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ABSTRACT

Quantum-limited microwave parametric amplifiers are genuine key pillars for rising quantum technologies and, in general, for applications that rely on the successful readout of weak microwave signals by adding only the minimum amount of noise allowed by quantum mechanics. In this Perspective, after providing a brief overview on the different families of parametric microwave amplifiers, we focus on traveling wave parametric amplifiers, underlining the key achievements of the last few years and the present open challenges. We also discuss possible new research directions beyond amplification such as exploring these devices as a platform for multi-mode entanglement generation and for the development of single photon detectors.

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I. INTRODUCTION

A low noise amplification chain is a key element of any system requiring the detection of low amplitude signals. In the microwave regime, a quantum-noise limited amplification chain is not only crucial, for example, for the readout of solid-state qubits, such as spin qubits[1] or superconducting qubits,[2] but also in a broader range of modern applications, such as nano-electromechanical systems,[3] electron spin resonance detection,[4] astronomy instrumentation,[5,6] or axionic dark matter detection.[7–10]

The noise performance of an amplification chain is established primarily by the noise and gain characteristic of the first amplifier in the chain.[11] For this reason, it is highly desirable to achieve quantum-limited added noise and high gain at the first stage of amplification.

State-of-the-art commercially available semiconducting low-noise microwave amplifiers are unable to reach quantum-limited added noise because of their dissipative nature. Superconducting parametric amplifiers are instead well suited for this task since they allow one to take advantage of highly non-linear interactions without introducing dissipation.[12] They can give access to quantum-limited noise performance with gain high enough to suppress the noise added by the following amplifiers in a detection chain.[13]

The development of superconducting parametric amplifiers recently became topic of intense research efforts and given the great value of quantum-limited microwave detection chains in many modern research and application areas.

In this Perspective, we briefly retrace the historic path that brought to the present state of the art for superconducting microwave parametric amplifiers, focusing, in particular, on traveling wave parametric amplifiers (TWPAs). For a more complete treatment of superconducting parametric amplification, we redirect the reader to a recent review.[14]

This Perspective is divided into two sections. In Sec. II, we provide a brief historical road-map and a summary of the main approaches adopted for the development of TWPAs and of the obtained figures of merit. In Sec. III, we present issues that are currently still open in the development of such devices and discuss possible new research directions that could be of great relevance within and beyond the purpose of low noise amplification.

II. A BRIEF HISTORICAL ROAD-MAP

Parametric amplification is a fundamental wave-mixing process in non-linear optics. When a material with non-linear polarization is excited/pumped with an intense electromagnetic field, a weak signal field can get amplified via the interaction with the medium. Energy gets transferred from the pump frequency mode f_p, to the signal frequency mode f_s via the creation of a third mode f_i dubbed idler.[15] Parametric amplification can occur either via a three wave-mixing process (3WM), where $f_s + f_i = f_p$, or via a four wave-mixing process (4WM), where $f_s + f_i = 2f_p$.

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The signal gain can be enhanced by increasing the interaction time in the non-linear medium via two main strategies. The first one, resonant-amplification, consists in placing the non-linear medium in a cavity; in this way, the interaction time will be as long as the inverse of the cavity linewidth. This approach, however, puts a constraint on the amplification bandwidth that depends on the cavity linewidth as well. The second strategy, traveling wave amplification, consists in optimizing the gain by increasing the physical length of the non-linear medium, removing the constraint on the amplification bandwidth given by the presence of a cavity.

For microwave parametric amplification, the role of the non-linear medium can be played by the non-linear inductance of a superconducting circuit composed, for example, of Josephson junctions. Josephson junction based amplification reaching near quantum limited operation was demonstrated for the first time by Bernard Yurke's pioneering works in the late 1980s.[16,17] In the early 2000s, with the arising of the field of circuit quantum electrodynamics (c-QED),[18–21] a renovated strong interest in microwave quantum limited amplification started and continued for the last two decades with intense research activity in many groups.

