom teaching

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Abstract

In classroom research on mathematics and language, analyses of teacher talk typically address features in this talk that make or do not make it productive with respect to the interactive communication of mathematical contents. These features are, nonetheless, much less often studied from the perspective of the explicit content-based communication of specialized mathematical meanings, and instead rather oriented to more general and equally important considerations regarding, for example, the kind of questions teachers ask their learners, or the kind of answers they provide to follow up learner contributions. In this conference paper, I take a mathematical-linguistic focus in order to present two verbal tools in language —i.e. naming and lexicalization— aimed at supporting content-based teaching of mathematics in school classrooms for the communication of specialized meanings related to the content at play. I argue that these verbal tools can be used to study content-based mathematics teaching, and particularly to conduct research-informed work on this teaching with groups of mathematics teachers in developmental settings.

Introduction

We can easily agree on the relationship between ‘good’ mathematics teacher talk and ‘good’ mathematics teaching in the school classroom, even regardless of the meanings considered for the quality of ‘good’. In mathematics education research, however, interest in the study of mathematics teacher talk is still in the beginnings of investigating the production of classroom talk to communicate specialized mathematical meanings for selected contents. While general considerations around the kind of questions teachers ask their learners, for example, or the kind of answers they provide to follow up learner contributions, there are a number of reasons for a shift in emphasis towards the study of and work on content specificity in mathematics teacher talk without compromising the research interest in context specificity. Firstly, the talk of the teacher in the classroom is a potential source of concrete mathematical meanings, and hence it becomes also a potential resource for the generation of opportunities of having access to, negotiating and learning these meanings. Secondly, since access to specialized mathematical meanings in classroom interaction is essential for school mathematics learning to happen, their communication needs to be offered in explicit and focused verbal ways —except for classrooms in which non-verbal languages are dominant—, alongside other semiotic modes in language and over the course of participation in mathematically rich discourse practices. Thirdly, developmental work with mathematics teachers could greatly benefit from research-informed demanding tasks on mathematics content teaching and language use oriented to the explicit communication of specialized mathematical meanings.

In what comes, I take a mathematical-linguistic focus in order to present two verbal tools in language —naming and lexicalization— aimed at supporting mathematics content teaching in school classrooms for the explicit communication of specialized meanings related to the content at play. I argue that these verbal tools can be used to study mathematics content teaching, and particularly to conduct research-informed work on this teaching with groups of mathematics teachers in developmental settings. The equation and the quadratic equation concepts are the content for illustrating my

arguments in the secondary school level, and the educational policies and mathematical pedagogies in Catalonia, Spain, are the context of the lesson and developmental data. Many of the reflections here can be found in three complementary texts (Planas, 2018, 2019, 2021), where details of analytical methods and theoretical framework building are provided.

A mathematical-linguistic focus in the study of mathematics teacher talk

There is a wide range of theoretical-analytical frameworks for the study of language, mathematical meaning making and communication in mathematics classrooms (Planas & Schütte, 2018), but not all of them offer tools easily applicable to the analysis of concrete mathematical meanings communicated through the resources of lexicon and grammar in the language system. The theory of Systemic Functional Grammar (SFG; Halliday, 1978, 1985) takes the realization of linguistic forms into meaning as mediated in practice, and fits in well with sociocultural views of teaching and of teacher talk that address the rules of both the language system and the relevant social context as mediators. Moreover, the adoption of SFG tools and reflections enables the integration of tools and reflections from other sociocultural frames to account for the production of mathematics teacher talk in content teaching, such as the Mathematics Discourse in Instruction frame (MDI; Adler, 2017, 2021). Although in my recent research I pay special attention to the mathematical-linguistic focus, I see this emphasis particularly in connection to some of the MDI tools and reflections, and hence close to mathematical-sociolinguistic stances. Despite not only verbal tools and spoken modes of communication are important in mathematics teaching and in the SFG and the MDI frames, these are the ones in focus in this paper.

