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# Model predictive control and moving horizon estimation for water level regulation in inland waterways

P. Segovia<sup>1,2,3\*</sup>, L. Rajaoarisoa<sup>3</sup>, F. Nejjari<sup>1</sup>, E. Duviella<sup>3</sup>, V. Puig<sup>1,2</sup>

<sup>1</sup> *Research Center for Supervision, Safety and Automatic Control (CS2AC), Universitat Politècnica de Catalunya (UPC), Terrassa Campus, Gaia Building, Rambla Sant Nebridi 22, 08222 Terrassa, Spain*

<sup>2</sup> *Institut de Robòtica i Informàtica Industrial, CSIC-UPC, Llorens i Artigas 4-6, 08028 Barcelona, Spain*

<sup>3</sup> *IMT Lille Douai, Univ. Lille, Unité de Recherche Informatique Automatique, F-59000 Lille, France*

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## Abstract

This work regards the design of optimization techniques for the purposes of state estimation and control in the framework of inland waterways, often characterized by negligible bottom slopes and large time delays. The derived control-oriented model allows these issues to be handled in a suitable manner. Then, the analogous moving horizon estimation and model predictive control techniques are applied in a centralized manner to estimate the unmeasurable states and fulfill the operational goals, respectively. Finally, the performance of the methodology is tested in simulation by means of a realistic case study based on part of the inland waterways in the north of France. The results show that the proposed methodology is able to guarantee the navigability condition, as well as the other operational goals.

*Keywords:* Inland waterways, control-oriented modeling, model predictive control, moving horizon estimation, time-delay systems.

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## 1. Introduction

Inland waterways are large-scale systems, composed of natural rivers and artificial canals, used mainly for transportation of passengers and freight. Indeed, fluvial transport constitutes an environmentally-friendly alternative to the traditional rail and road transport modes [1]. In order to facilitate the study of such systems, they are usually decomposed into reaches, which are parts of a stream of water between two hydraulic structures such as gates or locks. Reaches are

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\*Corresponding author: P. Segovia (pablo.segovia@upc.edu).

17 usually characterized by negligible bottom slopes, and therefore the backwater effect becomes of  
18 increasing importance. This effect takes place at the downstream hydraulic structure of a canal:  
19 when the water waves impact upon the structure, the water can flow back to the upstream end,  
20 resulting in a back-and-forth mass transport known as the resonance phenomena. Other applica-  
21 tions in the framework of free-surface water systems (e.g., sewage systems, irrigation and drainage  
22 canals) are generally not affected by the backwater effect. On the other hand, the dynamics of  
23 inland waterways are rather slow, which translates into large time delays in the network. These  
24 two features, typical of inland navigation networks, complicate its management.

25 Inland waterways management aims at allocating the available water resources to meet the  
26 desired objectives. The most important goal nonetheless consists in guaranteeing the **navigability** **R3.6**  
27 **condition**, i.e., ensuring that the water levels are such that vessels can travel safely. Indeed, a  
28 **setpoint** is defined for each reach, known as the Normal Navigation Level (NNL). The objective  
29 is to keep the water levels as close as possible to the setpoints. In addition, upper and lower  
30 level bounds known as the Lower and Higher Navigation Levels (LNL and HNL, respectively) are  
31 determined around the setpoint, thus defining the navigation rectangle. If the water level of a reach  
32 is outside of the rectangle, the navigability condition can no longer be guaranteed. Other common  
33 objectives regard, for instance, minimizing the operational cost and ensuring a long lifespan of **the** **R1.1**  
34 **actuators**, e.g., gates, weirs, pumps and valves.

35 Inland waterways are large-scale, complex systems affected by some phenomena that are not **R3.17**  
36 easy to account for, e.g., demands, uncontrolled inputs, rainfall and seepage, and thus it is not easy  
37 to assume that the water levels remain within the bounds in the absence of a control strategy. This  
38 fact motivates the design of monitoring and control approaches for inland waterways. Therefore, a  
39 suitable control policy is needed in order to fulfill the objectives. Optimal control techniques have  
40 been investigated for a long time to this end. In particular, Model Predictive Control (MPC) has  
41 received a lot of attention due to its adequacy to deal with these kinds of problems. Its underlying  
42 principle consists in using a dynamic model of the process to predict the effect of manipulable  
43 inputs, subject to operational restrictions, so that the performance of the plant is optimal regarding  
44 the chosen criteria. Its ease of understanding and application has fostered its use in many different  
45 domains. In the framework of water systems, MPC has also been widely used. For instance, in  
46 [2] it was applied at regional and national scales to protect against high river flows and sea tides,

47 to ensure navigation and to supply water during dry periods. This technique was also employed  
48 in [3], using a coalitional approach to find the best compromise between communication costs and  
49 control performance for irrigation canals. The combined water supply and navigability of river  
50 systems was tackled using MPC in [4, 5]. A nonlinear economic MPC was designed in [6] for water  
51 distribution networks, aiming at minimizing the economic costs associated to water treatment and  
52 pumping. A comparison of non-centralized MPC strategies for irrigation canals was carried out in  
53 [7], validating the benefits of cooperative control.

54 The use of MPC requires the vector of states to be known at current time. This information is  
55 used as the starting point to compute the set of control actions that must be applied during the  
56 future horizon. In general, the measurements of all states are not available, and thus estimates  
57 of unmeasured states must be provided to the MPC using observers. Although there exist many  
58 possibilities to estimate the states, this work employs the Moving Horizon Estimation (MHE),  
59 which is considered as the dual problem of MPC. Their combination is especially attractive since  
60 the MHE formulation corresponds also to an online optimization problem that can explicitly handle  
61 constraints [8]. Unlike MPC, this estimation technique started receiving wider attention only in  
62 the recent years [9]. Indeed, the combination of MPC and MHE has been applied in diverse fields  
63 such as autonomous agricultural vehicles [10], unmanned aerial vehicles [11], preventive sensor  
64 maintenance [12], airborne wind energy systems [13] and blood glucose regulation [14]. Concerning  
65 water systems, the combination of these techniques is not so common, although it has been used  
66 for flood prevention in rivers [15] and pollution mitigation for combined sewer networks [16].

### 67 *Summary of the paper and contribution*

68 This work regards the development of MPC and MHE for inland waterways, aiming not only at  
69 guaranteeing the navigability condition of the network, but also at ensuring the rest of operational  
70 goals, linked to optimizing the control effort and reducing wear and tear of the equipment. The  
71 first steps, although focusing only on the controller design and leaving aside the topic of state  
72 estimation, were carried out in [17]. Since a model of the process is at the core of MPC and MHE,  
73 a control-oriented methodology based on the classical Integrator Delay Zero (IDZ) model [18] is  
74 also developed. This new modeling formulation takes into account the challenging features of these  
75 kinds of systems, and provides flexibility in terms of adding reaches to or removing them from the  
76 case study.

77 The contributions of this work to the current state of the art are listed below:

R3.1

- 78 • Inland waterways are large-scale systems with slow dynamics, which result in large time  
79 delays. A common approach to model a system with delays consists in augmenting the  
80 system with more states, with as many new states as delayed samples the system exhibits  
81 [19]. Thus, the time delays have a direct influence on how large the resulting augmented  
82 system is. By contrast, this paper proposes a delayed representation by means of additional  
83 matrices where this augmentation procedure is not necessary.
- 84 • In order for distribution networks such as inland waterways to be completely described, it  
85 is not enough to represent their dynamic behavior. Indeed, these systems are only fully  
86 described when their static behavior is characterized. In the case of inland waterways, the  
87 mass balances at the junctions must be satisfied. This leads to a model representation  
88 that falls under a particular family of systems known as *descriptor* or *differential-algebraic*  
89 systems. The classical control theory was originally formulated for dynamic systems (without  
90 the static part), and these results are usually more involved for descriptor systems.
- 91 • Therefore, this paper proposes a new model formulation for inland waterways, for which the  
92 use of first principles yields a delayed descriptor formulation. These two features cause that  
93 the resulting model cannot be represented using the standard state-space formulation. To  
94 the best of the authors' knowledge, the formulation derived in this work constitutes a novel  
95 result.
- 96 • In addition, although this work employs only one linear model (this fact is duly justified in  
97 Section 3.4), the extension to a linear-parameter varying (LPV) formulation is rather direct  
98 using the results in [20], which allows to retain the nonlinear behavior of the system in the  
99 necessary cases.
- 100 • Standard tools for control and state estimation such as the Linear Quadratic Regulator  
101 (LQR) and the Kalman filter need to be extended to deal with delayed descriptor systems.  
102 Moreover, they cannot deal with input and state/output constraints. This work proposes  
103 instead the combined use of MPC and MHE, which can be easily adapted for this model  
104 formulation. Furthermore, constraints on the inputs and the states/outputs are dealt with by

105 these techniques in a natural manner. The review of the literature shows that the combination  
106 of MPC and MHE for delayed descriptor systems, especially in the field of open-flow water  
107 systems, is a novel approach.

108 The rest of the paper is organized as follows: Section 2 introduces the general waterway man-  
109 agement problem. The control-oriented model is derived in Section 3. The MPC and MHE opti-  
110 mization problems are designed in Section 4. The proposed methodology is tested in Section 5 by  
111 considering a realistic case study, based on the inland navigation network in the north of France.  
112 Finally, Section 6 draws conclusions and outlines future steps.

### 113 *Notation*

114 Throughout this paper, let  $\mathbb{R}^\gamma$  denote the set of column real vectors of length  $\gamma$ , and let  $\mathbb{R}_{\geq 0}$   
115 denote the set of real non-negative scalars. Scalars are denoted by either lowercase or uppercase  
116 letters (e.g.,  $\alpha$ ,  $a$  and  $A$ ); vectors, by bold lowercase letters (e.g.,  $\mathbf{a}$  and  $\mathbf{b}$ ); and matrices, by bold  
117 uppercase letters (e.g.,  $\mathbf{A}$  and  $\mathbf{B}$ ). Furthermore, all vectors are column vectors unless otherwise  
118 stated,  $\mathbf{0}$  denotes a zero column vector of suitable dimensions and  $\mathbf{I}_\delta$  denotes the identity matrix  
119 of dimension  $\delta$ . Transposition is denoted with the superscript  $\top$ , and the operators  $<$ ,  $\leq$ ,  $=$ ,  $\geq$  and  
120  $>$  denote element-wise relations of vectors.

## 121 **2. The waterways management problem**

122 As mentioned before, the management of inland waterways aims at ensuring that the trans- **R3.3**  
123 portation of passengers and freight is carried out safely. To guarantee seamless transport chains,  
124 the water levels must be kept inside the navigation rectangle. Furthermore, it is important that  
125 the water resources are dispatched in an optimal manner, i.e., minimizing their losses.

126 To this end, cross structures are operated in the waterways to regulate the levels of the reaches.  
127 In particular, this work considers two kinds of structures: undershot gates and sharp crested weirs.  
128 An undershot gate is a bottom opening in a wall, whose height can be regulated. Conversely, in  
129 the case of a weir, the water flows over its crest, whose elevation is also adjustable. A schematic  
130 representation of an undershot gate and a weir is given in Fig. 1, and their equations are given in  
131 Appendix B1. Note that  $\mathbf{q}$  is the flow through the cross structure,  $\mathbf{u}$  is the opening/elevation, and  
132  $\mathbf{y}_1$  and  $\mathbf{y}_2$  are the upstream and downstream water levels, respectively.

