



Proceedings of EPNACS'2007 - Emergent Properties In Natural and Artificial Complex Systems, A satellite conference within ECCS'07

M A Aziz-Alaoui, Cyrille Bertelle, Gérard H E Duchamp, Michel Cotsaftis

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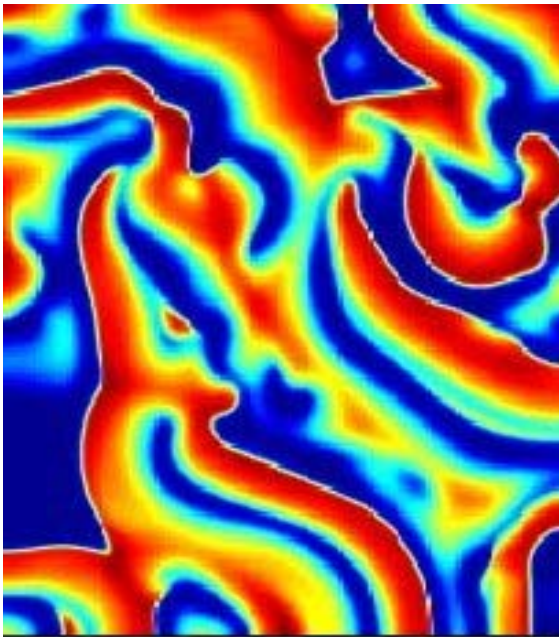
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October 4th, 2007

Dresden – Germany

EPNACS'2007

Emergent Properties In Natural and Artificial Complex Systems



A satellite conference

**within
ECCS'07**

**European Conference on
Complex Systems**

**Dresden, October 1-5,
2007**

Organization Committee

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Michel Cotsaftis
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EPNACS'2007

**Emergent Properties in Natural and
Artificial Complex Systems**

in

ECCS'O7 European Conference on Complex Systems

October 4th, 2007

Description

The aim in this satellite conference is to study emergent properties arising through dynamical processes in various types of natural and artificial systems. The session is concerned with **multidisciplinary approaches** for getting representations of complex systems and using different methods to extract emergent structures. Equations formulation can lead to the study of emergent features such as self organization, opening on stability and robustness properties. Invariant techniques can express global emergent properties in dynamical evolution systems. Artificial systems such as a distributed platform for simulation can be used to search emergent placement during simulation execution.

Le Havre & Paris
September 15th 2007

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PART 1

Concepts and Tools for Complexity Modelling

WHAT MAKES A SYSTEM COMPLEX?

AN APPROACH TO SELF ORGANIZATION AND EMERGENCE

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“Men in their arrogance claim to understand the nature of Creation, and devise elaborate theories to describe its behaviour. But always they discover in the end that God is more clever than they thought

Sister Miriam Godwinson

The fast changing reality in technical and natural domains perceived by always more accurate observations has drawn the attention on a new and very broad class of systems mainly characterized by specific behaviour which has been entered under the common wording “complexity”. Based on elementary system graph representation with components as nodes and interactions as vertices, it is shown that systems belong to only three states : simple, complicated, and complex, the main properties of which are discussed. The first two states have been studied at length over past centuries, and the last one finds its origin in the elementary fact that when system performance is pushed up, there exists a threshold above which interaction between components overtake outside interaction. At the same time, system self-organizes and filters corresponding outer action, making it more robust to outer effect, with emergence of a new behaviour which was not predictable from only components study. Examples in Physics and Biology are given, and three main classes of “complexity” behaviour are distinguished corresponding to different levels of difficulty to handle the problem of their dynamics. The great interest of using complex state properties in man-made systems is stressed and important issues are discussed. They mainly concentrate on the difficult balance to be established between the relative system isolation when becoming complex and the delegation of corresponding new capability from (outside) operator. This implies giving the system some “intelligence” in an adequate frame between the new augmented system state and supervising operator, with consequences on the canonical system triplet {effector-sensor-controller} which has to be reorganized in this new setting. Moreover, it is observed that entering complexity state opens the possibility for the function to feedback onto the structure, ie to mimic at technical level the invention of Nature over Her very long history.

1. Introduction

After a very long period of observations, it became very slowly clear to first Human observers since prehistoric age that the phenomena they were distinguishing were due to a specific order in Nature and not to the capricious will of mysterious Gods. Since then, they have been over the centuries patiently organizing the most visible ones, and, proposing hypotheses to find this order, they finally end up on laws to represent the simplest phenomena first which have been verified in the framework of actual observations. These elementary phenomena were mainly concerning “simple” systems which can reasonably be isolated in their dynamics and their observation. In the mean time, the advance in technology and observation accuracy was driving the attention on more complicated systems with always larger number of elements, which in the 17th century have been shown by R. Descartes to be reducible in their study to the former class of simple ones[1]. Finally, the result today of this long quest was an “adapted” representation of the Universe with a broad “classical” part for “human size” phenomena, corrected by “quantum” effects at very small atomic level, and by “relativistic” ones at very large galactic level. This basic “1-dimensional” picture emerging from millennium long effort of

human kind, and leaving fundamental questions only at infinitely large universe and infinitely small super elementary particle levels, is however not sufficient to represent correctly enough the present situation where systems exist over an extremely large parametric domain now easily explored with the recent development of always more performing technology over the last decades.

First observed from time to time in different domains, there is now evidence of existence of new huge class of systems, natural as well as artificial in all scientific and technical human activities, which have reached their own status by the corner of the millennium under the name of “*complex*” systems. They are in some sense opening a “2-dimensional” more global picture of the universe by being located along a direction “perpendicular” to previous 1-dimension line going from infinitely small to infinitely large size, see Fig.1. There is today a strong questioning about their origin and their formation[2]. Some have even suggested that previous “mechanistic” approach to the universe by researching “ultimate” fully unifying law[3] is just mistaking their very and irreducible structure and that a new “revolution” is necessary to grasp all aspects of known universe today[4]. To address these questions, a very pedestrian approach is proposed here, based on elementary source-sink model applied to the graph representing the aggregate of possible components of a system. Then it evidently appears that system structure can be divided into three different groups, simple, complicated and complex, corresponding to specific properties. The first two groups are usual ones nicely approachable by the methods of scientific reductionism[5]. The third group, by its very global nature, cannot be just reduced to the effect of its components[6] and requires some adjustment for being correctly handled, because now the key point is the way the system behaves under (or against) the action of its environment. It will appear that the mechanistic notion of individual “trajectory” is losing its meaning and should be replaced by more general “manifold” entity corresponding to accessible “invariants” under environment action. System self-organization and emergence properties are deduced and discussed. Different grades of complexity are fixed depending on specific system properties which relate to well known classes of observed phenomena. Advantages for application of complex structure to man made systems are stressed.

2. System State Analysis

Let consider first a system with a finite number N of identified and distinguishable components which can be represented by a graph with N nodes N_i . On this graph, three types of vertices can be drawn in between the N components i and outside sources e whenever an exchange exists between them. They correspond respectively to free flight state for vertex V_{ii} , to driven state from outer source for vertex V_{ie} , and to interactive state with other system components for vertex V_{ij} , see Fig.2. System dynamics are usually resulting from a combination of previous three different exchanges to which, on a very general setting, there can be associated three characteristic fluxes for each system component. Their specific nature (power, information, chemical,...) is depending on the problem at hand, but is here unnecessary as long as they unambiguously characterize system components status. The first flux corresponds to the “free” dynamics of i -th component p_{ii} along vertex V_{ii} , the second one p_{ie} to the transfer flux between outer source and system i -th component along vertex V_{ie} , and the last one to inter components effects $p_{i,int} = \inf_j \{p_{ij}\}$, where p_{ij} is the characteristic and oriented flux exchange between components i and j along vertex V_{ij} , see Fig.2. Depending on the various possible ordering between these three fluxes, the system behaves in different ways. When (A) $p_{ii} \gg p_{i,int}, p_{ie}$ the i -th component is weakly coupled to other components. Its dynamics are mainly the ones of an isolated element slightly perturbed by the other actions, and the system reduces to a set of almost independent one-component sub-systems. In this sense it can be considered as a **simple** one. When (B) $p_{ie} \gg p_{i,int}, p_{ii}$ the i -th component is mainly depending on outside sources. The action of the other components creates a (weak) coupling between the components, but the system can nevertheless be decoupled, at least locally, into a set of sub-systems which can be acted upon and controlled by as many exterior sources as there are components in the system because they can still be identified. In this sense the system can be termed as **complicated** from etymologic sense (from Latin *cum pliare* : piled up with). Finally when (C) $p_{i,int} \gg p_{ie}, p_{ii}$ the i -th component is very strongly coupled to other ones and its dynamics are now mainly determined by components interaction satisfying the inequality. If apparently there does not seem to exist a dramatic difference with the previous case as concerns their dynamics, there is however a fundamental one as concerns the effect of outside action. In previous

case input from the source can be tracked to the concerned component so this degree of freedom can be completely controlled from outside, whereas in present one the nature of interactions dominates and shields this tracking. As a result, control action can only be a "global" one from other system components satisfying condition for second case (B) and system dynamics are now also driven by internal action. Consequently a totally new situation occurs for all system components passing in third state in the sense that their control cannot as in second case be fixed by only outer source action. Effectively, it is no longer possible as in second state to determine completely all system components dynamics from outside because of stronger interaction effect which now dominates the dynamics of concerned components. In a sense a *self-organization* has been taking place inside the system which leads to an internal control replacing the classical one from outside. So it is no longer possible to continue to manipulate inputs with as many degrees of freedom as the system initially has because of the mismatch which would result from the conflict with the internal control due to components interaction satisfying condition for the third case. External system control dimension is thus *reduced*. A system in this case will be termed as **complex** in agreement with etymology (from Latin *cum plexus* : tied up with). According to this definition, a very elementary test for determining if a system is passing to complex state is thus to verify that its control requires the manipulation of less degrees of freedom than the system has initially. There exists an immense literature about complexity, its definition and its properties covering an extremely wide range of domains from Philosophy to Technology[7], especially in recent years where its role has been "discovered" in many different fields such as networks now playing a crucial role with the ascent of Information Technologies[8]. Here emphasis is more modestly put only on more restricted complex state compared to complicated state as concerns action from outside environment (ie from control point of view) onto the system. For a system **S** with a finite number N of identified components, one can then define its *index of complexity* $\mathcal{C}_S = 0$ if it is only in complicated state, and $\mathcal{C}_S = n/N$ if $n < N$ components cluster in complex state by satisfying inequality (C) and become insensitive to outer action. \mathcal{C}_S measures the grasp on the system from outside action/control and the limit $\mathcal{C}_S = 1$ corresponds to totally autarchic system, the most complex possible structure with this number of components. This may look paradoxical as the definition of a simple system is precisely that it can be isolated. In fact this apparently contradictory statement is resulting from the very nature of internal interactions effect which reduces the number of invariants on which system trajectory takes place. An elementary example is given by the particles of a neutral gas for which their initial $6N$ positions and velocities (the mechanical invariants of motion) are reduced to the only energy (or temperature), justifying thermodynamic representation. In other words, when represented from outside, a system is the less complicated as it is more complex inside. Differently said, a complicated system remains complicated whether observed from inside or from outside, whereas when becoming complex it is less complicated when observed from outside, on top of the fact that being depending on a restricted number of parameters, it is more insensitive to outer action. However, when observing from outside a system with apparent $\mathcal{C}_S = n/N$, if N is not fixed when the system is not well known, there is no one-to-one relation in general between apparent outer behaviour of the system and its initial internal structure. It is only possible to check relative variation of \mathcal{C}_S when crossing condition (C) for some components. On a general setting, it is very easy from its definition to envision complex state as a powerful way set up by Nature to generate more sophisticated entities able to gain at the same time more independence with respect to their environment, or, in other terms, to become more robust to its variations and yet, to appear much less complicated outside, so that they can in turn link more easily to similar entities and generate a chain of nested clusters. This seems to be an underlying trend in evolution on Earth which has been allowing the elaboration of living beings, the up-most advanced form of complex systems[9].

Summarizing, exactly like matter does appear in three states : solids, liquids and gas, depending on conditions, systems are exhibiting three states : simple, complicated and complex. A very simple way to locate a system with respect to them is to plot for each component i the three values $\{p_{ii}, p_{i,e}, p_{i,int}\}$ in a three dimensional space where each direction represents a single state. Each component is then represented by a point C_i and the system by a cluster of N points, see Fig.3. Depending on where they are located in this space, it is evident to figure out the status of the system by inequalities (A),(B) and (C). Moreover, it is easy to check what will be the consequence of parameter variation, especially when crossing inequality (C), a point extremely important for control of man-made systems as seen later. In this picture, components for which inequality $p_{i,int} \gg p_{i,e}, p_{ii}$ holds are in fact an internal sub-

cluster which cannot be further split from outside observation and action. As indicated above, the first two states have been observed and studied along past centuries. The fact is that with modern and very detailed observation diagnosis, complex state has been now very often observed in natural systems, and becomes the most common one in a broad range of phenomena. For artificial man-made systems, even if it were sometimes crossed in the past, it is only with recent progress of advanced technology that, when pushing systems performance for higher efficiency, the threshold for overtaking of internal interactions effect corresponding to third case is also very routinely over passed now. So complex behaviour is seen in many situations, and becomes an object of study in itself, with considerable consequences in technical and industrial applications it implies in conjunction with the corresponding demand of more delegation of decisional nature to the systems for better efficiency.

From previous and relatively rough source-sink representation always applicable in a first instance, it is already possible to stress a deep difference existing between the first two states and last complex one. In first two states, the possibility exists to split the system into as many independent one-component systems in a first approximation, whereas this is impossible in third state where all interacting components have to be taken as a whole. In mathematical terms the consequence is that usual approximation methods developed for the first two states do not straightforwardly apply and have to be revised in order to handle the global aspect of the coordinated response of components in complex state. This difficulty is at the origin of today important computerized research undertaken on the problem.

3. Emergence

When going to more specific situations, other elements are also entering the description and it is interesting from elementary global description above to recover various situations which have been observed and analytically studied for different parameters value. First it is likely that in general the system is not always in a "pure" state and often exhibits a mixed structure where some components are in one state and others in another one. An important example is the case of natural inhomogeneous but continuous natural systems such as fluids with non zero gradients in a domain. In these systems, fluctuations are universally observed the source of which is the free energy available in between this stationary equilibrium and complete (homogeneous) thermodynamic one. In present case, the free energy is coming from the space gradients related to medium non homogeneity. They have in general a very large range of fluctuations (roughly because the system has a very large number of components) which can be split into two groups depending on their wavelength compared to system characteristic gradient length. Typically, the fluctuations with large wavelength excited in the medium are in complicated state and, because they are sensitive to boundary conditions, can be observed and possibly acted upon as such from outside. Fluctuations with small wavelength on the other hand are generally in complex state due to parameter values. So under their strong interactions, and because they are much less sensitive to boundary conditions, they are losing their phase and globally excite a leak out of the fluid (usually called a *transport*) manifested by an out-flux expressing the non equilibrium situation of the system, and which counteracts the input flux responsible of medium non homogeneity. The determination of these transports is a very important element for qualifying the behaviour of the system and is an active research problem studied worldwide. This feature is observed for all natural systems when they enter the *dissipative* branch.

A natural system is called dissipative[10] when it is exchanging (particles and energy) fluxes with outside environment. Evidently the channels by which internal energy sources are related to these fluxes are playing a privileged role because they regulate the energy ultimately available for the system and finally determine its *self-organized* state[11]. Dissipative systems more generally only exist to the expense of these fluxes, and they evolve with parameter change – such as the power input – along a set of neighbouring states determined by branching due to bifurcations where internal structure changes in compatibility with boundary conditions and by following the principle of largest stability. So the picture of such systems is a transport system governing flux exchanges guided by the bifurcation system which, as a pilot, fixes the structure along which these exchanges are taking place. Finding the branching pattern thus entirely defines the possible states of the system and determines the fluctuation spectrum. Branching is found as nontrivial solutions of variation equations deduced from general system dynamics equations. Despite the physical origin of phenomena is here well identified,

the analysis is still in progress in a very large number of situations, such as in thermonuclear plasma to determine correct parameter sizing for realizing “burning” conditions[12], and in neutral fluids where turbulence is not fully explained yet[13], whereas the problem has been cleared up for fluctuations in deformable solid bodies[14]. In most cases the difficulty stems from finding an adequate representation of the global effect of small wavelength fluctuations, especially when due to parameter values, they feedback on longer time scale large wavelength oscillations and modify their dynamics. In this case, there is a significant change of initial system dynamics due to passage in complex state of an internal part of the system generated by branching phenomenon. Moreover, it is very likely that modification of system dynamics is the more important as the non homogeneity zone is the less “transparent” between the two domains above and below along power flow. This has been extended to the extreme with living cell systems which have been able to completely encapsulate within a filtering membrane (ie a steep gradient) a space domain where very specific “memory” DNA molecules are fixing the dynamics of inside system they control, with corresponding exchange across the membrane. It is easy to understand that conjunction of a gate and of adequate information is the most efficient way to deeply modify locally the effect of regular physics laws, as exemplified by the counter streaming motion of a piece of wood in a river with one stage locks when regulating their opening and closing.

However, dissipative behaviour does not exhaust all possible situations and many other ones do not follow this pattern. Natural systems so far are exhibiting components with relatively elementary features (charge, mass, geometrical structure, chemical activity, wavelength, frequency..), but there are also cases where complex state occurs in systems with more sophisticated components, usually called “agents”. Examples are herds of animals, insect colonies, living cell behaviour in organs and organisms, and population activity in an economy. In all cases, when observed from outside the systems are exhibiting relatively well defined behaviours but a very important element missing in previous analysis is the influence of the *goal* the systems are seemingly aiming at. Very often the components of these systems are searching through a collective action the satisfaction of properties they cannot reach alone, and to represent this situation the specific word “*emergence*” has been coined[15]. The point is that it is now possible to return back to previous case and in a unified picture to envision the laws of Physics themselves as emerging phenomena. For instance for an ensemble of neutral particles with hard ball interactions, and beyond the threshold of rarefied gas, (ie when the Knudsen number $Kn = \lambda/L$ is decreasing to 0 from the value 1, with λ the particle mean free path and L a characteristic length scale), the particle system is suddenly passing from a complicated to a complex state (due to the huge value of Avogadro number). Consecutive to overtaking of collective interaction by collisions (expressed by decrease of Knudsen number), it could be said that there is emergence of a pressure and a temperature, which, from a point of view outside the gas, summarizes perfectly well the representative parameters (the invariants) describing it at this global level and amply justifies the thermodynamic representation in this case, see next §. The elementary reason is that here the fast particle dynamics described by Boltzmann equation relaxes within extremely short collision time to its equilibrium Maxwell-Boltzmann-Gibbs distribution only depending on temperature (and the potential of applied forces if any), which justifies usual stochastic methods[16]. Of course it is tempting at this point to “forget” the underlying mechanical base and to decide that perfect gas law, if any, is the emergence directly resulting from billiard ball “properties” of interacting (agents) molecules. The difficulty in this case is that application of “ergodic” hypothesis[17], even if it can mathematically be invented from scratch, rests here heavily upon thorough analysis of molecule ensemble dynamics, ie on mechanical laws. Similar problem does exist at atom level itself where, after baryons are assembled from primitive quark particles below some threshold energy, protons and neutrons assemble in turn themselves below another lower energy threshold into ions with only mass and charge parameters, able to combine finally with electrons to create atoms. Speaking of “emergence of atoms” from underlying components does not add any more information to classical approach. So in an opposite direction, a more conservative and probably more appropriate way to account for the new situation is to just state that when becoming complex, systems are exhibiting emergence of self-organization out of which it has been verified that their new behaviour in complex state is always a consequence, without going into the details of this behaviour which is depending on specific system parameters. In fact it is quite elementary from previous source-sink model to understand that a key point is in the accuracy of modelling the components in complex state, as long

as the resulting “invariants” which will grasp all system information for interaction with environment are directly depending on this modelling. This has been at the origin of a computer “blind” search where the agents are given properties and “emerging” behaviour is obtained in a bottom-up approach, sometimes in surprising compatibility with experimental observations[18], in parallel to theoretical analysis[19]. Of course following classical science reductionism, there remains to justify the choice of agent properties, and to show that they are in one-to-one relation with observed behaviour. As this does not seem always possible, a different holistic approach[20] has been proposed where the research of ultimate underlying elements from which everything is constructed by as many layered shells on top of one another, is considered as unnecessary, to the price of losing prediction power. Finally an extremely important remark is that the logical chain :

{stimuli/parameter change} → {higher interactions between some system components} → {passage to complex state} → {system self organization} → {emergence of new behaviour}

discussed here is nothing but the sequence leading to the final step of system evolution toward more independence, and which is the feedback of “*function*” onto “*structure*”, a specific property of living organisms explaining their remarkable survival capability by structure modification.

4. Mathematical Analysis of Complex Systems

The few previous examples from common sense observation illustrate the elements which have been described above and which are providing a general base for complex system paradigm. From atomic nucleus to galaxy natural systems are seen to be constituted by aggregates of identifiable components (which, as already stressed, can be themselves, at each observation level, aggregates of smaller components) with well defined properties. These aggregates have been said to exhibit a complex behaviour when interaction between the components –or some of them– is overtaking their interactions with exterior environment. Similarly living beings are exhibiting the same behaviour, as observed with gregarious species, and in artificial man-made systems the same phenomenon is occurring when the overtaking conditions are satisfied. This is for instance the case for high enough performance level systems because the components are then tightly packed, as for high torque compact electrical actuators. Despite an extremely large variety of possible situations there are few base interactive processes leading to complex self-organization. Three main types will be discussed: the reducible case, the instantaneous differential case, and the integro-differential case with past neighbourhood memory.

4A – Reducible case :

It is first intuitively expectable that the approach to full problem gets easier when time-space scales are very different for components going to complex state as compared to the ones of other system components. This is mostly the situation in dissipative systems where characteristic frequency and wavelength of the spectrum of complex system part are respectively much larger and much smaller than the ones of the other (non complex) part of the system. The main reason is as explained above that components in complex state are produced in the initial system by the effect of bifurcation determined from variation equations of the system and are corresponding to high mode numbers (and have small enough wavelength). For such modes, their saturation amplitude is fixed by their nonlinear interaction which overtakes the level they would have reached under the only effect of the source and, as well known, exhibit globally a fluctuation spectrum usually very difficult to determine analytically. Let τ_s and τ_{cp} be respectively the characteristic response time of the non complex system part and of complex modes part with $\tau_{cp} = \varepsilon^n \tau_s$. The simplest situation $n > 1$ corresponds to the case where the interactions are strong and frequent enough to kill corresponding modes dynamics. In this case and because there are usually many modes with small amplitude, complex system part dynamics can be reduced to “thermodynamic” approximation with a random distribution of fluctuation spectrum. Then the initial system of equations describing the full system splits into a restricted set of identified modes perturbed by a random term. This unifying and powerful model is quite successful in science and

engineering[21] but it does not cover all situations. A more difficult and yet common case is occurring when $n = 1$, for which small $O(\varepsilon)$ modes dynamics have long enough coherence characteristic time comparable to τ_s . To account for this coherence, asymptotic expansion of small modes dynamics from complex part dynamic equations is required up to first order to be injected in non complex system part dynamics, because ε order term acting coherently over a time ε^{-1} produces a non negligible finite order term. The system again reduces to its non complex part but now modified by coherent effects of complex system part dynamics, see Fig.4. The unsolved situations in [12,13] are belonging to this case. However it will nevertheless be termed as *reducible* because full system dynamics can be projected onto non complex system subpart. It clearly illustrates the fact that, given a system with a fixed number of degrees of freedom and associated initial and boundary conditions, this number should increase at bifurcation crossing so other conditions would be required to determine the solution. In fact this is not the case here as new complex modes dynamics can be solved by asymptotic procedure and re-injected into non complex system part which obviously keeps the same number of degrees of freedom. The only constraint is on initial complex modes amplitude which, due to their smallness, obey central limit theorem and are randomly distributed. So the difference with previous case $n > 1$ is that only initial amplitudes values are random now and follow afterward their own dynamics whereas they were previously random for all time. In both cases the number of final degrees of freedom remains the same, so despite their number has apparently increased at bifurcation, in reality the system is still of the same dimension. The extremely important consequence is that for controlled man-made systems, when their dynamics cross a bifurcation point, the same number of controllers is still required but they should be adapted to explore a new larger function space. The main effect of the bifurcation has been to augment system response to a wider function space, and the motion takes place on a *manifold* of this new space so that at the end the effective number of degrees of freedom is remaining the same. As stressed earlier, this behaviour is the manifestation of system *self-organization*. The double mechanical-thermodynamic feature is extremely general and characterizes complex reducible systems where complex part is added by bifurcation crossing, see Fig.5. Again control of such systems (whether natural or man-made) is not classical because for $n \geq 1$ the new modes created after bifurcation crossing, though sometimes observable, are not accessible from outside. Because of their definite effect on system dynamics, a new more balanced approach respecting internal system action has to be worked out, which, very generally, is aimed at replacing inability of control action by insensitivity to variation for inaccessible complex modes. Though apparently loosing some hand on such systems, it has been surprisingly possible along this line to find explicit conditions in terms of system parameters expressing somewhat contradictory high preciseness (by asymptotic stability condition) and strong robustness (against unknown system and environment parts)[22]. In this way system dynamics are finally controlled and asymptotic stability can be demonstrated, but in general the price to pay is a not necessarily decreasing exponential asymptotic type.

4B – Instantaneous Differential Case :

The reducible case mainly deals with systems in partly or totally complex state, as with bifurcation crossing by some varying parameter, and with small parameter ordering between time and space scales, so it is possible to proceed to asymptotic expansion. More generally, a system may be in complete complex state, examples of which are atomic nucleus, herd of animals, and galaxies. Despite their very different space sizes, the systems exhibit always the same base characteristic feature to finally depend on an extremely restricted number of parameters as compared to the aggregate of their initial components. Searching the way to extract directly the remaining ‘‘control’’ parameters of such systems from their dynamics is a fundamental issue which today motivates a huge research effort worldwide, especially in relation with information networking. Extensive analytical and numerical study has been developed for differential systems of generic form

$$\frac{dX}{dt} = A(t, X, v)X + \lambda.F(t, X, u) + \mu.S(t, X) \quad (1)$$

where $X = \text{col}(X_1, X_2, \dots, X_n)$ is system state space, λ, μ are n-vector coupling parameters, and $A(\dots), F(\dots), S(\dots)$ are three specific $q_1 \times q_2$ -matrix terms ($q_1, q_2 \leq n$) corresponding to isolated free flight, nonlinear internal interactions and source terms respectively (the linear and source terms in the right hand side of eqn(1) are here split apart to indicate their respective role). The other variables $v=v(X, t)$ and $u(X, t)$ in A and F function account more generally for the possibility to feedback evolution of $X(t)$ onto their own dynamics as it often occurs in systems when splitting parameters into given and manipulated control ones. For fixed $u(\dots), v(\dots)$, depending on the values of λ, μ components the system will be in simple, complicated or complex state described in §1. When increasing the components of λ the system runs into complex state, and it has been repeatedly observed, especially on systems close to Hamiltonian ones[23], that system representative point in n-dimensional state space follows a more and more chaotic trajectory when crossing bifurcation values and at the end fills up a complete domain[24]. Of course sensitivity is largest when the system exhibits resonances, ie is close to conservative, and adapted mathematical expansion methods have to be worked out[25]. Because systems are basically non integrable[26], this is a direct evidence of increasing effect of internal interactions which reduce system dynamics to stay on attractor manifold of degree $p < n$, so that system dynamics are now layered on this manifold. This also expresses the fact that trajectories on the remaining $n-p$ dimension space are becoming totally indistinguishable (from outside) when taken care of by internal interactions of $n-p$ components going to complex state. So system trajectories reorganize here in equivalence classes which cannot be further split, a dual way to express the fact that there exists an invariant manifold on which system trajectories are lying. It is easy to understand that continuing to control these components by regular previous control[27] worked out for complicated state and specially designed for tracking a prescribed trajectory, is no longer possible and that a new approach is required which carefully respects internal system action due to complex state self-organization. More global methods of functional analysis[28] related to function space embedding in adapted function spaces[29] by fixed point theorem[30] are now in order as shown for reducible case[31], because they are providing the correct framework to grasp the new structure of system trajectory which cannot be fully tracked as before. Basically the method is again to counteract impreciseness in an element by robustness to its variation, a method very largely followed by living organisms.

4C – Integro-Differential Case with Past Neighbouring Memory :

Another important dimension relates to the properties system components are given, and a very influential one is the range of inter-component interaction, because this determines completely the build up of system clustering when becoming complex. Obviously long range interactions are leading to more intricate response with more difficult analysis. Examples are stars in a galaxy, electromagnetic interactions between ions and electrons in a plasma, animals in a herd and social behaviour of human population in economic trading such as stock market with internet link. In all cases a new element is coming from the size of the neighbouring domain each system component is sensitive to, and implies a time extension to past neighbouring components trajectories, see Fig.6. So the resulting complex behaviour is more generally determined by interactive component effects over a past time interval and weighted according to their importance. In this case systems are obeying for $t \geq t_0 > 0$ delayed equations of the form

$$\frac{dX}{dt} = F(t, X(t), I(t), u, d) \quad (2)$$

$$\frac{dI}{dt} = \int \rho(\lambda, t) G \left(t, X(t), \int_{-\infty}^t \varphi(t') X(t - h(t', X(t'))) dt', \lambda \right) d\lambda \quad (3)$$

with $X = \text{col}(X_1, X_2, \dots, X_n)$, $I = \text{col}(I_1, I_2, \dots, I_p)$, which clearly indicates the role of the past through intermediate “moments” $I(t)$ gathering a set of elements G with a weight $\rho(\lambda, t)$, each G being itself constructed on the past history of system trajectories $X(t)$ with fading weight $\varphi(t)$, and where $F(\dots) :$

$\mathbf{R}_1^+ \times \mathbf{R}_n \times \mathbf{R}_p \times \mathbf{U} \times \mathbf{D} \rightarrow \mathbf{R}_n$ and $G(.,.,.,.) : \mathbf{R}_1^+ \times \mathbf{R}_n \times \mathbf{C}^0(\mathbf{R}_1^-, \mathbf{R}_n) \times \mathbf{A} \rightarrow \mathbf{R}_p$ are specific vectors. Their expression generally include control parameters $u(.) \in \mathbf{U}$ which have to be stated explicitly by control law to fix the structure of eqns(2,3) and uncontrolled disturbing effects often represented by a noise $d(.) \in \mathbf{D}$. Here the delay $h(.,X(.)) < t$ can be state-depending as it appears in the analysis of natural (biological) systems where information from the past can be shielded by the state itself. The mixing from the breath of trajectory coverage and from past memory basically creates a “double” complexity in time and space for system components, slightly reduced by averaging effect of acting moments $l(t)$. It indicates that component trajectory is more constrained as it is constantly depending on neighbouring ones, and a possible way is to represent this collective action as an average potential through a distribution function equation of which eqns(2,3) are the characteristics. Apart few existence theorems[32], study of eqns(2,3) is at very preliminary stage today. Like for eqn(1), their numerical study can be undertaken by fixing design and control parameters in F and G such that some trajectory properties are met. Here also a useful companion approach is in application of embedding theorems by fixed point methods, especially when F and G are satisfying generalized Lipschitz conditions, which often provides an embedding property in Sobolev spaces by use of substitution theorems[33]. More precise statement can be obtained when there exist upper bounding estimates in norm of the RHS of eqns(2,3). For example, if the right hand sides of eqns(2,3) satisfy inequalities of generalized Lipschitz type[34]

$$|F|, |G| \leq a(t) + \sum_{i=1,n} [b_i(t)g_i(Z(t)) + c_i(d_i(t))k_i(Z(d_i(t)))] \quad (4)$$

where $Z = \|\text{col}\{X, l\}\|$, $d_i(t) = \text{Inf}_X\{t - h_i(t, X(.))\}$, and the various functions $g_i(.)$, $k_i(.)$, $a_i(.)$ and $b_i(.)$ are positive in their definition intervals, there exists the following bound on the solution

$$Z(t) \leq \mathbf{H}^{-1} \left\{ \mathbf{H}_0 + \text{sgn}(\mathbf{H}) \int_{t_0}^t \Phi(s) ds \right\} \quad (5)$$

with $H(Z) = \|\text{col}\{1, g_i(Z), k_i(Z)\}\|$, $\Phi(t) = \|\text{col}\{a'(t), b_i(t), c_i(t)d_i'(t)\}\|$, $' = d/dt$ and $\mathbf{H}(v) = \int v dv / H(v)$, $\mathbf{H}^{-1}(.)$ its inverse function. The solution $X(t)$ is bounded with $Z(t)$ defined over the complete time interval $t \in \{t_0, +\infty\} \subset \mathbf{R}_1^+$ and for all $Z_0 = Z(t_0) \in \mathbf{R}_n^+$ when $\mathbf{H}^{-1}(.)$ is bounded which generally occurs when the function is sub-linear, whereas there is a conditional bound inside the domain limited by $H_0 + \text{sgn}(H) \int_{t_0}^T \Phi(s) ds = 0$ in $\{H_0, T\}$ -space, ie in $\{X_0, T\}$ -space, when the function is super-linear. When there exists monomial bounds $g_i(x) = \lambda_i x^p$, $k_i(x) = \mu_i x^p$ and $a(t) = 0$, one obtains the explicit bound

$$Z(t) \leq \left[Z(t_0)^{1-p} - (p-1) \sum_{i=1,n} \left\{ \lambda_i \int_{t_0}^t b_i(s) ds + \mu_i M_i \int_{d_i(t_0)}^{d_i(t)} c_i(s) ds \right\} \right]^{\frac{1}{1-p}} \quad (6)$$

with $M_i = \text{Max}\{1/d_i'\}$, showing a limit in $\{t, Z(t_0)\}$ -plane for $p > 1$ when the term between bracket is 0. Bounds in eqns(5,6) exhibit double advantage of being analytically meaningful and tractable, and to naturally create equivalence classes within which system dynamics are globally the same. This is exactly the expression of system natural robustness resulting from the passage of some components to complex state which can be here easily exploited. An interesting observation from such an approach is again the emphasis on importance of the manifold on which system trajectories are staying, much more than one specific trajectory which has been seen above not to contain enough information for generating an outer action to control it. Another point is the way stability is now perceived in present setting: rather than researching usual specific decay by classical Lyapounov method (for asymptotic stability), it is more generally the belonging of solution manifold to a given function space determined by its properties, see Fig.7. Obviously the bound in eqn(6) exists over the entire time interval only if the term between bracket on the right hand side of eqn(6) is > 0 , which implies a relation between bounds parameters λ_i , μ_i and initial values $Z(t_0)$. So for a given initial ball $\|Z(t_0)\|$, there exists an

uncertainty ball $\| [\Delta\lambda_i, \Delta\mu_i] \|$ on system parameters within which the solution belongs to a manifold \mathcal{M} defined by the RHS of eqn(6), the embedding of which in required function space \mathcal{S} can be studied in a second step. This opens the way to get at the same time asymptotic stability and robustness, both useful properties for control of complex systems. The approach can be extended to the opposite and more difficult situation where system uncertainty ball is larger than robustness ball[35].