The fervent effort in developing such devices was strongly motivated by the pursuit of obtaining single-shot and high fidelity qubit readout in c-QED. This breakthrough result was initially achieved with the Josephson bifurcation amplifier (JBA)[22–24] that exploited the bistable regime of an rf-driven Josephson junction. Subsequently, Castellanos-Beltran and Lehnert successfully implemented what then became an established circuit scheme for resonant Josephson parametric amplifiers, resonant-JPAs.[28] Since then, resonant-JPAs reaching the quantum limit of noise have been successfully demonstrated and operated in many labs. Remarkable progresses have been obtained regarding their bandwidth,[29,30] saturation point,[31–33] pumping schemes,[34–38] frequency tunability,[39] and non-reciprocity.[40–43]

Despite such notable improvements, the bandwidth constraint imposed by the presence of the cavity in the circuit scheme of resonant-JPAs still represents a significant limitation for some applications such as multiplexed readout of high number of qubits or broadband squeezing generation. This strongly motivates the development of TWPA devices, superconducting non-linear metamaterials that exhibit parametric amplification in a traveling wave configuration. TWPAs allow us to achieve, with superconducting circuits, what can be achieved at optical frequencies with non-linear optical fibers[44] with the advantage to operate over a much larger fractional bandwidth.

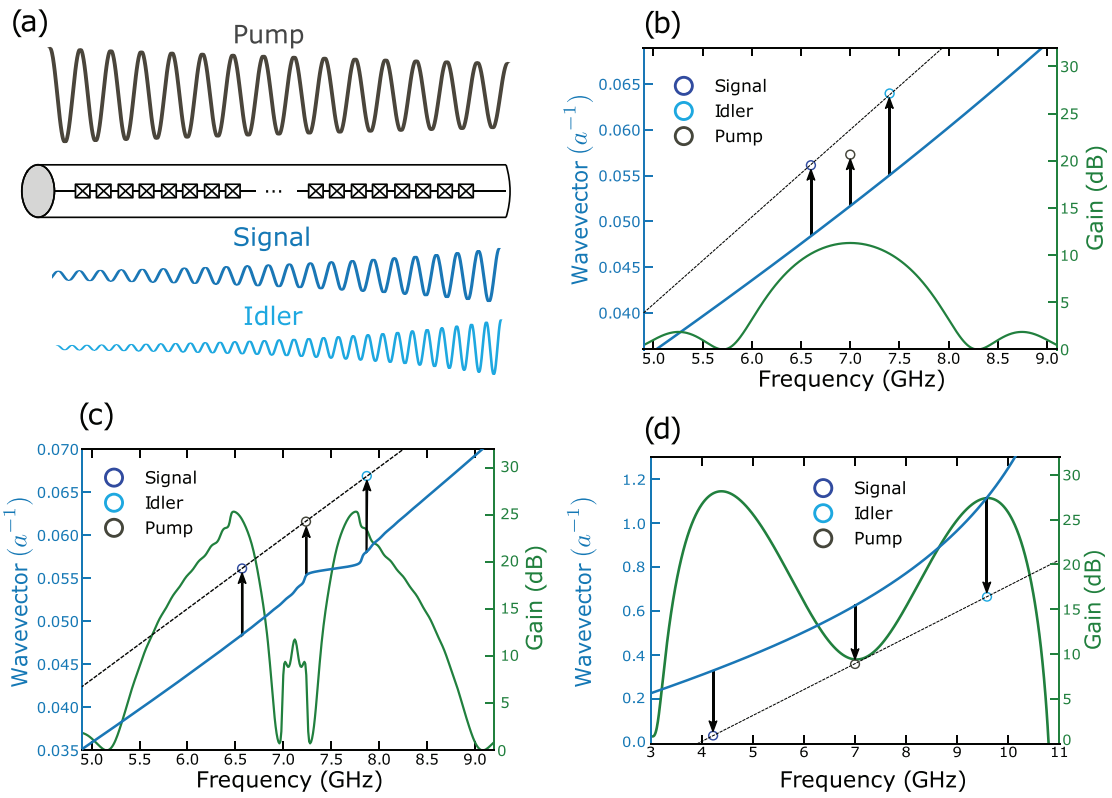

FIG. 1. Cartoon of TWPA physics and phase matching approaches. (a) Cartoon of a TWPA composed of a chain of Josephson junctions with propagating pump, signal, and idler fields. Simulated gain (green curve) and the chromatic dispersion relation (blue curve) for (b) a bare TWPA (no-phase matching approaches adopted), (c) a TWPA with dispersion engineering phase matching, and (d) a TWPA with reversed Kerr phase matching. The wave-vector is in units of $1/a$, where a is the size of a unit cell. The arrows indicate the Kerr phase shift. The circles indicate the wave-vectors for the pump, the signal, and the idler. Phase matching is reached when the three wave-vectors lay on the same line. Dashed lines are guides to the eye. Dispersion relation curves and Kerr phase shifts are exaggerated for pedagogical purposes. Gain curves are simulated using standard coupled mode equations and typical TWPAs parameters.[25,26] The code used for simulations of gain profiles and dispersion relations is available in Ref. 27.

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TWPA devices[45–48] are more challenging to develop in comparison to their cousins resonant-JPAs. The necessity to engineer a long non-linear medium requires typically more complex nano-fabrication methods. In addition, efficient traveling wave amplification requires the fulfillment of two challenging conditions: the impedance matching of the amplifier with the typical 50 Ω impedance of the external environment and the attainment of the phase matching condition for the pump, the signal, and idler fields.