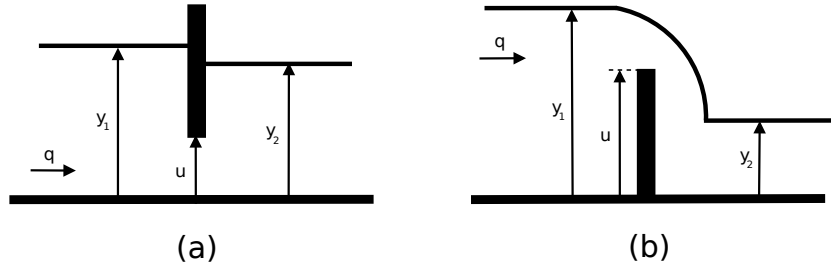


Figure 1: Water level regulation with: (a) an undershot gate. (b) a sharp crested weir.

133 A two-level control architecture is typically considered in this environment [4]: the global control  
 134 level (represented by the MPC) determines the setpoints and sends them to the local controllers  
 135 available at each control structure. In turn, these local controllers are in charge of ensuring that  
 136 the adequate flows are supplied through the actuators. However, this work deals only with the  
 137 global control level, thus assuming that the local controllers are able to perform as desired.

138 The control strategy must also reject the disturbances that affect the system and interfere  
 139 with the control objectives. In the framework of inland waterways, these disturbances refer to the  
 140 request of lock operations by the boat masters. Indeed, boats navigate along the network until their  
 141 final destination, probably along several reaches in their way. The access from one reach to the  
 142 adjacent one is granted by means of locks, which typically consist in enclosures that enable boats  
 143 to overcome the difference in elevation between the reaches. Lock operations require large water  
 144 volumes to be withdrawn from the origin reach, which are then discharged into the destination  
 145 reach. The reason for considering lock operations as disturbances is that they cannot be postponed  
 146 for a long time from the moment a boat reaches a lock, and it is therefore not possible to schedule  
 147 them in an optimal manner.

148 Figure 2 depicts a navigation canal consisting of three reaches, separated from one another by  
 149 cross structures. The red solid arrows represent uncontrolled inputs and outputs such as natural  
 150 bifurcations. On the other hand, the green dashed arrows indicate the flows generated due to the  
 151 lock operations. Finally, the blue dotted arrows represent controlled actions, carried out by gates  
 152 and weirs in order to regulate the water levels. Notice that the locks are often built next to a  
 153 control structure.

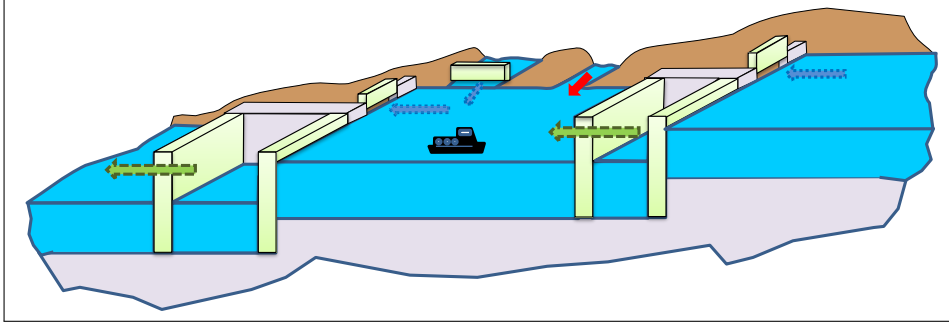


Figure 2: Navigation canal and its water-resource exchanges

### 154 3. Control-oriented modeling

155 An inland waterways model can be regarded as composed of a set of elements, which are intro-  
 156 duced and described below. Note that the physical nature of the variables, e.g., water levels, flows,  
 157 openings and elevations, as well as other elements in the waterways, constrain the performance of  
 158 the system.

#### 159 3.1. Actuators

160 Gates and weirs are used to regulate the water levels in the reaches. Discharges and open-  
 161 ings/elevations can be used in the automated control of canals, as shown in Fig. 1. The global  
 162 control level must compute the optimal action and send it to the slave controller that operates  
 163 the gate or weir. If the discharge is used as the control variable, the slave controller must convert  
 164 the given discharge into an equivalent opening or elevation, which is not as straightforward as  
 165 inverting the discharge equation [21]. Furthermore, choosing the openings and elevations allows to  
 166 link them with the local discharges and the upstream and downstream water levels at the struc-  
 167 ture, thus taking into account such complex dynamics [22]. For these reasons, the openings and  
 168 elevations are chosen as control variables in this work. The conversion is carried out following the  
 169 methodology described in [23], which basically consists in using linearized equations that describe  
 170 the relationship between openings and discharges.

171 These elements have lower and upper operating limits

$$\underline{\mathbf{u}}^m \leq \mathbf{u}_k^m \leq \bar{\mathbf{u}}^m, \quad m = 1, \dots, N_m, \quad (1)$$

172 where  $\underline{\mathbf{u}}^m$  and  $\bar{\mathbf{u}}^m$  are the lower and upper opening/elevation limits of the  $m$ -th actuator, and  $N_m$



173 is the total number of actuators in the system.

174 The type of flow at the structure determines the general linearized equation to be used:

- 175 • The *free-flow* case is characterized by critical or super-critical flow at the structure, which  
176 overrides the effect of the downstream water level on the gate discharge. The linearized  
177 expression reads as

$$q_2^{(1)}(s) \approx q_1^{(2)}(s) = k_y^{(1)} y_2^{(1)}(s) + k_u u(s). \quad (2)$$

- 178 • The *submerged flow* case is characterized by sub-critical flow at the structure. In this case,  
179 the discharge is affected by the downstream water level:

$$q_2^{(1)}(s) \approx q_1^{(2)}(s) = k_y^{(1)} y_2^{(1)}(s) + k_y^{(2)} y_1^{(2)}(s) + k_u u(s). \quad (3)$$

180 In both cases,  $q_2^{(1)}$  is the inflow of the structure at the downstream end of reach 1,  $q_1^{(2)}$  is the  
181 outflow of the structure at the upstream end of reach 2,  $y_2^{(1)}$  is the water level upstream of the  
182 structure,  $y_1^{(2)}$  is the water level downstream of the structure,  $u$  is the opening/elevation and  $k_y^{(1)}$ ,  
183  $k_y^{(2)}$  and  $k_u$  are the coefficients obtained in the [linearization of the nonlinear equations of the gates](#) **R3.4**  
184 [and weirs](#), given by (32).

### 185 3.2. Disturbances

186 Systems and processes are usually affected by disturbances, denoted in this work by  $\mathbf{d}_k$ . There-  
187 fore, the control strategy must minimize their effect on the system. As mentioned in Section 2,  
188 these disturbances correspond to lock operations, which makes it more difficult to stay close to the  
189 setpoints. Although lock operations are rather unpredictable and cannot be postponed for a long  
190 time, they can be somewhat anticipated. Indeed, when a boat passes through a lock, its manager  
191 informs the rest of the managers. In this way, the arrival time of the boat to the adjacent locks  
192 can be predicted, taking into account the distance and the average speed of the boat, which yields  
193 a close approximation, with an error of only several minutes. This allows the lock managers to  
194 elaborate lock operation time-series profiles ahead of time.

195 *3.3. Nodes*

196 Inland waterways are characterized by distributaries, i.e., streams that branch off from the main  
197 stream and flow away. When water streams flow into larger streams or lakes, they are referred to  
198 as tributaries. The locations in which these splittings and mergings take place are called nodes.  
199 They are regarded as mass balance relations modeled as equality constraints given by:

$$\mathbf{0} = \mathbf{E}_u \mathbf{u}_k + \mathbf{E}_{un} \mathbf{u}_{k-n} + \mathbf{E}_d \mathbf{d}_k + \mathbf{E}_{dn} \mathbf{d}_{k-n}. \quad (4)$$

200 Matrices  $\mathbf{E}_u$  and  $\mathbf{E}_{un}$  have as many rows as nodes are in the studied system, and as many **R2.5**  
201 columns as controlled inputs are available. Therefore, each equation in (4) establishes a link  
202 among the variables involved (mass balance at the node), and thus reduces one degree of freedom.  
203 Note that both the controlled inputs and the disturbances have an immediate and a delayed effect  
204 on the system. The delayed effect must be taken into account at the controller and estimator  
205 design stages. This issue is conveniently addressed in Section 4.

206 *3.4. Reaches*

207 An accurate mathematical representation of the dynamics of inland waterways is required in  
208 order to apply the MPC and MHE techniques. Indeed, a model of the system is needed in the  
209 control design stage to compute the predicted output at future time instants. Likewise, it is used  
210 to align measured and predicted values of the process, which results in the optimal state estimates.  
211 Therefore, a special effort has to be put in the computation of a precise model.

212 The Saint-Venant nonlinear partial differential equations allow the accurate representation of  
213 the dynamics of open-flow water systems [24]. However, the lack of a general analytical solution,  
214 as well as their extreme sensitivity to geometry errors and unmodeled dynamics, render them inad-  
215 equate for control purposes. **The nonlinear behavior can be retained, for instance, by considering **R1.2****  
216 **LPV models, which describe a class of nonlinear systems that can be modeled as parametrized lin-**  
217 **ear systems, each of them designed at a different operating point (average flow along the canal). A**  
218 **simpler solution is to use only one linear model (obtained by linearizing the original Saint-Venant**  
219 **equations around an operating point and considering simplifying assumptions), provided that a**  
220 **single operating point is enough to describe the system dynamics.** Examples of linearized models  
221 are the Integrator Delay (ID) model [25], the Integrator Delay Zero (IDZ) model [18], the Integrator

222 Resonance (IR) model [26], and grey-box [27] and black-box [28] models. **Given that the average** **R1.4–R1.7**  
 223 **flows in this work do not deviate too much from the operating point, it is not necessary to resort**  
 224 **to nonlinear modeling strategies such as LPV models.** Instead, it is enough to consider one linear  
 225 model. Although the IR model might appear as the most suitable option since it explicitly takes  
 226 into account the resonance phenomena, the IDZ model is chosen over the IR model for a number  
 227 of reasons. First, the reflecting waves in the resonance phenomena are especially predominant in  
 228 short and deep channels. Instead, the navigation reaches considered in this work are rather long.  
 229 Moreover, the IDZ model has proved to perform well when used for control purposes for these  
 230 kinds of systems [29].