5. Applications to man made systems

Evidently previous properties are of up-most importance when applied to man made systems now appearing in industry. Under economic competition, more advanced systems are conceived and worked out which include an always increasing number of heterogeneous components to be operated all together for production of higher value objects. This would imply to keep complete system mastery by efficient control, which becomes the more illusory as system dynamics cover a larger number of elements escaping from one single centralized control structure, especially by becoming complex due to higher value of coupling parameters. In parallel, to reduce system behaviour fragility, it is also interesting to reduce the number of input control parameters by transforming the system into a (partially) complex one by clustering some components into bigger parts. A trivial illustration of this approach is given by the way shepherd dogs are acting on a herd of cattle in the meadows. If there are n animals wandering around, they represent a system with $2n$ degrees of freedom ($3n$ when counting their orientation). Clearly the dog understands that it is hopeless with his only 2 degrees of freedom to control all the animals, so his first action is to gather all of them in a restricted space so that by being close enough they have strong enough interactions transforming their initial complicated system into a complex one with dramatically less degrees of freedom, in fact only two like himself. Then he can control perfectly well the herd as easily observed. The astonishing fact is that dogs are knowing what to do (and they even refuse to do anything with animals unable to go into this complex stage, ie to develop gregarious potential) whereas today engineers are not yet able to proceed in similar way with their own constructions and to get corresponding benefits in terms of global mastery. This is an immense challenge industrial civilisation is facing today justifying if any the needs to study and to create these complex systems. On the representation of system in Fig.3, this would mean to vary adequate parameters to move the representative points along complex axis in order to decide exactly new system status. In any case, internal non controlled dynamics are taken care of by system self organization resulting from passing to complex state, implying that precise trajectory control is now delegated to system. The challenging difficulty is that comply with new structure, some “intelligence” has also to be delegated to the system, leading for the operator to a more supervisory position[36]. In present case, this is contribution to trajectory management by shifting usual (imposed) trajectory control to more elaborated task control [37], a way followed by all living creatures in their daily life to guarantee strong robustness while still keeping accuracy and preciseness. This illustrates if any the limited possibility of expressions from laws of Physics because they are tightly linking *information flux* related to the described action to *power flux* implied in it. Thanks to the discovery of “memory” DNA molecules, Nature has been very early able to “escape” from the constraint of solely following Physics laws strict causality, by inventing sufficiently isolated systems manipulating information flux as well (ie in storing and releasing it according to a timing fixed by survival goal). In fact resulting entropy decrease from created order cannot be maintained for ever due to their unavoidable transport and energy loss because they cannot exist in complete isolation, so Nature has been circumventing resulting isolated system finite life by inventing reproduction permitting species survival instead. In some sense, Human kind today is faced to a similar problem in the research of higher efficiency for industrial systems. After first development of tools, then to machines, and later to efficient control structure, leading to Mechatronics, a new step is now under way to give man made systems more efficiency and autonomy by delegating more “intelligence” to them, see Fig.8. This implies to search an adequate merging of information flux mastery from recent Information Technology development with power flux mastery resulting from classical long term mechanical development[38].

An elementary illustration is provided by the evolution of Robotics to comply with always more severe industrial constraints. It can be shown[14] that robotic systems are completely fixed by the three parameters $(Jr^2)_i$, ε_i and $(\omega_d \tau_R)_i$ for each link i , with J the inertia moment of the link (including actuator), r the gearing ratio, ε the compliance factor and $(\omega_d \tau_R)$ the “rigidity” factor with ω_d the

characteristic deformation frequency and τ_R the characteristic mechanical frequency. First N-link robots were build up so that parameters Jr^2 , ε and $(\omega_d \tau_R) \ll 1$ for each link i . The inequalities can be shown in full generality to decouple link interactions and to keep the system rigid, so the system was in simple state. However slowness (speed is multiplied by r) could not be compensated by higher actuator rotation speed, and passing to complicated state, use of decoupling method has been allowing direct drive actuation $r = 1$ but with preciseness problems from inadequate control. This has been improved by developing more robust controllers in broader Mechatronics, especially to improve mastery of compliance and vibrations. Recent orientation with higher nominal parameters is toward more autonomous and more intelligent systems with larger decision delegation shifting new robots toward complex state. This new step implies the introduction of “intention” into the system and not to stay as before at simple action level of following prescribed fixed trajectory dictated by classical control. In such scheme intention is represented by the “task” communicated to the system which now determines its own trajectory. Such evolution is following the heavy trend started from beginning of human activity, as described in Fig.8.

6. Conclusion

Thorough examination of natural systems in last fifty years has confirmed the existence of systems exhibiting behaviours which do not fit with main stream scientific laws established from patient observations of Nature over past centuries. These laws are based on mechanistic representation of the Universe born from celestial body motion, ie from identifiable simple objects with well defined trajectories. With the ascent of modern technology, new natural systems have been studied and artificial ones have been constructed, both with very intricate structure implying a large number of heterogeneous components in strong interaction. Application of usual laws is often unable to describe their dynamics, because they stay outside the domain of complicated multi component systems only covered by use of reductionism method. The main reason is in the overtaking of component interaction strength which dominates enough over other effects to force the system to close on itself and to manifest an internal self-organization responsible of its new behaviour. Differently said in elementary terms, the new paradigm is that “increasing interactions between components lead to their isolation” as easily verifiable. Such systems are termed as “complex” from etymology, and their main feature is that components in complex state are internally ruled through this self-organization so that at the end they are less depending on environment action. As a consequence, natural complex systems are structurally more robust than complicated ones as evidenced by observation of living organisms, the most complex existing systems. Similarly, artificial complicated systems can be made more robust by transforming them into complex ones when linking adequately and strongly enough some of their components. As always since human origin, the price to pay for this insensitivity is the needs to replace shielded outer action by larger delegation of “intelligence” to the system for its correct internal driving. Analysis of complex systems dynamics shows, for high enough ability of system components (now called “agents”), the possibility of “emergence” of a new behaviour which is not included into the set of initial components behaviour. Though it could be the way of approaching the new global and larger “2-dimensional” picture of the Universe proposed here, to avoid trivial rephrasing of well known classical results, emergence in this context appears to be more justified when restricting its domain to action of agents teaming up for collective goal they cannot individually reach.

The problem of representing complex system structure build up has been discussed, and three typical structures have been singled out. A– The broad class of reducible structure to which natural dissipative systems are belonging, usually formally amenable to be analytically projected back into complicated structure thanks to very large time and space scale difference between components. Application of asymptotic expansion methods allows to directly get important system quantities such as transport coefficients. B– The ordinary differential structure, thoroughly analyzed theoretically and by computer simulation, which shows when increasing component coupling interaction the transformation of system trajectories into finally fully chaotic and indistinguishable ones filling complete domains of state space, manifesting a layered organization of motion into equivalence classes which are the only reachable system parameters from environment. C–The most difficult delayed structure mainly due to long range component interactions, exhibiting a double complexity in space and time together, and grasping the fundamental aspect of component self determination by properly accounting for

neighbouring ones within the range of interaction potential. Origin and main properties of class A have been cleared up by Physics study, but transport problem is not completely solved for continuous systems such as Fluids and Plasmas. Class B has been carefully analyzed from Mathematics, especially in the difficult case of resonances for Hamiltonian systems, but still resists to a full picture from analytical reconstitution of trajectory manifold. Class C remains mainly open as its study did not really begin yet. Functional methods approach in adapted function spaces has been shown to be successful for reducible systems in class A, in that both robustness and asymptotic stability are obtained together. Extension to classes B and C will be discussed elsewhere.

As it is based on elementary source–sink description from Physics of interactions between elements of a system (the agents), and between the system and its environment, complexity paradigm discussed here can be tested through a falsifiable protocol. This bottom-up analysis is directly privileging a middle (control engineering type) approach along possible action on the system from environment in between purely (speculative) mathematical and purely (contemplative) physical classical approaches, because it fits immediately with global concepts of auto-organization and adaptive robustness resulting from transformation to complex, and allows direct application of powerful and particularly adapted modern functional analysis methods. Application to artificial man-made systems is particularly appropriate for highly performing ones, as entering complexity is opening the possibility for the “function” to feedback onto the “structure”, due to emergence produced by self-organization, a very specific property of the only living systems to date guaranteeing their remarkable survival ability. Finally, possible extension of this approach to “soft” sciences could open on orientation of economic and social sciences research towards more rigorous and potentially predictive models of human conduct.

7. References

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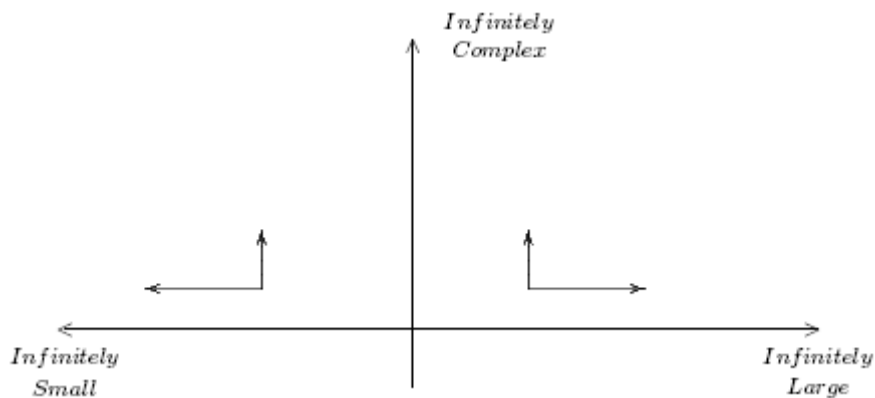
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**Fig. 1 : New 2-D Representation of Universe
with Arrows along Recent Trends**

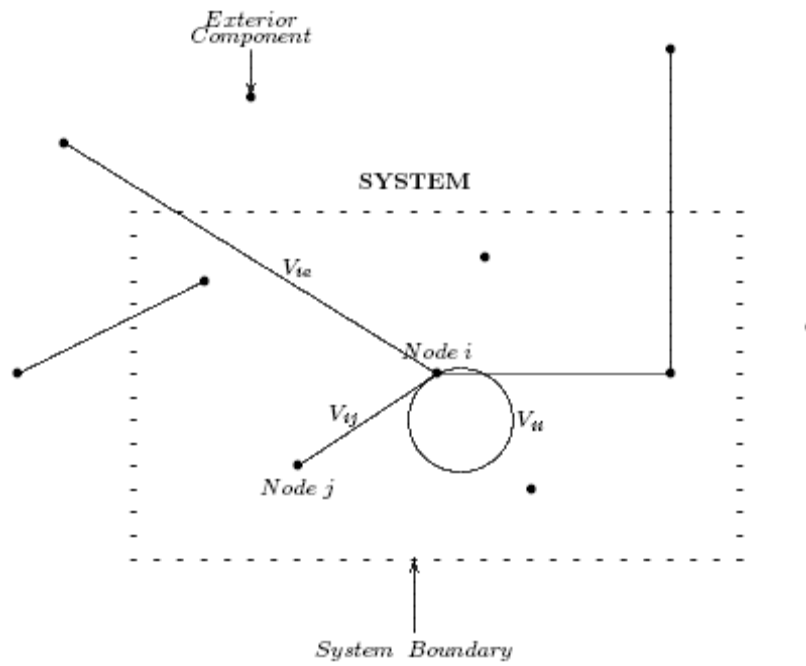


Fig. 2 : Graph Representation of System with its Three Exclusive Types of Vertices V_u , V_{ie} and V_{ij}

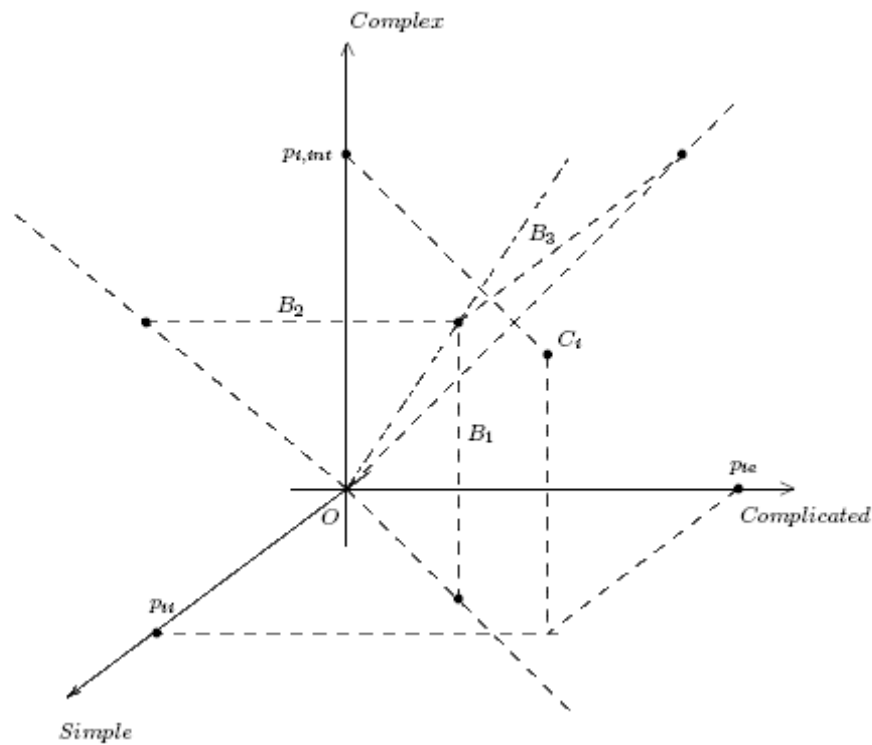


Fig. 3 : System Representation in [Simple-Complicated-Complex] State-Space Domain :

- Inequality (A) is satisfied in Tetrahedral Domain $(0, B_1 B_2)$
- Inequality (B) is satisfied in Tetrahedral Domain $(0, B_1 B_3)$
- Inequality (C) is satisfied in Tetrahedral Domain $(0, B_2 B_3)$

Component C_t is in Complicated State

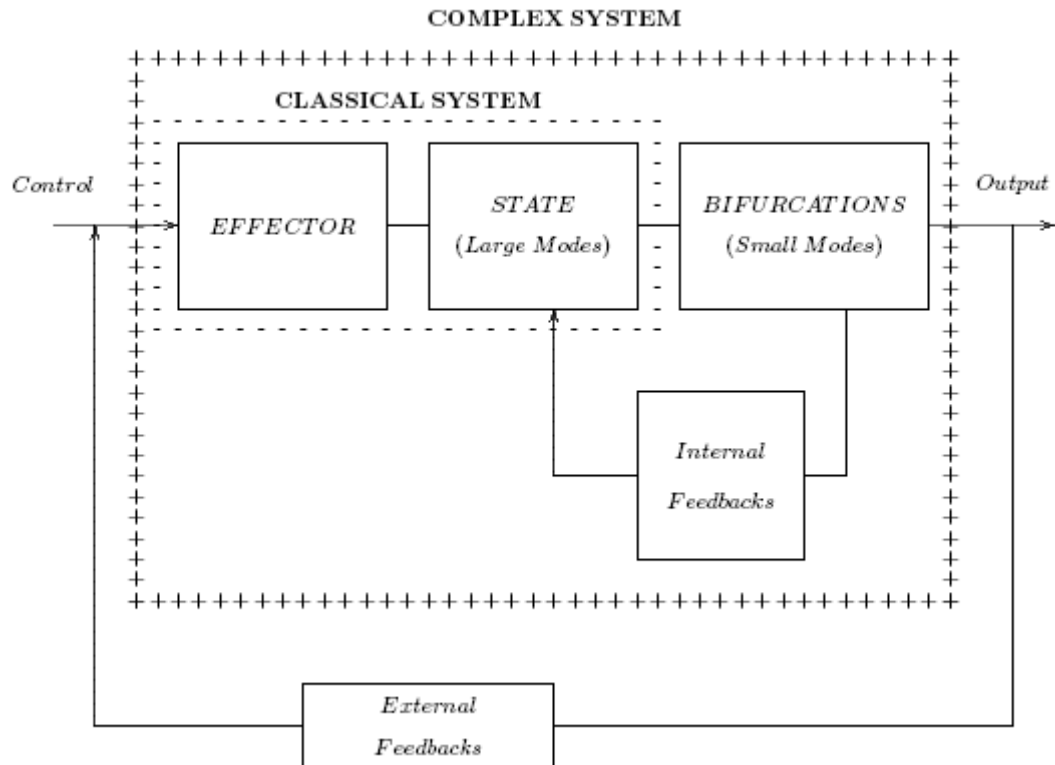


Fig. 4 : Block Structure of Typical Reducible Natural Complex System

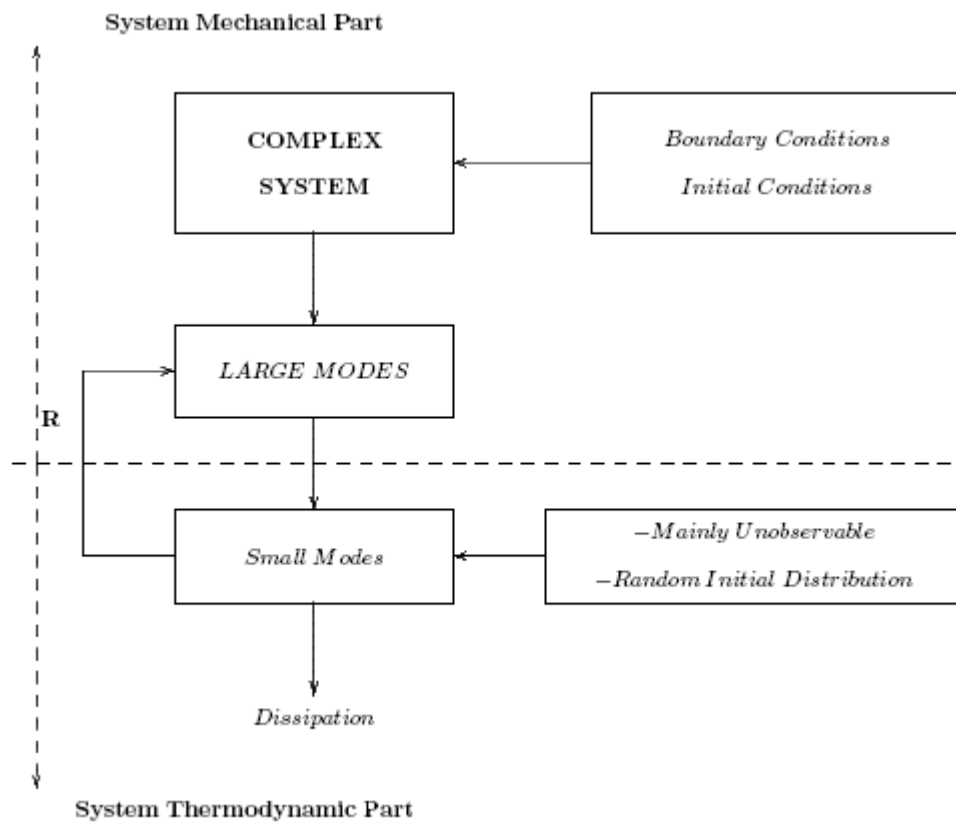


Fig.5 : Sketch of General Split Complex System Block Scheme

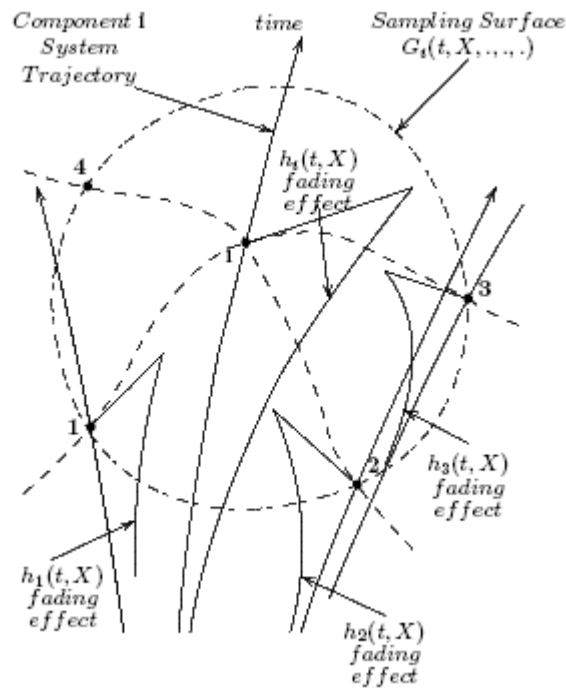


Fig. 6 : Domain Covered in Trajectory Space by Sampling Function $G_t(\text{---})$ with Components 1,2,3,4 and Normalized Past Trajectory Memory Effect Due to Delays h_j

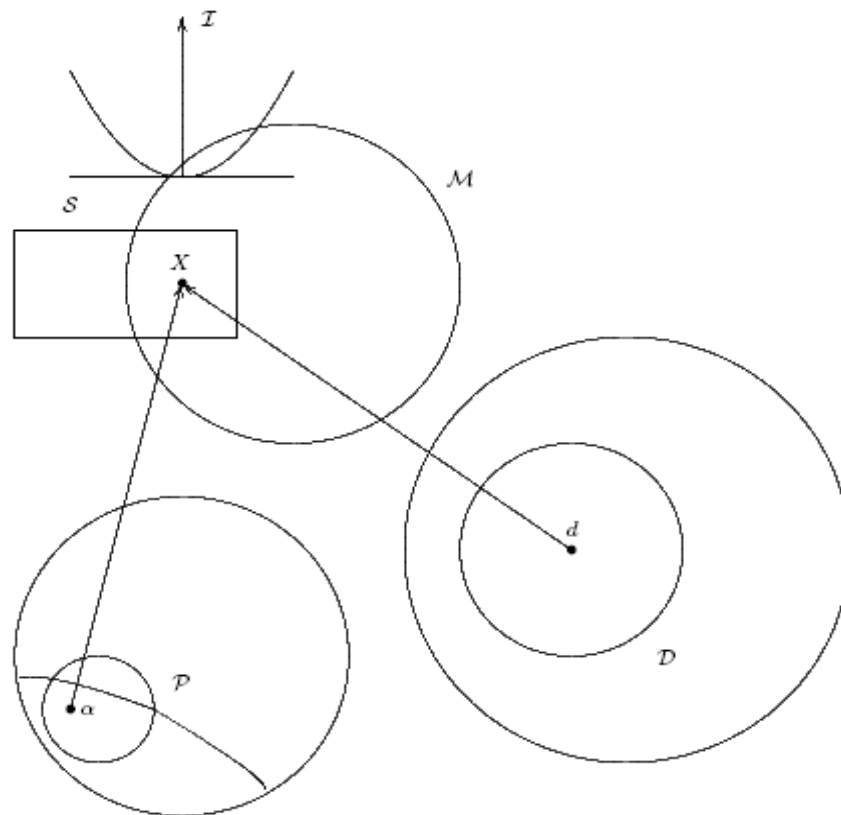


Fig.7 : Embedding of System Solution in Preselected Function Space S while Optimizing Trajectory Functional I

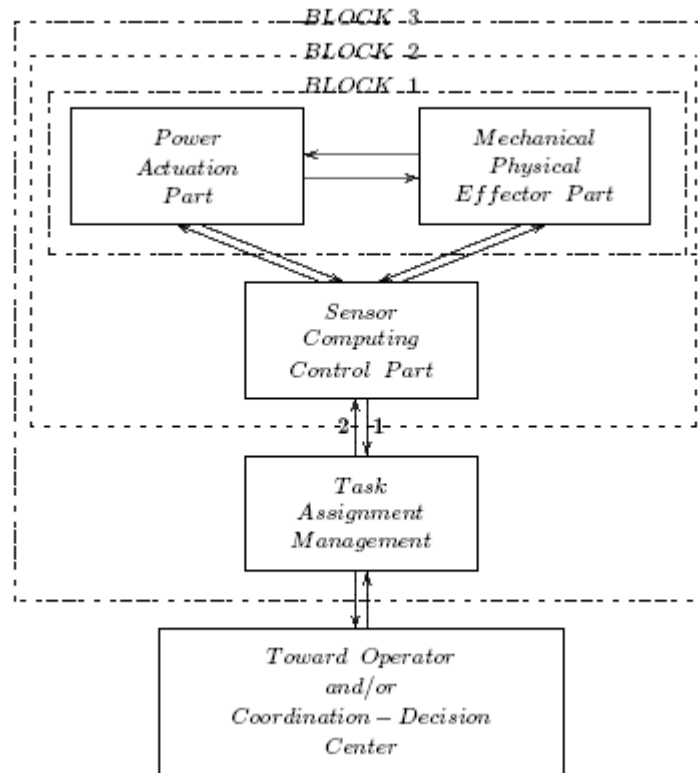


Fig.8 : System Structure Evolution with Main Added Component Parts. Flux 1 Corresponds to Useful Information Transfer. Flux 2 Corresponds to Intelligence Delegation to System

A calculus for multi-level emergent behaviours in component-based systems and simulations

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Summary. A major issue in Complexity Science is the formal description of emergent properties and behaviours in terms of lower level properties and behaviours. As a consequence, there are few techniques for empirically investigating specific emergent properties. In this paper, we introduce a general compositional approach to specifying such properties, using constraints to define representative sets of compositions. More specifically, we propose a calculus of complex events, which are compositions of events generated from component-level rule executions. Complex event types can be assembled hierarchically, giving a formal means of relating behaviours at different levels of abstraction. In being able to specify and then identify complex events of different types in systems and simulations, we have a method for empirically discovering relationships between behaviours defined at different levels. The formalism offers two important practical advantages. Firstly, higher level properties can be defined with different degrees of specificity so they can be defined with limited knowledge; we can then further sub-classify properties after they have been detected to discover differences in their constituent properties. Secondly, the formalism is related directly to the rules driving component behaviour so that all higher level behaviours can ultimately be decomposed into rule executions; this is particularly important for desirable and dysfunctional properties, and in circumstances where intervention at the component rule level is possible.

1 Introduction

Being able to understand and precisely analyse emergent behaviours in complex systems is a key goal in both Complexity Science and complex systems engineering. Emergent behaviours are those that arise from the organisational relationships and dynamic interactions between the system's components, and which are then able to drive the trajectory of the system in a particular direction. Information theory has provided a formal framework for defining emergence, and techniques for establishing the degree of emergence in a given system have been derived from this interpretation (see, for example (11), (26), (3), (19)). However, there is currently no universal formalism for describing

specific emergent properties in terms of their component properties (rather than the presence of emergence in general in a system). Our approach aims to address this while still assuming the information theoretic interpretation of emergence. We also work on the assumption that there may be emergent properties that can not be modelled or simulated because they are substrate-dependent and hence not computable (at least in the classical sense) (6), (1); these are not included in the current discussion.

The motivation for our work comes primarily from the growing use of agent-based modelling and simulation in studying complex systems (25). However, the formalism itself is general enough to describe any system in which higher level macro-properties (of which behaviours are a subset) can arise from the lower level properties of system components and their relationships. Many of the issues discussed here therefore apply equally to the engineering of robotic and swarm systems, where system behaviour is dependent on the collective behaviour of the system's components. There already exist several formalisms for specifying emergent behaviours in such systems e.g. temporal logics (27) but these tend to ignore the multi-level, compositional nature of many emergent properties.

In this paper, we first introduce a general approach for describing emergent properties at multiple levels. We then propose a calculus for describing emergent behaviours that allows them to be decomposed into lower level behaviours. The paper assumes familiarity with the elementary aspects of set, category and type theories (good introductions to these topics can be found in (20) and (16)).

1.1 Emergent Properties

As there is a lack of consensus as to what constitutes 'emergence' (see for example (5), (8)), we will avoid an explicit definition and instead give the assumed attributes that emergent properties are deemed to have in terms of a multi-component system.

- Emergent properties are 'macro'properties with respect to their constituents⁴, i.e. they have either:
 - a greater **scope** than its constituent properties i.e. for the property to be present, it has to include more than one of the constituent properties; or
 - a lower **resolution** i.e. the property makes fewer distinctions between its constituent properties than can possibly be made. (within the information theoretic framework, we say that it has lower statistical complexity (2)); or
 - both.

⁴ The converse does not hold however; not all 'macro' properties are necessarily emergent.

A more detailed account of these terms is given in Section 4.1 and formal definitions can be found in (23).

- An emergent property arises as a result, not only of the sum of its constituents (as formalised in (9)), but also as a result of their interactions or relationships with one another.
- In a multi-component system, emergent properties are those that are not explicitly defined in the component specifications. This is based on the design/observation distinction introduced in (21)

We will elaborate on these assumptions when we introduce the formalism.

1.2 Locating and composing properties

We define a property as anything that can be measured in a system. There are four central ideas behind our proposed calculus, all relating to way that properties can be located in a system or simulation.

1. Every property in a system consists of one or more (micro-)properties that can be located in an n -dimensional (hyper)space. For the lifetime of the system or simulation, properties can be located in this space by specifying the coordinates in each of the dimensions. The coordinate system used to specify the location can be global (from a whole system point of view) or local (where locations are in relation to a particular constituent *within* the system). For example, if the global coordinate (12, 1, 4, 2) represents the location of a state transition in the 12th time step (first tuple item holds time), located in coordinate (1, 4) of physical space (second and third tuple items hold space) in component with ID 2 (final tuple item holds component identity); the equivalent coordinate using a local coordinate system defined with respect to component 2 at time step 11 in the same spatial location would be (1, 0, 0, 0).
2. If two macro-properties consist of constituents of the same types and constituents of the same type have the same configuration with respect to each other in the two properties, we can say the two properties are of the same type.
3. We can describe **regions** as well as point locations in a system or sub-system space using propositional statements about the location in the system/simulation's various dimensions. For example, in a system with only time and identity represented, (before 3, 4) stands for all the states or state transitions that occur in component 4 before time step 3.
4. Higher level properties can be composed by defining organisational relationships between their constituents i.e. their **configuration**. This idea is generalisable to any dimension.

If a property is defined atemporally⁵, we call that property a **state** while if the property has temporal extension, we call it a **behaviour**. In this paper, we focus mainly on the latter of these.

1.3 Outline of the paper

The remainder of the paper is organised as follows:

- Section 2 introduces the main constructs that are used in our calculus of complex events and illustrates them by using them to construct a formalism for macro-states. Our main assumptions about multi-component systems is also given.
- Section 3 introduces the calculus and describes how complex event types can be used to specify behaviour at any level of abstraction that can be realised by the system or simulation.
- Section 4 revisits the criteria for emergence we outlined in Section 1.1 and relates them explicitly to complex events. We also briefly discuss emergent laws and top-down ‘causation’ in terms of our formalism.
- Section 5 suggests some promising directions for further work.
- Section 6 summarises and concludes the paper.

2 Assumptions and Fundamental constructs

In this section, we begin by stating our assumptions about the way state transitions occur in the multi-component systems we are concerned with. We then introduce two key concepts: configuration and compositionality and illustrate how they can be applied to subsystems, components and states. These two concepts are also fundamental to the temporally extended version of our calculus for temporally extended properties. Key terms are also more precisely defined.

2.1 State transition rules and state transition functions in multi-component systems

We assume that a multi-component system is composed of a set of components, each of which instantiate a defined component type⁶. A component type specifies the component’s behaviour with a set of state transition rules (many formalisms exist for specifying component behaviour e.g. state charts (12),

⁵ An atemporal definition is one that does not *internally* refer to time e.g. being blue but the property itself can persist through time, in which case it is defined as part of a behaviour e.g. stay blue.

⁶ Unencapsulated state variables representing continuous media, spatially dependent quantities etc. are also components under our definition.

Petri nets (18), communicating X-machines (13) but these ultimately reduce to a set of rules). State transition rules are defined relative to a source component and are made up of a *condition* or *trigger* and a set of state transition functions. The *condition* typically takes into account both the component's own state q_C and the state of its environment $q_{Env/C}$, where the environment of a component *for a given rule* is a set of n components $Env/C = C_1, \dots, C_n$ in some configuration λ . C and Env/C together make up a subsystem Sub/C (see Definitions 5, 6 and 7). The *condition* is then the set of Sub/C states sub_1, \dots, sub_m under which the state transition functions of the rule are executed (if the rule is non-deterministic, there is a further guard that determines whether or not the functions are executed). A state transition function is a mapping from a source state q_{source} to a target state q_{target} , where q_{source} and q_{target} are both subsystem states defined relative to the component.

Definition 1. *State transition.* A state transition is a transformation of one subsystem state q to another subsystem state q' . The state q before the transformation is applied is called the source state and is denoted q_{source} , while the state q' after the transformation has been applied is called the target state and denoted q_{target} . (The definition for subsystem state is given in Definition 7).

Definition 2. *State transition function.* A state transition function is a function that maps the source subsystem state q_{source} to the target subsystem state q_{target} .

$$STF :: q_{source} \rightarrow q_{target} \quad (1)$$

Definition 3. *State transition rule.* A state transition rule is a rule that restricts the applicability a set of state transition functions to cases where the propositional formula representing the realisation of condition evaluates to true.

$$STR :: (\{STF\}, condition) \quad (2)$$

2.2 Describing the organisation of system constituents using configurations and constraints

Definition 4. *Configuration.* In general, a configuration ω_x of a set of n properties x_1, \dots, x_n is a set of constraints Θ_x that together describe the organisation of the properties in an m -dimensional (hyper)space. At the lowest level, each constraint is itself a configuration of a pair of properties $\Theta = \theta_{x_1-x_2} \times \dots \times \theta_{x_1-x_n}$, where θ_{a-b} locates the property b relative to a .

If the constraints in a configuration result in every constituent being point located in the m -dimensional (hyper)space, we say the configuration is *fully determined*. If any constituent location is defined as a region (after taking into account all the constraints) e.g. given a range or left undefined in some dimension, the configuration is said to be *partially determined*. Two configurations ω_x and ω_y are said to be *identical* when their constraint sets have exactly the

same members; this is denoted by the identity operator \equiv . They are said to be *equivalent* when they apply to the same number of constituents and the application of ω_x 's member constraints is able to give every organisation ⁷ that satisfies every one of ω_y 's constraints and vice versa; equivalence is denoted by the operator \simeq . It is also worth noting that a partially determined configuration defines a set of fully determined configurations that satisfy all its constraints. This also means that a partially determined configuration ω_{PartA} can be a subset of another partially determined configuration ω_{PartB} i.e. every fully determined configuration in ω_{PartA} has an equivalent fully determined configuration in ω_{PartB} .

$$\omega_{PartA} \subseteq \omega_{PartB} \quad (3)$$

$$\omega_{PartA} = \{\omega_A1, \dots, \omega_An\}, \omega_{PartB} = \{\omega_B1, \dots, \omega_Bm\}, \quad m \leq n \quad (4)$$

$$\forall \omega_A \exists \omega_B (\omega_A \simeq \omega_B), \quad \omega_A \in \omega_{PartA}, \quad \omega_B \in \omega_{PartB} \quad (5)$$

From Definition 4, we can define what it means for a set of components to be in a configuration at a given point in time. This is then used to define subsystem states in Definition 7.

Definition 5. *Component configuration.* A component configuration ω_C at time point t_i of a set of components C_0, C_1, \dots, C_n is the set of constraints p_i that together describe the organisation of the components at t_i , $\omega t_i = (\theta_{0-1}t_i \times \theta_1t_i \dots \times \theta_nt_i)$.

Two (sub)systems $sysA$ and $sysB$ have equivalent component configurations if $sysA$ and $sysB$ have the same number of components and for every component $C1_{sysA}$ in $sysA$, there exists exactly one corresponding component $C1_{sysB}$ in $sysB$ which satisfies the same location constraint to a second component $C2_{sysB}$ as $C1_{sysA}$ satisfies to $C2_{sysA}$, and vice versa.

Definition 6. *Subsystem.* A subsystem Sub of a multi-component system is a subset of the system's components. If a system consists of a set of n components $Sys = C_1, \dots, C_n$, $Sub \subseteq Sys$ (a system is therefore its own subsystem).

Definition 7. *Subsystem state.* A subsystem state of a multi-component system Sys at a given time point t_i is the composition of the states of any subset of a system's components and a configuration ω . $q_{Sub}t_i = q_{SubComponents}t_i \times \omega_{Sub}t_i$.

⁷ in the case of fully determined configurations, there will only be one possible organisation

2.3 Compositionality of configurations, subsystems, components and states

The constructs and terms defined above imply compositionality in that they can be described by the recursive syntax:

$$A :: a \mid A_1 \bullet A_2, \quad (6)$$

where A is the construct, a is a primitive and \bullet is the compositional operator.

In the case of configurations for example, a configuration can be defined as either a single location or a two configurations satisfying a set of constraints with respect to one another:

$$\omega :: \nu \mid \omega_1 \bowtie \omega_2, \quad (7)$$

where ν is a location and the \bowtie means that the right hand side configuration satisfies a set of location constraints with respect to the left hand side configuration.