Early theoretical and experimental TWPA investigations emerged already in the 1970s–1990s.[49–52] However, it was in 2010s that TWPAs started to be conceived as we know them today and successfully realized with modern nano-fabrication techniques.[53,54,56] Key progresses were obtained with the demonstration of TWPA devices exploiting the high kinetic inductance of disordered superconductors[55] or Josephson junctions,[64] and in 2015, the first demonstration of a near quantum-noise-limited TWPA device was achieved.[58]

These breakthrough achievements demonstrated the possibility to go beyond the bandwidth constraints of resonant-JPAs and opened the road for the exploration of TWPAs based both on kinetic inductance of superconducting thin films (KTWPAs) and on the Josephson junction based meta-materials (JTWPAs).

In Fig. 1(a), we show a sketch on a TWPA consisting in a chain of Josephson junctions. For the case of KTWPAs, the array of Josephson junctions is substituted with a long high kinetic inductance superconducting transmission line.

A. Challenges and adopted solutions

Several strategies have been adopted for addressing the issues of complex nano-fabrication, impedance matching, and phase matching. Among all, phase matching (momentum conservation) is perhaps the biggest challenge in TWPAs' development. Phase mismatch between the signal idler and pump fields in TWPA devices arises because of two reasons: (1) dispersion phase mismatch, caused by the curved chromatic dispersion relation[59] of the meta-material and (2) Kerr phase mismatch, caused by non-linear processes, such as selfphase modulation (SPM) and cross phase modulation (XPM),[44] making the pump traveling faster (with higher phase velocity) than the signal and the idler. Dispersion and Kerr phase mismatch typically sum up giving a total phase mismatch that limits the optimal gain.

The first strategy, dispersion engineering phase matching, that has been adopted to solve such problems is based on engineering a gap in the dispersion relation of the meta-material. Such modification of the dispersion relation allows us to fulfill the phase-matching condition at a specific pump frequency close to the gap. Dispersion engineering phase matching has been obtained with different experimental approaches: using periodic loading patterns[25,55] and via the introduction of resonant elements alongside the meta-material (resonant phase matching).[57]

Recently, an alternative to dispersion engineering phase matching based on the sign reversal of the Kerr non-linearity[60] has been experimentally demonstrated,[26] reversed Kerr phase matching. This phase matching mechanism is based on reversing the sign of the Kerr phase mismatch such that it compensates the dispersion phase mismatch, canceling the total one. The sign reversal of the Kerr phase mismatch is obtained by using a superconducting non-linear asymmetric element (SNAIL)[61] as a unit cell of the TWPA, allowing *in situ* tunability of the 3WM and 4WM non-linearities via an external magnetic flux. In a reversed Kerr TWPA, optimal gain can be reached for a large range of pump frequencies.