Halliday (1978, 1985) established that written and spoken languages follow rules of organization which can be distinguished in the lexicogrammar, and which have an effect on meaning generation. The forms and functions of lexicon and grammar are related in ways that provide the meaning potential in language as well as the possibilities of its realization for concrete meaning making. In the case of the teacher talk in the mathematics classroom, part of the meaning potential is expected to be realized to communicate certain mathematical meanings to be taught and learned. Whereas mathematics teaching is a broad practice within which a diversity of social, cultural, political and linguistic events take place and are enhanced, they can all be thought as mediated by the use of tools in language and the lexicogrammar peculiarities in them, specifically comprising functions situated into linguistic forms. The study of mathematics teaching can thus be approached through the study of some of these tools and their peculiarities in classroom contexts of school mathematics. I provide ideas for such an approach by presenting the verbal tools of naming and lexicalization, and interpreting them in relation to the teaching of the equation and the quadratic equation concepts. Rather than presenting a theorization for these tools, I describe them briefly, exemplify their potential, and comment on possibilities of realization in content-and-context-specific focused ways —i.e. the equation and the quadratic equation concepts, and some of the classroom and developmental settings in my part of the world—. This does not go without saying that naming and lexicalization in the talk of the teacher for the teaching of the concepts mentioned are just two amongst the many resources in language for making specialized mathematical meanings available to learners in classroom communication.

The power of naming or what is in a word name?

In the communication of the meanings of the equation and the quadratic equation concepts in classroom teacher talk: Which are the word names involved and what are the implications in teaching (and learning) of the choice of some word names over others? These are two basic questions that guide the exploration of the verbal tool in language called naming. Word names or the product of naming are linguistic units with the potential of encoding meanings for the objects or phenomena named. The process of naming is itself complex in how meanings are encoded to the names and the objects named; it is, for example, quite common to rely on first several letters to infer similarities amongst names such as equation, equality and equivalence, and amongst their various meanings even though there may be no clarity or precision on either similarities or differences. The introduction of the equation concept in school teaching is certainly linked to the presentation of a family of word names in the language of instruction —sometimes referred to as the vocabulary for that concept—, which in turn may differ across contexts of culture. Success in teaching the equation concept and some of its particularizations —linear equation, quadratic equation, system of linear equations, etc.— thus lies in success in teaching a family of word names and their specialized mathematical meanings with respect to the concept. This success, in turn, lies in the synergy established amongst verbal tools —e.g. lexicalization and naming—, amongst verbal, non-verbal and paralinguistic tools —e.g. naming, graphical representations and hand gestures—, and amongst linguistic and paralinguistic tools and mathematically rich discourse practices —e.g. naming, graphical representations and mathematical practices of modelling—.

Naming, as considered in the particular case of the spoken language of the mathematics teacher in the classroom during content teaching, can be understood in terms of the production of word names with the potential of encoding specialized mathematical meanings of the content in focus. Naming is then a verbal tool described in reference to the restrictive process of encoding precise mathematical meanings through individual words in a particular language —e.g. Catalan, French or English—. For Halliday (1978), this is a fundamental process in the system of language which can involve the construction of newer words, and newer meanings, as either substitution or expansion, for already known words. The word name for angle in Catalan is lexically meaningful in the everyday when talking, for example, of the car blind spot —*l'angle mort*—; however, for this familiar and already existing word, meanings other than the space in which someone's view is obstructed need to be produced in the languages of school geometry. If we take the word name for equation in Catalan, the situation is quite different because *equació* is not a common word in the everyday and learners may well hear it for the first time in classroom. Some of the problems with naming are then understandably complex. A name chosen in one language and context of culture may already be a word with precise meanings in another language and context of culture. That is, each naming points to a set of meanings depending on conditions suggested by the many contexts of culture and languages in which the word name may be placed. Either way, being the name *angle* or *equació*, the teaching of the mathematical concept encoded requires much more than teaching the words and gaining (newer) familiarity with them in the practice. Naming, therefore, can be conceived from the simultaneous perspective of both providing word names and encoding specialized meanings for the mathematical objects named.

In line with these reflections, the choice and use of word names and of their explicit interpretations is a central feature of the teacher talk in the classroom, with implications in the teaching and construction of the mathematical contents. Putting the names of

equation and equivalence together and repeatedly in sentences, or the names of equation and solution together and repeatedly instead, communicates different relational meanings for the equation concept as well as provides access to different more or less procedural approaches to secondary school algebra. The significance of the power of naming for producing meaning in mathematics teaching is very much outlined in the MDI frame (Adler, 2007, 2021), where the mathematics teacher holds the authority of naming what an equation is, for example, and of deciding the family of word names to be said, taught and assessed in the language of instruction. There is power both in the choice and use of certain word names over others, and in the often-subtle consequences that flow from these types of choices and use. Of course, the process of naming involves all the participants in the mathematics classroom but it is the teacher who owns the authority to establish the relevant names in school mathematics —see the tool of legitimating, sometimes based on principles of mathematics but not always, in Adler (2017, 2021)—.