Similarly, a subsystem is either a component or a set of two unordered subsystems:

$$Sub :: C \mid (Sub_1, Sub_2) \quad (8)$$

So a system can be decomposed into a hierarchy of subsystems, with the largest subsystem being the system itself and the smallest subsystem being a component. The greater the number of components in a subsystem, the greater the scope and the higher the level.

Example 1. A system with three components (C_1, C_2, C_3) can be decomposed into the following subsystems:

1. (C_1, C_2, C_3)
2. (C_1, C_2), (C_1, C_3), (C_2, C_3)
3. C_1, C_2, C_3

Again using the compositional syntax, a subsystem state is either a component state or two subsystem states satisfying the constraints of some configuration:

$$q_{Sub} :: q_C \mid q_{Sub1} \bowtie q_{Sub2} \quad (9)$$

So we can decompose a given subsystem state into lower level subsystem states, retaining the (sub)configurations of the components and the component states.

Example 2. A subsystem state exists in 2-dimensional space, where component C_1 is located at (0,1) relative to C_2 and C_3 is located at (3, 4) relative to C_2 , and C_1, C_2 and C_3 are in states q_0, q_2 and q_1 respectively. We can denote this: $[C_1 \leftarrow q_0] \bowtie (0, 1)[C_2 \leftarrow q_2] \bowtie (3, 4)[C_3 \leftarrow q_1]$.

This can be decomposed into the following subsystem states:

1. $[C_1 \leftarrow q_0] \triangleright (0, 1)[C_2 \leftarrow q_2] \triangleright (3, 4)[C_3 \leftarrow q_1]$
2. • $[C_1 \leftarrow q_0] \triangleright (0, 1)[C_2 \leftarrow q_2]$,
 - $[C_1 \leftarrow q_0] \triangleright (3, 5)[C_3 \leftarrow q_1]$ (from the location of C_3 relative to C_2),
 - $[C_2 \leftarrow q_2] \triangleright (3, 4)[C_3 \leftarrow q_1]$
3. q_0, q_2, q_1 .

This compositional approach can be generalised to any property. For example, a component C consists of a set of encapsulated state variables whose values at t_i together make up the component's state q_C at t_i , $q_C t_i = var_0 t_i \times var_1 t_i, \dots \times var_n t_i$. We can specify subsets and configurations for these variables within and between components and associate them with macro-properties. In fact, the component's state is itself a macro-property of its constituent variable values. Figure 1 illustrates this general compositional approach.

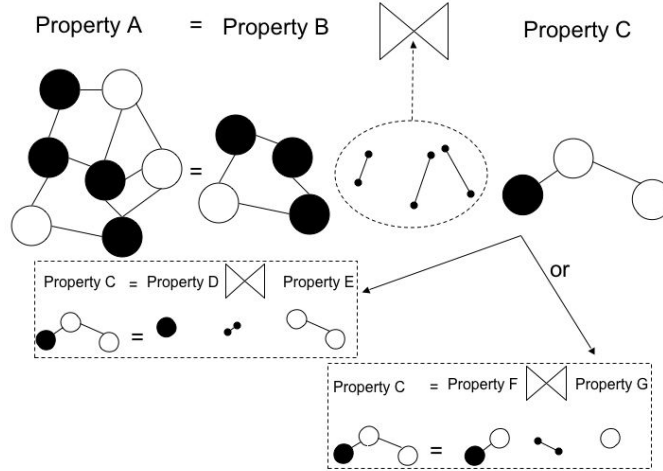


Fig. 1. Property A is a fully determined property (i.e. each of its lowest level constituent properties occupies a point location in the (hyper)space defined by a set of dimensions) composed of Property B and Property C in some configuration. B and C can be further decomposed, as can their constituents. \bowtie denotes the set of constraints that determine the configuration; x , y and z are used to distinguish between two different sets of constraints.

3 Complex events

In this section, we introduce the complex event calculus and describe how complex event types can be used practically to specify and then detect macro-

behaviours in systems. We then relate complex event type compositionality to scope and specificity to resolution. We take as a starting assumption the fact that behaviours are changes in state i.e. events (see Definition 8).

Definition 8. *Event.* An event is a state transition defined at a particular level of abstraction.

The compositionality of state means that state transition functions can also be decomposed into lower level state transition functions. For example, a state transition function that maps a source subsystem state q_{Sub} to another subsystem state q'_{Sub} (the target state) can be decomposed into a set of lower level subsystem state mappings, which can be further decomposed into a set of component state mappings. If variables are the lowest level of state representation, every state transition function can ultimately be reduced to a set of state transitions mapping a variable value to a new variable value.

$$q_{Sub} \rightarrow q'_{Sub} \equiv \{q_{Sub} \rightarrow q'_{Sub}\} \equiv \{q_C \rightarrow q'_C\} \equiv \{var \rightarrow var'\} \quad (10)$$

In a multi-component system, every event is the result of a state transition function being applied; we call these simple events (See Definition 9) and their level of abstraction depends on their state transition function.

Definition 9. *Simple event.* A simple event is a state transition that results from the application of a state transition function.

Two simple events that occur in a system are said to be of the same type if they result from the same state transition functions (state mappings) and the mappings are applied as a consequence of the same rule being executed.

Definition 10. *Simple event type.* Two simple events are said to be of the same type if (i) they are the result of the same state transition function and (ii) the function is applied as a result of the same state transition rule.

Figure 2 shows the relationship between component type and components and between state transition functions and simple events in the executing system.

Given these preliminaries, we can define a complex event CE as either a simple event SE or two complex events linked by \bowtie :

$$CE :: SE \mid CE_1 \bowtie CE_2 \quad (11)$$

\bowtie was introduced in Section 2.3. Here, it denotes the fact that CE_2 satisfies a set of location constraints with respect to CE_1 . Conceptually, complex events are a configuration of simple events where the configuration can be defined in a hyperspace that includes time, physical space and any other dimensions.

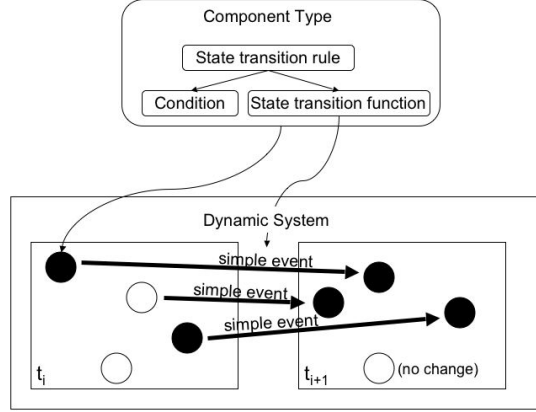


Fig. 2. Components instantiate component types in the real system or simulation while simple events can be said to instantiate state transition functions.

Definition 11. *Complex event (recursively defined).* A complex event is either a simple event SE or two complex events CE_1 and CE_2 satisfying a set of location constraints with respect to each other:

$$CE_1 \bowtie_x CE_2, CE_2 \bowtie_{x^{-1}} CE_1 \quad (12)$$

, where x^{-1} is the inverse of x so that $CE_1 \bowtie_x CE_2$ denotes satisfaction of the set of location constraints x by CE_2 with respect to CE_1 and $CE_2 \bowtie_{x^{-1}} CE_1$ denotes the satisfaction of the set of location constraints x^{-1} by CE_1 with respect to CE_2 .

Location constraints define relationships between complex events, which can be represented as a coloured multi-graph, where the coloured nodes stand for event types and coloured edges stand for the different relationship types (sets of location constraints) existing between events (4). It should also be pointed out that because location constraints can be defined at different levels of specificity (see also Section 3.2), different multigraphs can be drawn for the same complex event. These alternative multigraphs can be seen to represent different ‘views’ or descriptions of the same behaviour. Similarly, we can infer ‘hidden’ edges in a multi-graph when a relationship between one pair of events implies a relationship between another pair of events e.g. $e1 < e2$, $e2 < e3$ implies $e1 < e3$.

3.1 Specifying complex event types

We have already introduced the idea that events can be typed in our discussion of simple events (see Definition 10). We now extend this to complex events.

Definition 12. *Complex event type.* Two complex events CE_1 and CE_2 are said to be of the same type if, for each constituent event e_1 in CE_1 there is exactly one event e_2 in CE_2 satisfying the same location constraints, and e_1 and e_2 are events of the same type.

To specify a complex event type therefore, we need to specify the types for each of the constituent events and the location constraints that hold between them.

It is possible to use formal expressions from different formal systems such as temporal logics to specify the location constraints. This itself is a worthy subject for research since different applications are likely to have different semantic and expressivity requirements. However, a comprehensive review and analysis of these different formal systems remains outside the scope of the paper (good reviews can be found in (17) and (22)); here we only present a few illustrative examples.

Example 3. Temporal constraints: \parallel concurrently, $;$ immediately after, $<[\leq, <, \geq, >]$ t $]$ [[less than or equal to, less than, greater than or equal to, greater than] t time units] after .

E.g.

- $E_1 \prec [< 4] E_2$ means that E_2 occurs within 4 time units of E_1 .
- $E_1 (\prec [< 4] \vee (\prec [> 8])) E_2$ means that E_2 occurs either within 4 time units of E_1 or it occurs more than 8 time units after E_1 .

Example 4. Spatial constraints: within distance $v \circ (v)$, at location $(x, [y, \dots, z])$ *triangleright* $(x, [y, \dots, z])$.

E.g.

- $E_1 \triangleright (-3, 2)E_2$ means that every component in which a state changes occurs as a consequence of E_2 is $(-3, 2)$ with respect to the corresponding component in E_1 .
- $E_1 \prec [< 4] \wedge \circ(3)E_2$ means that E_2 occurs within 4 time units and within a distance of 3 units from E_1 .

Example 5. Token identical component $[C_A/C_B]$.

E.g.

- $E_1[C_A/C_B]E_2$ means that E_2 occurs in the same component (with the same ID) as the component in which E_1 occurs.

3.2 Specificity of complex event types

Just as partially determined configurations define sets of fully determined configurations (see Section 2.2), complex event types can differ in specificity with a fully determined complex event type CET_{Full} being defined as one whose constituent events are in a fully determined configuration. A partially

determined complex event type CET_{Part} is then one with a partially determined configuration and therefore defines a set of complex events with fully determined configurations.

$$CET_{Part} = \{CET_{Full}\} \quad (13)$$

The dimensions in which configurations are not fully specified lower the resolution of the complex event, with weaker constraints (greater ranges of possible values) implying a lower resolution in that dimension (this is defined more precisely in Section 4.1). More generally, the greater the number of complex event types with fully determined configurations that a complex event type contains, the lower its resolution (see Figure 3).

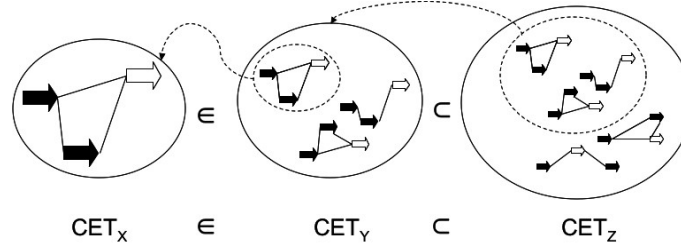


Fig. 3. CET_X is a fully determined complex event type which is an element in CET_Y (a partially determined complex event type). CET_Y is a subset of CET_Z , which is the least specific (lowest resolution) of the three complex event types.

4 When do complex event types represent emergent behaviours?

In Section 1.1, we listed three main criteria that emergent properties should fulfil, namely, (i) they should be macro-properties with respect to their constituents, (ii) they should not be reducible to the sum of their constituents and (iii) they have not been explicitly specified. In this section, we return to these criteria and explicitly link them with categories of complex event types, which can then be said to represent emergent behaviours. We also show how emergent ‘laws’ and top-down ‘causation’ can be formalised.

4.1 Scope and Resolution of complex event types

The criteria for a property being a macro-property with respect to another are that it has a greater scope, a lower resolution, or both. Complex event types represent behaviours, so the scope and resolution of a complex event

type is the scope and resolution of the behaviour represented by that complex event type.

Definition 13. A behaviour M is a macro-behaviour with respect to another behaviour m if the following is true of the complex event type CET_M representing behaviour M and the complex event type CET_m representing behaviour m :

$$Scope_{CET_M} \geq Scope_{CET_m} \quad (14)$$

where a complex event type CET_A has a greater scope than another complex event type CET_2 if the minimum number of simple events at the lowest possible level required for CET_1 is greater than that required for than in CET_2 .

$$Resolution_{CET_M} \leq Resolution_{CET_m} \quad (15)$$

where a complex event type CET_1 has a lower resolution than another complex event type CET_2 if the set of CET_{full} defined by CET_1 contains more elements than CET_2 .

$$(Scope_{CET_M}, Resolution_{CET_M}) \neq (Scope_{CET_m}, Resolution_{CET_m}) \quad (16)$$

4.2 Irreducibility of emergent behaviours

The second criterion required for us to say that a particular property is emergent with respect to another is that the first can not be reduced simply to the sum of the second. Applying this to behaviour, a behaviour M is emergent with respect to another behaviour m if M can not be reduced simply to the sum of ms . In complex event type terms, we can say that a complex event type CET_M is reducible to the sum of CET_m s if:

$$CET_M = CET_m[\bowtie CET_m]^* = CET_m^+ \quad (17)$$

where $*$ denotes 0 or more occurrences and $+$ denotes 1 or more occurrences. Hence, we can say that the emergent behaviour M is emergent with respect to m when their complex event types CET_M and CET_m satisfy:

$$CET_M = CET_m[\bowtie CET_m]^* \neq CET_m^+ \quad (18)$$

This is consistent with the grammar-based definitions given in (7), (9) and (15) e.g. Demazeau's pseudo-equation:

$$MAS = A + E + I + O = emergence$$

, which corresponds to:

$$L(MAS) \supset \sum_{v \in vowels} (L(v))$$

, where *vowels* is a micro-partition of a macro MAS (8).

The final criterion for a behaviour to be classified as emergent is that it is not explicitly specified in the component rules. This is the case when a complex event type is not a simple event type.

4.3 Emergent ‘laws’, top-down ‘causation’ and multi-functionality in terms of complex events types

A popular application of the multi-component paradigm is in multi-agent simulations of complex systems to understand their properties. Scientists working in this area are particularly interested in establishing rules at higher levels, which they often call emergent laws. Top-down constraints (also known as top-down ‘causation’⁸) and feedback are also effects that are sought in simulations. While simple statistical techniques can be used to detect these effects at the global level, a hierarchical approach has yet to be established. Definitions 14 and 15 support such an approach by stating in terms of complex event types the criteria for behaviours at any two levels to be related in these two ways.

Definition 14. *An emergent law exists between two complex event types CET_x and CET_y when a complex event type CET_x exists such that the occurrence of CET_x implies the occurrence of some complex event of the type CET_y i.e. $CET_x \rightarrow CET_y$, and CET_x is not a simple event.*

Definition 15. *A top-down constraint effect exists between two complex event types CET_M and CET_m when an emergent law $CET_M \rightarrow CET_m$ holds and CET_m is a lower level complex event type with respect to CET_M .*

Finally, we can use the complex event type formalism to express multi-functionality. A complex event type can be decomposed into different combinations of constituent events (see also the decomposition of property C in Figure 1), e.g.:

$$CET_A = CET_{x1} \bowtie_1 CET_2 = CET_{x1} \bowtie_2 CET_3 \quad (19)$$

where \bowtie_1 and \bowtie_2 are different sets of configuration constraints.

The overlapping of constituent properties gives us a criterion by which to establish multi-functionality.

Definition 16. *A complex event type CET_x is multi-functional if it is a constituent event in more than one complex event type.*

Example 6. If

$$CET_A = CET_x \bowtie CET_y \quad (20)$$

and

$$CET_B = CET_x \bowtie CET_z \quad (21)$$

we say that CET_x is multi-functional and that it has a role in CET_A and CET_B

⁸ Here, we make no assumptions about the metaphysical status of top-down constraints i.e. we take an agnostic stance on whether these constraints have real causal power, supervene on lower level laws, or are epiphenomena. This has been a long-standing debate in the Philosophy of Science, see, for example (10), (24), (14)

5 Directions for further investigation

In this section, we propose several directions for extending the formalism and enhancing its practical application.

- We have assumed that every fully defined complex event type in a partially defined complex event type set representing a behaviour is an equally good exemplar of that behaviour. However, an extended formulation in terms of fuzzy set membership functions would be worth investigating for some behaviours that can not be crisply defined. The same holds for the specification of subsystem states any other emergent properties.
- The specification of configuration constraints for complex event types and subsystem states should be further formalised. A sound approach would be to use the same formalism as used to specify the system (statecharts, petri nets etc.) to specify states and execution trajectories.
- In this paper, we have used the dimensions of physical space, time and component identity to ‘locate’ events and states. However, our approach could be generalised to other dimensions and to other properties so that other types of inter-level relationships can be identified.
- Quantitative techniques are required to precisely express and evaluate the participation of lower level properties in higher level ones since they often overlap.
- The implementation of complex event detection can be extremely costly from a computational point but the compositional nature of complex events could provide a promising means of optimising the search.

6 Summary and Conclusion

In this paper we have introduced a compositional approach to formalising and analysing emergent properties. In general, emergent properties can be described by constituent properties that are organisationally related in a set of dimensions. Organisational constraints can result in fully specified properties, where the configuration of the constituent properties is fully determined, or they can be partially specified so that the property is represented by a set. More specifically, we have proposed a practical formalism that can be used to categorise different system sub-trajectories so they can usefully be used to understand the mechanisms at work at multiple system levels. This allows us to analyse an emergent behaviour at any level and establish relationships between behaviours at multiple levels. Furthermore, by reducing behaviours at higher levels to smaller and smaller ‘motifs’ of behaviour, with simple events (and hence state transition rules) at the lowest level, we have a sound method for engineering and modifying multi-component systems to exhibit or prevent certain emergent behaviours.

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Emergence of Growth, Complexity Threshold and Structural Tendencies During Adaptive Evolution of System

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Summary. Adaptive evolution of a functioning network (e.g. Kauffman network) may force growth of this network. We consider random addition and removing of nodes in a wide range of networks types, including scale-free BA networks. Growth of a network leads to reaching a certain complexity threshold which appears suddenly as phase transition to chaos. It can be observed in the distribution of damage size of system outputs after damage spreading initialised by a small modification of the system. Over this threshold the adaptive condition as condition of network growth is a source of structural tendencies which were observed e.g. in developmental biology as regularities of ontogeny evolution, but still were not explained. Using specific algorithm the simulation has shown mechanisms of these tendencies. Our model describes living and human designed systems. We remark that the widely used two equally probable variants of signal (Boolean network, Ising model, spin glasses) are in many cases (especially outside physics) not adequate. They incorrectly lead to extreme structural stability instead of chaos. Therefore more than two equally probable variants of signal should be used. We¹ define fitness on systems outputs, which allows us to omit the local extremes and the Kauffman's complexity catastrophe.

1 Introduction

This article is a short overview of some new approach to emergence of basic properties like growth, complexity threshold and structural tendencies in natural and artificial complex systems. This approach needs a much more detailed description which is currently in press [8]. Complexity threshold appears suddenly during system growth in distribution of change size of output signals vector. In this form it can be observed in nature. It is a phase transition to chaos caused by growing number of nodes in the network when other parameters are constant. It is connected with damage spreading initialised by a small network disturbance, e.g. random addition or removing of a node.

¹ Research financed by government grant.

This study was directed to explain some known but not popular biological regularities in ontogeny² evolution. We observe in the simulation of different network types including scale-free BA networks some structural tendencies and their mechanisms corresponding with these regularities as an effect of random adaptive evolution of system over the above mentioned complexity and chaos threshold. We estimate that the typical living or human designed system is chaotic and grows under adaptive condition therefore we should expect such structural tendencies in these systems.

This interpretative expectation of chaos and independently the basic principle of Shannon theory lead us to treat the widely used assumption of $s = 2$ - two equally probable variants of signal (typical for Boolean network or for popular Ising and spin glasses models) as not adequate and dangerous in lots of cases other than physical spin. This assumption is extreme and can incorrectly lead to Kauffman's 'structural stability' [14]. We show, (fig.1) that using $K > 2$ - more than two input links per node cannot replace assumption of $s > 2$ (more than two equally probable variants of signal).

For simulation we use our simplified algorithm which allows us to obtain one particular vector of output signals instead of a periodic attractor. It gives correct answers for our statistical questions and practically allows to investigate structural tendencies which were known only on an intuitive level.

2 Model of system evolving under adaptive condition

Mechanism of emergence of regularities in ontogeny evolution is the main concern of this investigation. Therefore, we concern random adaptive evolution as the main source of desired statistical properties. Adaptive condition defined in the most simple way describes Darwinian mechanism of natural selection. Not only living systems grow under adaptive condition - most of human designed system as well, therefore adaptively evolved systems which are considered here constitute the main part of interesting systems.

We define adaptive condition on the effects of system function visible outside of system. They can be only output signals of system. We interpret them as system properties and we compare them to some arbitrary defined ideal. Similarity of the system's output signal vector and the ideal vector is treated as fitness. Adaptive condition does not allow the fitness to decrease.

This implicates that our system function and as network of nodes transform some signals. We use known Kauffman networks [14, 13, 11] but we use $s \geq 2$, mainly $s > 2$ (s - number of equally probable variants of signals) unlike Kauffman and Iguchi *et al.* $s = 2$, therefore our networks are generally not Boolean networks any more. There are two variants of signal for Boolean network but they do not have to be equally probable. We prefer the useful

² Simplifying - sequence of stages of organism development from zygote to adult form.

assumption of equally probable signal variants therefore for two variants but not equally probable e.g. $1/4$ and $3/4$ we would use $s = 4$ and the more probable original variant is represented by 3 new variants. It is rather not typical that two alternatives have identical probability. It would be strange, if in a real Boolean system each signal carried full available information optimally. Computer is a good sample of complex and real Boolean network. Note, that typically using zip you get a smaller file. This is the basis of Shannon's information theory [17], which considers this difference between information available to transmission by channel and real exploitation of them. We cannot use $s = 2$ as simplification because this value of s , especially when $K = 2$, is an extreme which can lead to different effects - i.e. it can place the system outside of chaos. In area of chaos using $s > 2$ we obtain different behaviour of network than using only $K > 2$ with $s = 2$. It is shown in fig.1 where we compare average damage size for different types of autonomous networks and for $s = 2$ and 4 , $K = 2$ and 3 .

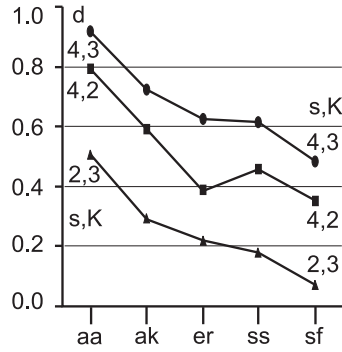


Fig. 1. Average damage size for different network types: *aa*-aggregate of automata $k = K$, Kauffman networks: *ak* ($k = K$), *er*-random Erdős-Rényi network, *ss*-single-scale and *sf*-scale-free; and different parameters $s, K=4,3; 4,2$ and $2,3$. Note that for $4,2$ and $2,3$ the coefficient $w = 1.5$. We cannot limit ourselves to one of K, s and w parameters or one network type. The shown points have 3 decimal digits of precision. These networks are autonomous, they are built without adaptive condition. The *er* network is the only one with $k = 0$.

In the system with feedbacks there is no single output signal vector, but a cyclic attractor instead - a finite sequence of output signals vectors. It is not easy to compare such attractors, but for statistical questions considered in this approach we construct some simplified algorithm which answers this question correctly although it generates single particular vector of output signals after each change of system. This algorithm is dedicated for statistical simulation of damage spreading in synchronous mode. We perform calculations only for nodes with at least one changed input signal, not for entire two systems - changed A and undisturbed B - as in classic method [12]. We do not care what the remaining input signals are, they can be e.g. effects of feedback loops. A node is calculated only once [6]. Such an algorithm works fast and gives correct statistical effects.

We consider random additions and removals of nodes as changeability. Unbalance of them can lead to network growth. Only shown in fig.1. random Erdős-Rényi networks [5] does not grow. We denote it as *er*, in [11] the name RBN is used. Kauffman works using the *er* network [14, 13]. Patterns of

connection of a new node define the remaining network types, which can be scale-free [3, 2] (*sf*, *se*) and single-scale [1] (*ss*) where number k of outgoing links is flexible or ak and aggregate of automata [6, 8] (*aa*, *an*, *lx*) where k is constant. We assume constant number K of node inputs (unlike in [11]).

The condition for adaptive evolution is that after each change the system's fitness (similarity of the system's output vector and the ideal output vector) must not decrease. We define tendencies as difference in changeability distribution between free process and adaptive evolution. These differences are generated by elimination - effect of adaptive condition when fitness is significantly higher than random. We use minimum $s = 4$ for simulation of adaptive evolution, for such s and 64 outputs of system we use 48 identical output signals as fitness level. Generally for any s (e.g. 8, 16, 64) we use such fitness on a level of $3/4$ of its maximum value. It is important to keep this level stable because otherwise the process quickly reaches the maximum fitness value. There is no local extreme in such an optimisation. In order to achieve this we change the ideal output vector each time fitness crosses an assumed level.

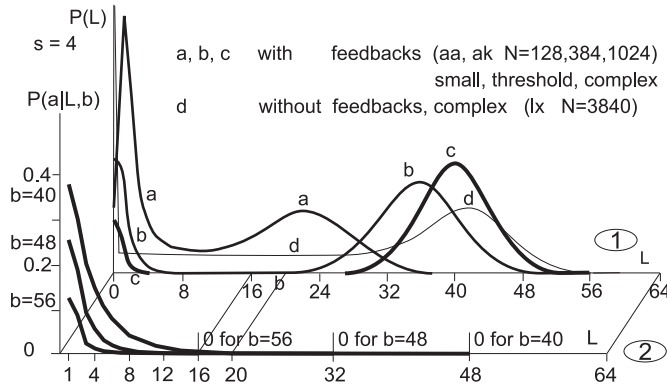


Fig. 2. Complexity threshold (1) and small change tendency (2) for 64 output signals and $s = 4$. Over the complexity threshold the main right peak is out of range of acceptable cases for higher fitness b , therefore all which can help for damage to fadeout in the first few steps, creates tendencies. For simulation of adaptive growth the fitness was fixed on $b = 48$ level by changes of ideal vector. 1- $P(L)$ distribution from simulations as the effect of random changes in the networks using *aa* networks as an example. Curves for *ak* are identical, for other networks are very similar, they differ with regard to the size of first peak. The base curve *c* - for complex network with feedbacks. It has two peaks and exact zero in-between. The same network at complexity threshold - curve *b*, and below it, when it was still small - curve *a*. For network *lx* like *aa* but strongly ordered and without feedbacks - curve *d*, space between peaks has a small but nonzero value. 2 - Theoretically calculated distribution of probability $P(a|L, b)$ of acceptance by adaptive condition, for three higher values of fitness b . For interesting cases of higher fitness the probability of acceptance is significantly different from zero only for very small change size ($L < 20$ for $b = 40$). To the right of indicated points the probability is equal exactly zero.

The main tendency, a simple but important one, is the tendency to prefer only small changes in adaptive evolution. Size of change L can be measured as number of changed output signals. This tendency is shown in fig.2.1 for three values of fitness b .

3 Growth and Complexity Threshold

Typically growth of a network is assumed as e.g. in the pattern of preferential attachment for Barabási-Albert scale-free network which concerns only addition of new node to the network like is used also in [11]. We concern random addition and removing of nodes and using equal probability of them before imposing the adaptive condition, we obtain total predominance of addition in set of changes accepted by adaptive condition. This is one of the structural tendencies. However, the mechanism of this growth does not always work. It needs some range of parameters of system to work. They are not yet introduced, we come back to growth tendency later. For a chaotic system the growth mechanism works better like other structural tendencies mechanisms.

Complexity threshold emerges during system growth. Complexity has as many descriptions and definitions [16, 9] as different aspects and meanings. If similar states of system create very different effects, we must know much more to predict these effects. We connect such a feature with the complexity of system but this is exactly chaos. A good phenomenon to observe this feature is damage spreading [12, 14]. For big chaotic systems damage typically explodes but for ordered or small systems it fades out. Using k and s we define coefficient of damage propagation: $w = k(s - 1)/s$ which means an average number of changed output signals of node if one input signal is changed. It is a good, intuitive indicator of damage explosion expectation, and ‘chaotic’ feature of system. However, only big systems can become chaotic (how big - that depends on the network type). We have investigated this dependency and we have found some properties of damage size distribution e.g. $P(L)$, which we define as complexity threshold.

During system growth the distribution $P(L)$ of damage size evolves: It starts as one peak, similar to distribution for an ordered system, when damage quickly fades out. Next, second peak appears. It shifts (during system growth) to the right and stops at a certain position of damage equilibrium. For networks with feedbacks, before the peak stops, between peaks there appears a large period of practically exactly zero frequency. This process is shown in fig.2.2 where curves a, b and c are for consecutive stages of network growth. We define complexity threshold as reaching a stable position (80% of maximum value of L which can be calculated [14]) by the right peak and appearance of zero in-between peaks. This phenomenon can be easily observed and measured in the reality and in the simulation. It is also strongly connected with different mechanisms of structural tendencies observed in adaptive evolution of complex and functioning systems.

4 Terminal Modification and Terminal Predominance of Addition Tendencies

To discuss structural tendencies (except growth) some parameter indicating place in the network is needed. We introduce depth D - in the simple case it may be the shorter distance to the outputs of system. It is a structural, sequential, approximated measure of functional order of signal flow through the network. ‘Terminal’ means: near output, and ‘deep’ - far from output.

The first structural tendency is ‘terminal modification tendency’. It means that most of changes constituting adaptive evolution take place near assessed outputs of an evolving system. This is a known regularity of ontogeny evolution introduced by Naef in 1917 [15]. It has a few explanations [4, 14, 18]. The mechanism observed in simulation in details is a quantitative confirmation of the Darwinian suggestion that changes in early stages of ontogeny usually yield monstrosities which are typically lethal. De Beer explanation, still used in biology, is extremely poor and naive. This tendency is observed and known on an intuitive level in human activity, but has not been formulated or investigated so far. It is easy to observe in computer programme development.

If coefficient $w > 1$ (system is chaotic, we assume it is big), then on average on each time step of damage spreading the damage initiated by a network change should grow. This change has much higher probability of acceptance by adaptive condition when it is very small (small change tendency, fig.2.1). If damage does not fade out in the first few steps, then it explodes and achieves equilibrium level - right peak in damage size distribution (fig.2.2). All cases forming this peak have no chance for acceptance (compare fig.2.1 and fig.2.2) therefore all which help damage to fade-out create structural tendencies. One way of such ‘help’ occurs at the output of system where a node does not send damaged signal to another node of system but ends its propagation. Therefore changes of network near the output of system have a higher probability of acceptance - this is terminal modification tendency.

Simulation effects of terminal modification tendency checked for different networks are shown in fig.3 in upper row. There are three simulation series summarized: aggregate of automata ($K = k = 2$, each outgoing link has its own signal which may be different than other output signal of this node) fig.3.1 with feedbacks (*aa* network), fig.3.3 without feedbacks (*an*) and fig.3.5 single-scale (*ss*) and modified scale-free networks (*se*). For *aa* and *an* networks the depth D has a more complex definition [6] than for *ss* and *se* where simply the shorter way to output is used. This tendency needs a higher s especially for the case without cost (described below) and networks *ss* and *se*.

Another structural tendency is ‘terminal predominance of addition over removal’ and complementary: ‘simplification of deeper part of system’. These tendencies correspond to Weismann’s ‘terminal addition’ regularity [19] and effect in old biological dispute on classic basic problem, nowadays forgotten

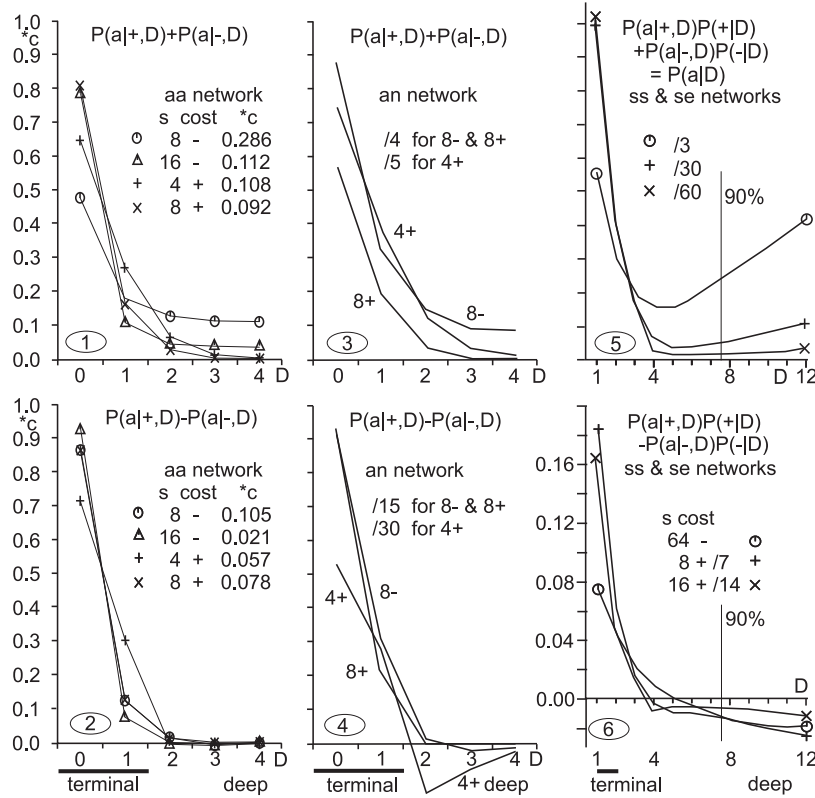


Fig. 3. The main result of simulations, (scaled by c for comparison). On the left (1,2) for *aa*, in the middle (3,4) for aggregate without feedbacks '*an*' and on the right (5,6) for Kauffman networks of variable k - scale-free *se* and single-scale *ss*. The results for *ss* and *se* are so similar, that one curve for both of them is used. Dependency on depth D as structural measure of functional order. Definition of D for 1-4 is more complex [6], for 5 and 6 it is the shortest way to outputs, here the boundaries 90% of nodes are shown. On the top (1,3,5): Terminal modifications and conservation of early area tendency. On the bottom (2,4,6): Balance of addition and removing: terminal predominance of additions (over removing) tendency and tendency of simplification (predominance of removing over addition) of early parts.

and especially not popular [10, 20] - recapitulation of phylogeny³ in the ontogeny. This tendency did not have any explanation and was therefore negated.

The sets of possibilities for random addition and removing of node is not the same. Removing can only be drawn from nodes currently present in the network, but addition has a much larger set of abilities. This difference creates difference of probability of acceptance in different areas of network, which differ on modification tempo because of terminal modification tendency. In

³ Simplifying - sequence of adult form during evolution.

a more constant area between two accepted changes it is possible to check all nodes that they cannot be removed, but in more often changed area new changes will change checked situation on time and create new possibilities to remove. Such phenomena have influence on balance of addition and removing in different areas and in the total system growth.

Fig.3 in lower row shows these tendencies observed in the same simulation series as the tendency of terminal modifications.

One of typical removing cases occurs when a ‘transparent’ node is added, one which does not change signals of remaining nodes. Some of interpretations of such a case suggest forbidding transparent addition. We introduce such forbidding using strict inequality in the adaptive condition for additions, and weak inequality for removals. This is equivalent to cost function for additions of new automata. Cost gives much stronger tendencies but also gives some limitation for network growth [6] which are moved away using more states for fitness for *aa* and *an* networks. For *ss* and *se* networks cost forces only 1% of removing because for more removing networks do not grow (fig.3.6 and fig.3.5). For these networks using more states for fitness also helps, but still does not allow equal probability for addition and removing.