231 The IDZ input-output model links the discharges and the water levels at the boundaries of a  
 232 reach and is given by:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \underbrace{\begin{bmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{bmatrix}}_{\mathbf{P}} \begin{bmatrix} q_1(s) \\ q_2(s) \end{bmatrix}, \quad (5)$$

233 where the subscripts 1 and 2 indicate the initial (upstream) and final (downstream) ends of the **R2.3**  
 234 reach,  $y_1(s)$  and  $y_2(s)$  are the upstream and downstream water levels,  $q_1(s)$  and  $q_2(s)$  are the  
 235 upstream inflow and downstream outflow, and  $p_{ij}(s)$  are the different IDZ terms

$$p_{ij}(s) = \frac{z_{ij} \cdot s + 1}{A_{ij} \cdot s} e^{-\tau_{ij} \cdot s}. \quad (6)$$

236 Remark: In this work,  $q_1(s) \in \mathbb{R}_{\geq 0}$  and  $q_2(s) \in \mathbb{R}_{\geq 0}$ . However, the latter is an outflow, which  
 237 means that it causes the water levels to diminish. Therefore,  $p_{12}(s)$  and  $p_{22}(s)$  are negative. **R2.1**

238 As it can be seen in (6), the IDZ model contains **an integrator whose gain is given by  $1/A_{ij}$ , a** **R2.1**  
 239 **time delay  $\tau_{ij}$  and a zero given by  $-1/z_{ij}$ , for  $i, j = \{1, 2\}$ .** The system can be characterized by  
 240 two different behaviors in the frequency domain. In low frequencies, the behavior of the system is  
 241 similar to a tank that is being filled and/or emptied. In this situation, the integrator gain and the  
 242 time delay have a predominant role. The former reflects how the volume changes according to the  
 243 water level variation, whereas the latter expresses the minimum time that a perturbation requires

244 to travel from one end of the canal to the other one. Two different time delays are defined:

$$\begin{aligned}\tau_{12} &= \frac{L}{C_w - V}, \\ \tau_{21} &= \frac{L}{C_w + V}.\end{aligned}\tag{7}$$

245 Equation (7) corresponds to the case in which both the wave celerity  $C_w$  and the wave velocity  
246  $V$  are constant. More precisely, the celerity is defined as the relative velocity of a wave with **R1.3**  
247 respect to the fluid in which it travels, whereas the velocity measures the variation of the particles'  
248 position of a fluid with respect to time. In particular,  $\tau_{21}$  is measured from the upstream end to the  
249 downstream end, while  $\tau_{12}$  is measured in the inverse direction. Note also that  $\tau_{11} = \tau_{22} = 0$ , since  
250 the discharges are assumed to have an immediate effect at the locations where they take place.

251 On the other hand, the high frequency phenomena is approximated by the zero of  $p_{ij}(s)$ . More  
252 specifically, its constant gain approximates the oscillating modes caused by the gravity waves,  
253 which are predominant in the high frequencies.

254 The parameters of the first equation in (5) are linked to the upstream water level, while those  
255 in the second equation are linked to the downstream water level. The notation of the parameters  
256 is modified based on this fact, and is as follows:  $A_{11} = A_{12} = A_u$ ,  $A_{21} = A_{22} = A_d$ ,  $\tau_{12} = \tau_u$  and  
257  $\tau_{21} = \tau_d$ . Note also that the complete model is taken into consideration. Indeed, it is common  
258 practice to design only downstream water level controllers [18]. Instead, the full model allows to  
259 take into account the backwater effect in the upstream water level, which is of relevance due to the  
260 negligible bottom slope of the reaches.

261 The navigability condition restricts the water levels in the reaches. This constraint might be  
262 relaxed for a short period of time, depending on factors such as the weather condition. Thus, a  
263 relaxation parameter  $\alpha_k$  is considered in the constraint, and a quadratic penalty on this parameter  
264 is included in the objective function.

265 The navigability condition is formulated as

$$\underline{\mathbf{y}}_r - \alpha_k \leq \mathbf{y}_k \leq \bar{\mathbf{y}}_r + \alpha_k,\tag{8}$$

266 with  $\underline{\mathbf{y}}_r$  and  $\bar{\mathbf{y}}_r$  the lower (LNL) and upper (HNL) bounds of the NNL values, respectively. These

267 relaxation parameters  $\alpha_k$  must satisfy

$$\alpha_k \geq 0. \quad (9)$$

268 Figure 3 depicts a waterway composed of several reaches for a better understanding of the **R2.2**  
 269 variables introduced to formulate the problem, and how they are linked to one another. Note that  
 270 the locks are not depicted in this figure, but their operations are indicated using the variable  $\mathbf{d}$ , as  
 271 defined in Section 3.2.

272 Remark: The subscripts  $u$  and  $d$  mean *at the upstream end* and *at the downstream end*, re-  
 273 spectively.

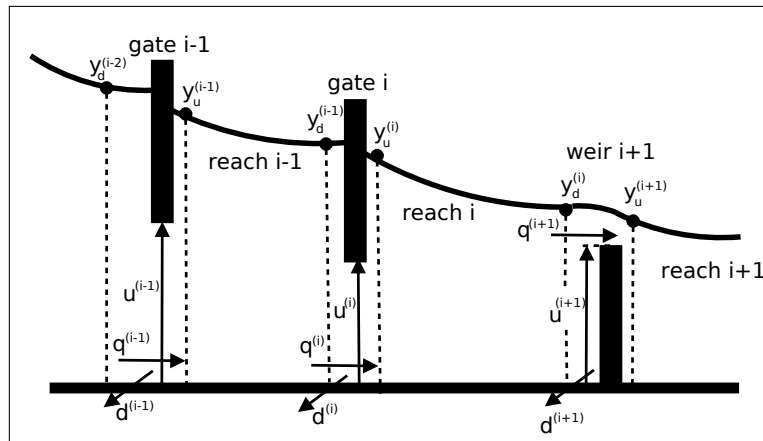


Figure 3: Navigation canal schematic with the variables involved

### 274 3.5. Final control-oriented model

275 The final, equivalent state-space representation is derived step by step. This new formulation **R3.5**  
 276 allows to coordinate current and delayed information in a systematic manner. Indeed, the model  
 277 is described by variables with an immediate and a delayed effect. Those variables with a delayed  
 278 effect are provided to the control and state estimation algorithms as parameters, ensuring that  
 279 their values are taken into account adequately, which is crucial for a satisfactory performance of  
 280 the algorithms. Remark: In the state-space model formulation, the notation  $\mathbf{q}$  represents the  
 281 discharges, whereas the variable  $\mathbf{u}$  is saved for the openings and elevations, and will be used later  
 282 on.

283 Model (5) can be rewritten as

$$\begin{aligned} y_1(s) &= p_{11}(s) q_1(s) - p_{12}(s) q_2(s), \\ y_2(s) &= p_{21}(s) q_1(s) - p_{22}(s) q_2(s). \end{aligned} \tag{10}$$

284 Then, (6) is substituted in (10), taking into account the parameter naming adopted in Section  
285 3.4, leading to

$$\begin{aligned} y_1(s) &= \frac{z_{11}s + 1}{A_us} q_1(s) - \frac{z_{12}s + 1}{A_us} e^{-\tau_us} q_2(s), \\ y_2(s) &= \frac{z_{21}s + 1}{A_ds} e^{-\tau_ds} q_1(s) - \frac{z_{22}s + 1}{A_ds} q_2(s). \end{aligned} \tag{11}$$

286 In order to simplify the task, the delays are initially dropped, and then reincorporated when  
287 the state-space representation is obtained. Additionally, a convenient manipulation of (11) leads  
288 to

$$\begin{aligned} y_1(s) &= \left( \frac{1/A_u}{s} + \frac{z_{11}}{A_u} \right) q_1(s) - \left( \frac{1/A_u}{s} + \frac{z_{12}}{A_u} \right) q_2(s), \\ y_2(s) &= \left( \frac{1/A_d}{s} + \frac{z_{21}}{A_d} \right) q_1(s) - \left( \frac{1/A_d}{s} + \frac{z_{22}}{A_d} \right) q_2(s). \end{aligned} \tag{12}$$

289 A standard transformation of (12) (*see* Chapter 2.5 in [30] for more details) yields the state-  
290 space representation

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \mathbf{q}(t), \\ \mathbf{y}(t) &= \begin{bmatrix} \frac{1}{A_u} & 0 \\ 0 & \frac{1}{A_d} \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} \frac{z_{11}}{A_u} & -\frac{z_{12}}{A_u} \\ \frac{z_{21}}{A_d} & -\frac{z_{22}}{A_d} \end{bmatrix} \mathbf{q}(t). \end{aligned} \tag{13}$$

291 Model (13) is discretized with a sampling time  $T_s$  as follows:

$$\begin{aligned} \mathbf{x}_{k+1} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} T_s & -T_s \\ T_s & -T_s \end{bmatrix} \mathbf{q}_k, \\ \mathbf{y}_k &= \begin{bmatrix} \frac{1}{A_u} & 0 \\ 0 & \frac{1}{A_d} \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} \frac{z_{11}}{A_u} & -\frac{z_{12}}{A_u} \\ \frac{z_{21}}{A_d} & -\frac{z_{22}}{A_d} \end{bmatrix} \mathbf{q}_k. \end{aligned} \tag{14}$$

292 The time delays are re-incorporated into (14), which yields

$$\begin{aligned}
\mathbf{x}_{k+1} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} T_s & 0 \\ 0 & -T_s \end{bmatrix} \mathbf{q}_k + \begin{bmatrix} 0 & -T_s \\ T_s & 0 \end{bmatrix} \mathbf{q}_{k-n}, \\
\mathbf{y}_k &= \begin{bmatrix} \frac{1}{A_u} & 0 \\ 0 & \frac{1}{A_d} \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} \frac{z_{11}}{A_u} & 0 \\ 0 & -\frac{z_{22}}{A_d} \end{bmatrix} \mathbf{q}_k + \begin{bmatrix} 0 & -\frac{z_{12}}{A_u} \\ \frac{z_{21}}{A_d} & 0 \end{bmatrix} \mathbf{q}_{k-n},
\end{aligned} \tag{15}$$

293 with  $\mathbf{q}_{k-n}$  the vector of discharges delayed  $n$  samples ( $n = \lceil \tau/T_s \rceil$ , with  $\lceil \cdot \rceil$  the ceiling function).

294 In practice,  $\tau_d \approx \tau_u$ , which leads to a single value of  $n$ .

295 Finally, the disturbances introduced in Section 3.2 are incorporated to the model. Since these **R2.4**  
296 lock operations are also flows, and the locks are next to the actuators, their effect on the system is  
297 the same as the controlled discharges. Thus, the matrices for controlled discharges and disturbances  
298 are the same, leading to

$$\begin{aligned}
\mathbf{x}_{k+1} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} T_s & 0 \\ 0 & -T_s \end{bmatrix} \mathbf{q}_k + \begin{bmatrix} 0 & -T_s \\ T_s & 0 \end{bmatrix} \mathbf{q}_{k-n} + \begin{bmatrix} T_s & 0 \\ 0 & -T_s \end{bmatrix} \mathbf{d}_k + \begin{bmatrix} 0 & -T_s \\ T_s & 0 \end{bmatrix} \mathbf{d}_{k-n}, \\
\mathbf{y}_k &= \begin{bmatrix} \frac{1}{A_u} & 0 \\ 0 & \frac{1}{A_d} \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} \frac{z_{11}}{A_u} & 0 \\ 0 & -\frac{z_{22}}{A_d} \end{bmatrix} \mathbf{q}_k + \begin{bmatrix} 0 & -\frac{z_{12}}{A_u} \\ \frac{z_{21}}{A_d} & 0 \end{bmatrix} \mathbf{q}_{k-n} + \begin{bmatrix} \frac{z_{11}}{A_u} & 0 \\ 0 & -\frac{z_{22}}{A_d} \end{bmatrix} \mathbf{d}_k + \begin{bmatrix} 0 & -\frac{z_{12}}{A_u} \\ \frac{z_{21}}{A_d} & 0 \end{bmatrix} \mathbf{d}_{k-n}.
\end{aligned} \tag{16}$$

299 As mentioned before, (16) must be obtained for each reach in the case study. Then,  $\mathbf{q}_k$  and  
300  $\mathbf{q}_{k-n}$  must be substituted in each case by either (2) or (3) accordingly. It can be anticipated that  
301 this substitution will cause delayed states to appear in the model. Indeed,  $\mathbf{q}_k = f(\mathbf{y}_k, \mathbf{u}_k)$ , and  
302 thus  $\mathbf{q}_{k-n} = f(\mathbf{y}_{k-n}, \mathbf{u}_{k-n})$ , with  $\mathbf{y}_{k-n} = g(\mathbf{x}_{k-n})$ , and  $f$  and  $g$  are the corresponding relationships  
303 among the variables.