Rather than going into the relationship between naming and legitimating —which I see interestingly examined in several ongoing applications of the MDI frame, e.g. Mousvold & Fauskanger, 2018—, in the next subsection I address a much less studied relationship. I connect naming with lexicalization of sentences with the potential to provide precise mathematical meanings behind the names for the mathematical objects, its qualities and relationships with other objects, so that both verbal tools can be viewed as contributing to and interacting in the communication of specialized meanings intended in teaching.

The power of lexicalization or what is in a sentence?

In the communication of the meanings of the equation and the quadratic equation concepts in classroom teacher talk: Which are the sentences involved and what are the implications in teaching (and learning) of the choice and use of some sentences — possibly made up through the choice and use of some word names— over others? Once more, these are two basic questions that guide the exploration of the verbal tool in language now called lexicalization —or lexicalizing if we wish to emphasize the process—. Sentences or the product of lexicalization are compound linguistic units or forms of grammar made of words and eventually word names, with the potential of encoding meanings for objects and phenomena. In particular, there are sentences with the potential to communicate and explain mathematical meanings specialized in the contexts of a certain school content and culture and, thus, encoded in the form of word names and grammars to be taught, learned, and assessed. The teaching of the equation and the quadratic equation is then not only linked to the introduction of a family of word names in the language of instruction and the specialized mathematical meanings behind, but also to the presentation of forms of grammar or sentences aimed at scaffolding these meanings through mathematically rich discourse practices, other tools available in language and various communication modes.

Lexicalization, as considered in the spoken language of the mathematics teacher in content teaching, can be understood in terms of the production of sentences with the potential of encoding specialized mathematical meanings of the content in focus. Lexicalization is then a verbal tool described in reference to the process of encoding precise meanings through forms of grammar in a particular language —e.g. Catalan, French or English—. Here, I am using ‘form of grammar’ in a wide sense to imply a variability of grammars including for example combinations of verbal languages and mathematical symbols, Hallidayan grammatical-semantic processes —i.e. material,

relational, mental, verbal, behavioural, and existential—, or switches between two or more languages within the same sentence and hence the predictable interaction between lexical elements and grammatical rules from these languages, amongst other options. Rather than looking at them linguistically, any of these forms of grammar are taken here as to the ways in which they are relevant to the production of specialized mathematical meanings in certain contexts of culture like secondary school mathematics.

For Halliday (1978), and like it happens with naming, lexicalization is a complex process in the system of language which involves the production of newer forms of grammar like those made up of letters and mathematical symbols, newer meanings beyond those encoded in the individual words conforming a given form of grammar, and even the encoding of newer meanings for already known forms of grammar existing in the everyday such as ‘being equal to.’ Like it occurs with naming again, lexicalization can be conceived from the simultaneous perspective of both providing sentences and encoding specialized meanings for the mathematical objects being named or referred to within the forms of grammar. The sentence in English ‘Add or subtract both sides of the equation by the same number’, for example, names the word for the equation concept but it does not provide specialized mathematical meanings for this concept because it is a form of grammar with material verbs primarily used to encode procedural solving routines. Conversely, ‘ $x+5 = 3x^2-2$ is a quadratic equation’, or even the rather problematic inverse version, ‘A quadratic equation is $x+5 = 3x^2-2$ ’ —or still ‘A quadratic equation is x plus five equals three x square minus two’— with forms of grammar including relational verbs and compound word names like x *square*, can function to encode specialized meanings for the concept of quadratic equation into school algebra, and hence to more generally represent equations as more than symbolic statements that need to be solved.

Importantly, the more precise teaching of the relationship between understanding the concept and solving an equation requires sentences in teacher talk encoding specialized meanings for the relationship between the concepts of equation or quadratic equation and of equivalence/equivalent —in the following paragraph, I address the criticality of renaming quadratic equations or linear equations simply as equations, which is a common practice presumably of economy of talk in school lessons I have observed over many years across countries—. In the talk for teaching the equation concept —or possibly the linear equation or the quadratic equation concepts—, a variety of sentences or forms of grammar are to be offered in the communication with learners for the realization of a variety of specialized meanings. These sentences are resources in language with the potential of scaffolding the processes that learners need to develop in order to work out the mathematical meanings in the teaching. That said, while some of these sentences may not be ‘wrong’ in itself, neither grammatically nor mathematically, the meanings encoded may not be those intended by the teacher, or it may become a challenge to interpret them because they are not made explicit or remain unfocused in the talk.