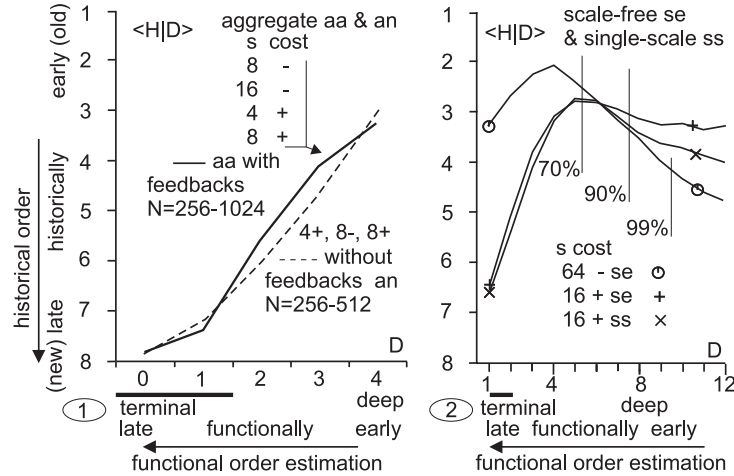


Fig. 4. Similarity of historical H and functional D order. H is a sequence of addition of given node to the network. The same set of simulations as in fig. 3. On the left (1) for aggregate with feedbacks (*aa*) - continuous lines, and without feedbacks (*an*) - dashed lines. On the right (2) for Kauffman networks with feedbacks (*se* and *ss*), containing $k = 0$ and $k = 1$. Here D is the shortest distance to outputs and can be large, but with small probabilities, therefore the boundaries 90% of nodes are shown.

Similarity of functional (measured using depth D) and historical sequence (sequence of addition of currently present nodes in the network) is an effect

of these two main tendencies (terminal modifications and terminal predominance of additions). This similarity observed in simulations is shown in fig.4. These two main tendencies are clearly observed in network with fixed number k of outgoing links for each node in network. If we allow k to be flexible, like in BA scale-free network, then in the adaptive evolution of network we observe much more different mechanisms creating some tendencies. One of them, competitive to terminal modification tendency, is the tendency to fade out change (damage) in deep areas, where nodes with $k < 2$ are concentrated. On a node with $k = 1$ damage cannot multiply but if $k = 0$ damage fades out obligatorily. In a deep area the coefficient of damage propagation can be therefore locally $w < 1$ like for ordered systems. Deep fade out effect is visible in fig.3.5 especially for the case without cost.

In these investigations there appear a few problems and additional assumptions are made to solve them. One of such problem was a big part of 'transparent' changes which do not change signals. They cloud other interesting effects and are interpretatively questionable - we introduce cost to remove such additions. Other problem is that nodes with $k = 0$ (without outgoing links) appear during removing of nodes for single-scale and scale-free networks. For scale-free network *sf* such nodes cannot come back into play and their number can grow creating a dummy system. We add drawing a node to the pattern of addition where only links were drawn and we use proportionality to $k + 1$ instead to k . Such network we named *se*. For application to description of processes like ontogeny an interpretation of the feedbacks is discussable. We have checked the main conclusion for *aa* network without feedbacks (*an* network) - terminal modifications and terminal predominance of additions as well as similarity of functional and historical orders do occur. In the simulation we observe many more interesting parameters and phenomena which will be described in more detail [8].

Summarise: adaptive condition as condition of network growth is a source of many important properties of complex systems.

5 Conclusion

Thanks to a specific algorithm these investigations open a new, large and still unexplored area of important phenomena (structural tendencies) waiting for mathematical description. In addition to structural tendencies we observe the emergence of complexity threshold upon which the mechanisms of these tendencies work. Structural tendencies are effects of adaptive condition as condition of network growth like preferential attachment for BA scale-free networks. Living and most of human designed networks grow under adaptive condition. Main application is, however, to show the mechanism of controversial recapitulation phylogeny in the ontogeny in developmental biology.

In this paper we only show the basic results (more in [8]) of a significant extension of an investigation published earlier [6, 7]. We check the conclusion

for a wide range of network types which increases the importance of this conclusion. An explanation of the complexity threshold is proposed. We also argue why more than two equally probable signal variants should be used.

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GraphStream: A Tool for bridging the gap between Complex Systems and Dynamic Graphs

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Summary. The notion of complex systems is common to many domains, from Biology to Economy, Computer Science, Physics, etc. Often, these systems are made of sets of entities moving in an evolving environment. One of their major characteristics is the emergence of some global properties stemmed from local interactions between the entities themselves and between the entities and the environment. The structure of these systems as sets of interacting entities leads researchers to model them as graphs. However, their understanding requires most often to consider the dynamics of their evolution. It is indeed not relevant to study some properties out of any temporal consideration. Thus, dynamic graphs seem to be a very suitable model for investigating the emergence and the conservation of some properties.

GraphStream is a Java-based library whose main purpose is to help researchers and developers in their daily tasks of dynamic problem modeling and of classical graph management tasks: creation, processing, display, etc. It may also be used, and is indeed already used, for teaching purpose. *GraphStream* relies on an event-based engine allowing several event sources. Events may be included in the core of the application, read from a file or received from an event handler.

1 From Complex Systems to Dynamic Graphs

Biology, Physics, Social Sciences, Economy, *etc.* yield all examples of systems where a large number of entities, homogeneous or heterogeneous, mutually interact. The emergence of global properties leads the community to qualify these Systems as Complex. This emergence comes from the dynamic of interactions. This dynamic is responsible not only for the building of the properties, which may be consider as a morphogenic process, but also for its maintaining, comparable to a morpho-static process, and as such it should be present in any relevant model. So, regarding Complex Systems, dynamic can be observed at two levels: at the level of the system formation and at the level of system evolution. Both processes may be distinct or may be the same, as Web graphs for instance. To sum up, we may transpose Dobzhansky's famous maxim: *Nothing in biology makes sense, except in the light of*

* The authors names are sorted alphabetically.

Evolution to Complex Systems: Nothing in Complex Systems makes sense, except in the light of Dynamic of Interactions.

Then, performing a relevant study of such Complex Systems requires a dynamic representation, and dynamic graphs impose themselves as the more suitable way of modeling them.

Figure 1 illustrates how the main processes leading to the formation and the maintaining of groups within a system composed of many entities in interaction may be observed using a dynamic graph model. In this representation, the vertices of the graph represent entities and the edges represent interactions. During the evolution of the system, both kinds of elements, vertices and edges, may be subject to variations and their characteristics as well.

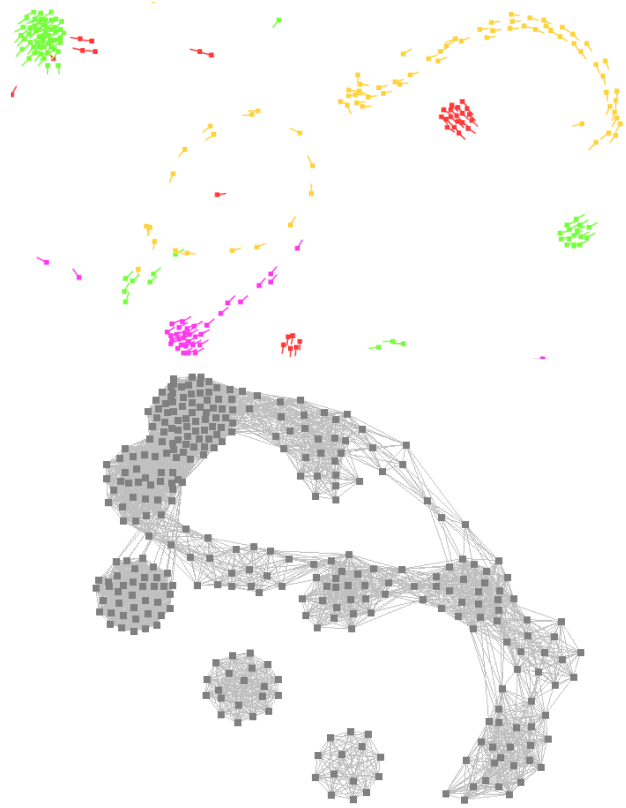


Fig. 1. This picture represents one zoom in part of a boids-based simulation (on the top) and its representation with a dynamic graph (on the bottom). On the left part, a snapshot of the simulation is proposed. The boids (interacting entities) can only perceive and interact with fellow creatures located in a limited area. During the process, some groups appear corresponding to global structures obtained from local interactions. On the right part of the figure, the graph represents at a given moment the interaction graph corresponding to the state of the system made of boids.

A Dynamic Graph $G(t) = (V(t), E(t))$ where t represents the time, is composed of:

- a set of vertices $V(t)$. This set may change with t . Each vertex v owns a set of characteristics $C_v(t)$ that may vary with time.
- a set of edges $E(t)$. $E(t)$ may change with t . Each edge $e_{ij} = v_i, v_j$ is defined as a pair of vertices, and owns a set of characteristics $C_{e_{ij}}(t)$ that may vary with time.

Therefore a dynamic graph $G(t)$ may change with the time t . According to the underlying modeled system, a dynamic graph may be considered as a family of graphs or as a complete graph where, according to t some vertices or edges can be zero weighted. There exist however another way of defining a dynamic graph, based on a discrete-time representation, and thus very well suited for most simulations.

A dynamic graph can be defined as a finite or infinite ordered set of couples (date, {events}). Every set of events may modify the graph structure/composition/topology and/or characteristics of some graph elements. The different snapshots of Figure 2 correspond to the series of sets of events listed below:

- (0, { v_1 creation, v_2 creation, (v_1, v_2) creation})
- (1, { v_3 creation, v_4 creation, (v_1, v_3) creation})
- (2, { v_2 suppression, (v_3, v_4) creation})
- (4, { v_2 creation, v_5 creation})
- (5, { v_6 creation, (v_5, v_6) creation, (v_4, v_6) creation, (v_2, v_4) creation, (v_1, v_3) suppression})

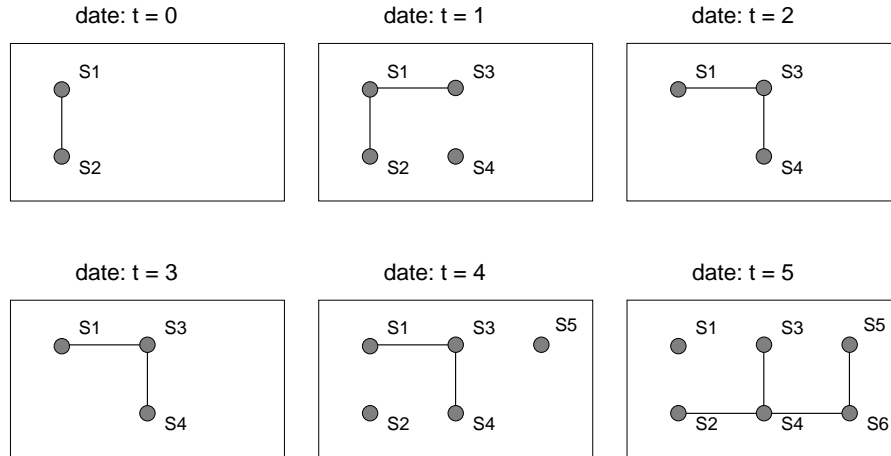


Fig. 2. Snapshots of a Dynamic Graph on the time interval $[0, 5]$. During a period of time the graph may not change as it is the case during interval $[2, 3]$. Some vertices may appear and disappear and appear again, as illustrated by vertex s_2 .

This approach was chosen for the implementation of *GraphStream* as explained in the forthcoming Section.

2 Graphs streams

GraphStream [3] is a Java library whose purpose is to create and manipulate dynamic graphs in order to study them and to use them in simulations. The whole library has been devised with the idea that graph will evolve during time. It is also naturally well suited for the manipulation of "static" graphs.

The central notion in *GraphStream* is that graphs are not static objects, but streams of events, and these streams can flow from producers (simulations, graph generators, graph readers, the web, etc.) passing by filters (a graph structure, a converter, etc.) to outputs (graph writers, a network, a display, etc.). There also exist a file format called DGS allowing the expression of graph changes as series of events that can be read by *GraphStream*.

Around this notion, *GraphStream* provides several tools.

2.1 Packages

Graph Representation Tools The main package provided by *GraphStream* allows the representation of graphs in memory. It is made of a graph interface and several implementation fulfilling different needs or usages. All share the property to handle nicely the dynamics of what they represent. Some of them are able to store arbitrary data in addition to the sole graph dynamic topology, whereas others are made to be small and fast. The library does not force the creation of a graph structure in memory. Several of its tools can handle a flow of events without ever constructing a whole representation of it in memory. For example it is possible to write a filter that will transform a very large graph written on disk into another graph on disk without necessitating a growing amount of memory.

Graph Generation Tools Most often, a simulation or observation tool will be used to produce graph events. However the library proposes a bunch of graph generators going from regular graphs like grids and tores, to scale-free graphs, random graphs, etc. It makes it easy to test algorithms on graphs having specific properties.

Graph I/O Tools The library can produce streams of events from several file formats read either from a file system or from the internet. This flow of event is not a graph and therefore, it offers the ability to process graphs that are larger than the available memory if no global representation is needed. Similarly, it is possible to feed a stream of graph events toward a file to save it in several formats. A specific format has been created to manage the complete set of graph events, not only the fact that nodes and edges exist, but also that they evolve.

Graph Theory Tools The library offers several well known algorithms on graphs and tries, where possible, to propose version of them that can handle the graph dynamics (this is the part of the library that still needs the more work, though, and can provide some interesting problems). For example, the connected components count algorithm not only is able to count the number of connected components of the graph, but is able to track it without having to recompute all the connected components count as the graph evolves. An important field of research named re-optimization aims at maintaining a previous result and recomputing parts of a solution when a changes occur in the environment. The

idea being that the re-optimization of a previously obtained result is more efficient than re-computing of the solution from scratch. An example of successful re-optimization algorithm can be found with the Dynamic Shortest Path Problem [2].

Graph Visualization Tools The library is equipped with display capabilities. It makes it easy to display any graph structure either on the local machine or even on a distant one. The viewer understands various style attributes (colors, dashes, arrows, labels on nodes and edges, icons). It is equipped with an optional automatic layout engine that can compute optimal node positions in order to make the graph as visible as possible (by avoiding edge crossing). This engine handles the graph dynamics and constantly reorganises as the graph evolves.

2.2 Example

Creating and visualizing a graph using *GraphStream* is more than easy. Indeed, the following piece of code creates a graph made of 3 vertices and 3 edges and open a window for visualizing it, the whole in only 8 lines of code. The result is shown Fig. 3). The positions of vertices are automatically computed in order to enhance the display. This helper can however be avoided by replacing the "true" boolean value by "false" in the parameters of `Graph()` constructor.

```
import org.miv.graphstream.graph.*;

public class Easy1
{
    public static void main( String args[] )
    {
        Graph graph = new Graph( false, true );
        graph.addEdge( "AB", "A", "B" );
        graph.addEdge( "BC", "B", "C" );
        graph.addEdge( "CA", "C", "A" );
        graph.display();
    }
}
```

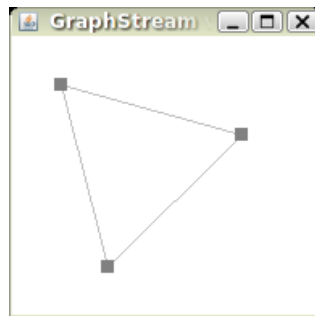


Fig. 3. The GraphStream viewer.

2.3 Suitability for Complex Systems Simulations

GraphStream is built upon an event-based engine and its whole conception is object-oriented, each element of the graph (nodes and edges) but also each attribute that qualify these elements are objects. These features allow an easy prototyping and simulation of systems made of sets of interacting entities. In particular:

- discrete-time simulations are easy to implement by associating an event to each time increment,
- the object approach allows a decentralized point of view that prevails over centralized approaches for Complex Systems modeling since interactions between entities are mainly characterized by their locality,
- finally, the library offers a lot of classical graph theoretic algorithms that may help for the analysis of interactions graphs.

In order to illustrate these claims, in the sequel an example of application in the fields of wireless telecommunication networks is developed.

3 Case study: Wireless Mobile Networks

3.1 Mobile Ad Hoc Networks

Nowadays almost all notebooks are equipped with wireless communication materials. These devices, that are also present in many smartphones and PDAs, are able to perform communications in two modes: *infrastructure* or *ad hoc*. In the former mode, the stations (computers, mobiles phones, PDAs, etc) communicate with each others via an access point. This access point manages the communications in the Wireless LAN and acts as a gateway for communications outside of the local network. This mode is usually used at home or in public places. In *ad hoc* mode, no access point is necessary for performing communications, stations "simply" broadcast their messages in their neighborhood. Such networks are called MANETs (Mobile Ad hoc NETWORKs) and when the assumptions allow the network to be partitioned, we call them DT-MANETs, meaning *Delay Tolerant* or *Disruption Tolerant* MANETs. Although it is not commonly used, *ad hoc* mode presents some advantages. In particular neither infrastructure nor network administration are required, and this mode is especially well-suited for mobility [5]. However, classical network problems like broadcasting or routing have to be revisited because stations may move in their environment, they may also be turned on or off at any moment such that the topology of the network changes while the operation is performed. The complexity of these systems comes from the mobility of stations. Indeed, as stations move according to the behavior of humans, situations occurring in real life (traffic jams, groups of people at one moment at one place, etc) also occur within such networks. The suitability of *GraphStream* for that task is obvious since the communication network is dynamic and can be modeled with a *dynamic graph* and from a station's point of view the network as a whole does not exist, each station only knows the stations that are within their neighborhood, that is the network as a whole is only a representation, it is actually decentralized.

3.2 Implementation choices

In the context of this example we are interested in testing algorithms that build spanning forests within DT-MANETs. Indeed, spanning forests may be used as a basic structure for different operations like routing or broadcasting messages between stations. There exist at least two ways for performing such a task. The first option consists in relying on an existing DT-MANET simulator and to gather for each time slot events like: station s_i turns on, stations s_j and s_k are in mutual communication range, etc. The second option consists in rebuilding a simulator from scratch using *GraphStream*. Because we already have at our disposal such a simulator (MADHOC [4]), we choose the first solution.

The whole process for testing algorithms first recover events from a run of MADHOC and keep them in a file. The IO part of *GraphStream* is used to read this file and generate the events accordingly by associating to each station a node and to each communication link an edge. *GraphStream* is also used to simulate the Spanning Forest Algorithm. So as to benefit the Object oriented approach of the library, the algorithm was modeled consequently. Before entering in more details, let us described the method for building and maintaining a spanning forest in a dynamic graph.

3.3 Building and Maintaining a Spanning Forest

The Spanning Forest Algorithm [1] is a mechanism based on a set of rules that are applied on each station of the network in a fully decentralized way. The application of these rules, locally by each station, results in the marking of a set of edges of the graph that form a spanning forest. In the best case, the process leads to one single tree for each connected component of the graph. Since the process is repeated (the rules) all the time, it adapts to the dynamics of the network. The root of each tree corresponds to a node owning a *token*. Each token moves along the constructed tree. After a local synchronization process with its neighbors, each station executes 4 rules:

- Rule 1: An edge belonging to the tree has been removed. At this moment, the tree is split in two. This rule applies to the node belongs to the part of the tree where there is no token (no root). So the node creates a token on itself (a new root).
- Rule 2: An edge is removed on the tree and the node is part of the tree where the token is.
- Rule 3: Two nodes with tokens meet. One of the two tokens disappears and the two trees merge.
- Rule 4: This last rule makes the token move along the tree.

For the purpose of the simulation with *GraphStream*, an agent-based model as been used. The 2 first rules of the above model can be implemented as a reaction to edge events. Rule 4 is pro-active and defines the movement of the token. Rule 3 is executed as a consequence to rule 4. The concept of token is modeled as an agent that traverses the network. Fig. 4 shows a graphical representation of such a network with the Spanning Forest Algorithm running on it.



Fig. 4. A graphical view of a MANET. Black thick links are colored by the Spanning Forest Algorithm and belong to the forest. Bigger node are the ones owning a *token*, they are the roots or the spanning trees.

3.4 Measurements

The real gain in using *GraphStream* with such an algorithm is that it allows both the decentralized point of view necessary for the algorithm and the centralized measurements that give an interesting analysis of its behavior.

An example of the measurements made thanks to the library can be observed on Figure 5. This chart represents the evolution of some measures made on the trees constructed with the defined algorithm. This evolution is observed here according to the size (number of nodes) of the trees.

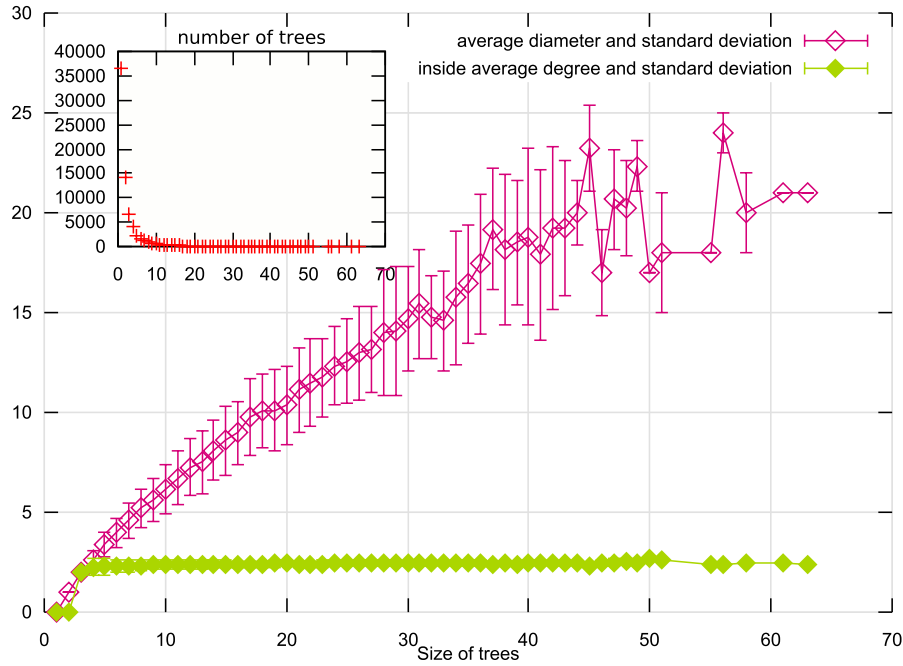


Fig. 5. Evolution of the average diameter and the average degree of inner nodes of trees. The small top left chart represents the number of trees w.r.t. their size.

4 Conclusion

A new dynamic graph representation and manipulation library was presented in this paper. The originality of the approach rests in the way dynamics is handled in graphs. The idea of a stream of events for the dynamics and the general object oriented outlook of the library give it a special ability to be used as a distributed and dynamic environment simulator. *GraphStream* also advertises more classical tools for analyzing static and dynamics graphs.

The usefulness of the library was shown with the modeling and the simulation of a decentralized algorithm. In this simulation *GraphStream* was used as a simulator. It was also use to perform an analyze of the behavior of the algorithm with graph theory measurements.

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PART 2

Geographical Systems Complex Modelling

THEORY OF REACTION-DIFFUSION AND EMERGENCE OF THE GEOGRAPHICAL FORMS

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Geography studies the organisation of territories in their physical dimensions as well as in their social dimensions. This is thus a knowledge about emergence of forms such as the scattering of fallows and then forests over abandoned agricultural soils, the growth of a city or of a transport network, or the spreading of a pioneer front in Brazil or Siberia.

In order to account for the emergence of spreading or stationary forms, a lot of scientists from other disciplines often refer to a theory simultaneously suggested by Fisher, Kolmogorov, Petrovskii and Piskunov, and usually designated by the acronym FKPP. Numerous papers and books dedicated to this theory and to the models derived from, such as Turing activator-inhibitor model or their generalisation by Meinhardt. As the creation of a chemical, its production and growth in time, is usually obtained by reaction, this theory is generally called « theory of reaction-diffusion ». We have highlighted the interest of this theory since 1985, at the European Colloquium on Theoretical and Quantitative Geography. But it was not a source of inspiration for the geographers.

We suggest then to insist on the components that should be added to make this theory effective in geography. Beyond the long distance interactions designated as convection, advection, turbulence in nature sciences or transport in the societal sphere, it is advisable to think about the initial conditions that strongly influence the emergence of the geographical forms and more about the introduction of human actors adaptability.

1. A theory very quickly enriched

1.1. A centenary theory

The theory of reaction-diffusion resumes Heraclite's idea for who every creation of a form is the product of the struggle between two principles. In its simplest form, this theory is illustrated by an equation which is written as following in usual language:

$$\textit{Variation of } a = \textit{Growth of } a - \textit{Diffusion of } a$$

In a given point of space, the law of reaction, production or growth integrates the creation and the disappearance of new « individuals », molecules in chemistry, cells in biology, individuals in ecology, or persons in social sciences. Frequently, this growth process is auto-catalytic: the fabrication of a product « x » depends on the presence of « x » and usually on its density. The autocatalyse is actually synonym of positive retroaction. For example, the demographic growth is function of the number of reproductive couples and, then, of the population size.

In this theory, the role of diffusion becomes anti-intuitive. As a rule, diffusion restores equilibria in a gradient field. It homogenizes this field and tends to erase forms. Coupled to a growth mechanism, diffusion has therefore a reverse effect and generates new forms. These forms are created and remained because diffusion is constantly feeded by the growth mechanism.

Following the considered discipline, this equation or system of equations, when at least one of both components is considered, has different names. In chemistry, it stands for a process of reaction-diffusion illustrated by the experiment of Belousov-Zabotinski (BZ). In ecology, this theory is named by different terms because ecologists use different growth models. They mention a KISS model (Kierstead, Slobodkin et Skellam) when growth is represented by an exponential law. The corresponding model is often solicited to explain the emergence and size of marine phytoplankton. In biology, this theory originates the activator-inhibitor model first figured out by Alan Turing in 1952. Actually, this theoretical model is well adapted to every statistical population composed by individuals that reproduce and diffuse in space.

1.2. Three types of generalisation

This theory, illustrated by an equation which integrates both mechanisms of growth and diffusion, can be generally applied. To make it applicable to the study of more realistic issues, three ways have been explored: modifying terms of growth or diffusion, taking into account several populations in interaction and finally, studying a mechanism additional to the initial equation.

A first form of generalisation consists in modifying the growth or diffusion mechanism. Indeed, there is not one but several growth laws. Thus, the observed growths in human societies are rarely exponential. The logistical Verhulst law, more realistic than the exponential growth, simulates a growth slowed down by density. This slowing down applies to most of the living populations which have a restricted food available. In ecology, the model of reaction-diffusion, built from this logistical growth, was qualified as the Fisher model because of the name of the mathematician who introduced it since 1934. This model will be then applied to represent the agriculture emergence in Europe in the Neolithic.

This first generalisation sometimes applies on the second mechanism, the diffusion. In its simple form, diffusion depends on the difference of the concentration of the considered product in space. Diffusion transfers the product from high densities to low densities. It is a slow process because it acts only gradually by contiguity. And the diffusion coefficient is often assimilated to a constant. But, it is possible to make the diffusion coefficient vary, for example in function of the density. Indeed, high densities fasten the diffusion process as testify the studies carried out on migratory fluxes: migrants rather leave very peopled spaces.

A second type of generalisation is obtained by adding new terms to the elementary equation in order to take other factors into account. Thus, a city grows up and spreads by the growth of its population and its diffusion in suburbs. But, in the same time, the centre, as well as the peripheral spaces, receives a demographic excess from the rural environment, other cities or further countries. A migratory phenomenon is superimposed on auto-growth and diffusion and is easily assimilated to an advection. To insert this new mechanism, it is sufficient to insert this advection mechanism in the initial equation, which then can be written as following:

$$\text{Variation of } a = \text{Growth of } a - \text{Diffusion of } a + \text{Advection of } a$$

This advection is symbolised, as diffusion, by a partial derivative affected of an advection coefficient which obviously does not have the same value as the diffusion coefficient. Thus, both elementary mechanisms are conserved but they are amplified or thwarted by these larger moves.

A third generalisation, already announced for the BZ model, consists on coupling two or several equations. Indeed, in the reality, a species never lives alone in its environment or its ecological niche. In physics and chemistry, molecules and products mix together. And, in geography or in social sciences, these interactions between different human groups are often more frequent. In a city, several ethnic groups or social classes cooperate or confront each other. In numerous disciplines, these interactions concern two populations. The ecologist analyses relations between preys and their predators from Volterra studies. Biologists who are specialists of animal forms development use the same equations. But they talk about activator-inhibitor model (Hans Meinardth, 1982). For two populations, the Turing theory is then expressed by a model which includes both of the following equations:

$$\begin{cases} \text{Variation of } a = \text{Growth of } a - \text{Diffusion of } a + \text{Interaction of } a \text{ and } b \\ \text{Variation of } b = \text{Growth of } b - \text{Diffusion of } b + \text{Interaction of } b \text{ and } a \end{cases}$$

Each equation includes both mechanisms of the basis model, simulating growth and diffusion and also an interaction mechanism. In this system of two equations, it is easy, in function of the positive or negative value attributed to interactions, to represent an effect of competition or, inversely, an effect of mutualism, of cooperation between the activator and the inhibitor, between both species. Fighting is not always the rule even if it was considered as essential by Malthus, Darwin and Marx.

This form of generalisation can be applied beyond two populations. Epidemiologists also use these equations by distinguishing sane and infected populations and populations which recover or disappear. They build then SIR (Susceptible, Infected, Release) models which simulate the behaviour of three populations. Recently, Michaël Batty (1999) adopted this model to represent dynamics of urban growths.

This theory of reaction-diffusion generates an infinity of forms. The combination of the simple laws is sufficient to simulate the emergence of linear forms, networks, gradient forms, repetitive forms, spiral forms of cellular textures, hexagonal for example. This set of two laws succeeds in creating homogeneous or heterogeneous forms, isotropic or anisotropic, punctual or areolar. Actually, as highlighted by James Murray (1990): « The theory of reaction-diffusion predicts a very rich diversity of complex forms, from the simplest as invasion fronts and solitary waves, to the most complex as spirals and spatio-temporal chaos ». The first works were about fronts dynamics. More recent studies insist on the persistence and stable condition of some forms. Thus, strips and hexagons are more stable than other forms. This attests of their higher frequency in Nature. Biologists are the ones who best studied the laws which govern forms generated by these mechanisms. Usually, forms stabilise when the diffusion coefficient of the inhibitor is seven times higher than the activator's one. In function of the size of the considered space and conditions of production, diffusion and interaction, several similar forms can appear at more or less regular space intervals.

2. Two geographical constraints: initial conditions and role of the actors

To apply the theory of reaction-diffusion in geography, a double enrichment is imperative. It concerns the initial conditions and the role of the social actors who have an amazing capacity of adaptation.

2.1. Initial conditions often determining

The initial conditions considered by physicians are usually simple. These mechanisms act on homogeneous spaces, isotropic or random. But, in geography, the terrestrial space is always structured and heterogeneous. Agents do not interact from random or homogeneous distributions. In a river, a pollutant will spread in function of the pressure gradient (diffusion) and the associated wind (advection) but also in function of the underlying topography. In a city, people diffusion takes road networks. All preliminary structures of the terrestrial space, physic or human, channel diffusion and advection mechanisms and they also intervene on growth.

It is then imperative to take into account this ground reality by introducing this terrestrial pre-structure in the initial conditions and then by qualifying the associated effects. Works of that type, developed by physicians, can guide geographers such as, for example, the book of Ben-Avraham and Havlin (2000) dedicated to the study of the mechanisms of reaction and diffusion in heterogeneous and fractal environments. They show how pre-existing structures have a direct influence on the emergence of new forms. This is a source of inspiration for geographers.

2.2. The agents of human societies are adaptive

A second specificity of geographic forms lies in the originality of the human action. Humans are not simple molecules. They pursue goals and their actions tend to reach economical, social or cultural objectives. It is then necessary to include these rules of behaviours in the theory of reaction-diffusion, rules which will depend on the scientific issue considered. These rules of behaviours are very numerous but can be reduced to some generic laws.

Humans act by comparing themselves to others but also by learning, by taking the past into account and anticipating the future. Thus, in a research about the emergence of segregation forms inside a city, Thomas Schelling showed that diffusive moves of each person are made after comparison of his cultural values with his neighbours' ones. Beyond a threshold of dissimilarity which can be modified in simulations, people leave their home and move to people sharing the same cultural values. In the course of time, people who have similar values gather and homogeneous quarters emerge from a random initial distribution. Then, the setting up of a city with its segregation phenomena needs no chief, no villain as suggested by some theories.

But, this initial model deserves to be deepened. Originally, both populations have a fixed and constant size. It is thus necessary to introduce the real growth of these populations to obtain a simple model of reaction-diffusion. Then, it is desirable to include, not one but several types of ethnic or social classes. Geographers make then a double segregation emerge which show homogeneous quarters but also gatherings always similar to these homogeneous quarters.

Segregation thus appears at two different spatial scales : green with green but also quarters which became green still stay in contact with blue quarters. This pattern corresponds to the urban organisation of great American cities.

Besides, in contact with a population which shows different value, a person can evolve, learn and adopt these values. This learning is frequent when persons in contact have different social status. It is assumed that classes said to be « underprivileged » tend to adopt quite easily values or behaviours of classes qualified as « well-off ». More globally, numerous socio-cultural criteria evolve under the neighbourhood pressure whereas others, such as religious beliefs, are very stable.

Contrarily to gaseous molecules, humans can even determine neighbours' choice with who they compare each other and interact and they can even take into account the past interactions whether negative or positive. This is namely the case of the models built on the prisoner's dilemma. Rules of decision and interaction are then implemented in the models of reaction-diffusion. Numerous simulation models illustrating the results of these behaviours are already available (Richard Gaylord and Louis d'Andria, 1998). They generally include a moving process by diffusion under the form of a brownian move.

Eventually, numerous human moves are made on long distances and are not assimilable to diffusion any more. Couples leave their city, their region or their country. An advection mechanism should be added to the diffusive moves and the brownian move should be replaced by a Levy flight.

Obviously, if this generalised cultural process of reaction-diffusion generates socio-spatial segregation, the accurate location of the quarters is often arbitrary and changes in function of the initial conditions in the simulation models. This is not the case in real cities where land-prices often determine these locations. In French Mediterranean cities, Italian quarters of the Fifties and actual North African quarters are rather located in wet glens devoid of a large view over the sea.

CONCLUSION

In order to understand the emergence of forms in geography, the theory of « Reaction-Diffusion » is an excellent starting point. However, the geographer must choose the most relevant growth (reaction) model. Moreover, he must add an advective component because only diffusive moves of human order are exceptional. Humans as albatross have daily moves which obey more generally to the formalism of a Levy flight, integrating short moves and further moves. Besides, the weight of the initial conditions is essential. Diffusion said to be normal is exceptional. Whatever the fluxes of matter, energy, people, money or information which generate territorial forms, they diffuse over a non-homogeneous space which imposes its structure. Real forms are not « free » but their emergence is channelled by these initial conditions and at the limits. In addition, human actions and interactions obey to a principle of adaptation including memory, learning and then evolutionary behaviours. This diversity does really not condemn an approach by the model of reaction-diffusion. It adds a source of complexity in the research in social sciences.