304 Although the step-by-step derivation of the final model is given in Section 5, it is convenient  
305 to present its final structure at this stage, since it will be used in Section 4 to design the controller  
306 and the estimator. Therefore, the general model formulation of a system with  $n_x$  states,  $n_u$  inputs  
307 and  $n_y$  outputs is

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{A}_n\mathbf{x}_{k-n} + \mathbf{B}_u\mathbf{u}_k + \mathbf{B}_{un}\mathbf{u}_{k-n} + \mathbf{B}_d\mathbf{d}_k + \mathbf{B}_{dn}\mathbf{d}_{k-n}, \quad (17a)$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{C}_n\mathbf{x}_{k-n} + \mathbf{D}_u\mathbf{u}_k + \mathbf{D}_{un}\mathbf{u}_{k-n} + \mathbf{D}_d\mathbf{d}_k + \mathbf{D}_{dn}\mathbf{d}_{k-n}, \quad (17b)$$

308 with  $\mathbf{x}_k \in \mathbb{R}^{n_x}$ ,  $\mathbf{u}_k \in \mathbb{R}^{n_u}$ ,  $\mathbf{y}_k \in \mathbb{R}^{n_y}$ , and  $\mathbf{A}$ ,  $\mathbf{A}_n$ ,  $\mathbf{B}_u$ ,  $\mathbf{B}_{un}$ ,  $\mathbf{B}_d$ ,  $\mathbf{B}_{dn}$ ,  $\mathbf{C}$ ,  $\mathbf{C}_n$ ,  $\mathbf{D}_u$ ,  $\mathbf{D}_{un}$ ,  $\mathbf{D}_d$  and  $\mathbf{D}_{dn}$   
 309 are time-invariant matrices of suitable dimensions. The state equation is given by (17a), and (17b)  
 310 is the output equation. The mass balances given by (4) can be formulated by means of constraints,  
 311 as it is now, or they can be incorporated into (17a) as shown in [31].

312 Model (17) corresponds to the case of only one delay in the network. The general case for a  
 313 system with multiple delays given by the set  $S = \{n_1, n_2, \dots, n_p\}$  reads as

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}_u\mathbf{u}_k + \mathbf{B}_d\mathbf{d}_k + \sum_{n_i \in S} (\mathbf{A}_{n_i}\mathbf{x}_{k-n_i} + \mathbf{B}_{un_i}\mathbf{u}_{k-n_i} + \mathbf{B}_{dn_i}\mathbf{d}_{k-n_i}), \quad (18a)$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{D}_u\mathbf{u}_k + \mathbf{D}_d\mathbf{d}_k + \sum_{n_i \in S} (\mathbf{C}_{n_i}\mathbf{x}_{k-n_i} + \mathbf{D}_{un_i}\mathbf{u}_{k-n_i} + \mathbf{D}_{dn_i}\mathbf{d}_{k-n_i}). \quad (18b)$$

314 Furthermore, (4) must be modified as

$$\mathbf{0} = \mathbf{E}_u\mathbf{u}_k + \mathbf{E}_d\mathbf{d}_k + \sum_{n_i \in S} (\mathbf{E}_{un_i}\mathbf{u}_{k-n_i} + \mathbf{E}_{dn_i}\mathbf{d}_{k-n_i}) \quad (19)$$

315 in order to account for multiple time delays.

316 Finally, note that the delayed terms in (17) and (18) cause these models to not be representable  
 317 using the standard state-space formulation. While the theory of other classical state feedback  
 318 control techniques might not be used for this representation, the combination of MPC and MHE  
 319 can deal with these models in a suitable manner. Furthermore, this formulation also allows a flexible  
 320 and more compact notation of a system with delayed variables. Indeed, a common approach to  
 321 represent such systems consists in the augmentation procedure described in [19], where the delay  
 322 effect is incorporated as a dead-beat dynamic to obtain an undelayed representation. However,  
 323 a downside of this methodology lies in the large dimensionality of the resulting description. By  
 324 contrast, no augmented model needs to be derived in the case of MPC and MHE.



## 325 4. Control design and state estimation

326 This section deals with the control design and state estimation, for which the model of the  
327 system is needed. An approach is presented for each of them, namely MPC for control and MHE  
328 for state estimation, for which the complete structure of the resulting multi-objective optimization  
329 problems is given. The last part of the section provides some insight on the combination of MPC  
330 and MHE in simulation.

### 331 4.1. Control design: the MPC approach

332 Modern inland waterways are complex, multivariable systems whose management requires the  
333 use of advanced control methods [32]. MPC is characterized by several interesting features that  
334 are very suitable for these kinds of systems [33, 34]:

- 335 • The model of the system captures the dynamic and static interactions between input, output  
336 and disturbance variables.
- 337 • The physical constraints on inputs and outputs can be handled in a systematic manner.
- 338 • Multiple operational goals can be taken into account simultaneously.
- 339 • It is particularly suitable for those systems for which the disturbances can be forecasted.

340 The main principle of MPC resides in computing a sequence of inputs that causes the predicted  
341 response of the system to move to the desired setpoint in an optimal manner while respecting the  
342 constraints. The constraints imposed by the elements that make up the model have already been  
343 defined in Sections 3.1–3.4. On the other hand, the set of operational goals is defined below.

#### 344 4.1.1. Operational goals and multi-objective function

345 One or more operational goals are expected to be achieved during the process. To this end, a  
346 certain criterion is optimized in the computation of the control signals. This criterion is usually  
347 built as the weighted sum of several terms, where each of them represents an operational goal.  
348 Note that the set of operational goals that can be taken into account is not unique. In this work,  
349 the following are considered:

350 • *Maintaining the water levels close to the setpoints:* This is the most important objective to  
 351 be fulfilled. Its mathematical formulation reads as

$$J_k^1 = (\mathbf{y}_k - \mathbf{y}_r)^\top (\mathbf{y}_k - \mathbf{y}_r), \quad (20)$$

352 with  $\mathbf{y}_r$  the vector of NNL values.

353 • *Cost reduction:* This term reflects the economic costs derived from operating the available  
 354 equipment. It can be formulated as

$$J_k^2 = \boldsymbol{\gamma} \mathbf{u}_k^\top \mathbf{u}_k, \quad (21)$$

355 with  $\boldsymbol{\gamma}$  the vector of known costs associated to the equipment operation.

356 • *Smoothness of the control signal:* In order to avoid wear and tear, and increase the lifespan  
 357 of the equipment, it is a common practice to penalize the control signal variation between  
 358 consecutive time instants:

$$J_k^3 = \Delta \mathbf{u}_k^\top \Delta \mathbf{u}_k, \quad (22)$$

359 with  $\Delta \mathbf{u}_k = \mathbf{u}_k - \mathbf{u}_{k-1}$ .

360 • *Penalty in the relaxation parameter:*  $\boldsymbol{\alpha}_k$ , which was introduced in (8), is penalized to ensure  
 361 that the water levels are outside the navigation interval as little as possible:

$$J_k^4 = \boldsymbol{\alpha}_k^\top \boldsymbol{\alpha}_k. \quad (23)$$

362 The multi-objective function  $J$  that gathers the control objectives can be described by

$$J(\mathbf{u}_k, \mathbf{y}_k, \boldsymbol{\alpha}_k) = \sum_{k=1}^{H_p} \sum_{j=1}^4 \beta^j J_k^j, \quad (24)$$

363 where  $H_p$  is the prediction horizon and  $\beta^j$  are the weights of the  $j$ -th objective. Note that  $H_p$  must  
 364 be chosen according not only to the system dynamics (settling time), but also to take into account

365 the system delays. Therefore,  $H_p > t_s + \max S$ , where  $t_s$  is the settling time (in samples), and  $S$   
366 was defined for (18). Moreover, in order to set the weight of each objective in a multi-objective **R3.8**  
367 optimization problem, the procedure described in [35] can be used.

#### 368 4.1.2. MPC formulation

369 Gathering the control-oriented model, the system constraints and the multi-objective function,  
370 the design of the MPC follows classical approaches [33, 34]: an optimization problem is solved  
371 over a prediction horizon, minimizing a cost function while respecting the system constraints. The  
372 first component of the vector of control inputs is extracted from the solution and is applied to the  
373 system, and the rest are disregarded. This procedure is repeated at each time instant, following a **R2.8**  
374 receding-horizon strategy.

375 The optimization problem is given by

$$\min_{\{\mathbf{u}_{i|k}\}_{i=k}^{k+H_p-1}, \{\mathbf{y}_{i|k}\}_{i=k}^{k+H_p-1}, \{\boldsymbol{\alpha}_{i|k}\}_{i=k}^{k+H_p-1}} J(\mathbf{u}_{i|k}, \mathbf{y}_{i|k}, \boldsymbol{\alpha}_{i|k}) \quad (25a)$$

subject to:

$$\mathbf{x}_{i+1|k} = \mathbf{A}\mathbf{x}_{i|k} + \mathbf{A}_n\mathbf{x}_{i-n|k} + \mathbf{B}_u\mathbf{u}_{i|k} + \mathbf{B}_{un}\mathbf{u}_{i-n|k} + \quad (25b)$$

$$\mathbf{B}_d\mathbf{d}_{i|k} + \mathbf{B}_{dn}\mathbf{d}_{i-n|k}, \quad i \in \{k, \dots, k + H_p - 1\},$$

$$\mathbf{y}_{i|k} = \mathbf{C}\mathbf{x}_{i|k} + \mathbf{C}_n\mathbf{x}_{i-n|k} + \mathbf{D}_u\mathbf{u}_{i|k} + \mathbf{D}_{un}\mathbf{u}_{i-n|k} + \quad (25c)$$

$$\mathbf{D}_d\mathbf{d}_{i|k} + \mathbf{D}_{dn}\mathbf{d}_{i-n|k}, \quad i \in \{k, \dots, k + H_p - 1\},$$

$$\mathbf{0} = \mathbf{E}_u\mathbf{u}_{i|k} + \mathbf{E}_{un}\mathbf{u}_{i-n|k} + \mathbf{E}_d\mathbf{d}_{i|k} + \mathbf{E}_{dn}\mathbf{d}_{i-n|k}, \quad i \in \{k, \dots, k + H_p - 1\}, \quad (25d)$$

$$\underline{\mathbf{u}}^m \leq \mathbf{u}_{i|k}^m \leq \bar{\mathbf{u}}^m, \quad i \in \{k, \dots, k + H_p - 1\}, \quad (25e)$$

$$\underline{\mathbf{y}}_r - \boldsymbol{\alpha}_{i|k} \leq \mathbf{y}_{i|k} \leq \bar{\mathbf{y}}_r + \boldsymbol{\alpha}_{i|k}, \quad i \in \{k, \dots, k + H_p - 1\}, \quad (25f)$$

$$\boldsymbol{\alpha}_{i|k} \geq \mathbf{0}, \quad i \in \{k, \dots, k + H_p - 1\}, \quad (25g)$$

$$\mathbf{x}_{j|k} = \hat{\mathbf{x}}_j^{MHE}, \quad j \in \{k - n, \dots, k\}, \quad (25h)$$

$$\mathbf{u}_{l|k} = \mathbf{u}_l^{MPC}, \quad l \in \{k - n, \dots, k - 1\}, \quad (25i)$$

376 where  $k$  is the current time instant,  $i$  is the time instant along the prediction horizon and  $k + i|k$

377 indicates the predicted value of the variable at instant  $k + i$  using information available at instant **R1.8**  
 378  $k$ . Remark:  $j$  and  $l$  are used to indicate the use of past information, for which the considered  
 379 time intervals are different than the one described by  $i$ . Equations (25b) and (25c) correspond to  
 380 the model described by (17), (25d) are the mass balances given in (4), and (25e)–(25g) are the  
 381 constraints given in (1), (8) and (9), respectively. Equation (25h) sets the values of the delayed **R2.10b**  
 382 states according to the solution provided by the MHE (noted as  $\hat{\mathbf{x}}_i^{MHE}$ ) in past iterations. Note  
 383 that the MHE will be introduced in Section 4.2. These delayed values are provided to the MPC  
 384 as parameters. Similarly, the delayed control actions obtained by the MPC (noted as  $\mathbf{u}_i^{MPC}$ ) in  
 385 previous iterations are also provided as parameters by means of (25i).