While words, word names, sentences and forms of grammar are material and visible in teacher talk, the contexts of culture shaping their choices and uses are usually not visible or at least not visibly expressed. A context of culture can be given, for example, by the school curricular guidelines regarding the topic of equations in a world region. It is interesting that the secondary school mathematics curriculum in my part of the world states the teaching of the equation and the linear and quadratic equation concepts and of the resolution procedures for linear and quadratic equations, that is, those involving a constant and first-/second-order linear terms. In this context, it can happen that the

general name of equation is used for both linear and quadratic equations and the sentences for explaining what an algebraic equation is are not sufficiently general to include those types with more than one variable. Specialized meanings for the equation concept can thus be lexicalized through sentences that are actually talking about the cases of linear or quadratic equations, and even accompanied by graphical representations of slopes or parabolas and tasks for discussion of x - and y -intercepts in coordinate axes. ‘ $x+5 = 3x-2$ is an equation’ or ‘an equation is $x+5 = 3x-2$ ’, for example, in written English and symbolic mathematics are grammatically and mathematically correct, and at the same time restrictive meanings for the equation concept are encoded particularly in the second sentence. While these descriptive forms of grammar are ‘good’ and do not explicitly state for generality, teaching what an equation is requires the choice and use of additional sentences with explanatory forms —e.g. ‘this is (not) an equation because...’— in order to make sure that linear equations do not cover all the meanings involved.

A mathematical-linguistic focus in the work with mathematics teachers

Over the last decade, in order to improve the impact of mathematics professional development on classroom practice, increasing attention has been given to work with teachers guided by their teaching needs (e.g. Kazima, Jakobsen & Kasoka, 2016). It is an assumption that work with teachers on teaching is especially productive in terms of professional learning when the teachers take responsibility for the identification of challenges. These challenges may indeed vary across contexts of culture and groups of teachers as is explained elsewhere (e.g. Civil & Planas, 2004; Essien, Chitera & Planas, 2016; Setati & Planas, 2012). The two teachers, Jana and Maia, in the first round of the research and developmental project partially reported in Planas (2019, 2021) expressed various concerns with the teaching of equations. Besides, Jana and Maia mentioned the role that equations played as an icon of mathematical knowledge in their secondary schools with some of the families especially interested in test results on this content. They had several years of mathematics teaching experience, and worked in secondary schools in the area of Barcelona at the time of the collaboration. Their professional knowledge and my mathematical-linguistic view of the MDI and the SFG frames were the points from which we explored content teaching of equations. The results of learners of Jana and Maia in the annual tests for the past years had shown poor conceptual understanding of equations, on the one hand, and good performance in the resolution of linear and quadratic equations and in the translation of verbal texts into algebraic expressions, on the other hand. This poor conceptual understanding was revealed, for example and as explained by the two teachers, in the beliefs that: 1) two different linear/quadratic equations can have the same numerical solution(s); and 2) a linear/quadratic equation can be simplified into some numerical solution(s) without an operation sign. One of Maia’s learners had in fact written in a recent test, “... and so the equation is now +2 and -2.”

My response to the demand of the teachers was to interrogate their talk when teaching equations in the school lessons. In most of my prior collaborative experiences of work with teachers, they did not normally feel that the mathematical richness of the classroom practices can be hampered by under specificity in talk, nor do they tended to feel that language was a content in mathematics teaching (e.g. Planas & Civil, 2014; Planas & Setati, 2009). Hence, my response was in a sense a surprise for Jana and Maia who were, as said by them, expecting to engage in developmental work oriented to learn and practice mathematically demanding tasks of explanation and modelling around the qualities and types of equations considered in the local curriculum. We finally agreed on

exploring the possibilities of improving content teaching of equations through improving teacher talk and, from there on, situating this talk in relation to mathematical discourse practices that went beyond the training of operational procedures. For this, five 90-minute developmental sessions were carried out. Even though the two teachers graduated in mathematics, there was initial time for revising mathematical knowledge on the meanings for the equation concept and preceding the work driven by language-based tasks. In what comes, I present and reflect on the task about the revision and elaboration of sentences with equation-related word names, forms of grammar, and specialized meanings. Analytical and theoretical details can be found in journal publications already cited.