TABLE I. Summary of TWPA devices and respective approaches used for addressing the challenges of nano-fabrication, impedance matching, and phase matching.

	Nano-fab	Impedance matching	Phase matching
Ho Eom *et al.*[55]	NbTiN coplanar waveguide (CPW)	Tapered impedance transformers	Periodic loading pattern
Bockstiegel *et al.*[6]			
Vissers *et al.*[62]			
Ranzani *et al.*[63]			
Macklin *et al.*[58]	Nb/Al-AlO_x/Nb JJ-array Nb trilayer process	Lumped-element capacitors	Resonant phase matching (RPM)[57]
White *et al.*[64]	Al/Al_2O_3/Al JJ-array double angle evaporation	Lumped-element capacitors	Resonant phase matching (RPM)[57]
Adamyan *et al.*[65]	NbN CPW	Quasi-fractal interdigitated capacitors	Periodic loading pattern
Chaudhuri *et al.*[66]	NbTiN Lumped-elements	Fish-bone interdigitated capacitors	Periodic loading and resonance phase shift
Zorin *et al.*[67]	Nb/AlO_x/Nb JJ-array Nb multilayer process	Lumped-element capacitors	…
Miano n*et al.*[68]			
Planat *et al.*[25]	Al/Al_2O_3/Al JJ-array double angle evaporation	Top ground plane, microstrip geometry	Periodic JJs modulation
Goldstein *et al.*[69]	WSi microstrip	Top ground plane, microstrip geometry	Periodic loading pattern
Malnou *et al.*[70]	NbTiN CPW	Fish-bone interdigitated capacitors	Periodic loading pattern
Ranadive *et al.*[26]	Al/Al_2O_3/Al JJ-array double angle evaporation	Top ground plane, microstrip geometry	Reversed Kerr phase matching[60]
Shu *et al.*[71]	NbTiN microstrip	Top ground plane, microstrip geometry	Periodic loading pattern

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In Figs. 1(b)–1(d), we elucidate the physics of phase matching mechanisms in TWPAs with pictorial sketches. In panel (b), we shown gain and chromatic dispersion relations for a bare TWPA with no phase matching approach, giving a low gain limited by the total phase mismatch. The arrows indicate the Kerr phase mismatch. In panel (c), we show the same for a TWPA with dispersion engineering phase matching and in panel (d), reversed Kerr phase matching.

In Table I, different experimental approaches adopted so far for the phase matching are summarized, together with the techniques implemented for addressing the other, not less difficult, challenges of nano-fabrication and impedance matching. TWPAs' performances are typically evaluated on the basis of some key figures of merit: gain, bandwidth, added noise, saturation power, and required pump power. The typical gain target is of the order of 20 dB. Such values guarantee that, when the TWPA is placed at the beginning of a standard microwave detection chain, the system noise of the entire chain is mostly dominated by the added noise of the TWPA.[11] The total system noise target is the standard quantum limit (SQL). The bandwidth and the saturation power are typically targeted to be as large as possible to allow the broadest range of possible applications. Finally, the required pump power should ideally be as low as possible to avoid saturation of the next stages of amplification.

To give the reader a feeling of the last progresses in improving such figures of merit, in Table II, we report a comparison for different TWPA devices.

III. OPEN CHALLENGES AND OUTLOOK

Despite the remarkable improvements of the last few years in TWPAs development, several challenges are still open. In the following, we list what we think are the most relevant open issues, discussing possible approaches to address them and potential future research directions.

A. Reaching standard quantum limit (SQL)

Improving added noise in the TWPA is certainly one of the major open challenges. So far, the system noise of microwave detection chains that employ a TWPA as a first stage of amplification has been reported to be at best two times larger than the SQL (Table II). Although full understanding is still missing, there are two main candidate phenomena that could explain it: capacitive dielectric losses and sidebands generation.