386 The optimal solution is given by the sequences  $\{\mathbf{u}_{i|k}\}_{i=k}^{k+H_p-1}$ ,  $\{\mathbf{y}_{i|k}\}_{i=k}^{k+H_p-1}$ ,  $\{\boldsymbol{\alpha}_{i|k}\}_{i=k}^{k+H_p-1}$ . As **R3.9**  
 387 it was stated before, only  $\mathbf{u}_{k|k}$  is applied to the system, according to the receding philosophy

$$\mathbf{u}_k^{MPC} \triangleq \mathbf{u}_{k|k}. \quad (26)$$

#### 388 4.2. State estimation: the MHE approach

389 The control strategy presented in Section 4.1 uses the states to compute the set of optimal  
 390 control actions. The system states oftentimes are not directly measurable, and therefore they need  
 391 to be estimated from the available data using a state estimator.

392 Thus, the problem to be solved is that of designing an observer that fully reconstructs the  
 393 system states. In this work, the MHE is used for this purpose. The main principle of this technique  
 394 consists in formulating the estimation problem as a quadratic program using a moving estimation  
 395 window of a fixed size [36, 37]. Indeed, it is assumed that only part of the available information  
 396 of the system (inputs and outputs) is considered, which is shifted in time to consider the most  
 397 recent information. Otherwise, the computational burden renders the full-information problem  
 398 impractical to solve, as more and more data are processed with time. In this way, a truncated  
 399 sequence of state estimates is computed at each time step instead of the full-state sequence to make  
 400 the problem tractable [38].

401 The formulation corresponding to the optimization problem solved by the MHE reads as

---

<sup>1</sup> $\{\mathbf{u}_{i|k}\}_{i=k}^{k+H_p-1} \triangleq \{\mathbf{u}_{k|k}, \mathbf{u}_{k+1|k}, \dots, \mathbf{u}_{k+H_p-1|k}\}$ ;  $\mathbf{y}_{i|k}$  and  $\boldsymbol{\alpha}_{i|k}$  are defined in the same manner

$$\min_{\{\hat{\mathbf{x}}_{i|k}\}_{i=k-N}^k} \left( \hat{\mathbf{x}}_{k-N|k} - \mathbf{x}_{k-N} \right)^\top \mathbf{P}^{-1} \left( \hat{\mathbf{x}}_{k-N|k} - \mathbf{x}_{k-N} \right) + \quad (27a)$$

$$\sum_{i=k-N}^{k-1} \left( \mathbf{w}_{i|k}^\top \mathbf{Q}^{-1} \mathbf{w}_{i|k} + \mathbf{v}_{i|k}^\top \mathbf{R}^{-1} \mathbf{v}_{i|k} \right)$$

subject to:

$$\mathbf{w}_{i|k} = \hat{\mathbf{x}}_{i+1|k} - \left( \mathbf{A} \hat{\mathbf{x}}_{i|k} + \mathbf{A}_n \hat{\mathbf{x}}_{i-n|k} + \mathbf{B}_u \mathbf{u}_{i|k} + \mathbf{B}_{un} \mathbf{u}_{i-n|k} + \mathbf{B}_d \mathbf{d}_{i|k} + \mathbf{B}_{dn} \mathbf{d}_{i-n|k} \right), \quad i \in \{k-N, \dots, k-1\}, \quad (27b)$$

$$\mathbf{v}_{i|k} = \mathbf{y}_{i|k} - \left( \mathbf{C} \hat{\mathbf{x}}_{i|k} + \mathbf{C}_n \hat{\mathbf{x}}_{i-n|k} + \mathbf{D}_u \mathbf{u}_{i|k} + \mathbf{D}_{un} \mathbf{u}_{i-n|k} + \mathbf{D}_d \mathbf{d}_{i+|k} + \mathbf{D}_{dn} \mathbf{d}_{i-n|k} \right), \quad i \in \{k-N, \dots, k-1\}, \quad (27c)$$

$$\mathbf{0} = \mathbf{E}_u \mathbf{u}_{i|k} + \mathbf{E}_{un} \mathbf{u}_{i-n|k} + \mathbf{E}_d \mathbf{d}_{i|k} + \mathbf{E}_{dn} \mathbf{d}_{i-n|k}, \quad i \in \{k-N, \dots, k-1\}, \quad (27d)$$

$$\underline{\mathbf{x}}_r \leq \hat{\mathbf{x}}_{i|k} \leq \bar{\mathbf{x}}_r, \quad i \in \{k-N, \dots, k-1\}, \quad (27e)$$

$$\hat{\mathbf{x}}_{j|k} = \hat{\mathbf{x}}_j^{MHE}, \quad j \in \{k-N-n, \dots, k-N-1\}, \quad (27f)$$

$$\mathbf{u}_{l|k} = \mathbf{u}_l^{MPC}, \quad l \in \{k-N-n, \dots, k-1\}, \quad (27g)$$

$$\mathbf{y}_{m|k} = \mathbf{y}_m, \quad m \in \{k-N, \dots, k-1\}, \quad (27h)$$

402 with (27b) accounting for the system disturbances and (27c) for the measurement noise. Addi-  
403 tionally, (27d) describes the static part of the model, (27e) defines the valid interval of the state  
404 variables, and (27f), (27g) and (27h) set the values of the delayed states, inputs and outputs, re-  
405 spectively, following the same ideas as in (25). The value  $\mathbf{x}_{k-N}$  in (27a) corresponds to the most **R3.10**  
406 likely initial state vector, and is chosen based on the available knowledge of the system, whereas  
407  $\hat{\mathbf{x}}_{k-N|k}$  is the first value of the optimal state sequence computed by the MHE at time instant  $k$ .  
408 The error in this initial guess, given by  $(\hat{\mathbf{x}}_{k-N|k} - \mathbf{x}_{k-N})$ , is weighted by means of the matrix **R3.11**  
409  $\mathbf{P}^{-1}$ , which indicates the confidence into the initial state, and its tuning allows to guarantee the  
410 boundedness of the estimation, as discussed in [39]. On the other hand,  $\mathbf{Q}^{-1}$  and  $\mathbf{R}^{-1}$  are the  
411 weighting matrices inverses of suitable dimensions linked to the confidence in the quality of the  
412 model and the measurements, respectively. The larger these matrices are, the lesser the confidence  
413 in the associated term is, as the matrices are inverted. These inverses are directly related to the co-

414 variance matrices only in the case of linear systems with zero-mean uncorrelated random variables  
 415 for unknown disturbances [40]. In any other situation, e.g., constrained states, this connection is  
 416 only an approximation.

417 The MHE problem (27) is formulated as follows: at the current time instant  $k$ ,  $N$  input-output  
 418 pairs  $[(\mathbf{u}_{k-N}, \mathbf{y}_{k-N}) : (\mathbf{u}_{k-1}, \mathbf{y}_{k-1})]$  shall be available. Therefore,  $N$  is the length of the moving  
 419 estimation window, which bounds the size of the problem. The resulting least-squares problem is  
 420 solved, yielding the optimal sequence  $\{\hat{\mathbf{x}}_{i|k}\}_{i=k-N}^k$ . However, as is the case in the MPC problem,  
 421 only one value in the sequence is considered, and the rest are discarded. In the MHE problem, this  
 422 corresponds to the last value, that is,  $\hat{\mathbf{x}}_{k|k}$ . Therefore,

$$\hat{\mathbf{x}}_k^{MHE} \triangleq \hat{\mathbf{x}}_{k|k}. \quad (28)$$

423 In the next iteration, for  $k' = k + 1$ , the truncated data sequence is updated, and becomes  
 424  $[(\mathbf{u}_{k'-N}, \mathbf{y}_{k'-N}) : (\mathbf{u}_{k'-1}, \mathbf{y}_{k'-1})]$ , which is equivalent to  $[(\mathbf{u}_{k-N+1}, \mathbf{y}_{k-N+1}) : (\mathbf{u}_k, \mathbf{y}_k)]$ . Then, note  
 425 that the oldest measurement pair  $(\mathbf{u}_{k-N}, \mathbf{y}_{k-N})$  is dropped, and the newest measurement pair  
 426  $(\mathbf{u}_k, \mathbf{y}_k)$  is incorporated, following the moving horizon philosophy.

### 427 4.3. Simulation

428 Once the MPC and MHE are designed, they must be integrated in the simulation loop. The  
 429 solution of the controller (the optimal control inputs), together with the measurements, are fed  
 430 into the estimator. In turn, the estimator computes the optimal state estimates, which are used in  
 431 the next time instant by the controller to compute the new set of control inputs.

432 The real system is equipped with sensors, which provide water level measurements. Unfortu- **R1.9**  
 433 nately, these real values are not available in this work. Since the MHE requires the measurements  
 434 to estimate the states, these must be generated in simulation, using the output equation (17b).  
 435 The effect of this limitation is that the estimator cannot be used for the first time at  $k = 1$ , but  
 436 at  $k = N + 1$ . Indeed, an input-measurement pair  $(\mathbf{u}_k, \mathbf{y}_k)$  will be generated at each time instant  
 437  $k$ , using the solution of the controller. Thus, the necessary data to compute the state estimates  
 438 will not be available until  $N$  samples have elapsed. In addition, the system must be manually **R2.11**  
 439 initialized by selecting any feasible state vector.

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<sup>2</sup> $\{\hat{\mathbf{x}}_{i|k}\}_{i=k-N}^k \triangleq \{\hat{\mathbf{x}}_{k-N|k}, \hat{\mathbf{x}}_{k-N+1|k}, \dots, \hat{\mathbf{x}}_{k|k}\}$

440 Finally, note that, at time  $k$ , the MPC yields the sequence  $\{\mathbf{u}_{i|k}\}_{i=k}^{k+H_p-1}$ . Thus, it is necessary  
441 to know the disturbances until, at least, the instant  $k + H_p - 1$ . By contrast, the MHE sets its  
442 starting point  $N$  samples in the past, and reconstructs the optimal sequence of state estimates until  
443 the current time instant  $k$ . This requirement in terms of available information is fulfilled based on  
444 the policy introduced in Section 3.2, which allows to anticipate future lock operations.

## 445 5. Case study

446 This section illustrates the performance of the MPC and the MHE by means of a realistic case  
447 study system, which is based on part of the inland waterways in the north of France. First, the  
448 system is described, emphasizing its physical features, so that the modeling step yields a model  
449 as close as possible to the real system. Then, the experimental design step is regarded, describing  
450 the considered scenarios, which try to represent faithfully an average navigation day. Finally, the  
451 results coming from the experimental design are presented and discussed.