Developmental workshop on mathematics content teaching and teacher talk

In the developmental experience with Maia and Jana, there was a session organized to include the presentation of the task in Table 1, followed by the discussion of the teachers, and the final reflection with me. The English version of the task offered in this paper, with only some of the underlined examples of lexical elaborations produced during the session, does not pretend to reproduce these sentences as if they were exactly equal in meaning to those originated and discussed in Catalan; in fact, I avoid those whose ‘translation’ makes the communication of mathematical-linguistic arguments more difficult in that finer details of the source and target language systems seem to be of necessary knowledge. Overall, the sentences selected in Catalan for its representation in English in Planas (2021) and herein, show choices in language that can inform mathematics teachers in the use of meaning-focused sentences for the teaching of equations in the secondary school. Despite the original sentences taken from the lessons of Maia or Jana made good sense and could be said to work in the talk of the respective teacher (T), they were not followed or preceded by complementary sentences adding or particularising specialized meanings for the quadratic equation and the equation concepts, and were not placed into pedagogic general talk or application of routines either. Even so, by presenting the sentences separated from the immediate lesson context in which were said, and whereas this was done intentionally in the design of the developmental task, the meaning potential regarding (newer) specialized meanings became bigger.

Task. Sentence use into equation-focused talk	
Which are the meanings for the equation concept behind these sentences?	
<i>What does T say?</i>	<i>What could T say?</i>
We can solve a quadratic equation with formula.	We can solve a quadratic equation with formula. <u>That is, we can obtain the numerical values for x that solve the equation.</u>
We will modify the written initial equation.	We will modify the written initial equation. <u>In other words, we will look for ways of writing the same equation for the final application of the formula.</u>
Get a sequence.	Get a sequence, <u>which is to say, get a sequence of equivalent equations, or equations with the same solutions.</u>
Every equation, you change it a bit.	Every equation, you change it a bit. <u>By changing it a bit, I mean adding, subtracting, multiplying or dividing both sides with the same numbers so that the solutions do not change.</u>
You have to use the transposition rules.	You have to use the transposition rules. <u>That is, the rules for the generation of equivalent equations.</u>
You go mapping one written form to another up to the	You go mapping one written form to another up to the general formula on the board. <u>All the equations will be the</u>

general formula on the board.	<u>same because the same numerical values solve all them.</u>
All this around the equal sign makes the equation.	All this around the equal sign makes the equation. <u>I mean these two expressions with numbers, coefficients and one variable, one on each side of the equal sign.</u>

Table 1. English version of the task and some elaborations in the right column

For each given sentence —left column, Table 1—, the written practice was organised into individual writing, group discussion of the two individual proposals, and final shared writing on the worksheet. In this process, Jana and Maia decided which were the specialized mathematical meanings for equation and quadratic equation whose communication they wanted to prioritise and facilitate in the creative re-elaborations of the original sentences —right column, Table 1—. I participated by pushing them to think the individual writing as an opportunity for referring to the specialized meanings that they missed most in common conversations with school learners and test results from them, and that could seemingly remain implicit or unfocused in the original sentences though the meaning potential was somehow suggested. These same examples of shared writing for equation-related sentences below were already published in Planas (2021, pp. 282-3), and I have added one more unpublished example in the last line of Table 1. The alternative sentences rewritten by the teachers are not solutions in the sense of totally adequate or perfect; they are just more ideal in the sense of being closer to the idea of communicating specialized mathematical meanings within languages of equations and quadratic equations beyond the representation of solving and operational routines. They are also more ideal in the sense of taking the opportunity of naming relevant equation-related specialized terms such as the word names for the variable and the coefficient concepts. Not only the naming but also some lexicalization is still needed for preventing the common confusion between unknown coefficients and variables that Jana reported.