Dielectric losses are coming from the dielectric in the capacitive elements that are typically inserted in the non-linear metamaterials to reach impedance of 50 Ω. Enhancing the quality of such dielectrics is one of the main strategies to reduce losses. Dielectrics with a low number of defects, such as amorphous silicon[72] or silicon nitride,[73] reported loss tangent between 10^{-6} and 10^{-5} and are promising candidates for reducing losses and possibly improve added noise in TWPAs. Another interesting approach would be the complete removal of the dielectric material and the use of vacuum gap capacitors.[74]

Further source of non-ideal added noise in TWPAs is the generation of sidebands, given by the intrinsic broadband/multi-mode nature of TWPA devices. A recently suggested strategy to mitigate the effect of sidebands generation on the added noise is based on a Floquet mode TWPA implementation.[75] Sidebands could also be mitigated via dispersion engineering approaches.

B. Saturation power improvement

Saturation power defines the maximum signal input power. Increasing the signal input power handling capability of TWPAs is a key goal especially for the commercial adaptability of these devices. The 1 dB compression point (signal power at which the gain is reduced of 1 dB) in the state-of-the-art TWPAs is still considerably inferior

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TABLE II. Comparison of TWPA figures of merit. For each reported TWPA, we list the kind of wave-mixing process, total system noise of the amplification chain having the TWPA as first stage of amplification (SQL corresponds to 1 photon total system noise: half photon plus additional half photon added by the amplified idler vacuum), best gain value, bandwidth (defined considering the frequency region with gain within 3 dB from the maximum gain), saturation power (−1 dB compression point) and pump power necessary to reach the maximum gain. When not explicitly stated, gain/bandwidth values are approximated from the published gain plots.

	Process	System noise (photons)	Gain (dB)	Bandwidth (GHz)	Saturation (dBm)	Pump power (dBm)
Ho Eom *et al.*[55]	4WM	···	10	4.5	−52	−9
Bockstiegel *et al.*[6]	4WM	···	20	7	···	−10
Vissers *et al.*[62]	3WM	···	18	4	−35[a]	−15[a]
Ranzani *et al.*[63]	3WM	3.5	10	5	−40	−30
Macklin *et al.*[58]	4WM	2	21.6	3	−99	−63
White *et al.*[64]	4WM	4	12	4	−90	···
Adamyan *et al.*[65]	4WM	···	6	···	···	−15
Chaudhuri *et al.*[66]	4WM	···	15	3	···	···
Zorin *et al.*[67]	3WM	···	11	3	···	···
Miano n*et al.*[68]	3WM	···	12.5	4	···	···
Planat *et al.*[25]	4WM	3	18	2.3	−100	−70
Goldstein *et al.*[69]	4WM	···	15	2	···	−53
Malnou *et al.*[70]	3WM	3	16.5	2	−63	−29
Ranadive *et al.*[26]	4WM	4	20	3	−97	−75
Shu *et al.*[71]	3WM	···	18	19	−57	−29

[a]Measured for 10 dB gain.

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with respect to commercially available semiconductor-based amplifiers. A possible source of this limitation is pump depletion, coming from the inevitable conversion of pump power into power at the signal and idler frequency or from possible leaks of power to higher harmonics.[59] This explains why KTWPAs, which work at pump powers much higher than JTWPAs, present higher saturation powers. However, the necessity of high pump powers (typically 4–5 orders of magnitude higher than JTWPAs) have the disadvantage of requiring pump cancelation methods to avoid saturation of the next stages of amplification. Another possible sources of limited saturation power could be Kerr phase modulation and higher order processes.[32,76]

C. Pumping schemes

Wave mixing mechanisms, 3WM and 4WM, determine the pumping scheme of a TWPA. For 4WM amplification, the pump frequency lies in the middle of the amplification band, while, for 3WM amplification, the pump frequency is far away from the amplification range since it is set to be equal to the sum of signal and idler frequencies. KTWPAs have been demonstrated experimentally both in the 3WM and in the 4WM configurations, while JTWPAs are so far only in the 4WM configuration. The advantage of 3WM amplification is the absence of the pump frequency in the amplification band. However, one needs to adopt strategies to mitigate unwanted 3WM processes like second harmonic generation, which is typically more efficient than parametric amplification.