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Spatial risks and complex systems : methodological perspectives

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Abstract: The reflections regarding risk and industrial catastrophes question the complexity theories. Besides the new concepts which lead us to question older concepts (risk, hazard and vulnerability), we propose to think about more practical aspects, for example the modelling of human behaviour and the confrontation in crisis situations. The link between concepts as: critical self-organization, emergence, bifurcation, and the methods in the Distributed Artificial Intelligence used to model them is however difficult. In this paper, we present ongoing analysis on the key concepts of risk science, such as the hazards and the catastrophes. We propose to enrich them with complex systems theories. First, we present methodological perspectives of the DAI, for example multi-agent systems, and compare them with other simulation methods used in the context of the risks. Secondly, we present the MOSAIIC model (Modelling and Simulation of Industrial Accidents by Individual-Based methods) which gives possibilities to simulate the behaviour of individuals during an industrial accident. The project and the MOSAIIC model aim to explore the effects of a major industrial accident on public health. For instance, the emission and the spread of a toxic gas in an urban environment may be a serious danger for the human health. Thus we propose to study the consequences of this type of event in order to reduce the vulnerability of the populations. In the model, we emphasize both on spatial and behavioral dimensions (ie.

mobility and perception of risk) All these questions lead us to use different methodologies of analysis. For example, concerning mobility, the daily traffic can be simulated at a meso scale: a road axis for example. In that way, we aim to simulate the global dynamics of the network from the modelling of flows on arcs of the network (modulated according to the time of day and the day of week). Yet, we use classical models (for instance equilibrium models) because they give an "average image" of the flows of vehicles on the arcs. Based on this first structural mobility, it is then possible to consider "a change of level" regarding both the representation and the analysis: if a risk occurs or if a specific context disrupts the structure. As a consequence, from a management of flows on the arc, we turn to an analysis of the individual behaviours in a multi-agent system.

1 Introduction

We considered risks and hazards from the point of view of complexity, and especially through the theory of self-organization and critical behaviour. Our case of study is the diffusion of a toxic cloud caused by the explosion of an industrial plant and its spread in an urban area. The aim of this research is to study, through modelling, the consequences of this kind of events on the population and especially on their mobility behaviours, in order to reduce the vulnerability of networks. If some models, such as fluid dynamics, seem to be efficient to handle the structural traffic in a network, they seem to be less useful if we aim to model a disrupted situation. Agent-based models offer a solution. They allow, during the time of the event, to switch from a management of flows at the level of arc, i.e. the road of a network, to a formalization of behaviours at the level of the individual. This methodology is more appropriated to deal with different situations, and especially for the analysis of panic for which nonlinear dynamics are important. Nonlinear means that very small variation on the model parameters (for example, the number of population susceptible to panic) have significant effects on the evolution of the system (from non-panic to collective panic), which is qualified as a bifurcation. In this paper, we will first go back over the concepts of the science of risks, such as hazard and disasters, that we will reformulate and enrich through the theory of complex systems. Second, we will present briefly the MOSAIIC project designed to simulate the behaviour of individuals facing an industrial accident.

2 Risks and complexities

Hazard and vulnerability are two concepts mobilized to define the risk. After delimiting them briefly, we will question them in relation to the theories of complexity, especially that of self-organized criticality [1].

2.1 The risk, a compound of hazard and vulnerability

Natural, societal or technological risks (R) are essentially tackled through two key concepts: hazard (A) and vulnerability (V). The risk is a measure crossing hazard and vulnerability according to a function $R = f(A, V)$. In this acceptance, and as A. Dauphiné emphasizes [2] it, a disaster is a reality whereas a risk is a probability. Hazard is generally defined both as a probability to occur and as an intensity. Regarding environmental risks, hazard is a measure which results from the probabilities observed on a long time scale. On the contrary, regarding technological risks, the probability to occur is less relevant because of the infrequency of these risks and of the theoretical absence of another occurrence in the future, except if we consider a natural cycle of Human mistake. In order to put off this limit, one tries to find the conditions liable to trigger a harmful event for people and equipments. An event tree analysis can be implemented to identify these causes and effect sequences as well as to determine theoretical probabilities. The intensity of the phenomena is the second dimension of hazard. It depends on the duration of this phenomena and of the considered area. This intensity is often employed to define the areas of protection surrounding industrial sites for example. The probability to occur and the intensity are thus the two key elements of studies which deal with hazard, but both of them become significant if the stakes, also called here the targets, have an interest recognised by a society at a particular time. This leads us to the second concept linked with risks: the vulnerability. Vulnerability generally expresses both the measure of a damage to equipments and people and the ability of a society to resist a disaster. The determination of the vulnerability at a global level thus depends on the stakes exposed to a hazard and on factors of vulnerability (sensitivity of population, resistance of houses and premises, but also the quality of risk prevention, the management of disasters with supervision and alarm systems). So, the identification of the stakes potentially important for the system in question is needed: people for an epidemic, buildings and population for an earthquake, biological factors for an oil slick...However, the existence of domino effects add difficulties to analyse the stakes. For instance, an earthquake destroying buildings may also cause victims among their inhabitants and may provoke an industrial accident whose consequences may be the flow of toxic products in a nearby river. These multi-risk scenarios are feared, especially in countries like Japan. In order to take into account this complexity, we can either use synthetic indicators based on qualitative studies or model and simulate the dynamics of such systems [3].

2.2 Complexity, bifurcation and resilience

In order to integrate the theories of complexity in the study of risks, we have to face a major challenge: that of time of processes and their interactions.



- Hazard, as the probability for a source to change its state in a qualitative and eventually quantitative way, at a time t . This change of state depends on the problem: for instance, in the case of an epidemic and at the individual level, it refers to the transition from a susceptible to an infected state;
- Intensity, as a measure of the quantity of energy released by a source (output), from its change of state and towards the outside. If the reference period of the study of intensity is relatively short, then we can comprehend it, not as a simple result, but as a process which describes the behaviour of energy in time and space. For example, the study of the diffusion of a toxic cloud after its release in the atmosphere;
- Vulnerability, as the probability for a stake to be disrupted by the energy released by a source, and thus to change its state (qualitatively or quantitatively). This probability is the result of a process which describes how the stake behaves to protect its entrances from the exits of the source, and the quantity of energy which gets into the stake. For example, the level of individual's sensitivity to panic behaviours which are displayed around him, and thus the probability for him to panic;

- Resilience, as a measure which describes the ability of the stake to adapt to change after a disturbance. For example, how individuals recover from a situation of individual, and eventually collective panic?

This last concept, richer than that of resistance already mentioned, was recently introduced in the literature. Some researchers have suggested, in order to reduce the damages of disasters and thus to minimize the vulnerability of exposed elements, to adopt a management strategy of risks based on this concept of resilience [4, 5, 6]. It can be defined as the ability of a system to return to a single steady or to a cyclic state after a perturbation. This concept can thus be useful to analyse the dynamic part of the system, especially its capacity to find new trajectories and rebound after a disturbance. Such a definition of risk gives the complexity of the objects considered: they may be successively sources and targets, and damages may have most of the time indirect origins with the first phenomena. In addition, according to the type of event, a same structure of interactions between a source and a stake may be a positive or negative part of vulnerability. The traffic network is a good example. If it is known to be a driving force for an epidemic hazard, it is however considered as useful for a fire hazard. Therefore, an analysis of network vulnerability in the field of risk studies is fundamental. In the case of panic phenomena, we find this same kind of complexity. In a situation of panic, the most vulnerable individuals to the intensity of the phenomenon (for example the diffusion of a toxic cloud) can quickly change their behaviours: from a state of non-panicking population to that of population in panic. If for the French school on the one hand, the terrified individuals are submitted to their gregarious instincts and irrational behaviours, for the American school on the other hand, the individuals keep forms of lucidity, abilities to analyse the situation and to take decisions: copying the neighbours, escaping... [7]. How irrational individuals may be or not, the non-linear interactions are very important in the diffusion of a panic. A few individuals may spread a panic among a whole group. The crowd as a whole can then become a source for other human and technological stakes. The self-organization theory is well adapted to give an account of the emergence of such phenomena for which little local disruptions may product global and unpredicted events. The self-organization theory identifies the principles which allow us to describe how a system creates his own behaviour at a global level, persisting in time and space, from the numerous interactions among entities displaying at one or several lower levels. These interactions are generally local ones, develop in the vicinity of each other, and such systems are characterized by the absence of planning: no global control which would pilot such structure, such behaviour, or such form. These kinds of systems are called "emergent systems" because their developments are not fully explained by the properties of entities at inferior levels. As most of natural or technical systems, self-organized systems are not systems whose equilibrium is permanent. Impermanence is the only reality of the living world. The activity of a system, dynamic and open to

the outside like all the living systems, is in evolution. Self-organized systems arrange their behaviour in relation to certain points in their environments. Order in an interconnected system of element arises in the vicinity of attractors, which create and maintain patterns within the system. Evolution between attractors can be cyclic like prey-predator systems. Such systems are characterized by phases of intense activities: the curves of population linked to the two groups reversing more or less regularly along the time. Otherwise, the system can evolve towards a stationary state, converged on a point of attraction and absorbing progressively its activity. The activity of the system can lead it through different states through the time. This switch from a state to another is situated close to a point of bifurcation that may lead towards chaos. In an earlier work, we explored the different phases of activity of a system from the example of the logistic function often linked with diffusion processes [8, 9]. When the system evolves from a bifurcation threshold, the transition from one state to another qualitatively similar refers us to the concept of resilience. The stability of self-organized systems goes hand in hand with a possibility of change which explains that all living systems go through different phases during their activities. These phases are theorised by the criticality [1 op. cit.], which shows that all self-organized systems evolve towards a critical state and that a small and local disruption is sufficient to make the system change. This phase is characterized by a system which goes into a phase of mutual and global interaction during which the level of connections and interdependences of the elements of the system is maximal. If they are useful in a heuristic context, such concepts are however difficult to use when one wants to apply them or to spot them in an empirical way. How knowing if a particular system comes from a decentralised context or not? How qualifying, identifying the emergent phenomena in such systems? How evaluating the intensity of relationships between elements at the same level and between the elements at different levels? How measuring resilience in a system? From measures based on particular methodologies? Systemic measures? Indicators? These uncertainties lead us to propose simple models of the complexity. This way of modelling is based on a constructivist approach for which the principle of parsimony is the crucial point for scientists who wish to tame the "artificial creatures" they build.

3 Simulating panic phenomena: methodological orientations

The aim of the MOSAIIC project is to study individual behaviours and their consequences during an industrial accident, and mainly through an analysis of the traffic network vulnerability. The hypothesis of this project is that any traffic system, made of numerous mobile entities in mutual and environmental interactions, tends to evolve towards a critical state. A small fluctuation may thus disturb the system toward a phase which considerably increases the

vulnerability of the persons if the origin of the disturbance, even indirect, is a technological or natural event. The experimentation in this field is of course either hardly conceivable or difficult to realise. Thus, the modelling methods and computer simulation offer an interesting option.

3.1 An environment of risk where hazard and vulnerability are statistically weak

According to Dauphine's classification of risks [2 op. cit.], our case study belongs to the category of local and short time events. The analysis of technological disasters is especially relevant today in a context of urban growth leading to a proximity between residential areas and industrial plants, and after recent industrial events (AZF in Toulouse, Mède in Marseille...). The gap between the dynamics of an observed disaster and the structure of the risk generally estimated in cities is partly explained by environmental factors. In urban areas, the variety and the number of sources and targets may grow in the course of the event. This fact is partly the result of the proximity of elements and of the growing interactions between them. However, the main characteristics of these environments for the individuals are both the quality of buildings to confine and the traffic network in order to escape and to be rescued. As a consequence, based on the conception of risk developed in 2.2, we view global risk as a measure of all the local and contextual risks that can be observed in a situation, and for which the sequence of interactions is determining for the magnitude of the risk and of the disaster. Thus, the source is an object partly submitted to hazard and probabilities, whose outputs are a quantity of energy released (virus, toxic cloud, individual aggressiveness, physical pressure...). This energy moves in space depending on the nature of the released energy. In order to counter the flows and to prevent the target stakes from being reached, preventive measures can be taken to limit the vulnerability upstream: alarms, buffer zones, educational measures etc. The aim is to reduce the inputs to the target objects (stakes). However, if the quantity of energy coming in the stakes is important, then these last may have a high probability to become targets. These last will then determine the outputs towards new stakes (see Figure 1). In this conception of global risk as a dynamic process in which the sequences of phenomena may be numerous, the MOSAIC project proposes to focus on individual behaviours and on vulnerabilities linked to them. Thus in this project stakes and sources are humans.

3.2 The vulnerability of traffic networks during disasters

Studies dealing with vulnerability of systems need to take into account the constraints that both time and space present. The spatial level has to be defined. If a disaster may destroy a system, this last is - most of the time - a subsystem belonging to a bigger system which may not be disturbed by this

disaster. Once the spatial level of the vulnerable system defined and analysed, elements and interactions of the system have to be recognized. In this paper, we will especially focus on the vulnerability of traffic networks in order to estimate its consequences on the exposed populations.

Self-organised and stable systems

The traffic networks may be directly or indirectly the origin of a disaster. For instance, the transport of dangerous materials in inner cities is a factor which tends to increase hazards. In this case, a mobile source (a truck) is not submitted to the same types of control and is not classified in the same category of risks as a static source (a factory). In addition, the nature and the quantity of stakes may vary during the shift of the object. An other aspect concerns the stakes and the vulnerability, and the need to qualify the role and to quantify the impact of traffic networks on a disaster. The urban environment is strongly restricted by its traffic network and may create use conflicts between the 'active' or 'passive' actors of the crisis. As one of the options to avoid epidemics is to isolate individuals [10], a sound management of technological disaster would propose to evacuate them out of a perimeter and then to confine them beyond a security line. However this strategy is rarely used and, in fact, is difficult to implement. Different individual strategies and behaviours coexist during an accident. We can summarize these strategies by considering two forces: a centrifugal one (moving away) and a centripetal one (moving closer) [11]. These two forces, constrained by the reticular environment which limits the possible paths may produce three types of movement or flows within the network: a flow in the opposite direction of the source, a flow towards the source, and a parallel one (the source is a front) or perpendicular one (the source is a point) to the source. According to the connectivity of the network¹ (see Figure 2) and the area where the event is located, the different types of mobility will be more or less possible in a given perimeter. How characterizing the "normal" regime of a network and how detecting the change of this regime toward an exceptional activity? This new regime could be considered as a precursory sign of dysfunctions in the system, and in our case of a possible disaster. The average regime observed in a traffic network can be written with a function A . It is an indicator of the level of the functioning of the system for a time t .

$$A_i = tmp_i - tp_i \quad (1)$$

where tmp_i is the average traffic on the network at a time i and tp_i is the instantaneous traffic on the same network. Within this framework, tmp correspond to a traffic modulation on different routes which are always the same in the network and at different moments in a day. The global tmp , as shown in

¹ we use the connectivity index β [12] which is based on non oriented graphs, and can be calculated by carrying over the number of nodes (s) to the number of links (l): $\beta = l/s$

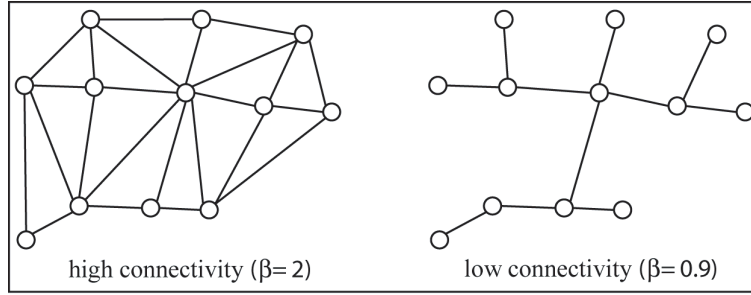


Fig. 2. Two graph models of a road network

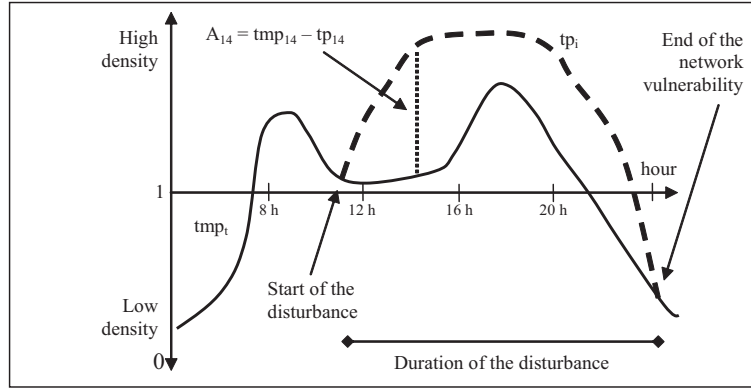


Fig. 3. Comparison between a diagram of disturbance and a series of daily average values

the Figure 3, is an average value at different hours of the day (continuous line on the graphic). For example, the tmp indicates the average value of density for a week day (for instance at 8am, in the graph from the southernmost point to the northernmost one) (see Figure 3). We can select a standard day to analyse the variability of situations, and thus to observe the consequences on the dynamic of a technological accident. Doing this way, we observe three very different types of daily modulations: the working days (JO), Saturday and the days before official holidays (SVF), Sunday and days before official holidays (DJF)². This typology emphasizes the intervals between different days at the same hour. In addition, it allows us to compare the average hourly values between them in order to give a qualitative level of the functioning of the network: a level of fluidity, for example (horizontal continuous line on the graphic). At the same level of analysis, the tpi is the traffic observed on

² This parameter (C) is equal to the hourly traffic car average on an axes (VMH) divided by the daily traffic car average (VMJ), divided by 24. Thus, $C = VMH/(VMJ/24)$, this parameter has an average equal to 1.

the network, measured 'in live' during the simulation. So if the tmp is an estimation calculated from a counting during a period of time, then the tpi is the traffic in the network, or in a path, observed at a given moment. For example, the $tp8$ value indicates that at 8am on the 2nd of August, n vehicles are in the graph from the southernmost point to the northernmost one. As a consequence, if the At value is positive, the traffic on the section is, on average, higher at this hour and during this type of day, than the traffic recorded at the moment i . On the contrary, if Ai is negative, the traffic, observed at the moment i on the network, is higher than the average traffic. The advantage of such a formalization is to show, at which moment the system enters in a disrupted phase, the duration of this phase, its intensity and at which moment the system recovers its stable regime. Such a diagram presents our conception of the traffic network: a complex, stable and partly self-organised system whose dynamic is independent from the entities which constitute it. Although the entities (cars, pedestrians), may change, the laws which organise it remain. Yet, there is an autonomy of the traffic in relation to its constituent elements. The existence of dynamic independent patterns at a meso level of the network will lead us to model this dynamic at a meso level: for example with differential equations. But how switching from this modelling of the dynamic at a meso level to a modelling at a level where micro-changes have impacts on the global functioning?

Phases of criticalities

Why a system may move from a attraction point to another one and thus globally move away from the average observed traffic? Our hypothesis is that changes of aims and motivation within a group of individuals favour this bifurcation. These individual changes will be all the more quickly transmitted to the whole people (a total correlation between the elements of the system [1 op. cit.]) since they will be constrained by a network and a territory whose capacities of adaptation on a so short period is nearly non-existent. Only few entities have to change their behaviour to produce feedback effects on a part or on the whole system. Yet, its properties may be changed partly, temporally or on a long term basis. This criticality of the system is all the more high since we are situated, in the space of parameters, in a zone of instabilities which are characterised for example by a level close to congestion. The possibility for the system to bifurcate is all the more high since the degree of freedom of the entities in the system is weak, and since interdependence becomes global. In our case, that means that hazard and vulnerability factors are all the more important since the number of individuals circulating in the network and in the urban area is high. The diagram in Figure 4 presents these ideas. It combines the density (number of vehicles on an x axis) and the flow (number of vehicles along an y period of time). This diagram is in general estimated and observed on small intervals of space and time, in order not to combine different states of traffic. So the density is low if the individuals adopt the

speeds of their choices only limited by statutory constraints. On the contrary, if both the density or the rate of occupation of the way rises, the traffic is more and more constrained and, beyond a certain limit - a critical density (K_c) - it reaches the congestion. We plan to model the micro level dynamics close to this threshold: for example, the variable of density will be converted into a list of agents on the relevant axes. In a next step, we will define the properties, methodologies, aims or strategies of these agents. The state of traffic thus

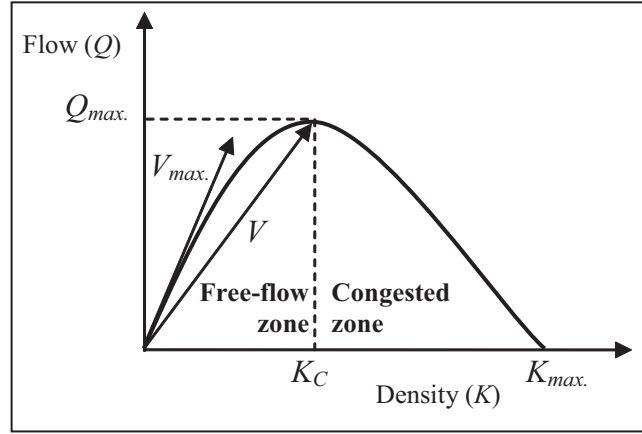


Fig. 4. Parabolic diagram: estimating congestion with flow and density

goes through different points, a possible trajectory leading to K_{max} : points of attraction which indicate a full saturation of the road, or even of the network. The network resilience is thus the ability of the network to move to a point of attraction below the critical threshold K_c : a point that we plan to discover thanks to the analysis of the behaviour of the agents.

4 Conclusion

The analysis of the network vulnerability, measured by its trend to maintain a phase of global disturbance, proposes to stress one of the elements of complexity of disasters: the interactions between individuals and territory as well as between individuals and individuals. Despite this paper as well as the MO-SAIIIC project both focus on a particular category of risk, these reflections may lead us to suggest a more general model dealing with human behaviours in situations of crises. Thus, this model may be applied to other categories of risks that imply population shifts in strongly constrained spaces. Such a model offers an opportunity to test a great variety of behaviour scenarios as well as to analyse the incidence of the network structure on behaviours. Besides these different tests, we plan to study to which extent individual behaviours are

likely to lead towards a critical point, even during situations when events occur far away from a critical phase. At last, this project plans to provide a cartography of the different strategies possible in contexts of crises to decision makers and people dealing with the problem of crisis management: evacuation strategies by getting closer to potentially disaster zones or strategies of intervention by mapping routes that allow to reach the targets.

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A proposition for a classification of the catastrophe systems based on complexity criteria

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Abstract. A classification system of catastrophic events is a methodology assembling all the catastrophes groups. It is possible to identify several catastrophes classifications. The most widely known are classified into: nature, consequences of the event, duration, affected territories and areas of the destroyed zone, and at last into the needed intervention measures. But these criteria of classification allow with difficulty to apprehend the complexity of the catastrophe. Thus we propose a classification based on the complex characteristics of the risks and catastrophes. Within the scope of this paper, we focus first on the complexity of the organization and the emergence of the phenomenon which result from it, and then, on the complexities resulting to the spatial and temporal scales of the catastrophe. The organization is considered as a central concept of the complexity. In the field of the catastrophe, the complexity of organization results essentially from the self-organization of the systems (the system develops its internal constitution and its behaviour thanks to the interactions between its various components and not thanks to an external strength). Phenomena as different as mantels of snow, seismic hazards, behaviours of people and population have characteristics of self-organization allowing the emergence of new events: snowslides, earthquakes, collective panic. A particular attention will be given to the emergence of this kind of panic in situation of disaster. There is indeed a double-way within two levels, a double action of the crowd on the individual and the individual on the crowd, without leader. It means that we need to take into account the multi-scales aspect in order to be able to study the behaviours. The complex systems of catastrophe have characteristics able to emerge at higher or lower levels of scales. It allows us to apprehend the complexity of the disasters through the scales. The disasters belong to the multifarious temporal- and space scales. First, the disasters can not be classified in one single category of spatial scale. Some of them appear on the scale of a territory, a region, a country or the planet. If we speak about a natural or technological disaster, none of them

will be automatically associated with a spatial scale. Furthermore, a local disaster can have large-scale impacts. Various events (attacks of September 11th, 2001 in New-York, the tsunami which ravaged the South of Asia in December 2004, the hurricane Katrina who destroys New-Orleans in 2005) remind us that the catastrophe is not always an event restricted at the affected area but can have consequences outside this area. The increase of the complexity of the disaster can result from the movement between different spatial levels and from systemic relations between these levels. The complexity also results from various temporal scales of the risks and the disasters. Three temporal phases can be found. The first one is relative to the temporality of the potential risk I mean what takes place before the disaster. The second phase refers to the temporality of the disaster I mean all what happens during the catastrophe. We show that during the disaster, the temporalities of the hazard, the vulnerability and the domino effects rarely happen together. The third and last phase refers to the time after the disaster and to the experience feedback for the risk management. These three temporal phases are based on two scales of time: a short time, I mean a time - action, inherent to the functioning of any dynamic system (Ch.-P. Péguy, 2001) and a long time. Thus, the catastrophe must be approached in various scales. And the study in each of the scales gives several information of the disaster in its whole, or about some of its components (hazard, vulnerability and domino effects).

1 Introduction

In the expression, "systems of catastrophe", the word "system" represents a set of elements in interaction, "an entity not reducible in its parts. (...) It [a system] implies the appearance of emergent qualities which didn't possess the parts" [1]. The word "catastrophe", in the sense of social and spatial disorganization of the territorial system affected by a disturbing event, implies that these interactions concern the various components of the catastrophe, namely the hazard, the vulnerabilities, and the domino effects I mean a chain of events, activated by hazard or vulnerability. Thus the catastrophe is at the interface of the nature-society relations, at the interface between the space of danger and the vulnerable space. In this article, we set up basis for a classification of the catastrophes established on complexity criteria. We focus first on the complexity of the organization and the emergence of the phenomenon which result from it, and then, on the complexities resulting to the spatial and temporal scales of the catastrophe. But first, we set up a panorama concerning the systems of existing classifications.

2 Classifications of the catastrophes based on a sector-related approach

A classification system of catastrophes is a methodology assembling all the disaster groups. It is possible to identify several catastrophe classifications based on a sector-related approach. The most widely known are classified into nature, it means the hazard being at the origin of the event. Therefore the catastrophes are either: -natural, technological or technical and can be as well the result of a social behaviour. Each of the groups will be again changed into subgroups. The natural catastrophes can be the result of an action of the earth, for example in case of earthquakes, volcanic eruptions, landslides, avalanches. It can be the result of an action of the water, for example in case of floods, tsunamis or drought, an action of the air and wind in case of storms and hurricanes, an action of the fire during fires caused by the lightning or the volcanic eruptions [2]. The technological risks have always anthropic origins. They include industrial, nuclear and technical risks or accidents of transport. These events can be again decomposed into subcategories. For example, the accidents of transport can happen: in the air, on the sea, on the road or by rail. Concerning the sociological catastrophes we can find 2 groups: the accidental ones and the ones which have been intentionally caused [3]. In the first case, they appear during crowd events, the crowd creating a disaster of appear with an external event: for instance the collapse of a building, of tier of seats. In the second case, the disasters are related to warfare or terrorism attacks. A second kind of classification is set up regarding the consequences of the event. These consequences are estimated mostly in term of losses, more rarely in terms of gains. They can be either material or human. The material consequences concern the deteriorations and the destructions of various infrastructures (houses, public establishment, industries, roads etc.). The categories of catastrophe also vary according to the number of victims. According to the amount of victims, the disaster will be considered as moderate, average or major. These data are mostly mixed with the nature of the disaster, in order to obtain the amount of injured and dead people for every type of catastrophe. The Emergency Disaster Data Base gives this type of information. The third classification can be realized with the needed intervention measures: local, regional, national or even international management measures to apprehend the disaster. Finally, scales of classification exist in the field of seismic or nuclear disasters. The Richter magnitude scale assigns a single number (from - 2 to + 9) to quantify the amount of seismic energy released by an earthquake and the earthquake effects. On the same way, the International Nuclear Event Scale gives 8 levels of severity: from the major accident to the most simple anomaly.

These classifications have an operational aim. They must allow the risk manager to anticipate the situations, to manage them during the catastrophic event, to increase the means of intervention and to ask for more support (international assistance for example during the tsunami in Asia in December, 2004), to identify the priority zones of intervention, and to realize rescue plans.

If these classifications have an operational aim, however, they present some limits. First, there are no common methodologies of disaster classification on a national or international scale. Furthermore, these classifications, based on a sector-related approach, are multifarious and rarely required in their variety to categorize a catastrophic event. It's thus difficult to realize comparisons between catastrophes of diverse previous history, if not in terms of human and material losses. Furthermore these classifications exist for a single type of risk. There is no multi-risks classification. Now, the urban societies, which are the most concerned by the disaster, are multi-risks societies. Finally, these criteria of classification allow us to apprehend the complexity of the catastrophe with difficulty. Thus we propose another way of reading of the catastrophes by setting up basis for a classification established on the complex characteristics of the catastrophes.

The complexity of a system of catastrophes is based on at least 4 criteria: those inherent to the organization of the system, those coming from the spatial and temporal scales, those resulting from geometrical forms of the risk and the disaster, and finally those resulting from the non-linearity and the unpredictable dynamics of the systems. These various types of complexity don't exclude each other but can be observe together during a disaster [4]. Within the scope of this paper, we focus first on the complexity of the organization and the emergence of the phenomenon which result from it, and then, on the complexities resulting to the spatial and temporal scales of the catastrophe.

3 Organization and emergence of phenomenon

The organization is a key concept of the complexity. We can differentiate two types of organizations: the structural organization (that is an organization in sub-systems or in modules) and the organization in levels. In the first case, the complex system of the disaster is constituted from sub-systems in interaction. These sub-systems are from now on well identified: it is about hazard, about vulnerabilities (human, building, network) and domino effects [5]. We will not mention it in details here. In the second case, the organization in levels interacting together creates new phenomena: some activate, strengthen or weaken the catastrophe. In the field of the catastrophes, the complexity of organization results essentially from the self-organization of the systems (*i.e.* the system develops its internal constitution and its behaviour thanks to the interactions between its various components and not thanks to an external strength [6]). Phenomena as different as mantels of snow, seismic hazards, population behaviours have characteristics of self-organization allowing the emergence of new events: snowslides, earthquakes, collective panic [6 op.cit.] [7, 8, 9]. The emergence is a polysemous notion. We shall use here the term of emergence in the sense of S. Kauffman [10]: the emergence implies that collective phenomena can't be explained by the properties of their constituents. In this paper, we propose a simulation of the emergence of the collective panic

from individual panic. The study of panic enables to put human vulnerability in the foreground of our analysis of disasters.

3.1 A model of panic

For the American school of thought, the mechanisms of panic propagation are based on behaviour of imitation, contagion or suggestion [9 op. cit.][11]. The crowd is then the support of phenomenon of contagion [12]. In the same way, the collective panic is characterized by an absence of coordination and dialogue between the individuals. The collective panic would thus appear from the diffusion of individual panic, without the attendance, the domination of a leader who would call to the panic.

In this article, we have chosen the system dynamics modelling and the Stella Research software to simulate the behaviour of panic during a catastrophe [13]. The mathematical formalism of this modelling is based on differential equations; the graphical formalism on stocks, flows, converters and connectors. System dynamics is a methodology used to understand how systems change over time. A dynamic system is therefore a system in which the variables interact to simulate changes over time. And, it is in the interaction that we have the source, the origin of the emergent phenomenon.

The proposed model represents the dynamics of transmission of the individual panic to the collective panic in a crowd. This model is based on the epidemiological models of W. Kermack and A. McKendrick. It is based on three simple hypotheses.

Hypothesis 1: we imagine a situation with a crowd constituted by three types of populations: the Population Susceptible to Panic (Psp), the Panicking Population (Pp), and the "Non-Panicking Population" (Npp). In a crowd, we can observe interactions between these three populations.

Hypothesis 2: Panic is a phenomenon of contagion, of collective imitation [14]. We use transmission rate to apprehend the contagion of the panic between both human populations in contact. Indeed interactions between human populations do not necessarily lead to contagion. This transmission rate is a coefficient which varies from 0 to 1, *i.e.* a low to a high contamination.

Hypothesis 3: After a certain period of time, people will stop panicking and resume normal behaviour. In the model, there is an outflow which "empties" the stock of the panicking persons. This outflow is proportional to the numbers of individuals in panic and the return time to normal behaviour (Rtn).

These hypotheses simplify sometimes the real situations. For example, we ignore any subdivisions of the population by age, social structure [15], or others factors, although such distinctions are obviously of importance.

Figure 1 shows a Stella version of a situation of panic behaviour. This model includes three stocks of population: the population susceptible to panic (Psp), the panicking population (Pp) and the "non-panicking population" (Npp), *i.e.* people will stop panicking and resume normal behaviour; inter-

actions between these three types of population, transmission rate of panic (Tr), return time to a normal behaviour (Rtn).

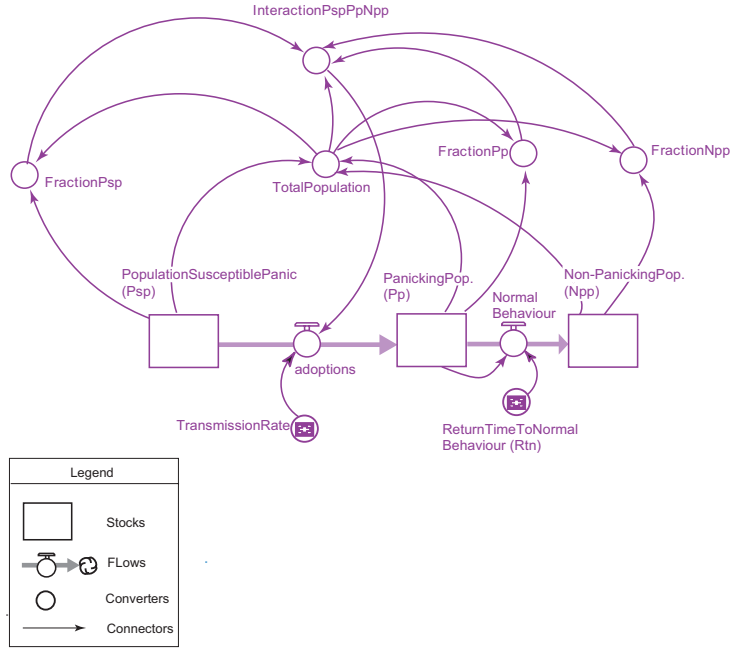


Fig. 1. A model of panic

The corresponding equations are:

$$Psp(t) = Psp(t - dt) - (adoptions) * dt \quad (1)$$

$$Adoptions = InteractionPspPpNpp * TransmissionRate \quad (2)$$

$$Pp(t) = Pp(t - dt) + (adoptions - Normalbehaviour) * dt \quad (3)$$

$$Normalbehaviour = Pp/Rtn \quad (4)$$

$$Npp(t) = Npp(t - dt) + (NormalBehaviour) * dt \quad (5)$$

The focus of this paper is a formal modelling, but not the prediction results of the panic behaviour.

3.2 Simulation results

We present results of simulation for different values of initial conditions of panicking population and of parameters related to the model. We will focus more particularly our attention on the spread and the emergence of panic. The population susceptible to panic is always equal to 500 individuals. Three cases are presented. The cases 1 and 2 present simulation results for initial different conditions of panicking population. In these two cases, the transmission rate of the panic is equal to 1 and the return time to normal behaviour to 24 units of time. The case 3 shows the evolution of the model when the transmission rate and the return time to normal behaviour vary. The representation of variables on a phase plan (Figures 2 to 6) allows us to show the various trajectories of the system.