### 452 5.1. System description

453 The inland waterways in the north of France is linked with the Belgian and Dutch inland  
454 waterways, and is managed by *Voies Navigables de France*<sup>3</sup> (VNF). Its main objective is that of  
455 guaranteeing the navigability condition, which is achieved by keeping the water levels inside the  
456 navigation rectangle defined by the LNL and the HNL, and as close as possible to the>NNL.

457 This inland navigation network consists of more than fifty reaches that are interconnected by  
458 locks, gates and weirs. Part of it is depicted in Fig. 4, which shows the two reaches considered  
459 in the case study. The  $i$ -th reach is labeled as  $\text{NR}_i$ , and its setpoint (NNL) is specified in red. In **R3.13**  
460 addition, the locks that connect adjacent reaches are labeled in black.

461 A more schematic view of the system is depicted in Fig. 5, resulting in a four-reach case study.

462 The case study choice is motivated by the following reasons:

- 463 • It features a distributary, which branches off from  $\text{NR}_1$  at an intermediate point and flows  
464 away, to the lock of Don. This topology is regarded as of special interest, since the mass  
465 balance at this natural bifurcation (not controlled) is not straightforward to model. Indeed,  
466 a possible approach for this situation is shown below.

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<sup>3</sup><http://www.vnf.fr>

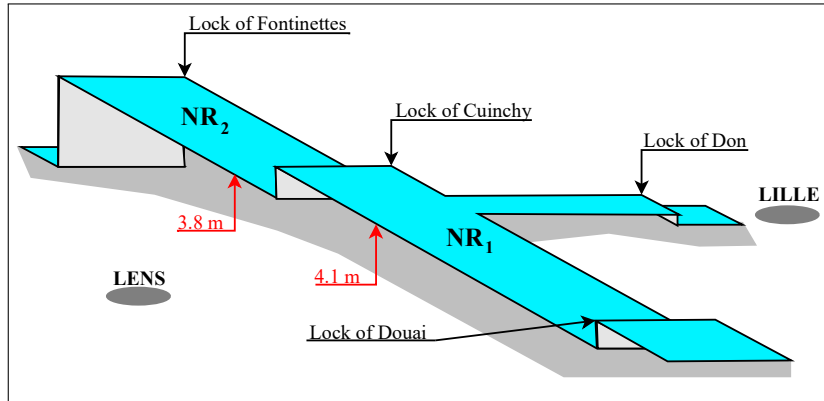


Figure 4: Part of the inland waterways in the north of France

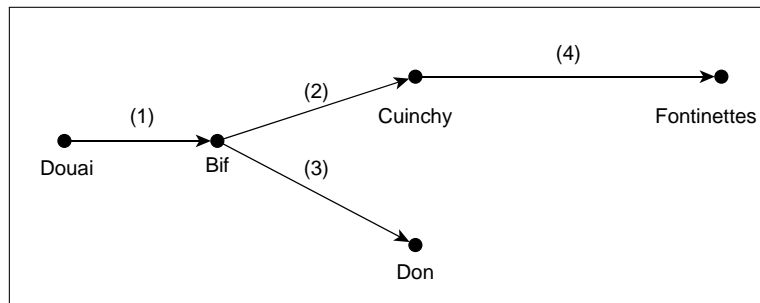


Figure 5: Schematic diagram of the case study

467 • Besides,  $NR_2$  is an important reach in this network for two reasons [41]: its strategic loca-  
 468 tion, which allows dispatching water among the three major catchments in the region; and  
 469 its downstream lock in Fontinettes, which performs the largest lock operations in terms of  
 470 volume, and is therefore responsible for the largest disturbances. [Being able to deal with R3.14](#)  
 471 [the worst-case scenario can give a feel for the magnitude of the disturbances that the control](#)  
 472 [strategy attempts to reject.](#)

473 Since the bifurcation is of natural type (uncontrolled), this node can be eliminated, based on  
 474 an estimation of the ratios of the total flow for each stream after the bifurcation. Indeed, it can be  
 475 considered, according to VNF, that the each of the flows after the bifurcation correspond to 50%  
 476 of the flow before the bifurcation. This yields the simplified, final three-reach case study scheme  
 477 given in Fig. 6. Note that the reaches are renamed for convenience, and also the nodes for labeling  
 478 purposes.

479 The physical parameters of the case study are summarized in Table 1. Note that the lengths  
 480 of the reaches are approximately the same, which results in the same time delay (in samples) for



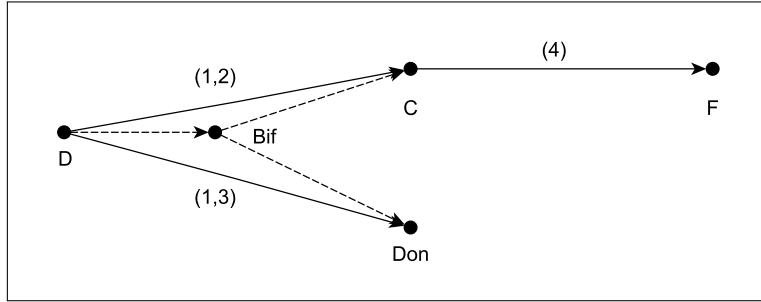


Figure 6: Simplified schematic diagram of the case study

481 the three reaches. A sampling time  $T_s = 20 \text{ min}$  has been considered in the model discretization,  
 482 which is deemed reasonable due to the slow dynamics of the system. This allows to use model (17),  
 483 and thus there is no need to resort to the more general case (18).

Table 1: Parameters of the reaches

Reach nr.	LNL [m]	NNL [m]	HNL [m]	$L$ [m]	$w_r$ [m]	$m_r$ [m/m]	$s_b$ [m/m]	$n_r$ [s/m <sup>1/3</sup> ]	$Q_s$ [m <sup>3</sup> /s]
(1,2)	3.95	4.1	4.25	39000	50	0	0	0.035	0.6
(1,3)	3.95	4.1	4.25	37000	50	0	0	0.035	0.6
(4)	3.65	3.8	3.95	42000	50	0	0	0.035	0.6

484 LNL, NNL and HNL are the relative lower, normal and higher navigation levels (with respect  
 485 to the bottom of the reach),  $L$  is the length of the reaches,  $w_r$  is the bottom width,  $m_r$  is the side  
 486 slope of the reach ( $m_r = 0$  for a rectangular cross section),  $s_b$  is the bottom slope ( $s_b = 0$  for a  
 487 flat reach),  $n_r$  is the Manning roughness coefficient and  $Q_s$  is the operating point considered when  
 488 linearizing the Saint-Venant equations. Indeed, it is considered that an average flow of  $1.2 \text{ m}^3/\text{s}$   
 489 comes from upstream of Douai, and that it is divided into two equal parts after it. The numerical  
 490 state-space matrices computed from these parameters are given in Appendix A.

491 In order to estimate the states, the measurements of the system are needed. Depending on the  
 492 structure of the problem, e.g., the topology of the network or the connections between reaches,  
 493 not all measurements are required. This statement can be realized by inspecting matrix  $\mathbf{C}$ , which  
 494 links the states and the measurements. In the present case, it is not necessary to consider the  
 495 measurement  $y_k^{C(1,2)}$ , since the associated state has an effect on the downstream level  $y_k^{C(4)}$ , given  
 496 by the off-diagonal, nonzero entry in the fourth row of  $\mathbf{C}$ . Thus, in order to show the effectiveness of  
 497 the approach, the six states will be reconstructed with only five measurements, i.e., assuming that  
 498 the water level  $y_k^{C(1,2)}$  is not available. Thus, this value will be obtained from the state estimates.

499 The lock operations that take place in Douai, Don, Cuinchy and Fontinettes disturb the system.  
 500 Their average magnitudes and durations are given in Table 2.

Table 2: Lock operations

Lock	Dispatched water volume [ $m^3$ ]	Duration [ $min$ ]
Douai	18000	20
Cuinchy	12000	20
Don	12000	20
Fontinettes	30000	20

501 On the other hand, the same four nodes are equipped with controlled devices which allow to  
 502 dispatch water to fulfill the control objectives. In particular, Douai and Fontinettes are equipped  
 503 with undershot gates, whereas Don and Cuinchy are equipped with weirs. It is considered that both  
 504 the gates and the weirs can deliver a maximum flow of  $10 m^3/s$ , which will have to be converted  
 505 into maximum gate openings and sill elevations, respectively. Their nonlinear expressions, as well  
 506 as their linearized equations, are given in Appendix B.

### 507 5.2. Experimental design

508 A 24-hour scenario, depicted in Fig. 7, is designed by considering a lock operation time-series  
 509 model for a typical navigation profile. In the real system, the following existing management  
 510 restrictions must be taken into account:

- 511 • A day is divided in two periods: navigation and stoppage. Boats are only allowed to navigate  
 512 during the navigation period, which starts each day at 6 a.m. and finishes after fourteen  
 513 hours, at 8 p.m. The navigation is interrupted until the next day at 6 a.m.
- 514 • The current policy allows a maximum of two lock operations per hour.

515 Besides, the scenario does not consider changes in the setpoints, thus assuming that the navi-  
 516 gation conditions do not change during the simulation. Such modifications typically occur due to  
 517 changes in the weather condition, e.g., flood and drought episodes, which might require to readjust  
 518 the LNL, NNL and HNL values.

519 In order to test this scenario, it is necessary to compute the IDZ model of each reach as shown  
 520 in [18], and then discretize them using  $T_s$ . The global model (17) is built by stacking the IDZ  
 521 model of each reach, so that the MPC and MHE can be used together.

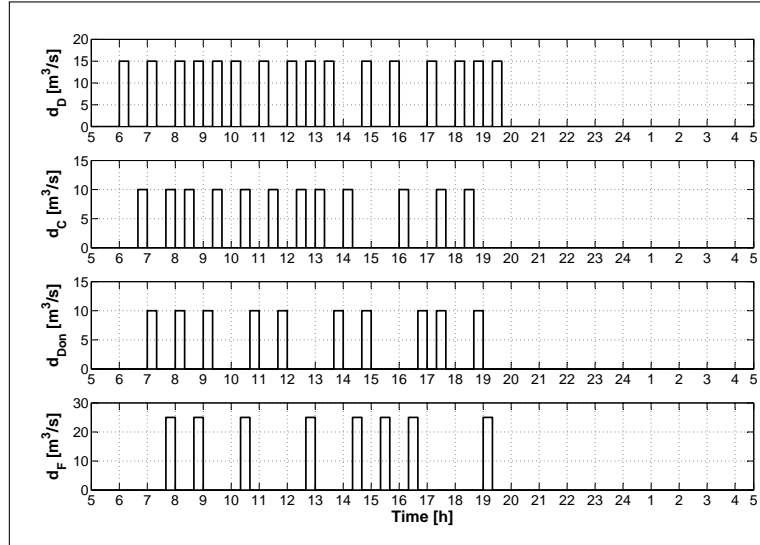


Figure 7: The considered lock operation profile

522 Furthermore, both the MPC and the MHE prediction horizon values are set equal to 12 samples  
 523 (4 hours). This dimensioning is aligned with the time delays of the reaches, which are in the **R3.15**  
 524 interval [6127, 6807] s, or equivalently [5.11, 5.67] samples. Thus, a unique delay of 6 samples can  
 525 be considered for all the reaches, according to the ceiling rule introduced in (15). In this way,  
 526 by setting the prediction horizon values equal to 12 samples, the waves are given enough time to  
 527 reflect at the downstream end and travel back.