In the final discussion of the session, the three of us talked about what we could possibly learn from the further elaborations of the original sentences told in the lessons. Jana said that even in the school lessons that are planned to practice the manipulation and resolution of equations, teacher talk can and must provide opportunities for learners to step back, and think and reflect upon what they are doing and why. This teacher gave special value to the elaboration and expansion of sentences with the word names for equation and solution through grammars of explanation such as “This is a solution of this equation because...” The question of what is meant by a solution of an equation is an important one, and Jana and Maia agreed that answers should be addressed explicitly throughout different lessons and teaching moments. Maia gave special value to the elaboration and expansion of sentences with the word names for quadratic equation, resolution and formula alongside grammars of description, such as “The formula for the resolution of a quadratic equation can be expressed in different ways using different letters for the variables.” From here on, the questions emerged of what is meant by the coefficients usually expressed with the letters a , b and c , and how this is overtly explained to learners while teaching and in relation to the difficult concept of variable, again expressed with letters such as x , y and z . Regarding the formula for the resolution of quadratic equations expressed in the form $ax^2+bx+c=0$, both teachers explained cases of learners for whom a , b , c and x mathematically represented the same: letters from which to generate numerical values. It became clear that the (ab)use of the name ‘letter’ instead of the specialized word names, ‘coefficient’ and ‘variable’, could be hindering access to the concepts, as well as deliberate classroom talk on conceptual distinctions and similarities.

These are only some examples of the engagement and reflections of the teachers emerging from one of the developmental tasks about content teaching and teacher talk. Throughout all the sessions, two levels of talk were demarcated. There was representation of talk to/with learners in lessons of Jana and Maia —where school teaching occurs—, and discussion of talk amongst teachers and teacher educators about lesson talk —where professional learning occurs—. The pieces of data in the present paper seek to show the extent to which the meta-level of talk about talk can be organized to support the level of teacher talk as it occurs in the school classroom. In my context of culture, this is at present a challenging approach to mathematics teaching and mathematics teacher education because pedagogic discourses on the reduction of teacher talk in the classroom, so as to provide more opportunities for learners to talk, are typical and interpreted in rather extreme ways in the developmental programs and policies. In this context, therefore, it is important to clarify that the relevance given to teacher talk is not at the expense of learner-centred pedagogies of mathematics teaching and learning. In these pedagogies, nonetheless, the talk of the teacher in the classroom continues to be a fundamental resource to convey specialized mathematical meanings, to model mathematical languages and to guide mathematically rich discourse practices in the communication with learners. Regardless of the quantity of teacher talk and the balance of teacher talking time and student talking time inside classrooms, the focus here is on the quality of this talk.

What is in word names and sentences that matters in mathematics teaching?

The focus throughout this conference paper has been the study of and the work on mathematics teacher talk with special attention paid to two verbal tools in language. The choice and use of word names —i.e. naming— and of sentences —i.e. lexicalization— are processes of providing verbal forms for encoding specialized mathematical meanings for particular contents in concrete contexts of culture. Work on these processes in mathematics teacher talk is simply work on how talk develops to support the communication of certain meanings for certain contents, in interaction with other tools in language, semiotic modes and mathematically rich discourse practices in the immediate classroom and the broader contexts. While mathematics teacher talk tends to presuppose the successful communication of mathematics meanings, naming —i.e. the choice and use of word names— and lexicalization —i.e. the choice and use of sentences made up with words and word names— are not processes always functioning to communicate the specialized mathematical meanings intended, or even functioning to communicate them in sufficiently precise or explicit ways. Some work on naming and lexicalization may therefore be necessary in preservice and developmental education of mathematics teachers aimed at supporting content teaching. The examples with lesson data on the equation concept in the prior section provide a hint as to how this work could be done.

Naming and lexicalization are ordinarily reduced to refer to the language teaching of the word names and the related forms of grammar for specific mathematical objects or phenomena. This is especially visible in the teaching of mathematics in multilingual classrooms where some of the learners if not all are beginning learners of the language of instruction. The learning of the word names and of the related forms of grammar in the relevant languages is indeed important, but it does not cover the learning of the mathematical meanings encoded behind in the particular culture and interaction. A learner can well know the word name for the equation concept and forms of grammar including the combination of words and symbols like the equal sign in the language of instruction —as well as its word name and related forms of grammar in her home

language—, and she can even know how to solve equations of the linear and quadratic types, and still do not know what an equation is. It can certainly happen that conceptual meanings have been taught and communicated through other tools available in language like visual representations of equations interpreted as graphs or like numerical representations of sets of paired or n-tuple ordered values. Despite the fact of talk not being the only potential resource in content teaching, unfocused teacher talk accompanying other ways and forms of communication remains an obstacle to the teaching of precise meanings. The implications of always talking about equations instead of linear/quadratic equations but also talking always sentences with material verbs instead of relational, for example, in the description of the graph of a line/parabola and of the table of values that goes with it, are many. Both Jana and Maia renamed *equació quadràtica* as *equació* in their classroom talk and did not overtly told the variability of types of equations and what they all had in common, namely, all the types amongst them and all the representatives of one equation.