Case 1 :

- $Psp = 500$
- $Pp = 13, Pp = 50, Pp = 100, Pp = 200, Pp = 300, Pp = 500, Pp = 600$
- $Npp = 0$ (at the beginning of the simulation, this stock is equal to 0 because the panicking persons have not found yet their normal behaviour)
- Transmission rate (Tr) = 1
- Return time to a normal behaviour (Rtn) = 24 units of time

Figure 2 shows that for the values of the panicking population ≥ 12 , and for $Psp > Pp$ or $Psp < Pp$, all the trajectories of evolution aim to the equilibrium point where Psp and Pp are zero. The panicking population and the population susceptible to panic tend to disappear. For different values of Pp , we do not observe qualitative modification of the model. Trajectories have the same shape. They spread on various points on the Y axis (as soon as the curve reaches the axis Y (Pp), then the values of Psp are zero for various values of panicking population), before converging all to the same equilibrium point. All these trajectories show that for a high transmission rate of panic ($Tr = 1$) and a return time to a normal behaviour equals to 24 units of time, there is emergence of the panic (upward slope), before reaching a equilibrium point. The emergence of the panic is particularly visible for low initial values of panicking population (13, 50 and 100). Beyond these values, the slope of the curve is smoother. This equilibrium point with coordinates (0,0) can be explained by the flow "normal behaviour" which tends to empty the stock "panicking population" and to feed that entitled "non-panicking population". Figure 3 shows the phase plan for "panicking population" and "non-panicking population". For every trajectory, in the similar profile, we can identify two breaking points: a first point of break indicates the transition from the decrease to the emergence of the panicking population; and a second breaking point where the trajectory of the system tends to the increase of the "non-panicking population". As soon as the curve reaches the axis Y (Pnp), then

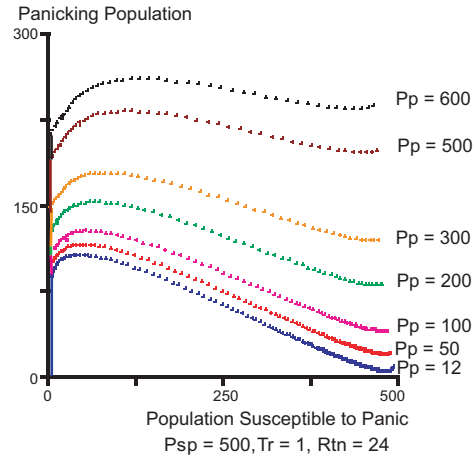


Fig. 2. Phase plan of population susceptible to panic and panicking population

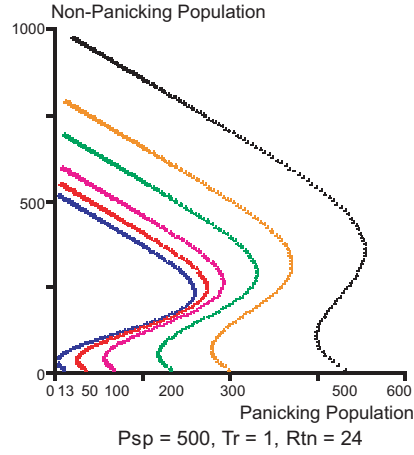


Fig. 3. Phase plan of panicking population and non-panicking population

Pp is zero. All the equilibrium points spread to the axis Y .

Case 2 :

- $Psp = 500$
- $Pp = 11$
- $Npp = 0$
- $Tr = 1$
- $Rtn = 24$ units of time

On the other hand, for values of $Pp < 12$, we observe a qualitative modification of trajectories (Fig. 4). The phase plan is completely different from the

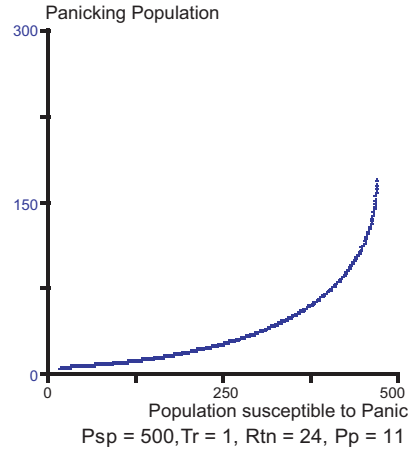


Fig. 4. Phase plan of population susceptible to panic and panicking population - An effect of threshold

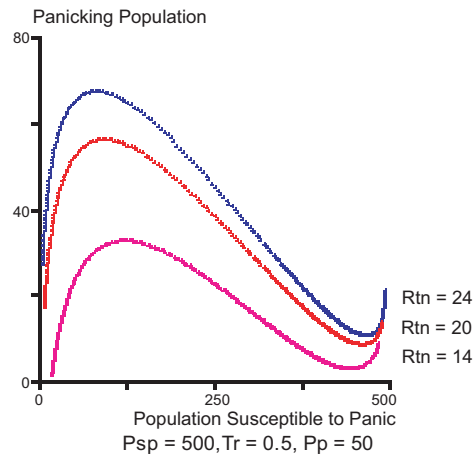


Fig. 5. Phase plan of the variables Psp and Pp for $Rtn > 13$

previous case. The trajectory converges to an equilibrium point (coordinated 0,0), but contrary to the previous case, there is no emergence phase of panic. The transition from 12 to 13 panicking persons with 500 persons susceptible to panic modifies qualitatively the dynamics of the system. There is a bifurcation, an effect of threshold, with $Pp = 12$, the "value threshold" beyond which there is effectively emergence of the panic.

Case 3 :

Finally, we study the system evolution by making vary the transmission rate of the panic ($Tr = 0.5$) and the return time to normal behaviour. Six tests

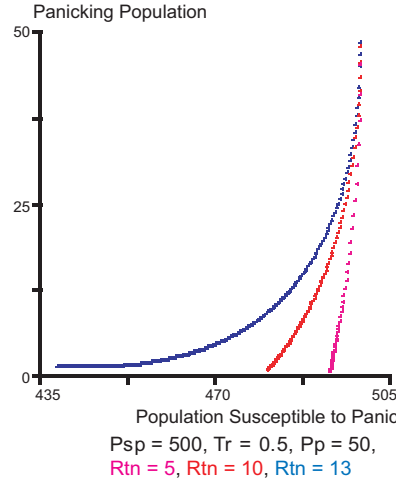


Fig. 6. Phase plan of the variables P_{sp} and P_p for $R_{tn} \leq 13$

are realized with values of R_{tn} equal to 14, 20 or 24 (Fig. 5) or equal to 5, 10 or 13 (Fig. 6). The population susceptible to panic is equal to 500, the panicking population is equal to 50. For a "return time to normal behaviour" (R_{tn}) equal to 14, 20 or 24 units of time, the phase plan is identical to the case 1 (Fig. 2). There is a first phase corresponding to the decrease of the population susceptible to panic and the panicking population, a bifurcation then the emergence of the panic before reaching a bifurcation bringing the system to a new state of equilibrium where P_{sp} and P_p are zero. On the other hand, trajectories are different for return time to normal behaviour, equivalent to 5, 10 or 13 units of time. The modification of R_{tn} influences the proportion of persons susceptible to panic and panicking people. The panic can't spread over.

The emergence of the panic does not appear in every situation. This emergence depends on the transmission rate, the return time to the normal behaviour, but also on the number of panicking population at the beginning of the simulation. The complex systems of catastrophe have characteristics able to emerge at upper levels of scales. It allows us to apprehend the complexity of the disasters through the scales.

4 The spatial and temporal scales of the disasters

The scale notion has a main place in the study of the catastrophes. Indeed, the disasters belong to the multifarious temporal- and space scales. First, the disasters can not be classified in one single category of spatial scale. Some of them appear on the scale of a territory, a region, a country or the planet. It allows

us to distinct the disasters which are considered as localized from the more diffuse ones. But, if we speak about a natural, a technological, a social or a sanitary disaster, none of them will be automatically associated with a spatial scale. For example, terrorism, present on a world scale, can affect a confined territory, and have something of an impact on vast territories. Furthermore, we can note that metropolis play a role of "spatial switch" which enables the disturbance to spread outside the initial impact zone and on multiple scales (agglomeration, region, country, planet) [16]. Thus, a local disaster can have multiple scales impacts. We can proceed from the same assumption for more diffuse risks. Various events (the tsunami which ravaged the South of Asia in December 2004, the hurricane Katrina which destroys New-Orleans in 2005, the climatic risk) remind us that the disaster is not always an event restricted at the affected area but can have consequences outside this area. Regarding this various spatial scales of disaster, we have to consider the interaction of the scales of intervention (not only the local and regional one, but also sometimes the international one). The increase of the complexity of the disaster can result from the articulation between different spatial levels and from systemic relations between these levels. The question of the interaction of various spatial scales is certainly a theoretical and methodological challenge for the researchers in science of the risk. Indeed, studies of risk are mostly limited to a single spatial scale. The multi-agent formalism or cellular automata give certainly possibilities to study the various scales of a catastrophe.

The complexity results from various temporal scales of the cycle of the disasters. Three temporal phases are identified [17]. The first one is relative to the temporality of the risk as well at the level of the physical or social processes as at the level of the regional planning and prevention policies I mean what takes place before the release of the hazard. Generally these events work on a long temporality. The second phase refers to the temporality of the disaster I mean all what happens during the catastrophe. Mostly the temporality of the catastrophe is of short duration. But we showed that during the disaster, the temporalities of the hazard, the vulnerability, the domino effects and the rescue operations rarely happen together. Models of simulation showed the existence of temporal gaps between these three constituents [18]. The third and last phase refers to the time after the disaster and to the resilience of the system. It is possible to classify the systems affected by a disaster according to their resilience that is to say according to the time needed to return to the initial situation after a disturbance, for example after an environmental disorder [19]. Resilience gives us the possibility to study the catastrophe not any more from the point of view of the event but rather from the angle of the crisis. This concept of resilience requires us to apprehend the system of the disaster in its whole. This return time to equilibrium depends on the extent of the disaster and the damage, on the adaptability of the society and on the type of properties [20]. These three temporal phases are based on two scales of time: a short time, I mean a time - action, inherent to the functioning of any dynamic system [21] and a long time. The short

time square with the rapidly developing disasters (some exceptions do exist: famine, drought are slowly developing disasters). It is also the time of the alert and of the management of the event. On the other hand the long time is the time of the prevention, the anticipation of the consequences resulting from these phenomena, the experience feedback. It can also be the time of the processes. For example, during an earthquake, the energy accumulates slowly in the fault networks (long time), before being brutally released (short time). Long phases of stability are juxtaposed to intense and brief fluctuations [22]. Furthermore, there are temporal relations not only between the various phases of the catastrophe cycle but also between the entities which constitute every phases of the cycle. Some events depend on the occurrence of other events (for example, vulnerability of the population, buildings etc. depend on the occurrence of the hazard, prevention measures are being often established after the disaster etc.), on some states of the system (the alarm system mobilize the evacuation forces).

It is thus necessary to study the cycle of the disaster in its various phases: the long time of the processes, the effects of threshold at the origin of the release of the hazard, the time of the catastrophic event and that of "after disaster". And the study in each of the scales gives several information of the disaster in its whole, or about some of its components (hazard, vulnerability and domino effects). Let us take an example: the floods of the Seine and the risk of inundation of Paris. The hazard can be studied on the scale of drainage basin (study of the precipitations on the whole basin and determination of the stream flow of the river, *i.e.* hazard), the catastrophe on the scale of the city (flooded surface and vulnerability of the population = vulnerability) and of the country (impact of the flood on the economy = domino effects) and the resilience on the city, national and world scale. Each of these scales is interesting. They offer different information on the catastrophe. Thus it's necessary in a modelling approach of phenomena to clarify the levels of observation and modelling [23].

5 Conclusion

In this article, two types of complexities were more particularly studied: the first one concerned the concept of organization and the emergence of phenomena; the second one is relative to the multi-scales character of the catastrophe. This proposal to classify the disasters according to criteria of complexity is certainly imperfect and must be further refined. However this classification leads to the conclusion that you have to apprehend the disaster on a different way, that is to say to exceed the disciplinary approaches and to establish comparisons between the various categories of disaster. The natural, technological or social disasters have similarities which can be identifies with the help of the sciences of the complexity. It leads us to compare disasters of different origins, different impacts (in terms of human and financial losses), different

duration, different required measures and different territories. It would not be in conflict with the already existing classifications, but so we can apprehend the catastrophes in another way, taking into account all the complex aspects. It enables us answering following question: is one system of catastrophe more complex than the other? It gives the possibility of more differentiated answers for the spatial and temporal management of the catastrophe.

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Community Swarm Optimization

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Summary. The development of distributed computations and complex systems modelling [11] leads to the creation of innovative algorithms based on interacting virtual entities, specifically for optimisation purposes within complex phenomena. Particule Swarm Optimisation (PSO) and Ant Colony Optimisation (ACO) are two of these algorithms. We propose in this paper a method called Community Swarm Optimisation (CSO). This method is based on more sophisticated entities which are defined by behavioral automata. This algorithm leads to the emergence of the solution by the co-evolution of their behavioral and spatial characteristics. This method is suitable for urban management, in order to improve the understanding of the individual behaviors over the emergent urban organizations.

Key words: swarm optimisation, community detection, self-organization, automata, evolutive methods, geographic systems, resource management.

1 Introduction

Artificial complex systems allow to implement distributed problems solving. Such systems are composed of interacting entities from where emergent properties appear. We focus, in this paper, on the artificial swarm or population methods allowing these emergent property computations. The artificial swarms or populations move on a representation of the environment or on a representation of the space of solutions. The emergent properties of these artificial systems are the collective meaning of the system itself, according to some objective functions. In many cases, we can express the control of the system by these objective functions as an optimization problem; the optimal configuration could be expected in advance or could be computed during the

evolution of the system in order to some adaptive properties.

In the following, we will present some of these modern methods concerning artificial swarm intelligence and we will propose a new one, called Community Swarm Optimization. This method is mainly based on the concept of spatial evolutive populations of behavioral entities. The concept of community is the basic property of this method.

Definition 1. (Community operational definition)

A community is a system or an organization which is characterized by a spatial property, a behavior property and the interaction between both.

Example 1. In ecology, a community is a group of plants or animals living in a specific region and interacting with one another.

Example 2. The spatial patterns generated by Schelling's segregation models [13] are some examples of communities and these spatial patterns are linked with some elementary behavioral rules implemented for each grid case. These rules describe, for each step, the movement of each individual according to its neighborhood.

In SCO method, we need to represent an efficient way to describe the behavior of each entity and we use algebraic structures called automata with multiplicities [14]. The main advantage of these automata is to be associated with algebraic operators leading to automatic computation. With these operators, we can define behavioral distances for the entities modelled with these automata. The behavioral distance is one of the major keys of this new method. Section 3 describes the algebraic basis for the automata management used in this method. In section 4, we describe the proposed method and in section 5, we discuss some applications which can be efficiently modelled by this method, according to their own complexity.

2 Swarm Optimisation Methods

Decentralized algorithms have been implemented for many years for various purposes. In this algorithm category, multi-agent systems can be considered as generic methods [17]. Agent-based programming deals with two main categories of agent concepts: cognitive agents and reactive agents. The first category concerns sophisticated entities able to integrate, for example, knowledge basis or communications systems. Generally, efficient computations, based on these cognitive architectures, implement few agents. The second category of agents, based on reactive architecture, is expected to be used inside numerous entity-based systems. The goals of programs using such architectures, is to deal with emergent organizations using specific algorithms called emergent computing algorithms. Swarm Intelligence is the terminology used to point

out such reactive agent-based methods where each entity is built with the same basis of behavior, but reacts in autonomous way. Swarm Optimization methods concern the problems of optimization where the computation of a function extremum is based on the concept of swarm intelligence.

Ant Colony Optimization (ACO) methods [3] is a bio-inspired method family where the basic entities are virtual ants which cooperate to find the solution of graph-based problems, like network routing problems, for example. Using indirect communications, based on pheromon deposits over the environment (here a graph), the virtual ants react in elementary way by a probabilistic choice of path weighted with two coefficients, one comes from the problem heuristic and the other represent the pheromon rate deposit by all the ants until now. The feed-back process of the whole system over the entities is modelled by the pheromon action on the ants themselves.

Particle Swarm Optimization (PSO) is a metaheuristic method initially proposed by J. Kennedy and R. Eberhart [10]. This method is initialized with a virtual particle set which can move over the space of the solutions corresponding to a specific optimization problem. The method can be considered like an extension of a bird flocking model, like the BOIDS simulation from C.W. Reynolds [12]. In PSO algorithm, each virtual particle moves according to its current velocity, its best previous position and the best position obtained from the particles of its neighborhood. The feed-back process of the whole system over the entities is modelled by the storage of this two best positions as the result of communications between the system entities.

Other swarm optimization methods have been developed like Artificial Immune Systems [5] which is based on the metaphor of immune system as a collective intelligence process. F. Schweitzer proposes also a generic method based on distributed agents, using approaches of statistical many-particle physics [15].

The method proposed in this paper, is called Communities Swarm Optimization (CSO) and it consists in the co-evolving of both the spatial coordinates and the behavior of each individual of a virtual population of automata. The feed-back process of the whole system over the entities is modelled by a genetic algorithm based on this co-evolving. The automata behaviors allow to define for each individual, a set of arbitrary complex transition rules. We develop the formalism needed to describe this method and the associated algorithm in the two next sections.

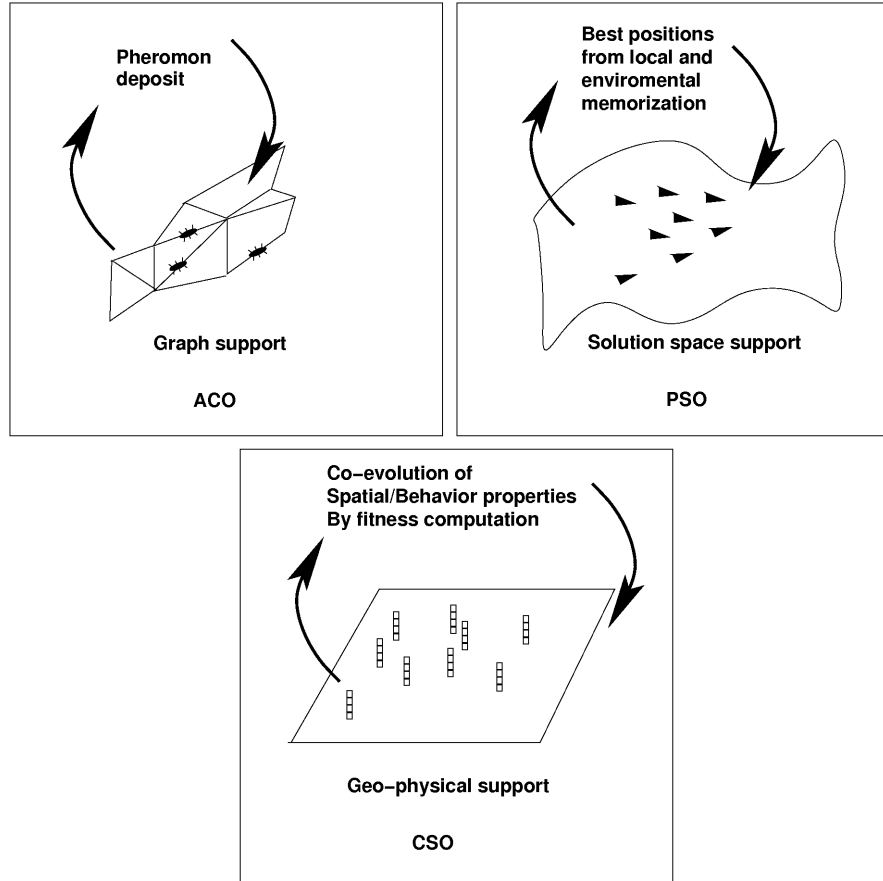


Fig. 1. Support and feed-back comparison from Ant Colony Optilization (ACO), Particule Swarm Optimization (PSO) and Community Swarm Optimization (CSO)

3 Spatial Behavioral Automata

3.1 Behavior modelling using automata

An automaton with multiplicities is an automaton with output values belonging to a specific algebraic structure, a semiring, including real, complex, probabilistic, non commutative semantic outputs (transducers) [8, 16]. In this way, we are able to build effective operations on such automata, using the properties of the algebraic structures which belong the output data. We are specifically able to describe automata by means of a matrix representation with all the power of the new (i.e. with semirings) linear algebra.

Definition 2. (Automaton with multiplicities)

An automaton with multiplicities over an alphabet A and a semiring K is the 5-uple (A, Q, I, T, F) where

- $Q = \{S_1, S_2 \dots S_n\}$ is the (finite) set of states;
- $I : Q \mapsto K$ is a function on the set of states, which associates to each initial state a value in K , called entry cost, and to each non- initial state a zero value ;
- $F : Q \mapsto K$ is a function on the set states, which associates to each final state a value of K , called final cost, and to each non-final state a zero value;
- T is the transition function, that is $T : Q \times A \times Q \mapsto K$ which to a state S_i , a letter a and a state S_j associates a value z of K (the cost of the transition) if it exist a transition labelled with a from the state S_i to the state S_j and zero otherwise.

Remark 1. We have not yet, on purpose, defined what a semiring is. Roughly it is the least structure which allows the matrix “calculus” with unit (one can think of a ring without the ”minus” operation). The previous automata with multiplicities can be, equivalently, expressed by a matrix representation which is a triplet

- $\lambda \in K^{1 \times Q}$ which is a row-vector which coefficients are $\lambda_i = I(S_i)$,
- $\gamma \in K^{Q \times 1}$ is a column-vector which coefficients are $\gamma_i = F(S_i)$,
- $\mu : A^* \mapsto K^{Q \times Q}$ is a morphism of monoids (indeed $K^{Q \times Q}$ is endowed with the product of matrices) such that the coefficient on the q_i th row and q_j th column of $\mu(a)$ is $T(q_i, a, q_j)$

Figure 2 describes the linear representation of a probabilistic automaton which is a specific automaton where output values are probabilistic values. For these probabilistic automata, the sum of the coefficients of each matrix row is equal to 1 (being the sum of outgoing and loop probability).

Definition 3. (Automata-Based Agent Behavior)

We represent the agent behavior by automata with multiplicities (A, Q, I, T, F) over a semiring K :

- The agent behavior is composed of a states set Q and of rule-based transitions between them. These transitions are represented by T ; I and F represent the initial and final costs;
- Alphabet A corresponds to the agent perceptions set;
- The semiring K is the set of agent actions, possibly associated to a probabilistic value which is the action realization probability (as defined in [6]).

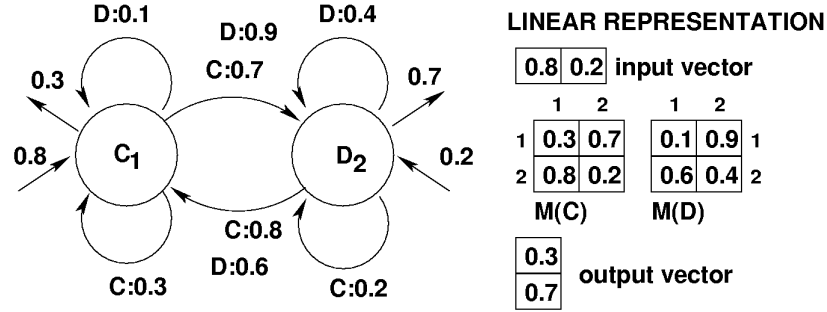


Fig. 2. Probabilistic automata over the alphabet $\{C,D\}$ and its linear representation

3.2 Spatial Automata and associated spatial distance

Definition 4. (Spatial Automata-Based Agent)

A spatial automata-based agent is defined by its structural representation:

- An automaton with multiplicities corresponding to its behavior as a whole processus managing its perceptions and its actions over its environment. They include its communication capabilities and so its social behavior;
- A spatial location defined on some specific metric space.

Remark 2. According with this previous definition, we define two metrics on the spatial automata-based agent. The first one concerns a spatial distance which is directly induced by the metrics of the spatial location (from any standard Hölder norm). The second one is more innovative and concerns a behavioral distance or semi-distance. One of the major interest of the previous automata-based modelling is to be able to define such behavioral distance which leads to powerful automatic processes dealing with self-organization. We detail this definition in the next section.

3.3 Metric spaces for behavioral distances

The main advantage of automata-based agent modelling is their efficient operators. We deal in this paragraph with an innovative way to define behavioral semi-distance as the essential key of the swarm algorithm proposed later.

Definition 5. (Evaluation function for automata-based behavior)

Let x an agent whose behavior is defined by \mathcal{A} , an automaton with multiplicities over the semiring K . We define the evaluation function $e(x)$ by:

$$e(x) = V(\mathcal{A})$$

where $V(\mathcal{A})$ stands for the vector of all coefficients of (λ, μ, γ) , the linear representation of \mathcal{A} , defined in remark 1.

Definition 6. (Behavioral distance)

Let x and y two agents and $e(x)$ and $e(y)$ their respective evaluations as described in the previous definition 5. We define $d(x, y)$ a distance or metrics between the two agents x and y as

$$d(x, y) = ||e(x) - e(y)||$$

a vector norm of the difference of their evaluations.

3.4 Genetic operators on spatial automata-based agent

We consider in the following, a population of spatial automata-based agents, each of them is represented by a chromosome, following the genetic algorithm basis. We define the chromosome for each spatial automata-based agent as a couple of two sequences:

- the sequence of all the rows of the matrices of the linear representation of the automata. The matrices, associated to each letter from the alphabet of the agent perceptions, are linearly ordered by this alphabet and we order all the rows following these matrices order [2]. The figure 3 describes how this sequence is created from a linear representation of two matrices;
- the sequence of all its spatial coordinates.

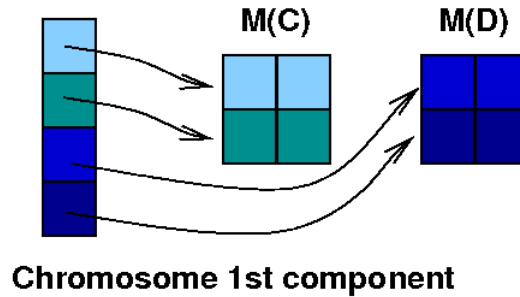


Fig. 3. Chromosome first component building from the matrix rows of the linear representation of an automaton over the alphabet $\{C,D\}$

In the following, genetic algorithms are going to generate new automata containing possibly new transitions from the ones included in the initial automata.

The genetic algorithm over the population of spatial automata-based agent follows an evolutive iteration composed of two main operators, as on adaptation of the classical genetic operators [9]:

- *Reproduction (Duplication and Crossing-over)*: This operator is a combination of the standard duplication and crossing-over genetic operators. For each couple of spatial automata (called the parents), we generate two new spatial automata (called the children) as the result of the chromosome crossings and we keep, without change, the parent spatial automata. To operate for the crossing-over operation, we have to compute
 - the automata of the behaviors of the two children. For this purpose, we consider a sequence of rows for each matrix of the linear representation of one of the two parents and we make a permutation on these chosen sequences of rows between the analogue matrix rows of the other parent;
 - the spatial locations of the two children. These children locations can be chosen from many ways: on the linear segment defined by the parent locations or as the nodes of the square obtained with the parent and the children as describe in the figure 4.
- *Mutation*: This operator deals only with the linear representation of the spatial automata-based agent. With a low probability, each matrix row from this linear representation is randomly chosen and a sequence of new values is given for this row (respecting some constraints if exist, like probabilistic values [2]).

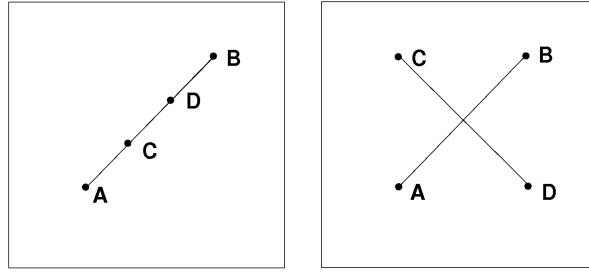


Fig. 4. Spatial locations for the children C and D from the parents A and B, after a reproduction step. Two possible locations computation are presented in the two sub-figures.

4 Community Swarm Optimization Algorithm

4.1 Adaptive objective function for community-based swarm optimization

The community swarm optimization method is based on a genetic algorithm over a population of spatial automata-based agents. The formation of the

community is the result of the population evolution, crossing by a selection process computed with the fitness function defined in the following.

For this computation, we deal with two distances defined on agent sets. The first is the spatial distance associated with the agent spatial location and the second is the behavioral semi-distance defined in the definition 6.

Definition 7. Community clustering and detection fitness

Let \mathcal{V}_x a neighbourhood of the agent x , relative to its spatial location. We define $f(x)$ the agent fitness of the agent x as :

$$f(x) = \begin{cases} \frac{\text{card}(\mathcal{V}_x)}{\sum_{y_i \in \mathcal{V}_x} d(x, y_i)^2} & \text{if } \sum_{y_i \in \mathcal{V}_x} d(x, y_i)^2 \neq 0 \\ \infty & \text{otherwise} \end{cases}$$

where $d(x, y)$ is the behavioral distance between the two agents x and y .

This fitness allows to implement a co-evolution process which generates an emergent set of community swarms. These community swarm formation is the result of both the emergence of the spatial location of the generated communities and the adaptive behavior of the communities as the result of the homogenisation of the behavioral automata of all agents which compose these emergent communities.

4.2 General CSO algorithm

CSO algorithm needs a initial step description which is the major step of the modelling process. The way of going from the problem formulation to the initial spatial automata-based agents must be realized in accuracy. The formal description of the methodology to use, for this initial step, is described in Algorithm 1.

Algorithm 1: Methodology to model the initial step of CSO

1. Problem formulation by the definition of a set of transition rules ;
 2. Building of the behavioral automata based on the previous set of transition rules, describing the sequences and the context of their applications ;
 3. Discretization of the spatial domain, according to its topological properties (Cellular automaton, network or graph, Geographical Information System) with the spatial location of the initial virtual population of spatial automata-based agents;
-

The core of the CSO algorithm is described by the iterative scheme defined in the Algorithm 2.

Algorithm 2: Iterative scheme of CSO

Building the initial virtual population of the spatial automata-based agents (following the methodology of Algorithm (1)) ;

repeat

for *Each couple of individuals in the population* **do**

 Reproduction step generating 2 new children as described in the section (3.4) ;

 Mutation step as described in the section (3.4) ;

 Selection of the half population of the individuals corresponding to the highest values of the agent fitness described in section (4.1) ;

until *(the sum of the fitness values of the whole population reaches a threshold) or (the maximum iteration number is reached)* ;

An example of fitness function computation output is illustrated by the figure 5 where we show, on the same population, an high level fitness individual which will be probably kept inside the population at the next iteration and a low level fitness individual which be probably removed from the population at the next iteration. The colors used describe graphically the chromosome composition, allows to appreciate the similarity of the individuals.

5 Conclusion and Perspectives

In this paper, we describe Community Swarm Optimization (CSO) method which can be described as a swarm optimization process. With the comparison of other methods from this category (ACO and PSO), CSO differs mainly on the modelling purpose. CSO deals with transition rules included in data structures (automata with multiplicities) for which algebraic operators allow to implement automatic computation for self-organizational phenomena.

Presented as a very generic method in this paper, CSO can be efficiently applied in engineering problems where the spatial characteristics are not only additional coordinates for the data but the spatial organization is the result of the self-organization process as the output of the method. Typically, urban and territorial management need this kind of modelling to improve the decision making within a sustainable development (including environmental, economic and social aspects).

Schelling's Segregation model [13] is one of the basic problem dealing with spatial self-organizations within urban or territorial management. In

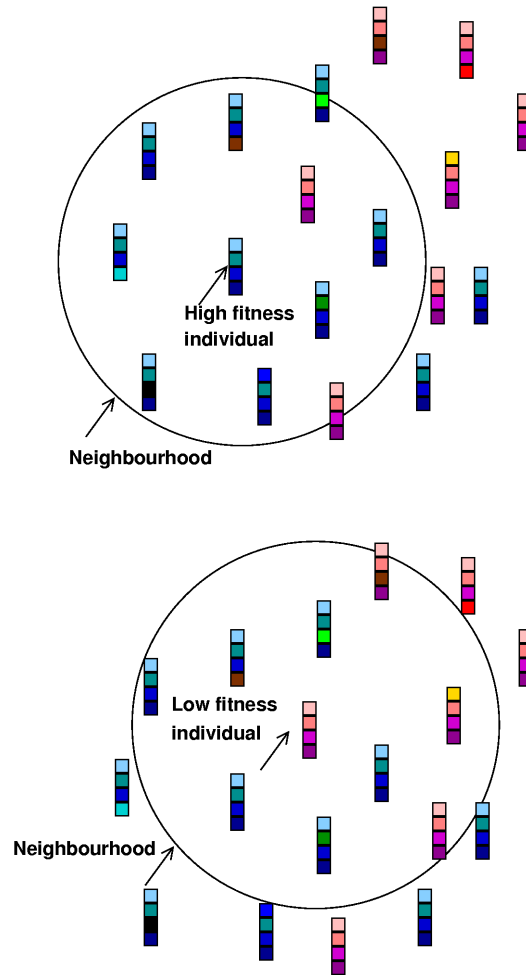


Fig. 5. Two examples of the result of selection operators within the CSO algorithm

this model, the spatial domain, based on cellular automata, is constrained and the rules are elementary. Another example concerns partitioning problem for territorial management which can be computed by genetic algorithms [18], using objective functions with various parameters to adjust. Following Benenson and Torrens [1], we have to deal nowadays with the understanding of the development of urban dynamics as collective phenomena. Models of election/voting (studying the political affiliation of citizen under the influ-

ence of their environment and neighbors), diffusion of innovation in the city (studying the acceptance or rejection of innovation from citizen depending on their environment and neighbors) are typically spatial complex systems with multi-dimensional (social, economic, ...) rule-based aspects which need accurate transition machines and efficient self-organization processes that CSO is expected to solve.

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PART 3

Natural Dynamical Systems

Tissular coupling and frequency locking

I finite population

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1 Introduction

Synchronization is an extremely important and interesting emergent property of complex systems. The first example found in literature goes back to the 17th century with Christiaan Huygens works [8, 1]. This kind of emergent behavior can be found in artificial systems as well as in natural ones and at many scales (from cell to whole ecological systems). Biology abounds of periodic and synchronized phenomena and the works of Ilya Prigogine showed that such behaviors arise within specific conditions: a dissipative structure generally associated to a non-linear dynamic [12]. Biological systems are open, they evolve far from thermodynamic equilibrium and are subject to numerous regulating processes, leading to highly non-linear dynamics. Therefore periodic behaviors appear (with or without synchronization) at any scale [13]. More generally, life itself is governed by circadian rhythms [6]. Those phenomena are as much attractive as they are often spectacular: from cicada populations that appear spontaneously every ten or thirteen years [7] or networks of heart cells that beat together [9] to huge swarms in which fireflies, gathered in a same tree, flash simultaneously. Furthermore, beyond biology one can find a wide source of examples in completely different fields of science (e.g. in behavioral psychology with the example of synchronizing applause [15]). For much more artificial and/or theoretical examples, one can consider the whole field of research that studies the coupling of smooth dynamical systems. Nowadays, it is one of the most important subject related to non-linear systems' dynamics, especially through the notion of chaotic systems' synchronization [10, 5]. This wide source of examples leads the field of research to be highly interdisciplinary, from pure theory to concrete applications and experimentations. The classical concept of synchronization is related to the locking of the basic frequencies and instantaneous phases of regular oscillations. Those questions are usually addressed by studying specific kinds of coupled discrete or differential systems, using classical tools of the field (see for instance [11]). Convinced that synchronization phenomenon is completely natural in a large

variety of coupled dynamical systems, we propose a new approach of the subject: firstly, we ask the question of synchronization differently than the usual way. Rather than trying to prove that synchronization actually takes place, we search conditions under which frequencies are locked as soon as the whole system oscillate. Secondly we enlarge the scope of handled models, by building a general framework for coupled systems called "tissular coupling". This framework is inspired by biological observations at cell's scale, but relevant at any scale of modeling. Under some general assumptions on the kind of interactions that constitute the coupling of the systems, we prove that for a wide class of tissular coupling systems, frequencies are mutually locked to a single value as soon as the whole population is oscillating.

This paper exhibits our model of tissular coupling and the frequency locking in the case of a finite number of coupled systems. In the first section we present some mathematical tools and the background we have used in order to study synchronization issue (the results exposed at the end of this paper is only a part of what we have fulfilled, and surely a really small part of what can be done using tissular coupling, this is why we state this framework in its general form). Then, we describe dynamical objects on which we focus, namely the tissular coupling and periodical motions of a population. In the second section we expose a useful way to reduce the problem to a structural one, with no more reference to the dynamics of the coupled systems. In the final section we exhibit some natural conditions under which we are able to prove the main result of this paper, a case of synchronization, in terms of frequencies locking. In a second paper [3] we expose the case of an infinite compact and connected population, which is processed with different mathematical tools.