### 528 5.3. Results

529 As mentioned before, any feasible initial state might be considered, as there is no available data  
 530 that can be used to estimate the initial conditions. Once the system reaches the steady state, it is  
 531 disturbed by considering the 24-hour navigation profile depicted in Fig. 7. The final state-space  
 532 model is computed, and then the results yielded by the estimator and the controller are shown.

#### 533 5.3.1. Control-oriented model

534 For the sake of convenience, the steps are outlined here, while the final system matrices as well  
 535 as the nonlinear and linearized equations are detailed in the Appendices A and B, respectively.

536 First, the methodology described in [18] must be applied for each reach, and then these partial  
 537 models (16) are stacked to build the global model. Next, the nonlinear equations of the gates and  
 538 weirs are linearized around the NNL and  $Q_s$  values given in Table 1. The linearized equations

539 must be substituted in the model. Note that  $q^D$  does not depend on its upstream water elevation.  
 540 Since this water level is outside of the scope of the control problem, it is considered that there is  
 541 enough water upstream of Douai, and thus that this level remains constant.

542 Next, it is necessary to substitute the water depths by the states in the linearized equations.  
 543 The relationship between these variables provided by matrix  $\mathbf{C}$  can be exploited to this effect.  
 544 Once this step has been completed, the linearized equations can be substituted in the model. As it  
 545 was mentioned in Section 3.5, the delayed expressions  $\mathbf{q}_{k-n} = f(\mathbf{x}_{k-n}, \mathbf{u}_{k-n})$  introduce the delayed  
 546 states in the final formulation. Note also that, since  $\mathbf{q}_k$  depends on the states, a rearrangement  
 547 and grouping of terms is required. The final model is then given by (17), with

$$\begin{aligned}
 \mathbf{x}_{k+1} &= \begin{bmatrix} x_{k+1}^{D(1,2)} & x_{k+1}^{C(1,2)} & x_{k+1}^{D(1,3)} & x_{k+1}^{Don} & x_{k+1}^{C(4)} & x_{k+1}^F \end{bmatrix}^\top, \\
 \mathbf{x}_k &= \begin{bmatrix} x_k^{D(1,2)} & x_k^{C(1,2)} & x_k^{D(1,3)} & x_k^{Don} & x_k^{C(4)} & x_k^F \end{bmatrix}^\top, \\
 \mathbf{x}_{k-n} &= \begin{bmatrix} x_{k-n}^{D(1,2)} & x_{k-n}^{C(1,2)} & x_{k-n}^{D(1,3)} & x_{k-n}^{Don} & x_{k-n}^{C(4)} & x_{k-n}^F \end{bmatrix}^\top, \\
 \mathbf{u}_k &= \begin{bmatrix} u_k^D & u_k^C & u_k^{Don} & u_k^F \end{bmatrix}^\top, \\
 \mathbf{u}_{k-n} &= \begin{bmatrix} u_{k-n}^D & u_{k-n}^C & u_{k-n}^{Don} & u_{k-n}^F \end{bmatrix}^\top, \\
 \mathbf{d}_k &= \begin{bmatrix} d_k^D & d_k^C & d_k^{Don} & d_k^F \end{bmatrix}^\top, \\
 \mathbf{d}_{k-n} &= \begin{bmatrix} d_{k-n}^D & d_{k-n}^C & d_{k-n}^{Don} & d_{k-n}^F \end{bmatrix}^\top, \\
 \mathbf{y}_k &= \begin{bmatrix} y_k^{D(1,2)} & y_k^{D(1,3)} & y_k^{Don} & y_k^{C(4)} & y_k^F \end{bmatrix}^\top.
 \end{aligned}$$

548 Since the bifurcation node can be eliminated as proposed in Fig. 6, (4) is no longer needed.

### 549 5.3.2. Estimator

550 The comparison between the optimal estimated states given by the MHE and the real states  
 551 obtained in simulation are depicted in Figure 8. At each time instant, the MHE is fed with the  
 552 corresponding sequence of  $N$  input-measurement pairs, which are used to compute the optimal  
 553 sequence of states.

554 In general, the values provided by the estimator match the real states with no significant error.  
 555 In addition, the real states are noisier than the estimated states. This behavior is in line with  
 556 the nature of the observer, which acts as a filter, smoothing the predictions. Furthermore, the

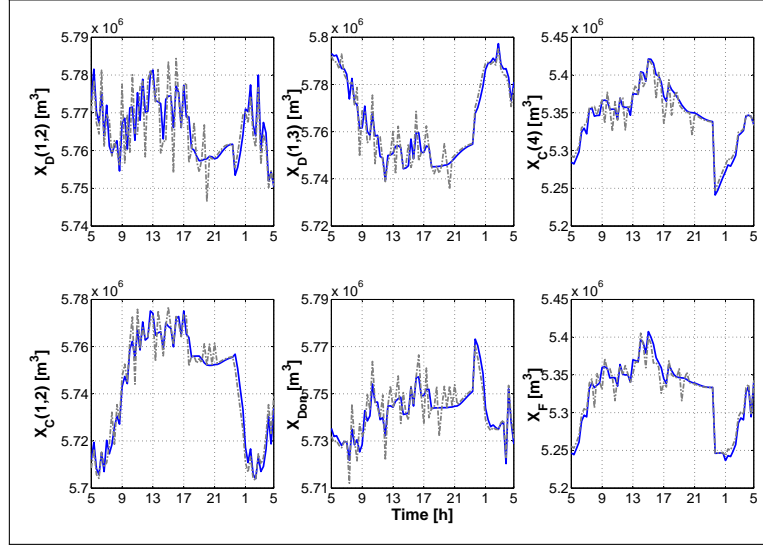


Figure 8: State estimates (blue solid lines) and computed states (dash-dot gray lines)

557 constraints on the state bounds are satisfied. Such bounds are not even depicted in Fig. 8 for the  
 558 benefit of a better visualization of the results, as the states are far from the bounds.

559 In order to ensure a quantitative comparison between the real and the estimated states, the  
 560 similarity of both signals is quantified by means of the correlation coefficient. Given a pair of  
 561 signals  $(m_t, n_t)$  with  $M$  observations each, the correlation coefficient is defined as

$$\rho_{m,n} = \frac{1}{M-1} \sum_{i=1}^M \left( \frac{m_i - \mu_m}{\sigma_m} \right) \left( \frac{n_i - \mu_n}{\sigma_n} \right), \quad (30)$$

562 where  $\mu_m$  and  $\sigma_m$  are the mean and standard deviation of  $m_t$ , respectively, and  $\mu_n$  and  $\sigma_n$  are  
 563 the mean and standard deviation of  $n_t$ . This coefficient is bounded between 1 and -1: the closer  
 564 this coefficient is to 1 (respectively -1), the stronger the positive (respectively negative) correlation  
 565 between the pair of signals is. The correlation coefficients between the real and the estimated states  
 566 are summarized in Table 3.

Table 3: Correlation coefficients

D (1,2)	C (1,2)	D (1,3)	Don	C (4)	F
0.8389	0.8856	0.8841	0.9276	0.9654	0.9403

567 It can be stated that the performance of the MHE is satisfactory, since all the correlation  
 568 coefficients are very close to 1, which indicates a strong, positive correlation. Indeed, the main goal

569 of the estimator is that of reconstructing an accurate state sequence that is not directly measurable.  
 570 This procedure is used to achieve the final goal of this work, which consists in fulfilling the control  
 571 objectives, so that the desired system performance is attained. Therefore, state estimation is  
 572 regarded as a tool employed by the controller in pursuit of the final goal.

### 573 5.3.3. Controller

574 The estimated states are used by the controller in order to compute the sequence of future  
 575 optimal inputs, applying only its first component. It must be recalled that the real system mea-  
 576 surements are not available in this work, and thus they must be obtained using the output equation.

577 Based on the control objectives defined in Section 4.1.1, two main results are looked at: the  
 578 water levels and the control signals, depicted in Figs. 9 and 10, respectively.

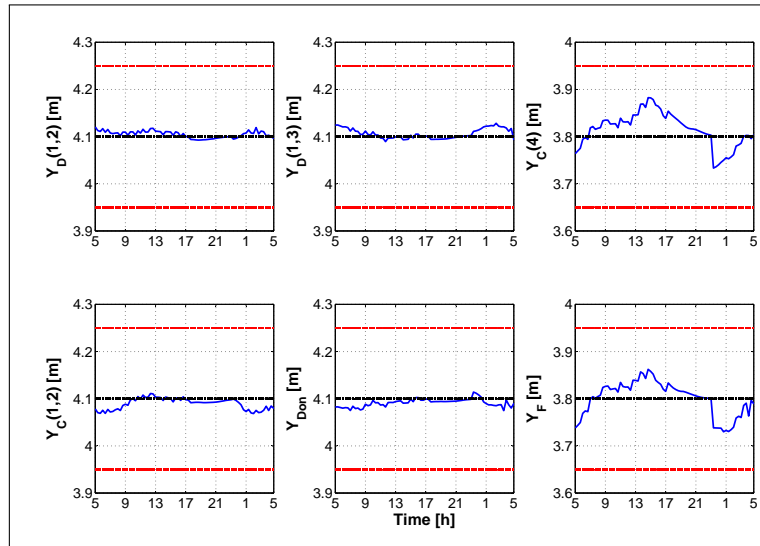


Figure 9: Water levels (blue solid lines), NNL (black dashed lines), and LNL and HNL (red dashed lines)

579 The modeling simplification introduced in Section 5.1 led to considering three reaches, where  
 580 their upstream and downstream water levels are arranged by columns in Fig. 9. The flow diversion  
 581 is considered to take place downstream of the gate of Douai, and thus the water levels in Douai for  
 582 reaches (1,2) and (1,3) might be different. Recall also that  $y_k^{C(1,2)}$  is not measured, but computed  
 583 from the state estimates.

584 Regarding the control objectives linked to the water levels, it can be stated that the MPC is  
 585 able to keep the levels very close to the setpoints despite of the disturbances. To quantify the

586 performance of the controller, consider the indices given by

$$TP = 1 - \frac{1}{H_p} \sqrt{\sum_{k=1}^{H_p} \left( \frac{\mathbf{y}_k - \mathbf{y}_r}{\frac{1}{2}(\bar{\mathbf{y}}_r - \underline{\mathbf{y}}_r)} \right)^2}. \quad (31)$$

587 Equation (31) was introduced in [17] as a modification of the standard root relative squared  
 588 error. These tracking performance indices are defined as the relative error between the predicted  
 589 levels  $\mathbf{y}_k$  and the setpoints  $\mathbf{y}_r$  (NNL values). The denominator, given by  $\frac{1}{2}(\bar{\mathbf{y}}_r - \underline{\mathbf{y}}_r)$ , equals the  
 590 semi-amplitude of the symmetric [LNL, HNL] interval, which is the maximum allowed variation  
 591 from  $\mathbf{y}_r$ . The squaring emphasizes larger differences, which is of interest in this case, since it  
 592 focuses on the water levels  $\mathbf{y}_k$  that are far from the setpoints  $\mathbf{y}_r$ .

593 The numerical values of the indices for each water level are summarized in Table 4. Note  
 594 that TP is bounded between 0 and 1, where 1 corresponds to the perfect tracking performance.  
 595 Therefore, it can be stated that the MPC provides satisfactory results in terms of keeping the  
 596 water levels close to the setpoints.

Table 4: Tracking performances

D (1,2)	C (1,2)	D (1,3)	Don	C (4)	F
0.9930	0.9877	0.9904	0.9907	0.9693	0.9717

597 Furthermore, Fig. 9 shows that the water levels are never outside of the navigation rectangle,  
 598 and therefore the penalty on this behavior, represented by  $\alpha_k$  in (23), equals 0.