The specialized mathematical meanings for equation, linear equation and quadratic equation are objects of teaching and learning in secondary school mathematics education, and hence the distinction amongst the mathematical meanings for each should also be taught and learned. Careful attention to processes of naming and lexicalization in the preparation and development of content mathematics teaching can uncover more or less abundant instances of teacher talk and of possible renaming that do or do not function to communicate the specialized meanings and relationships intended. Still one question is: What is in the word names and the sentences of the teacher talk that is left to the learners?

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References

- Adler, J. (2021). Content and context specificity matter in the ‘how’ of language-responsive mathematics teacher professional development. In N. Planas, C. Morgan, & M. Schütte (Eds.), *Classroom research on mathematics and language: Seeing learners and teachers differently* (pp. 77–100). London: Routledge.
- Adler, J. (2017). Mathematics Discourse in Instruction (MDI): A discursive resource as boundary object across practices. In G. Kaiser (Ed.), *Proceedings of the 13th International Congress on Mathematical Education* (pp. 125–143). Cham, Switzerland: Springer.
- Civil, M., Planas, N. (2004). Participation in the mathematics classroom: Does every student have a voice? *For the Learning of Mathematics*, 24(1), 7–12.
- Essien, A. A., Chitera, N., & Planas, N. (2016). Language diversity in mathematics teacher education: Challenges across three countries. In R. Barwell et al. (Eds.), *Mathematics education and language diversity: The 21st ICMI Study* (pp. 103–119). Cham, Switzerland: Springer.
- Halliday, M. A. K. (1978). *Language as social semiotic: The social interpretation of language and meaning*. London: Edward Arnold.
- Halliday, M. A. K. (1985). *An introduction to functional grammar*. London: Edward Arnold.
- Kazima, M., Jakobsen, A., & Kasoka, D. N. (2016). Use of mathematical tasks of teaching and the corresponding LMT measures in the Malawi context. *The Mathematics Enthusiast*, 13(1–2), 171–186.
- Mousvold, R., & Fauskanger, J. (2018). Opportunities and challenges of using the MDI framework for research in Norwegian teacher education. In E. Norén, H. Palmér, & A. Cook

(Eds.), *Proceedings of the 8th Nordic Conference on Mathematics Education* (pp. 209–218). Stockholm, Sweden: NORMA.

Planas, N. (2018). Language as resource: A key notion for understanding the complexity of mathematics learning. *Educational Studies in Mathematics*, 98(3), 215–229. <https://doi.org/10.1007/s10649-018-9810-y>

Planas, N. (2019). Transition zones in mathematics education research for the development of language as resource. In M. Graven, H. Venkat, A. Essien, & P. Vale (Eds.). *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 1–31). Pretoria, South Africa: PME.

Planas, N. (2021). How specific can language as resource become for the teaching of algebraic concepts? *ZDM—Mathematics Education*, 53(2), 277–288. <https://doi.org/10.1007/s11858-020-01190-6>

Planas, N., & Civil, M. (2009). Working with mathematics teachers and immigrant students: An empowerment perspective. *Journal of Mathematics Teacher Education* 12(6), 391–409. <https://doi.org/10.1007/s10857-009-9116-1>

Planas, N., & Schütte, M. (2018). Research frameworks for the study of language in mathematics education. *ZDM—Mathematics Education*, 50(6), 965–974. <https://doi.org/10.1007/s11858-018-0997-2>

Planas, N., & Setati, M. (2009). Bilingual students using their languages in the learning of mathematics. *Mathematics Education Research Journal* 21(3), 36–59. <https://doi.org/10.1007/BF03217552>

Setati, N., & Planas, N. (2012). Mathematics education across two language contexts: A political perspective. In O. Skovsmose, & B. Greer (Eds.), *Opening the cage: Critique and politics of mathematics education* (pp. 167–186). Rotterdam: Sense Publishers.