2 Basic material and notation

As our model is inspired by cellular tissues, several terms clearly come from those kinds of complex systems. Nevertheless, its scope is not limited to cells nor cell tissues.

Model of population

Here are the basic compounds of our model:

- a **population** \mathcal{P} is a measured space with finite measure. Typical populations are finite populations with counting measure or measurable subset of \mathbb{R}^m with Lebesgue measure. Every $p \in \mathcal{P}$ is called a cell;
- for sake of concision, we suppose that the systems we want to study are \mathbb{R}^n -valued. Thus, a **state** of \mathcal{P} is an element of $\mathcal{S} = \mathcal{M}_b(\mathcal{P}, \mathbb{R}^n)$, the space of measurable bounded applications from \mathcal{P} to \mathbb{R}^n . A state of \mathcal{P} will be denoted s , $s(p)$ representing the state of the cell p ;

- let I be a real interval and $r \in \mathbb{N}$. A **trajectory** of \mathcal{P} is an element of $\mathcal{F}(I, \mathcal{S})$. For sake of concision, any trajectory will be written s again, and $s(t, p)$ stands for the state of cell p at time t (which should be written $s(t)(p)$). We then define the space of trajectories as:

$$\mathcal{T}^r = \left\{ \mathcal{C}^r(I, \mathcal{S}), \forall \text{ compact } J \subset I, \sup_{J \times \mathcal{P}} \|s(t, p)\| < \infty \right\}$$

A trajectory s is regular only along the time t (excepted in terms of measurability), thus we use the following unambiguous notation :

$$s'(t, p) = \partial_1 s(t, p)$$

- as there is no ambiguity, all norms will be denoted in the same manner :

$$\begin{aligned} s \in \mathcal{S} : \|s\| &= \sup_{\mathcal{P}} \|s(p)\| \\ s \in \mathcal{T} : \|s\| &= \sup_{I \times \mathcal{P}} \|s(p, t)\| \\ \|s(t, \cdot)\| &= \sup_{\mathcal{P}} \|s(t, p)\| \\ \|s(\cdot, p)\| &= \sup_I \|s(t, p)\| \end{aligned}$$

With this kind of norm, \mathcal{T}^0 becomes obviously a Banach space.

Periodic motions and properties of periods on \mathcal{P}

Now, we define the notion of periodic trajectories on a population and some mathematical tools related to their study.

Definition 1. A **period on \mathcal{P}** is a map τ from \mathcal{P} to \mathbb{R}_+^* . A trajectory $s \in \mathcal{T}^r$ is said to be **τ -periodic** if for any $p \in \mathcal{P}$, $s(\cdot, p)$ is $\tau(p)$ -periodic. $\tau(p)$ is then called the **period of p** . The space of such trajectories is written \mathcal{T}_τ^r

For any $s \in \mathcal{T}_\tau^0$ we define the τ -mean $I^\tau(s)$ of s as:

$$I^\tau(s)(p) = \frac{1}{\tau(p)} \int_0^{\tau(p)} s(t, p) dt$$

Note that this quantity is \mathbb{R}^n -valued.

Here follows a corollary of the ergodic theorem [4] (and for references to ergodic theory see for instance [14, 2]):

Proposition 1 (Ergodic alternative). Let f a smooth real-valued map, τ -periodic and $\tau' > 0$. We define the τ' -average of f :

$$a_{\tau'}(f)(t) = \lim_{N \rightarrow +\infty} \frac{1}{N+1} \sum_{k=0}^N f(t + k\tau')$$

Then the following alternative holds:

- τ and τ' are \mathbb{Z} -free (non commensurable), then $a_{\tau'}(f)$ is constant and equals $\tau I^\tau(f)$;
- there's an irreducible fraction $\frac{a}{b}$ such that $b\tau = a\tau'$, then $a_{\tau'}(f)$ is $\frac{\tau'}{b}$ -periodic.

The synchronization issues we want to study are related to the following question: as soon as the whole system oscillates, which assumptions insure that cells oscillate according to the same period. We will see that showing that periods are commensurable is the first natural step to achieve an answer to this question. We need the following notion: a (finite) subset $\{\tau_1, \dots, \tau_k\}$ of \mathbb{R} is said to be **dependent** (over \mathbb{Z}) if there exists some integers l_1, \dots, l_k non all zeros and such that:

$$l_1\tau_1 + \dots + l_k\tau_k = 0$$

Let's go back to differential systems on \mathcal{P} now to complete our description of their coupling.

Differential systems on \mathcal{P}

As we want to reach the most general result, each cell p is supposed to behave according to an autonomous differential system given by an application F_p . Considering non-autonomous differential systems could be assimilated to the case of synchronization by an external force. Moreover, as we work with any kind of population, this external force could be assimilated to new cells and integrated in the population. For sake of concision, we will assume that F_p is smooth. Then, we have a family of applications $\{F_p\}_{p \in \mathcal{P}}$. We can define the extension $F_{\mathcal{P}}$ of this family to \mathcal{S} :

$$\begin{aligned} F_{\mathcal{P}} : \mathcal{S} &\longrightarrow \mathcal{S} \\ s &\longmapsto F_{\mathcal{P}}(s) \end{aligned}$$

with

$$F_{\mathcal{P}}(s)(p) = F_p(s(p))$$

The extension $F_{\mathcal{P}}$ is said to be \mathcal{C}^r if every F_p is \mathcal{C}^r and if for any bounded subset B of \mathbb{R}^n we have:

$$\sup_{x \in B \times p \in \mathcal{P}} \|dF_p(x)\| < \infty$$

Coupler on \mathcal{P}

Now, we consider how to couple the differential systems given by $F_{\mathcal{P}}$. Usually [11], the coupling is made by adding to $s'(t, p)$ a quantity that depends on $s(t, \cdot)$:

$$s'(t, p) = F_p(s(t, p)) + c(s(t, \cdot))(p) \quad (1)$$

where $c(s(t, \cdot))$, an element of \mathcal{S} , is a function of the global state of \mathcal{P} . This is what we naturally name a coupling **in the final space**. Another way of coupling is **in the initial space** :

$$s'(t, p) = F_{\mathcal{P}}\left(c(s(t, .))\right)(p)$$

Coupling in the final space or in the initial space may have different interpretations. The first coupling method could be seen as physical exchanges and the second one as an instantaneous exchanges (for instance an high speed information exchange which could be modeled as instantaneous with regards to the system time scale). But, in most situations, those two couplings are equivalent. Indeed, the right-hand-side term of the equation (1) has the following shape:

$$F(x) + \varepsilon$$

so that, as soon as this r.h.s. stays in the range of the map F , one can find a vector y :

$$F(x) + \varepsilon = F(y)$$

writing $\delta = y - x$ we have:

$$F(x) + \varepsilon = F(x + \delta)$$

In fact, this case is quite general as ε is usually small, and even if it is not, we will study only periodical trajectories occurring essentially around limit cycles surrounded by the range of F . For these reasons, we will concentrate on the last kind of coupling. Now we give technical details about our method of coupling, by the way of a **coupler** :

$$\begin{aligned} C : \mathcal{S} &\longrightarrow \mathcal{S} \\ s &\longmapsto C(s) \end{aligned}$$

This coupler is said to be **linear** if there is an application c satisfying:

$$\begin{aligned} c : \mathbb{R}^n \times \mathcal{P} &\longrightarrow \mathcal{L}(\mathcal{S}, \mathbb{R}^n) \\ (x, p) &\longmapsto c(x, p) \end{aligned}$$

such that for all $p \in \mathcal{P}$:

$$C(s)(p) = c(s(p), p) . s$$

In other words, depending on the cell p (*e.g.* its spatial position) and on its state $s(p)$, the coupling compute linearly (as a mean) the pseudo-state $c(s(p), p) . s$ dictating the evolution of this cell, instead of the state $s(p)$ which is used if the cell is isolated (without coupling).

Sometimes the following point of view, the operator one, is be useful:

$$\begin{aligned} L_c : \mathcal{S} &\longrightarrow \mathcal{L}(\mathcal{S}, \mathcal{S}) \\ s &\longmapsto L_c(s) \end{aligned}$$

where:

$$[L_c(s).s](p) = c(s(p), p) . s$$

If L_c is constant, the coupler is said to be **uniformly linear** and is naturally associated to an element of $\mathcal{L}(\mathcal{S}, \mathcal{S})$. The case of uniformly linear coupler is the simpler one: the way a cell is influenced by its environment does not depends on its proper state. It is an approximation of what happens in real systems, but this simplification allows us to reach very global results.

To familiarize with the notion of coupler, let's write how it transforms a trajectory of \mathcal{P} . If $s \in \mathcal{T}$, the effect of a coupler on this trajectory is:

$$C(s)(t, p) = c(s(t, p), p).s(t, .)$$

Tissular coupling

We are now able to define a tissular coupler. Given a population \mathcal{P} , a coupler C is said to be a **tissular coupler** if the associated application c satisfies:

$$c(x, p).s = \int_{\mathcal{P}} c_d(x, p, q).s(q) dq + c_a(x, p).s(p)$$

where c_d and c_a are continuous and are called diffuse part and atomic part of c (and of C), respectively. The diffuse part represents the influence of the environment (the whole population) on the behavior of the cell p . The atomic part models the self-influence of p . We could have merge this into an unique linear operator, containing a Dirac impulse, but for both comprehension and manipulation purposes we think that this shape is preferable and less theoretical.

Finally, we can define a tissular coupling by specifying the equation that any solution should satisfy. Given a population \mathcal{P} , a family of applications $\{F_p\}_{\mathcal{P}}$ and a tissular coupler C , the **tissular coupling** $S(\mathcal{P}, F_{\mathcal{P}}, C)$ is defined by the equation:

$$s' = F_{\mathcal{P}} \circ C(s) \tag{2}$$

in other words, any solution s satisfies for all $(t, p) \in I \times \mathcal{P}$:

$$s'(t, p) = \int_{\mathcal{P}} c_d(s(t, p), p, q).s(t, q) dq + c_a(s(t, p), p).s(t, p)$$

We naturally end this introduction of the tissular coupling with a result on the existence of solutions, which is proved in a classical way [4]:

Theorem 1. *Suppose $F_{\mathcal{P}}$ is \mathcal{C}^1 and c_d and c_a are locally lipschitzian in their first variable. Given any initial condition (t_0, s_0) in $I \times \mathcal{S}$, the tissular coupling given by equation (2) admits a unique maximal solution.*

We can then go further and begin to work on the heart of our matter.

3 Reduction of the synchronization problem

Cyclic application

The first reduction of the problem we have done was deciding to handle coupling in the initial space. Indeed, by the way of a natural assumption made on $F_{\mathcal{P}}$ we will reduce the problem to a structural one.

Definition 2. Let F be a map from \mathbb{R}^n to \mathbb{R}^n and U an open subset of \mathbb{R}^n . F is said to be cyclic on U if for any motion $x : \mathbb{R} \rightarrow U$ such that there exists τ satisfying:

$$F \circ x(t + \tau) = F \circ x(t) \quad \forall t \in \mathbb{R}$$

then:

$$x(t + \tau) = x(t) \quad \forall t \in \mathbb{R}$$

Before we expose what the problem of frequency locking becomes under this assumption, we should explain why considering cyclic maps is somewhat natural. For example, if one tries to build an elementary non-cyclic map, one can consider the following:

$$F : \mathbb{R}^2 - \{0\} \rightarrow \mathbb{R}^2 - \{0\}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \frac{1}{\|x\|} \begin{bmatrix} -x_2 \\ x_1 \end{bmatrix}$$

for which any map $x(t)$ satisfying :

$$x(t) = \begin{bmatrix} r(t) \cos(t) \\ r(t) \sin(t) \end{bmatrix}$$

leads to (fig. 1):

$$F \circ x(t + \omega) = F \circ x(t)$$

But this kind of applications is not of great interest in the scope of physical and biological phenomena, indeed if we want to consider a little more credible version of this map, we must look at the mathematical pendulum, which is defined by:

$$F : \mathbb{R}^2 - \{0\} \rightarrow \mathbb{R}^2 - \{0\}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} -x_2 \\ x_1 \end{bmatrix}$$

for which the preceding property does not hold any more (it's a one-to-one map). Moreover, even if the map isn't one-to-one, in a realistic model of a physical phenomenon, periodic trajectories are rare: given such a dynamical system, any specific periodic trajectory in the phase space corresponds to a limit cycle. Those limit cycles are indeed rare in non conservative systems (compare for instance the mathematical and the physical pendulum). In other words, there exists a neighborhood of those limit cycles in which the only maps $x(t)$ such that $F \circ x(t)$ is periodic are precisely the limit cycles themselves,

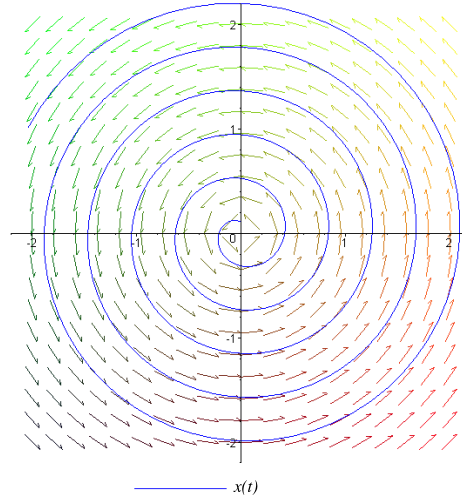


Fig. 1. Vector field associated to F and an example of non periodic map $x(t)$ such that $F \circ x$ is periodic.

which are effectively periodic applications.

Moreover, this condition of cyclicity is extremely useful and allows us to prove the following result:

Proposition 2. *Let $(\mathcal{P}, C, F_{\mathcal{P}})$ be a tissular coupling and τ a period on \mathcal{P} . Let's assume that there exists a family $\{U_p\}_{p \in \mathcal{P}}$ of open subsets of \mathbb{R}^n such that for all $p \in \mathcal{P}$ F_p is cyclic on U_p . If s is a τ -periodic solution of the coupling such that for all $p \in \mathcal{P}$, $s(I, p) \subset U_p$, then s belongs to the following set:*

$$A(\tau, C) = \mathcal{T}^\infty \cup C^{-1}(\mathcal{T}^\infty)$$

We then have drastically reduced the problem: under hypotheses of the proposition 2 we can throw $F_{\mathcal{P}}$ and only work with the coupler C and the space of periodic solutions.

This last section exhibits the main results of our work on finite populations.

4 Synchronization in finite population

Here we focus on a coupler in the case of a finite population \mathcal{P} :

$$\mathcal{P} = \{p_1, \dots, p_k\}$$

If C is a tissular coupling, the diffuse part is sufficient to define it, thus we can also write it for $x \in \mathbb{R}^n$ as:

$$c_d(x, p_i, p_j) = c_{ij}(x) \in \mathcal{L}(\mathbb{R}^n)$$

L_c adopts naturally a matrix shape:

$$L_c(x) = \begin{bmatrix} c_{11}(x) & \dots & c_{1k}(x) \\ \vdots & \ddots & \vdots \\ c_{k1}(x) & \dots & c_{kk}(x) \end{bmatrix}$$

We point out that each $c_{ij}(x)$ is a linear application, an element of $\mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ (to be precise and to anticipate on some theoretical generalizations of the tissular coupling, we note that $L_c(x)$ is the matrix of a linear application on $(\mathbb{R}^n)^k$ seen as a module on the ring $\mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$).

The image of the motion $s(., p_i)$ is thus given by:

$$C(s)(t, p_i) = \sum_{j=1}^k c_{ij}(s(t, p_i))s(t, p_j)$$

The kind of assumptions we are about to make on C involve linear properties. Especially we work with the rank of L_c and of sub-matrices. Thus, if $J \subset \{1, \dots, k\}$ contains l elements, we write $I^c = \{1, \dots, k\} - J$. If $M = (m_{ij})$ is an $k \times k$ matrix, we define M^I as the $l \times (l - k)$ matrix which coefficients are the $m_{i,j}$ for $(i, j) \in I \times I^c$. For $s \in \mathcal{S}$ or \mathcal{T} we write s^I the vector $[s(p_i)]_{i \in I}$ or $[s(., p_i)]_{i \in I}$ (see figure 2).

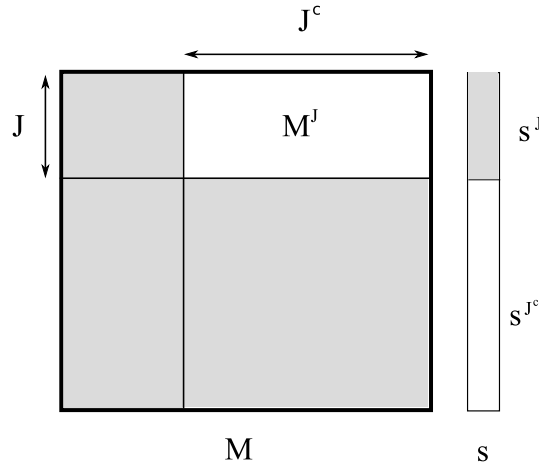


Fig. 2. Matrix and vectors associated to a subset J of $\{1, \dots, k\}$.

Definition 3. Let C a coupler on \mathcal{P} . C is said to be **reducible** if there exists $x \in \mathbb{R}^n$ and a non trivial part J of $\{1, \dots, n\}$ such that $L_c^J(x)$ has non-invertible coefficients (as elements of $\mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$). C is **non-reducible** if it's not reducible.

The next step consists in "cleaning" the space of periodic solutions in order to exclude some degenerated and/or trivial cases. First the trivial periodic solutions:

Definition 4. Any s in $A(\tau, C)$ is said to be non trivial if for all $p \in \mathcal{P}$, neither $s(\cdot, p)$ and $c(e)(\cdot, p)$ are constant maps.

Now we consider some trajectories that are degenerated in a stronger sense than the one of trivial trajectories: our aim is to avoid periodic trajectories which oscillating behaviors are the only consequences of the coupler's dynamic (we focus on the synchronization of interacting oscillating systems, not on some systems oscillating due to an external force, and we have already mentioned that this kind of external force can be included in the population itself). Such trajectories are those for which there exists a state b satisfying:

$$L_c(s(t, \cdot))(s(t, \cdot) - b) = 0$$

In terms of matrices, we then have for the cell p_i :

$$\sum_{j=1}^k c_{ij}(s(t, p_i))(s(t, p_j) - b(p_j)) = 0$$

In other words, the preceding equality shows that the vector $[s(\cdot, p_1), \dots, s(\cdot, p_k)]$ is not linearly free, according to the compounds of $L_c(e(t, \cdot))$. These observations lead us to the following definition:

Definition 5. Let \mathcal{P} be a finite population, C a tissular coupler on \mathcal{P} and τ a period on \mathcal{P} . A non-trivial element $s \in A(\tau, C)$ is said to be **C -free** if for any $b \in \mathcal{S}$ and for any non-trivial $J \subset \{1, \dots, k\}$ we have:

$$L_c(s(t, \cdot))^J \cdot (s(t, \cdot) - b)^{J^c} \neq 0$$

In fact, we can prove that, when C is non-reducible, a non C -free element is degenerated because it comes from a solution of a system on a population which cardinal is strictly lower than k .

Here is the first step to frequencies locking as it deals with dependency of periods [4]:

Theorem 2. If C is non-reducible and if $A(\tau, C)$ contains a non-trivial and C -free element, then $\tau(\mathcal{P})$ is a dependent set.

Finally we extend this result to a more powerful conclusion. In order to prove synchronization, we must add a last condition on C . This is done introducing the following definition:

Definition 6. Let C be a uniformly linear coupler on \mathcal{P} . C is said to be strongly non-reducible if it is non-reducible and if for all $x \in \mathbb{R}^n$ and all non trivial subset $J \subset \{1, \dots, k\}$ with more than $\frac{k}{2}$ elements, $L_c^J(x)$ is injective.

Remark 1. Because of its dimensions, $L_c^J(x)$ can't be injective if J contains less than $\frac{k}{2}$ elements.

We can now state our main result [4]:

Theorem 3. *Let \mathcal{P} be a finite population of cardinal k , C a tissular coupler, uniformly linear and strongly non-reducible and τ a period on \mathcal{P} . Suppose $s \in A(\tau, C)$ isn't trivial and is non-reducible. If more than $\frac{k}{2}$ cells have the same period then all cells have the same period.*

5 Conclusion

In this work we have built a general framework, the tissular coupling, to handle a wide variety of coupled systems, and therefore a wide class of complex systems. We focused on an emergent property of those dynamical systems: synchronization, and precisely frequencies locking. We used the notion of tissular coupling to show that the synchronization issue may be addressed differently. Usually one observes solutions of particular coupled systems and shows that within suitable conditions synchronization must occur. Those results are often qualitatively dependent of the studied systems and do not stand in the general case. We tried to change our point of view and to bring completing results. These are less precise than usual ones as we don't prove that synchronization ultimately happens. Instead, we consider the problem at its end: if one supposes that coupled systems oscillate, then they must be synchronized. The loss in time evolution informations is compensated by very general results, almost independent from the individual differential systems to be coupled. We believe that this complementary approach of frequencies locking will lead to future developments as it brings about many unused mathematical tools. This will be showed in a following paper in which we study the same question concerning an infinite compact connected populations but with very different methods [3]. Other examples of the use of tissular coupling can be found in [4] on emergence of spatial patterns. All those work and results are stimulating and seem to prove that this framework will be fruitful.

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Tissular coupling and frequency locking

II infinite population

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1 Introduction

In an accompanying paper [1], we addressed the issue of synchronization under the framework of tissular coupling and in the case of a finite population. This issue is one of the main dynamical emergent property of complex systems. Indeed, this phenomenon actually occurs in natural systems at many scales (from cell to whole ecological systems), as it has been shown in [9, 10]. Many examples can be found in various fields of experimentation and theoretical research ([4, 6, 5, 11]). It is in particular a key concept in the field of non-linear systems' dynamics, especially through the notion of chaotic systems' synchronization [7, 3]. This wide source of examples explain why this field of research is highly interdisciplinary, from pure theory to concrete applications and experimentations. The classical concept of synchronization is related to the locking of the basic frequencies and instantaneous phases of regular oscillations. Those issues are usually addressed by studying specific coupled systems, using classical tools of the field (see for instance [8]). Convinced that synchronization phenomenon is completely natural in a large variety of coupled dynamical systems, we proposed in a preceding paper a new approach to the subject: we built a general model of coupled systems, called "tissular coupling". Within natural assumptions on interactions of those coupled systems, we proved that, for a wide class of autonomous differential systems, as soon as a finite population exhibits oscillating behaviors, their frequencies are mutually locked to a single value [1]. This paper presents a new piece of answer to those synchronization issues as it addresses the issue of an infinite compact connected population. Even if the general questions addressed in this paper are similar to those addressed in the later one, this study is done with completely different techniques than in the finite situation and the results achieved require assumptions that are specific to the continuous situation. This paper stresses the fact that synchronization is a very natural phenomenon that can be find in many kind of coupled dynamical systems.

In the first section, we recall some parts of the framework already introduced and some mathematical tools specific to our problem. Then we address a sub problem which deals with diffuse coupling, concluding with a general result on synchronization. Finally, we go back to the general case of a uniformly linear tissular coupling and show how those dynamical systems, and based on which assumptions, exhibits the frequencies locking property.

2 Basic material and notation

The main background of this work can be found in the accompanying paper [1] and in [2], so we just recall here some notations and concentrate on the new materials specific to the case of an infinite population.

- \mathcal{P} is a measured space with finite measure λ . Typical populations are finite populations with counting measure or measurable subset of \mathbb{R}^m with Lebesgue measure, so that \mathcal{P} has a topology. Every $p \in \mathcal{P}$ is called a cell;
- A **state** of \mathcal{P} is an element of $\mathcal{S} = \mathcal{M}_b(\mathcal{P}, \mathbb{R}^n)$, the space of measurable bounded applications from \mathcal{P} to \mathbb{R}^n . A state of \mathcal{P} is denoted s , $s(p)$ standing for the state of the cell p ;
- let I be a real interval and $r \in \mathbb{N}$. A **trajectory** of \mathcal{P} is an element of $\mathcal{F}(I, \mathcal{S})$. For concision purpose, any trajectory will be written s again, and $s(t, p)$ stands for the state of cell p at time t . Then, the space of trajectories is:

$$\mathcal{T}^r = \left\{ \mathcal{C}^r(I, \mathcal{S}), \forall \text{ compact } J \subset I, \sup_{J \times \mathcal{P}} \|s(t, p)\| < \infty \right\}$$

A trajectory s is regular only along the time t (excepted in terms of measurability), thus we use the following unambiguous notation :

$$s'(t, p) = \partial_1 s(t, p)$$

- All norms are denoted in the same manner :

$$\begin{aligned} s \in \mathcal{S} : \|s\| &= \sup_{\mathcal{P}} \|s(p)\| \\ s \in \mathcal{T} : \|s\| &= \sup_{I \times \mathcal{P}} \|s(p, t)\| \\ \|s(t, \cdot)\| &= \sup_{\mathcal{P}} \|s(t, p)\| \\ \|s(\cdot, p)\| &= \sup_I \|s(t, p)\| \end{aligned}$$

- a **period on \mathcal{P}** is a map τ from \mathcal{P} to \mathbb{R}_+^* . A trajectory $s \in \mathcal{T}^r$ is said to be **τ -periodic** if for any $p \in \mathcal{P}$, $s(\cdot, p)$ is $\tau(p)$ -periodic. $\tau(p)$ is then called the

period of p . The space of such trajectories is written \mathcal{T}_τ^r . For any $s \in \mathcal{T}_\tau^0$ we define the τ -mean $I^\tau(s)$ of s as:

$$I^\tau(s)(p) = \frac{1}{\tau(p)} \int_0^{\tau(p)} s(t, p) dt$$

- If C is a coupler on \mathcal{P} and τ is a period on \mathcal{P} , the set $A(\tau, C)$ is defined as:

$$A(\tau, C) = \mathcal{T}^\infty \cup C^{-1}(\mathcal{T}^\infty)$$

In this paper, \mathcal{P} stands for an infinite compact connected population, with finite measure λ , mostly a measurable subset of \mathbb{R}^m with Lebesgue measure. Any period τ on \mathcal{P} is assumed to be measurable and bounded. On the contrary of the finite population case, to define a tissular coupler one needs to specify both its diffuse part c_d and its atomic part c_a . Such a coupler will be called **diffuse** if $c_a = 0$.

Partition associated to a periodic motion

Let suppose that s is a τ -periodic trajectory of \mathcal{P} , it divides \mathcal{P} into a partition according to the period of each cell. Let $p \in \mathcal{P}$ and $a \in \mathbb{R}^*$, we define the following application:

$$\tau_a(p) = a\tau(p)$$

and for any subset $A \subset \mathbb{R}$:

$$\mathcal{P}_p^A = \bigcup_{a \in A} \tau_a^{-1}(\tau(p))$$

In other words:

$$\mathcal{P}_p^A = \{q \in \mathcal{P} : \exists a \in A, a\tau(q) = \tau(p)\}$$

As soon as $1 \in A$, the set of all distinct \mathcal{P}_p^A realizes a partition of \mathcal{P} , we write \mathcal{P}^A this partition (with non-empty sets). The main examples are:

- if all the periods constitute a dependent set, then $\mathcal{P}^\mathbb{Q} = \{\mathcal{P}\}$;
- if all the periods are identical (\mathcal{P} is synchronized), then $\mathcal{P}^1 = \{\mathcal{P}\}$.

Measures associated to an uniformly linear coupling

The next tool we need is a method to measure the latter sets. But this cannot be done independently from the coupler itself. This is why we introduce the following family of measures on \mathcal{P} , one for each cell p :

$$\lambda_p(B) = \int_B \|c_d(p, q)\| dq$$

We recall that the support of a measure is defined to be the largest closed subset of \mathcal{P} for which every open neighborhood of every cell of the set has positive measure. Let S_p stands for the support of $c_d(p, \cdot)$, this support indicates where the measure λ_p lives and then which cells influence p in its evolution. In virtue of c_d 's continuity, we know that for any measurable subset B of \mathcal{P} , the following equivalence holds:

$$\lambda_p(B) > 0 \Leftrightarrow \lambda(B \cap S_p) > 0$$

Moreover, in the case of a diffuse coupler, this family of measures is uniformly absolutely continuous with respect to λ :

Lemma 1. *If C is an uniformly linear and diffuse coupler, then there exists a constant $m_C > 0$ such that for any measurable subset B of \mathcal{P} we have:*

$$\lambda_p(B) \leq m_C \lambda(B) \quad , \quad \forall p \in \mathcal{P}$$

Useful subsets of $A(\tau, C)$

The kind of systems we want to handle comes from differential equations studied near limit cycles, so, like in the finite population case, we need to avoid some degenerated trajectories. In order to do this, if $s \in \mathcal{T}^\tau$, we define the set of all cells on which the variations of s are not negligible. Precisely, if $\varepsilon > 0$, we define the following set:

$$V^\varepsilon(s) = \left\{ p \in \mathcal{P} \mid \inf_{c \in \mathbb{R}^n} \|s(\cdot, p) - c\| > \varepsilon \right\}$$

We call ε -kernel of C the set:

$$\text{Ker}^\varepsilon(C) = \{s \in \mathcal{T}^\tau, \quad V^\varepsilon(C(s)) \neq \mathcal{P}\}$$

An element in the ε -kernel of C contains, for at least one cell p , an application $s(\cdot, p)$ which image under C is uniformly near zero. Those solutions are out of interest here because they can be seen as solution of a system on a sub-population of C . Indeed for such a p $s'(\cdot, p)$ is negligible and $s(\cdot, p)$ nearly constant. In other words, we only consider trajectories which dynamic in each cell is "truly" periodic.

Given a period τ on \mathcal{P} , any element s of $A(\omega, C)$ and $\delta > 0$, the following set gathers cells which period are "not isolated", it will be naturally linked to $V^\varepsilon(s)$:

$$R_C^\delta(\tau) = \{p \in \mathcal{P} \mid \lambda_p(\mathcal{P}_p^\mathbb{Q}) > \delta \quad \}$$

If $p \in R_C^\delta(\tau)$, there is a set, which measure is at least δ , consisting of cells with periods commensurable with $\tau(p)$. Moreover, those cells are situated in the neighborhood of p defined by S_p .

Ergodic alternative

Like in the finite case, we will use this alternative within the following form [2]:

Proposition 1. *Let τ be a period on \mathcal{P} and C a tissular diffuse coupler, then for all s in $A(\tau, C)$ we have:*

$$C(s)(t, p) = \int_{\mathcal{P}_p^{\mathbb{Q}}} c_d(s(t, p), p, q) s(t, q) dq + \int_{\mathcal{P} - \mathcal{P}_p^{\mathbb{Q}}} c_d(s(t, p), p, q) \tau(q) I^{\tau(q)}(s)(q) dq$$

This results clearly shows that the ergodic theorem acts like a filter on periodic solutions, separating commensurable periods from the others. It is a key element of our proofs.

3 Synchronization with uniformly linear diffuse tissular coupler

As it is suggested in the title, in this section the kind of coupler we handle has the following shape:

$$C(s) = \int_{\mathcal{P}} c_d(., q) s(q) dq = L_C.s$$

Here is the first result explaining the link between the sets V and R . It will be generalized in the next result:

Proposition 2. *Let τ be a period on \mathcal{P} , for any s in $A(\tau, C)$ the following inclusion holds:*

$$V^0(C(s)) \subset R_C^0(\tau)$$

Noticing that the elements of $A(\tau, C)$ are uniformly bounded on I , we can then consider $\|s\| = \sup_{I \times \mathcal{P}} \|s(t, p)\|$ and use it to acquire the generalization:

Proposition 3. *Let τ be a period on \mathcal{P} , for any s in $A(\tau, C)$ we have:*

$$V^\varepsilon(C(s)) \subset R_C^{\frac{\varepsilon}{\|s\|}}(\tau)$$

In particular, as \mathcal{P} is a compact set, we can state:

Corollary 1. *Let τ be a period on \mathcal{P} and suppose there exists $\varepsilon > 0$ and $e \in A(\tau, C) - \text{Ker}^\varepsilon(C)$, then there exists a finite number of cells p_1, \dots, p_j such that:*

$$\mathcal{P} = \bigcup_{i=1}^j \mathcal{P}_{p_i}^{\mathbb{Q}}$$

The latter result leads us to this somewhat different result:

Corollary 2. *Let τ be a period on \mathcal{P} and suppose there exists $\varepsilon > 0$ and $e \in A(\tau, C) - \text{Ker}^\varepsilon(C)$, then there exists a sequence $\{p_i\}_{i \in \mathbb{N}}$ such that :*

$$\mathcal{P}_{p_i}^{\mathbb{Q}} = \bigcup_i \mathcal{P}_{p_i}^1$$

These results indicate a great reduction of the problem as they show that the periods cannot be arbitrary scattered. We are now able to prove the main result of this section. In order to handle only true cases of synchronization, we add an assumption on τ so that we won't have to deal with sub-period stuff:

Definition 1. *τ is said to be simple if $2 \text{Conv}(\tau(\mathcal{P})) \cap \text{Conv}(\tau(\mathcal{P})) = \emptyset$.*

Under this hypothesis, we have the following global synchronization result:

Theorem 1. *Suppose τ is a simple period on \mathcal{P} , if there exists $\varepsilon > 0$ and s in $A(\tau, C) - \text{Ker}^\varepsilon(C)$, then τ is a constant map on \mathcal{P} .*

Remark 1. To prove this result, we used the continuity of τ , deduced from the one of c_d . Nevertheless, leaving this assumption aside, we can get to an interesting result which could lead to generalization. We only need corollary 2 (we point that its validity does not directly depends on c_d 's continuity but on the uniform absolute continuity of the family λ_p and on the fact that $\|c_d(.,.)\|$ is uniformly bounded on \mathcal{P}^2).

Definition 2. *The set of point with isolated period, written \mathcal{P}^0 , is defined as:*

$$\mathcal{P}^0 = \{p \in \mathcal{P}, \quad \lambda_p(\mathcal{P}_p^1) = 0\}$$

The following proposition shows that under minimal assumptions, there are almost no points with isolated period:

Theorem 2. *Suppose there exists a partition as in the corollary 2 and that*

$$\mathcal{P} \subset \bigcup_{p \in \mathcal{P}} S_p$$

then $\lambda(\mathcal{P}_0) = 0$.

The latter result gives a global information that is verified in a general case: almost no cell has an isolated period. Those two results are really different, the first one is less theoretical but refers to stronger hypotheses than the second one, which comes under the field of measure theory.

4 Synchronization with uniformly linear tissular coupler

Now, we consider the general case, leaving aside the assumption that C is diffuse. Thus, in this last and short section, we consider a uniformly linear tissular coupler on \mathcal{P} (c_a needs no more to be zero). Like in the finite population case, the first step consists in detecting and eliminating degenerated solutions:

Definition 3. Let $s \in \mathcal{T}(I, \mathcal{E}_b)$ and $\varepsilon \geq 0$, s is said to be an ε -eigenvector (for C) if:

$$C(s) - c_a \in \text{Ker}^\varepsilon(C)$$

An ε -eigenvector is therefore a s which contains a map $e(., p)$ uniformly near a hypothetical eigenvector of C for the eigenvalue $c_a(p)$.

Finally, here is the generalization of theorem 1:

Theorem 3. Let τ be a simple period on \mathcal{P} and $\varepsilon > 0$. If there exists $s \in A(\tau, C)$ which is not an ε -eigenvector, then τ is a constant map.