599 On the other hand, the control objectives (21) and (22) are linked to the control signals, depicted  
 600 in Fig. 10.

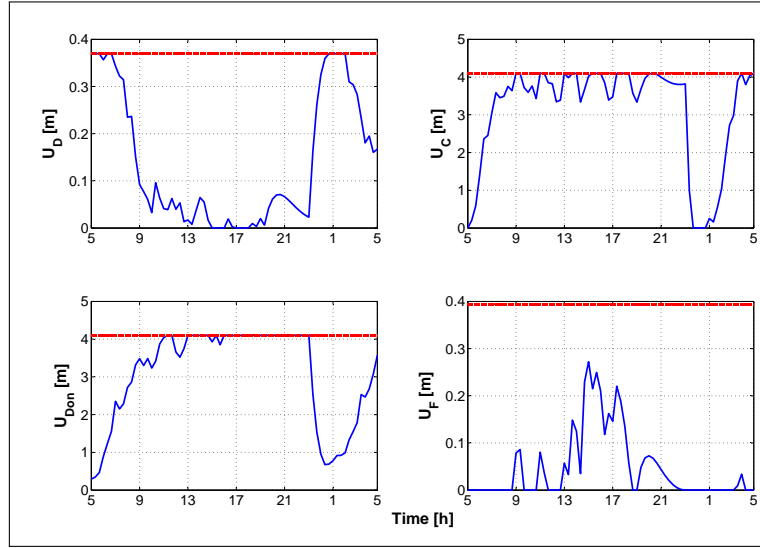


Figure 10: Gate openings (blue solid lines) and physical limits (dash-dot red lines)

601 The undershot gates in Douai and Fontinettes work at their maximum capacity for a maximal  
602 gate opening, whereas the weirs work at their maximum capacity when the sill elevation is zero.  
603 This can be realized by the minus sign in the linearized weir equations. When the water levels  
604 reach the setpoints, the flows delivered by the actuators should be minimum, i.e., minimum gate  
605 openings and maximum sill elevations. However, rejecting disturbances results in continuously  
606 operating the actuators, as these disturbances go against the control objectives. Therefore, it can  
607 be seen how the actuators work close to their maximum capacity only during short periods of time,  
608 and always within the equipment design range. During most of the simulation, the inputs are far  
609 from the physical limits of the actuators, thus taking into account the cost reduction objective  
610 (21).

611 Regarding the smoothness of the control signals given by the operational goal (22), the control  
612 actions present some peaks. However, there are no large variations between consecutive control  
613 actions, especially compared with the design range of the actuators. This behavior should result  
614 in a long lifespan of the equipment. Although the weight of this objective could be increased, this  
615 would probably interfere with the rest of objectives.



## 616 6. Conclusions

617 This work regarded the design of state estimation and control techniques in the framework  
618 of inland waterways, often characterized by negligible bottom slopes and large time delays. To  
619 this end, the MHE and MPC approaches were considered, respectively. This choice obeys to the  
620 appropriateness of these optimization techniques to solve the considered problem. In addition,  
621 the existing duality between both can also be exploited. Even though this work targets inland  
622 waterways as the field of application, for which the time-delay structure comes directly from the  
623 control-oriented modeling formulation, the general methodology can be applied to any kind of  
624 system characterized by the transportation of mass, energy or information.

625 The main objective of this work was that of fulfilling a set of operational goals, each of them  
626 linked to an aspect of the system performance. This task was taken care of by the MPC, which  
627 requires the knowledge of the states in order to determine the optimal predictive control law.  
628 However, as the states are not directly measurable in this problem, the MHE was used for this  
629 purpose. Both techniques were synchronized, using the previously derived control-oriented model  
630 formulation. Finally, the inland waterways in the north of France were used to build a realistic  
631 example in order to demonstrate the performance of the methodology. Although the simulation  
632 results are deemed satisfactory, the centralized architecture employed in this work can encounter  
633 implementation problems due to the large dimensionality of the network. Thus, non-centralized  
634 implementation approaches might be considered in the future to deal with this limitation, possibly  
635 continuing the first steps carried out in [42].

636 From the proposed approach, it has been shown that the MPC is able to reject the disturbances  
637 that are caused by the lock operations. These disturbances are assumed to be known in this work,  
638 but this seldom happens in real applications. Thus, strategies to estimate the effect of unknown  
639 disturbances, such as the unknown input observer (UIO), might be considered in the future. This  
640 class of observers assumes no *a priori* knowledge about such inputs, which is an interesting feature to  
641 be exploited for fault diagnosis purposes. Moreover, combining fault diagnosis and non-centralized  
642 control and state estimation could lead to implementable reconfiguration strategies, aiming at  
643 guaranteeing that the system exhibits an acceptable performance even in the presence of faults.

644 Both MPC and MHE use a dynamic model of the process. In this work, the IDZ model was  
645 used as the starting point to build a control-oriented model. The IDZ is a physical model, and is

646 originally formulated for a canal with two inputs and two outputs. This formulation was extended  
647 to consider larger portions of inland waterways that comprise more than one canal. To do so,  
648 the original model was manipulated, yielding a more convenient state-space representation, which  
649 also allows to handle the delays in the system in a more suitable manner. This work tackled the  
650 particular case in which the lengths of all the reaches are approximately equivalent, leading to  
651 only one delay in the network. In addition, this work considers only one operating point during  
652 the whole simulation. Although this is sufficient when the setpoints do not change during the  
653 simulation, this assumption is rather limited in the case of large operating ranges. In this regard,  
654 LPV models and Takagi-Sugeno fuzzy models could be considered.

## 655 **Acknowledgments**

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662 waterways management.

## 663 **Appendix**

664 All the numerical values used in this work are presented here for the purpose of reproducibility.

### 665 *A. Numerical model*

666 The final state-space matrices are:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9989 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9989 & 0 & 0 \\ 0 & 0.0011 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \mathbf{A}_n = 10^{-5} \cdot \begin{bmatrix} 0 & -112.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -115.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4.3 \\ 0 & 112.5 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_u = 10^4 \cdot \begin{bmatrix} 1.6533 & 0 & 0 & 0 \\ 0 & 0.2223 & 0 & 0 \\ 1.6533 & 0 & 0 & 0 \\ 0 & 0 & 0.2223 & 0 \\ 0 & -0.2223 & 0 & 0 \\ 0 & 0 & 0 & -3.1083 \end{bmatrix}; \mathbf{B}_{un} = 10^4 \cdot \begin{bmatrix} 0 & 0.2223 & 0 & 0 \\ 1.6533 & 0 & 0 & 0 \\ 0 & 0 & 0.2223 & 0 \\ 1.6533 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.1083 \\ 0 & -0.2223 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_d = \begin{bmatrix} 600 & 0 & 0 & 0 \\ 0 & -1200 & 0 & 0 \\ 600 & 0 & 0 & 0 \\ 0 & 0 & -1200 & 0 \\ 0 & 1200 & 0 & 0 \\ 0 & 0 & 0 & -1200 \end{bmatrix}; \mathbf{B}_{dn} = \begin{bmatrix} 0 & -1200 & 0 & 0 \\ 600 & 0 & 0 & 0 \\ 0 & 0 & -1200 & 0 \\ 600 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1200 \\ 0 & 1200 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C} = 10^{-6} \cdot \begin{bmatrix} 0.5061 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5197 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5197 & 0 & 0 \\ 0 & 1.3519 \cdot 10^{-9} & 0 & 0 & 0.4579 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4579 \end{bmatrix}$$

$$\mathbf{C}_n = 10^{-14} \cdot \begin{bmatrix} 0 & -0.2064 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2178 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0072 \\ 0 & 0.1863 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{D}_u = 10^{-7} \cdot \begin{bmatrix} 0.2196 & 0 & 0 & 0 \\ 0 & 0.0295 & 0 & 0 \\ 0.2255 & 0 & 0 & 0 \\ 0 & 0 & 0.0303 & 0 \\ 0 & -0.0267 & 0 & 0 \\ 0 & 0 & 0 & -0.3736 \end{bmatrix} ; \mathbf{D}_{un} = 10^{-7} \cdot \begin{bmatrix} 0 & 0.0408 & 0 & 0 \\ 0.3034 & 0 & 0 & 0 \\ 0 & 0 & 0.0419 & 0 \\ 0.3118 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5147 \\ 0 & -0.0368 & 0 & 0 \end{bmatrix}$$

$$\mathbf{D}_d = 10^{-8} \cdot \begin{bmatrix} 0.0797 & 0 & 0 & 0 \\ 0 & -0.1594 & 0 & 0 \\ 0.0818 & 0 & 0 & 0 \\ 0 & 0 & -0.1637 & 0 \\ 0 & 0.1442 & 0 & 0 \\ 0 & 0 & 0 & -0.1442 \end{bmatrix} ; \mathbf{D}_{dn} = 10^{-8} \cdot \begin{bmatrix} 0 & -0.2202 & 0 & 0 \\ 0.1101 & 0 & 0 & 0 \\ 0 & 0 & -0.2263 & 0 \\ 0.1131 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.1987 \\ 0 & 0.1987 & 0 & 0 \end{bmatrix}$$

667 Remark: The delay of the system is equal to 6 samples, as stated in Section 5.2.

668 *B. Equations of the gates and weirs*

669 *B1. Nonlinear equations*

670 The nonlinear equations that describe the dynamics of the gates and the weirs in the considered  
671 case study are:

$$\text{Undershot gate: } q = C_{dg} L_g u \sqrt{2gy_1} \quad (32a)$$

$$\text{Weir: } q = C_{dw} \sqrt{2g} (y_1 - u)^{3/2} \quad (32b)$$

672  
673 where  $q$  is the discharge through the structures;  $C_{dg}$  and  $C_{dw}$ , the discharge coefficients, equal to **R3.15**  
674 **0.6 and 0.4**, respectively;  $L_g$ , the gate width;  $u$ , the gate opening or elevation;  $g$ , the acceleration  
675 of gravity; and  $y_1$ , the upstream water level.

## 676 B2. Linearized equations

677 Using the NNL and  $Q_S$  values in Table 1, the linearized versions of (32) are:

$$\text{Douai: } q_k^D = 27.5553u_k^D \quad (33a)$$

$$\text{Cuinchy: } q_k^C = 1.8524y_k^{C(1,2)} - 1.8524u_k^C = 1.32 \cdot 10^{-6}x_k^{C(1,2)} - 1.8524u_k^C \quad (33b)$$

$$\text{Don: } q_k^{Don} = 1.8524y_k^{Don} - 1.8524u_k^{Don} = 1.32 \cdot 10^{-6}x_k^{Don} - 1.8524u_k^{Don} \quad (33c)$$

$$\text{Fontinettes: } q_k^F = 0.0791y_k^F + 25.9037u_k^F = 5.63 \cdot 10^{-8}x_k^F + 25.9037u_k^F \quad (33d)$$

## 678 C. Weighting values for the MPC problem

$$679 \quad \beta^1 = 20; \beta^2 = 2; \beta^3 = 5; \beta^4 = 1.$$

## 680 D. Weighting values for the MHE problem

$$681 \quad \mathbf{P}^{-1} = \mathbf{I}_{n_x} = \mathbf{I}_6; \mathbf{Q}^{-1} = \mathbf{I}_{n_y-1} = \mathbf{I}_5; \mathbf{R}^{-1} = \mathbf{I}_{n_x} = \mathbf{I}_6.$$

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