Another advantage of 3WM amplification is the possibility to be implemented in a theory with a flux-pumping scheme. TWPAs experimentally demonstrated so far are pumped by a microwave tone, which travels through the meta-material together with the signal. One limitation of this current-pumping scheme is that it is very difficult to separate the pump from the signal field, causing potential detrimental effects to the system under measurement. A flux-pumping approach[77] instead would have the advantage of physically separating the pump input line from the signal input line providing better isolation.

D. Non-reciprocity

TWPA devices are, in principle, directional since the signal gets amplified in the direction of the pump propagation. However, they can still transmit in the direction opposite to gain, and a careful analysis of wave propagation in non-linear materials shows that under certain conditions modes with opposite wave-vectors can also be coupled leading to backwards gain.[25,75] This means that vacuum fluctuations could be backward amplified and exposed to the device under test. Hence, even if technical problems such as pump leakage and impedance matching could be solved with better engineering, isolators between the device under measurement, for example, a qubit, and the TWPA would still be required.

The demonstration of truly directional TWPAs, with signal attenuation in the direction opposite to gain, would be a remarkable advancement in the field.

Directional amplification in narrow-band resonant-JPAs has been successfully achieved via reservoir engineering and multi-mode parametric interactions.[41,42,78] Directional isolation in Josephson transmission lines has been demonstrated by frequency conversion,[79] and recently, adiabatic mode conversion has also been proposed.[80] Combining traveling wave parametric amplification and broadband backward isolation would be of great impact for superconducting quantum technologies.

E. Operating frequency range and operating temperature

TWPA devices developed so far are usually operating in the typical c-QED microwave frequency range between 4 and 12 GHz. The operation of TWPAs in other frequency ranges recently became the subject of intense research. For example, TWPAs working at frequencies lower than 1 GHz are very appealing for the spin qubits community,[1] while higher frequencies are required for some applications such as the search of dark-matter axions.[7] In the context of astronomy instrumentation, a valuable achievement would also be the possibility to operate TWPAs at 4 K temperature, in order to substitute the dissipative semiconductor amplifiers in large detector arrays[81] with superconducting amplifiers, drastically reducing the power requirement for amplification. In this case, it will be necessary to employ materials that are still superconductors at 4 K, for example, NbTiN or also high T_C superconductors.

F. Beyond amplification

As we reported in this paper, superconducting non-linear meta-materials became an established solution to reach near quantum-noise-limited broadband microwave amplification and serve as optimal pre-amplifiers in many contexts. Nonetheless, they have the potential to be successfully adopted for implementations that also go beyond quantum-limited amplification. They could serve as a powerful platform for multi-mode entanglement generation[82,83] and potentially quantum information with continuous variables. Their broadband nature and the possibility of *in situ* customization of the non-linearities[26] are unique features for quantum optics investigations in the microwave regime. In addition, the intrinsic multi-mode nature of TWPA devices, which is considered detrimental for added noise and saturation power of an amplifier, could become instead a resource for the generation of multi-mode entanglement.

Moreover, inspired by JTWPAs, single-photon counters have been recently proposed.[84] By utilizing the tunability of strength and nature of non-linearity of Josephson meta-materials, these detectors could allow broadband and nondestructive counting of traveling microwave photons, opening new possibilities for single-photon measurement and control in c-QED.

IV. CONCLUSIONS

In this Perspective, we reviewed progresses in superconducting traveling wave parametric amplifiers. We presented the main obstacles to the development of such devices and the different approaches adopted so far to overcome them, reporting and comparing the corresponding figures of merit. We discussed open issues related to the achievement of the standard quantum limit, the improvement of saturation power, the adoption of different pumping schemes, the implementation of non-reciprocity, and the extension to different frequency ranges and higher working temperatures. Future progresses in all these areas would be of great value for a diverse range of applications requiring broadband and low-noise microwave amplification, from quantum computation with solid state qubits to astronomy detection. Finally, we reported examples, in which superconducting non-linear meta-

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materials could also be extremely valuable for purposes beyond amplification such as multi-mode entanglement generation and detection of single traveling photons, appealing tools for quantum optics and metrology in the microwave frequency domain.

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DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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