5 Conclusion

In this work, we have given another example of how the notion of tissular coupling can help to address synchronization issues within a wide variety of coupled systems. Our approach is completing the classical methods showing that within suitable conditions, synchronization must occur along time. Those results are often qualitatively dependent to the studied systems. After the first positive result of our accompanying paper [1], we hold to our approach and didn't prove that synchronization ultimately happens but demonstrated that if coupled systems oscillate, then they must be synchronized. As in the finite population situation, the loss in time evolution informations is compensated by very general results, almost independent from the individual differential systems to be coupled. Furthermore, conclusions are less restrictive than in [1] (there is no threshold like "half of the population is synchronized"). Moreover, we hope that this complementary approach to synchronization issues will lead to future developments as it brings about, also in this paper, many mathematical tools unfamiliar to this field of research. Furthermore, other examples of the use of tissular coupling can be found in [2] concerning emergence of spatial patterns. All those works and results are inspiring and seem to prove that tissular coupling will be a prolific framework, completing what already exists.

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Autonomous Dynamical Systems with Periodic Coefficients

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Summary. The solutions of autonomous dynamical systems with periodic coefficients mainly depend on the Floquet-Liapunov exponents of a Hill's associated equation. These exponents are computed without integration by a very fast algorithm which exponentially converges.

So, some important features of the solutions behaviour, such as the location of the temporal mean in the phase plane, funnelling phenomenon, period doubling, parametric resonances, can be specified. In this paper, the implementation of the method is shown on a parametric Van der Pol equation.

Key words: slow-fast dynamics; Floquet-Liapunov exponents; Hill's equation.

1 Some previous results

Consider the Van der Pol equation:

$$\begin{cases} \varepsilon \frac{dx(t)}{dt} = -\frac{x^3}{3} + x + y \\ \frac{dy(t)}{dt} = -x \end{cases} \quad (1)$$

where $\varepsilon = 0.05$. In previous works, we established that the location of the points where the curvature of the trajectory in the phase plane vanishes provides the following equation of a slow manifold [1]:

$$\phi = y - \frac{x^5 - 4x^3 + 3x(1 - \varepsilon)}{3(x^2 - 1)} = 0 \quad (2)$$

Moreover, the attractive part of this manifold, given by $\mathbf{V} \cdot \nabla \phi > 0$, is an invariant manifold.

2 Interpretation

Let $\xi(t)$ and $\eta(t)$ be small variations around any point (x, y) . The variation equation, which we call “tangent-system”, is the linearized equation:

$$\begin{bmatrix} \frac{d\xi(t)}{dt} \\ \frac{d\eta(t)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} & \frac{\partial f(x,y)}{\partial y} \\ \frac{\partial g(x,y)}{\partial x} & \frac{\partial g(x,y)}{\partial y} \end{bmatrix} \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{\varepsilon}(-x^2 + 1) & \frac{1}{\varepsilon} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} \quad (3)$$

If $\lambda_k, k = 1, 2$, are the eigenvalues, the eigenvectors are given by :

$$y_k = \begin{bmatrix} -\lambda_k \\ 1 \end{bmatrix}, \quad k = 1, 2$$

Each eigenvalues determine a mode. In the regions of the phase plane where the eigenvalues are real, one of them, say λ_2 , is proportional to ε^{-1} and its absolute value much larger. When it is negative, the associated mode is evanescent. Let λ_1 be the other eigenvalue. We also established that the exact equation of the invariant slow manifold, on which the slow trajectories of the dynamical system (1) lean, is obtained by writing that the velocity is parallel to the eigendirection y_1 associated to the non evanescent mode. So, we obtain another expression of the exact equation of the slow part of the Van der Pol equation limit cycle:

$$y = -\frac{x^3}{3} - x + \varepsilon x \lambda_1 = \frac{x^3}{3} - x + \frac{1}{2}x \left(1 - x^2 + \sqrt{(1-x)^2 - 2\varepsilon}\right) \quad (4)$$

3 Autonomous dynamical systems with periodic coefficients

Now, consider the parametric Van de Pol equation

$$\begin{cases} \frac{dx(t)}{dt} = \frac{1}{\varepsilon} \left(-\frac{x^3}{3} + x + \theta(t)y \right) \\ \frac{dy(t)}{dt} = -x \end{cases} = \begin{bmatrix} f(x, y, t) \\ g(x, y, t) \end{bmatrix} \quad (5)$$

where $\theta(t)$ is a T-periodic mapping from \mathbb{R} to \mathbb{R} , $t \mapsto \theta(t)$, supposed to have a Fourier development limited to p_M terms :

$$\theta(t) = \sum_{p=-p_M}^{p_M} \theta_P e^{ip\omega t}, \text{ with } \omega = \frac{2\pi}{T} \text{ and } \theta_{-k} = \theta_k^* \quad (6)$$

Let $\theta_0 = 1$. Now, the variation equation is a homogeneous linear equation with periodic coefficients, a Hill equation:

$$\begin{bmatrix} \frac{d\xi(t)}{dt} \\ \frac{d\eta(t)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{1}{\varepsilon} (-x^2 + 1) & \frac{\theta(t)}{\varepsilon} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} \quad (7)$$

According to the Floquet theory, each of the two solutions of this equation can be written as:

$$\eta(t) = e^{\mu t} \psi(t), \quad (8)$$

where the complex number μ is the Floquet exponent and $\psi(t)$ the T-periodic part of the solution

$$\psi(t) = \sum_{m=-\infty}^{\infty} \psi_m e^{im\omega t}, \text{ with } \psi_{-m} = \psi_m^*. \quad (9)$$

There are two modes corresponding to the two Floquet exponents. We use the same argument as previously: if the fast Floquet exponent has a real negative part, the associated mode is evanescent and the other mode, a slow solution, remains.

4 Floquet's exponents computing

A generalization of the G. W. Hill method [4] allows us to deduce a very fast algorithm that exponentially converges to the exact numerical value [5].

5 Behaviour of the temporal mean of the solutions

Proposition 5.1 *Let μ_1 the slow Floquet's exponent. We introduce the associated dynamical system as the dynamical system with constant coefficients having μ_1 as slow eigenvalue. The temporal means of the slow solutions of a parametric dynamical system are located on the slow curve of this associated system. In our example, the equation of this curve is given by:*

$$y = -\frac{x^3}{3} - x + \varepsilon x \mu_1 \quad (10)$$

Proof. According to the Floquet's theorem, let $\eta(t) = e^{\mu_1 t} \psi(t)$ be the particular solution of (6) associated to μ_1 , the slow value of μ , where $\psi(t)$ is a T-periodic mapping. Since $\psi(nT) = \psi(0)$, $\forall n$ we have $\eta(nT) = \eta(0) e^{\mu_1 nT}$. The temporal means are located on the eigendirection related to the slow solutions of a dynamical system having μ_1 as slow eigenvalue. Then the trajectories are given by (4), in which λ_1 is replaced by μ_1 .

6 Some considerations on stability

Proposition 6.1 *Let $\phi(x, y) = y - \frac{x^3}{3} + x - \varepsilon x \mu_1 = 0$ the equation of the curve where the means are located. This curve is attractive with respect to the trajectories of the system if the scalar product $\mathbf{V} \cdot \nabla \phi$ is positive, where v is the velocity. The values where the stability changes, corresponding to $\mathbf{V} \cdot \nabla \phi > 0$, are*

$$\mathbf{V} \cdot \nabla \phi(x, y) = \left(-\frac{x^3}{3} + x + \theta(t)y \right) (-x^2 + 1 - \varepsilon \mu_1) - \varepsilon x = 0 \quad (11)$$

$$\text{where } \theta(t)y = \frac{x^3}{3} - x + \frac{\varepsilon x}{-x^2 + 1 - \varepsilon \mu_1} \quad (12)$$

7 Funnelling

Proposition 7.1 *The amplitude of the oscillations of the slow mode decreases in the regions where the Floquet exponent μ_1 is negative. It is sensitive to initial conditions in the regions where μ_1 is positive.*

Proof. The amplitude of the oscillations of the slow mode being modulated by $e^{\mu_1 t}$, their amplitude decreases all along the mean curve where μ_1 is negative and exponentially increases all along the mean curve where μ_1 is positive. We have drawn the phase plane for several values of the amplitude and of the frequency for a sinusoidal parameter $\theta(t)$. In every picture, this funnelling phenomenon appears along the slow trajectories.

8 Parametric resonance, period doubling

The imaginary part of the Floquets exponent is related to the frequency of oscillations: some important features of the solution, such as parametric resonances and bifurcations by period doubling can be brought to light.

9 Discussion

Several features, like the mean location, the amplitude, the stability of solutions of parametric dynamical systems depend on the real part of the Floquet coefficients. The imaginary part is related to parametric resonances and bifurcations by period doubling. The same algorithm can be used to compute the Liapunov exponents of n^{th} - order Hill's equations associated to higher order parametric dynamical systems, for example predator-prey models that take in account the daily or the annual variations of parameters, or dynamical systems related to periodic biologic rythms.

10 Figures

In this section are presented some solutions of the Van der Pol parametric equation

$$\begin{cases} \frac{dx(t)}{dt} = \frac{1}{\varepsilon} \left(-\frac{x^3}{3} + x + \theta(t)y \right) \\ \frac{dy(t)}{dt} = -x \end{cases}$$

With $\theta(t) = \theta_0 + \theta_1 \cos(\omega t)$. For all pictures, $\varepsilon = 0.05$ and $\theta_0 = 1$. The period T and the amplitude θ_1 of the only harmonic are specified. The red line is the mean trajectory of the parametric dynamical system, say the invariant manifold of the associate dynamical system having the Floquet's exponent as eigenvalue. The green line is the nullcline, corresponding to $\frac{dx(t)}{dt} = 0$ and $\theta(t) = \theta_0 = 1$. The blue line is the invariant manifold of the dynamical system with the constant coefficient $\theta_0 = 1$, the mean of the periodic coefficient.

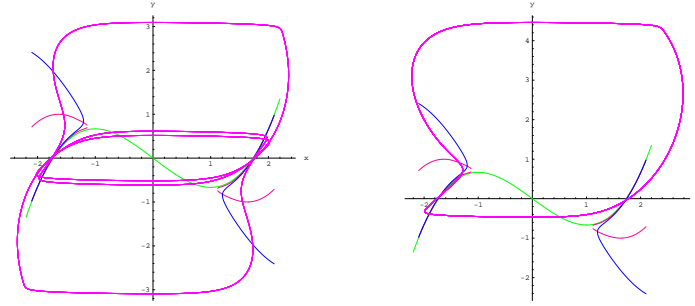


Fig. 1. Amplitudes 1, 1.5 - Period 5

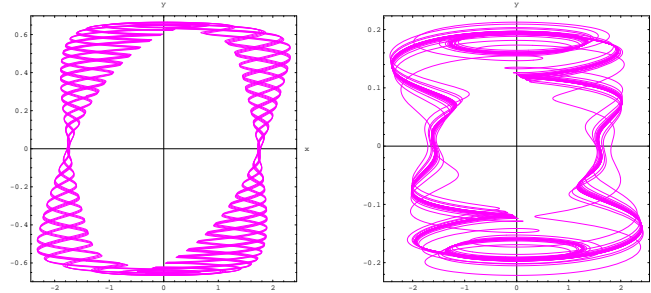


Fig. 2. Amplitudes 4, 30 - Periods 0.1, 0.089493

Table 1. The Floquet - Liapunov exponents role

	Tangent Linear System	Hill System
Characteristic exponent	Eigenvalue	Floquet exponent
Symbol	λ	μ
Fast evanescent modes	Large and negative values of $Re(\lambda)$	Large and negative values of $Re(\mu)$
Slow remaining modes	Smaller values of $Re(\lambda)$	Smaller values of $Re(\mu)$
Manifold $\phi = 0$ in the phase plane	The location of the points where trajectories are parallel to a slow eigendirection defines the slow manifold $\phi(\lambda) = 0$	To define the trajectory replace λ by μ in $\phi(\lambda)$. So we obtain $\phi(\mu) = 0$
Attractive part of the manifold: $\phi = 0$ and $\mathbf{V} \cdot \nabla \phi < 0$	Invariant manifold	Domain of the phase plane where the amplitude of the oscillation decreases (funnelling)
Repulsive part of the manifold: $\phi = 0$ and $\mathbf{V} \cdot \nabla \phi > 0$	If there is a fast positive eigenvalue: unreachable part of the manifold If they are all negative: weakly repulsive part of the manifold	Domain of the phase plane where the amplitude of the oscillation increases
Imaginary part	Frequency of oscillation, period doubling	Frequency of oscillation: period doubling (bifurcations) and parametric resonance

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Dynamics and complexity of Hindmarsh-Rose neuronal systems

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Summary. This work addresses the dynamics and complexity of neuron mathematical models. The aim is first the understanding of the biological meaning of existing mathematical systems concerning neurons such as Hodgkin-Huxley or Hindmarsh-Rose models. The local stability and the numerical asymptotic analysis of Hindmarsh-Rose model are then developed in order to comprehend the dynamic evolution of a single Hindmarsh-Rose neuron. This examination naturally comes to the study of neuron networks. The analysis of these networks uses the synchronization theory via connections between neurons and can give rise to emergent properties and self-organization. Our result leads to a classical law which describes many self-organized complex systems like earthquakes, linguistic or urban systems. This has been performed using numerical tools.

1 Introduction and neurophysiology

Also called *nerve cells*, *neurons* are the most important cells of the nervous system. They are composed of a cell body, or soma, extended by an axon, an axon terminal and some dendrites (see figure (1)).

These extensions are useful not only for the transmission of information through a neuron but also for the transmission from a neuron to another. Connections between neurons are possible thanks to synapses.

Ionic channels can be seen as macrocellular pores in neuron membrane. They enable molecules to pass through the membrane. They are the link between intra-cellular and extra-cellular space. There are many types of ionic channels. As some example, we may cite those which are always open, those which are voltage dependant or those which select the molecule allowed to cross...

Here are the ionic channels we will use in this paper. Two of them are voltage dependant.

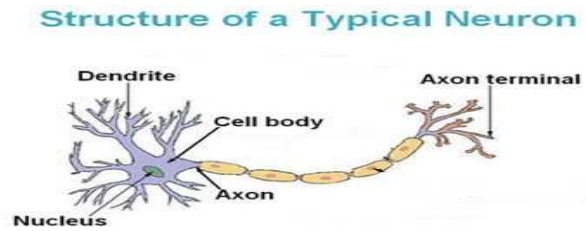


Fig. 1. Neuron.

Sodium channels are called so because they are specific for sodium ions. In fact, they have a particularity : they can be in an active state or in an inactive state. *Potassium channels* open and close with delay. *leak channel* are always open.

Neurons are enclosed by a membrane which separates the interior of the cell from the extracellular space. Inside the cell, the concentration of ions is different than in the surrounding liquid. The difference of concentration generates an electrical potential which plays an important role in neuronal dynamics.

When a neuron is not sending a signal, it is "at rest". In this case, the inside of the neuron is negative compared to the outside (see figure (2)). At rest, potassium ions (K^+) can cross through the membrane easily. Also at rest, sodium ions (Na^+) find this crossing more difficult. The negatively charged protein molecules inside the neuron cannot cross the membrane. When the difference in the voltage between the inside and the outside of the neuron is measured, we have the *resting potential*. The resting membrane potential of a neuron is about -70 mV, this means that the inside of the neuron is 70 mV less than the outside. At rest, there are relatively more sodium ions outside the neuron and more potassium ions inside that neuron.

At the *equilibrium potential* the flow of ions across the membrane does not stop, as the ions are always moving, but the same number of ions flow in both directions.

The equilibrium is predictable with the Nernst equation, which is given here in a form applicable to cell membranes:

$$E_S = \frac{RT}{ZF} \ln \frac{[S]_{out}}{[S]_{in}}$$

where :

E_S represents the Nernst potential for ion S (measured as for membrane potentials, inside with respect to outside),

$[S]_{out}$ represents the concentration of S outside the cell ,

$[S]_{in}$ represents the concentration of S inside the cell ,

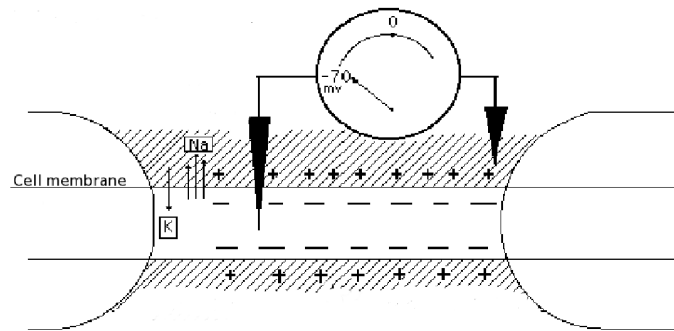


Fig. 2. Resting potential schematic representation.

Z represents the valence of ion S : $Z(K^+) = +1$, $Z(Na^+) = -1$,
 R represents the gas constant,
 T represents the absolute temperature,
 F represents Faraday's constant.

The resting potential informs us about what happens when a neuron is at rest. An *action potential* occurs when a neuron sends information down to an axon away from the cell body. Neuroscientists use other words, such as a "spike" or an "impulse" for the action potential. The action potential is an explosion of electrical activity that is created by a depolarizing current. This means that a stimulus causes the resting potential to move toward 0 mV. When the depolarization reaches about -55 mV a neuron will fire an action potential. This is the threshold. If the neuron does not reach this critical threshold level, then no action potential will fire. Also, when the threshold level is reached, an action potential of a fixed size will always fire. For any given neuron, the size of the action potential is always the same. Therefore, the neuron either does not reach the threshold or a full action potential is fired - this is the "*all or none*" principle.

Action potentials are caused by an exchange of ions across the neuron membrane. A stimulus first causes sodium channels to open. Because there are many more sodium ions on the outside, and the inside of the neuron is negative compared to the outside, sodium ions rush into the neuron. As sodium has a positive charge, the neuron becomes more positive and becomes *depolarized*. It takes longer for potassium channels to open. When they do open, potassium rushes out of the cell, reversing the depolarization. Roughly at that time, sodium channels start to close. This causes the action potential to go back toward -70 mV (a *repolarization*). The action potential actually goes past -70 mV (a *hyperpolarization*) because the potassium channels stay open a bit too long. Gradually, the ion concentrations go back to resting levels and the cell returns to -70 mV (see figure (3)).

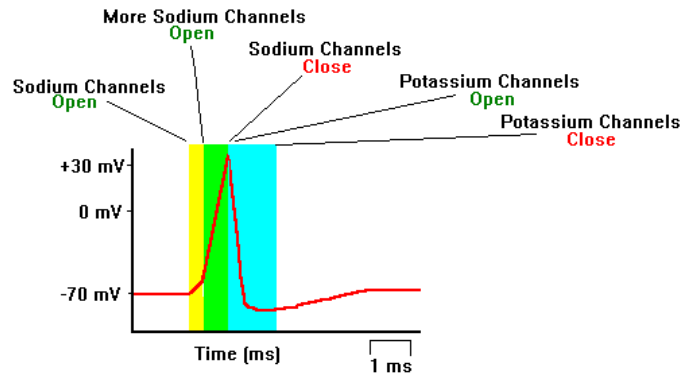


Fig. 3. Action potential.

2 Mathematical models

Alan Lloyd Hodgkin and *Andrew Fielding Huxley*'s goal was to account for ionic fluxes and permeability changes of the excitable membrane in terms of molecular mechanisms. They studied the squid giant neuron. After an intensive consideration of different mechanisms, they reluctantly concluded that they still needed to know more about it before a unique mechanism could be proven. Indeed, this conclusion is unfortunately still valid. They decided, instead, to develop an empirical kinetic description that would be simple enough to make practical calculations of electrical responses, yet sufficiently as to predict conduction correctly the major features of excitability. In this goal they succeeded. Their model not only comprises mathematical equations but also suggests major features of the gating mechanisms. For their whole work, they obtained the nobel prize in physiology or medicine in 1963.

Let us call this Hodgkin Huxley model the *HH model*.

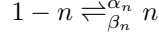
The computation is based on Sodium, Potassium and leakage ion flow.

This model has separated equations for g_{Na} and g_K . In each case, there is an upper limit to the possible conductance, so they are expressed as maximum conductances \bar{g}_{Na} and \bar{g}_K multiplied by coefficients representing the fraction of the maximum conductances actually expressed. The multiplying coefficients are numbers varying between 0 and 1. In the model, the conductance changes depend only on voltage and time. Because of their observations, Hodgkin and Huxley noted that such kinetic would be obtained if the opening of a K channel were controlled by several independent "particles". Suppose that there are four identical particles, each one with a probability n of being in the correct position to set up an open channel. The probability that all four particles are correctly placed is n^4 .

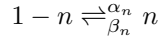
To express this in mathematical form, I_K is represented in the *HH model* by :

$$I_K = n^4 \bar{g}_K (E - E_K)$$

The voltage- and time- dependent changes of n are given by the reaction :



The voltage- and time- dependent changes of n are given by the reaction :



where the gating particles make transitions between the permissive and non-permissive forms with voltage-dependent rate constants α_n and β_n . If the initial value of the probability n is known, subsequent values can be calculated by solving the simple differential equation :

$$\frac{dn}{dt} = \alpha_n(1 - n) - \beta_n n$$

The HH model uses a similar formalism to describe I_{Na} , with four hypothetical gating particles making independent first order transitions between permissive and non permissive state to control the channel. However, because there are two opposing gating processes, activation and inactivation, there must be two kinds of gating particles. Hodgkin and Huxley called them h and m . Three m particles control activation, and one h particle, inactivation. Therefore, the probability that they are all in permissive position is $m^3 h$, and I_{Na} is represented by :

$$I_{Na} = m^3 h \bar{g}_{Na} (E - E_{Na})$$

As for the n parameter of K channels, m and h are assumed to undergo transitions between permissive and non-permissive forms with rates satisfying differential equations :

$$\begin{aligned} \frac{dm}{dt} &= \alpha_m(1 - m) - \beta_m m \\ \frac{dh}{dt} &= \alpha_h(1 - h) - \beta_h h \end{aligned}$$

The HH model for the squid giant axon describes ionic current accross the membrane in terms of three components.

$$I_i = m^3 h \bar{g}_{Na} (E - E_{Na}) + I_K = n^4 \bar{g}_K (E - E_K) + \bar{g}_L (E - E_L)$$

where \bar{g}_L is a fixed background leakage conductance.

The HH model is as follows :

$$\begin{cases} -C \frac{dV}{dt} = m^3 h g_{Na} (E - E_{Na}) + n^4 \bar{g}_K (E - E_K) + \bar{g}_L (E - E_L) - I \\ \frac{dn}{dt} = \alpha_n (1 - n) - \beta_n n \\ \frac{dm}{dt} = \alpha_m (1 - m) - \beta_m m \\ \frac{dh}{dt} = \alpha_h (1 - h) - \beta_h h \end{cases}$$

The parameter names are fixed variables :

I : the total ionic current across the membrane ,
 m : the probability that one of the three required activation particles has contributed to the activation of the Na gate (m^3 : the probability that all three activation particles have produced an open channel) ,
 h : the probability that the one inactivation particle has not caused the Na gate to close ,
 g_{Na} : maximum possible Sodium Conductance ,
 E : total membrane potential ,
 E_{Na} : Na membrane potential ,
 n : the probability that one of four activation particles has influenced the state of the K gate ,
 g_K : maximum possible Potassium Conductance ,
 E_K : K membrane potential ,
 G_L : maximum possible Leakage Conductance ,
 E_L : leakage membrane potential ,
 α_m : constant rate for particle not activating a gate ,
 β_m : constant rate for particle activating a gate .

In 1982, *Hindmarsh* and *Rose* have simplified the Hodgkin-Huxley model. The model they obtained is a FitzHugh-Nagumo type and can be a *slow-fast* system.

Indeed, they observed that some variables could be replaced by constants, they found relations between different variables, and therefore, they could simplify the Hodgkin-Huxley system into a more simple two equations one, as follows ;

$$\begin{cases} F(x) = ax^3 - bx^2 \\ G(x) = c - dx^2 \end{cases}$$

Two years later, *Hindmarsh* and *Rose* decided to add a third equation to their model, so that the motion of their neuron could be closer to the motion of a real one. The last equation enables the model to control the rest period between two action potential.

$$\begin{cases} \dot{x} = y + ax^2 - x^3 - z + I \\ \dot{y} = 1 - dx^2 - y \\ \dot{z} = \mu(b(x - x_e) - z) \end{cases} \quad (1)$$

With $a = 3$, $b = 4$ represents the influence of the membrane potential on the slow dynamics, $d = 5$ is a constant experimentally determined, $I = 3.25$ is the applied current, $\mu = 0.001$ is a recovery variable, $x_e = -\frac{1}{2}(1 + \sqrt{5})$ which is the equilibrium of the 2 dimensional Hindmarsh Rose System, corresponding to the threshold potential for the triggering of bursts.

A possible variable changing is as follows (see [8]) :

$$y = 1 - y, z = 1 + I + z, d = a + \alpha, c = -1 - I - bx_e.$$

Then we obtain the following system :

$$\begin{cases} \dot{x} = ax^2 - x^3 - y - z \\ \dot{y} = (a + \alpha)x^2 - y \\ \dot{z} = \mu(bx + c - z) \end{cases} \quad (2)$$

For this system, we choose the following parameters :

$$a = 3, \alpha = d - a = 2, b = 4, c = -1 - I - bx_e = -4.25 + 2(1 + \sqrt{5}), \mu = 0.001. \quad (3)$$

3 Local dynamics and bifurcations

Let us consider the Hindmarsh-Rose system :

$$\begin{cases} \dot{x} = y + ax^2 - x^3 - z + I \\ \dot{y} = 1 - dx^2 - y \\ \dot{z} = \mu(b(x - x_e) - z) \end{cases}$$

Equilibria are given by the following system :

$$\begin{cases} y + ax^2 - x^3 - z + I = 0 \\ 1 - dx^2 - y = 0 \\ \epsilon(b(x - x_e) - z) = 0 \end{cases}$$

We obtain the following polynomial equation :

$$x^3 + (d - a)x^2 + bx + (-bx_e - I - 1) = 0.$$

There is a unique equilibrium given by :

$$(x_0, y_0, z_0) \cong (-0.7221259058, -1.607329119, 3.583632333).$$

The nature of this equilibria is given by the study of the eigenvalues of the jacobian matrix J of this system :

$$J = \begin{pmatrix} -3x^2 + 2ax & 1 & -1 \\ -2dx & -1 & 0 \\ \epsilon b & 0 & -\epsilon \end{pmatrix}$$

which are : $\lambda_1 \cong 0.0020576373 > 0$, $\lambda_2 \cong 0.1835898203 > 0$, and $\lambda_3 \cong -7.083532689 < 0$.

Thus, the equilibrium is a saddle-node.

The chaotic behavior of the Hindmarsh-Rose system is shown by numerical analysis in different regions of the parameter space. A special period-halving route to chaos is observed, as show in figures (4, 5). These figures give bifurcations diagrams for the system fixed by (3). In figure (4), I is varied as the bifurcation parameter : $I \in [1.5 : 4]$. We have done the numerical calculations for different initial conditions and for different variations of parameters. In order to obtain reliable numerical results, the step size for the integration procedure has been choosen to be equal to 10^{-3} or less, and the first 10^5 steps are discarded to avoid the transient regime.

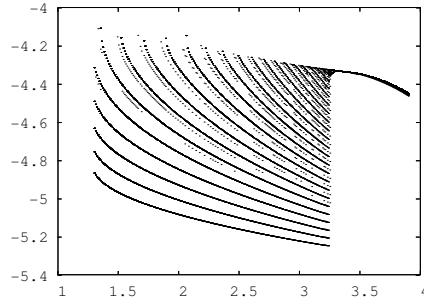


Fig. 4. Bifurcation diagram for parameter I .

We can also compare the bifurcation diagram for parameter I and the time series. Indeed, for a fixed I , the number of spikes we can observe in the time series (see figure (7)) is the same as the number of branch in the bifurcation diagram (see figure (4)).

Obviously, this figures show transitions from stable to periodic or quasiperiodic, then complex and finally chaotic behaviour.

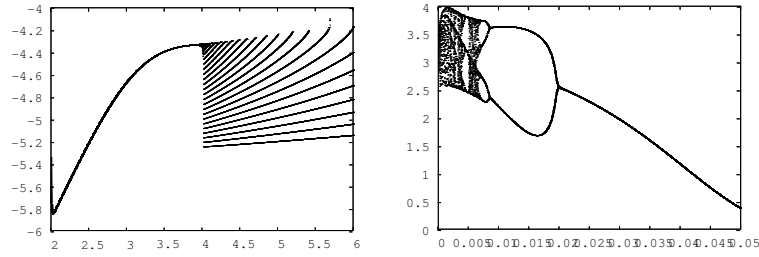


Fig. 5. (a) Bifurcation diagram for parameter b in the system (1) , (b) Bifurcation diagram for parameter μ in the system (2).

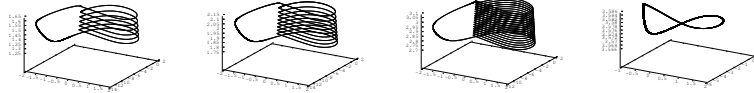


Fig. 6. Phases portrait for $I = 1.5, I = 2, I = 3, I = 3.5$.

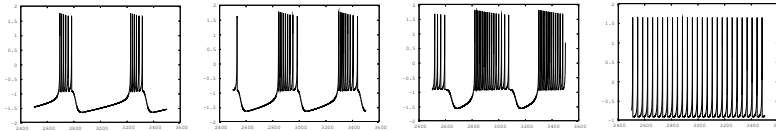


Fig. 7. Time series for $I = 1.5, I = 2, I = 3, I = 3.5$.

4 Synchronization

4.1 Generalities

Synchronization is a phenomenon characteristic of many processes in natural systems and non linear science. It has remained an objective of intensive research and is today considered as one of the basic nonlinear phenomena studied in mathematics, physics, engineering or life science.

This word has a greek root, *syn* = *common* and *chronos* = *time*, which means to share common time or to occur at the same time, that is correlation or agreement in time of different processes.

Thus, synchronization of two dynamical systems generally means that one system somehow traces the motion of another.

Chaotic oscillators are found in many dynamical systems of various origins. The behavior of such systems is characterized by instability and, as a result, limited predictability in time. Roughly speaking, a system is chaotic if it is deterministic, if it has a long-term aperiodic behavior, and if it exhibits

sensitive dependence on initial conditions on a closed invariant set. Consequently, for a chaotic system, trajectories starting arbitrarily close to each other diverge exponentially with time, and quickly become uncorrelated. It follows that two identical chaotic systems cannot synchronize. This means that they cannot produce identical chaotic signals, unless they are initialized at exactly the same point, which is in general physically impossible.

Thus, at first sight, synchronization of chaotic systems seems to be rather surprising because one may intuitively expect that the sensitive dependence on initial conditions would lead to an immediate breakdown of any synchronization of coupled chaotic systems, which led to the belief that chaos is uncontrollable and thus unusable.

Despite this, in the last decades, the search for synchronization has moved to chaotic systems. A lot of research has been carried out and, as a result, showed that two chaotic systems could be synchronized by coupling them : synchronization of chaos is actual and chaos could then be exploited. Ever since, many researchers have discussed the theory, the design or applications of synchronized motion in coupled chaotic systems. A broad variety of applications have emerged, for example to increase the power of lasers, to synchronize the output of electronic circuits, to control oscillations in chemical reactions or to encode electronic messages for secure communications.

Several different regimes of synchronization have been investigated. Here are some examples of synchronization regimes ;

- *Identical (or complete) Synchronization*, which is defined as the coincidence of states of interacting systems ;
- *Generalized Synchronization*, which extends the identical synchronization phenomenon and implies the presence of some functional relation between two coupled systems ; if this relationship is the identity we recover the identical synchronization ;
- *Phase Synchronization*, which means driving of phases of chaotic oscillators, whereas their amplitudes remain uncorrelated ;
- *Lag Synchronization* which appears as a coincidence of shifted-in-time states of two systems.

4.2 Coupling Hindmarsh-Rose neurons

Let us consider a network composed by n Hindmarsh-Rose neurons. These neurons are coupled by their first variable x . This network can be modeled by the system :

$$\begin{cases} \dot{x}_i = ax_i^2 - x_i^3 - y_i - z_i - h(x_i, x_j), & i \neq j, \quad i = 1, \dots, n, \quad j = 2, \dots, n \\ \dot{y}_i = (a + \alpha)x_i^2 - y_i \\ \dot{z}_i = \mu(bx_i + c - z_i) \end{cases} \quad (4)$$

The coupling function h is given by (see [8]):

$$h(x_i, x_j) = (x_i - V_s)g_s \sum_{j=1}^n c_{ij} \Gamma(x_j) \quad (5)$$

in which the synaptic coupling Γ is modeled by a sigmoid function with a threshold :

$$\Gamma(x_j) = \frac{1}{1 + \exp(-\lambda(x_j - \Theta_s))} \quad (6)$$

Θ_s is the threshold reached by every action potential for a neuron. Neurons are supposed to be identical and the synapses are fast and instantaneous. The parameter g_s corresponds to the synaptic coupling strength. The synapse is excitatory, that is why the reversal potential V_s must be larger than $x_i(t)$ for all i and all t .

$C_n = (c_{ij})$ is the $n \times n$ connectivity matrix :

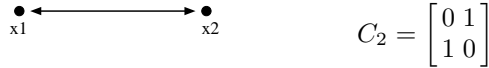
$$\begin{cases} c_{ij} = 1 \text{ if } i \text{ and } j \text{ are connected, } i = 1, \dots, n, \quad j = 2, \dots, n \\ c_{ij} = 0 \text{ if } i \text{ and } j \text{ are not connected.} \end{cases}$$

This matrix C can be symmetric or not, so that unidirectional or bidirectional coupling are possible.

The synchronous solution exists if and only if each neuron has the same number of inputs from other neurons (see [8], [9]).

Coupling two Hindmarsh-Rose neurons

The first step here is to adapt the previous method to two Hindmarsh-Rose neurons. In this case, we use a bidirectional connection since the number of inputs (that is one) has to be the same for all neurons to have a synchronous solution.



Bidirectional synchronization for two neurons and the connectivity matrix C_2 .

The following system represents our two Hindmarsh-Rose neurons bidirectionally coupled :

$$\begin{cases} \dot{x}_1 = ax_1^2 - x_1^3 - y_1 - z_1 - (x_1 - V_s)g_s \frac{1}{1 + \exp(-\lambda(x_2 - \Theta_s))} \\ \dot{y}_1 = (a + \alpha)x_1^2 - y_1 \\ \dot{z}_1 = \mu(bx_1 + c - z_1) \\ \dot{x}_2 = ax_2^2 - x_2^3 - y_2 - z_2 - (x_2 - V_s)g_s \frac{1}{1 + \exp(-\lambda(x_1 - \Theta_s))} \\ \dot{y}_2 = (a + \alpha)x_2^2 - y_2 \\ \dot{z}_2 = \mu(bx_2 + c - z_2) \end{cases} \quad (7)$$

Parameters are fixed as follows : $a = 2.8$, $\alpha = 1.6$, $c = 5$, $b = 9$, $\mu = 0.001$, $\Theta_s = -0.25$, $V_s = 2$, $k = 1$.

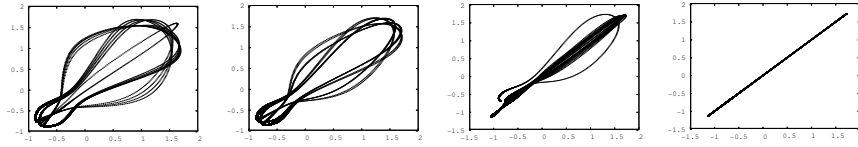
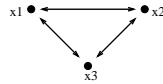


Fig. 8. x_2 according to x_1 for the coupling strength : (a) $g_s = 0.80$, (b) $g_s = 1.00$, (c) $g_s = 1.22$, (d) $g_s = 1.26$.

We observe that the synchronization numerically appears for a coupling strength $g_s \geq 1.26$. This phenomenon is also observed for the variables y and z , for the same numerical values of parameters.

Coupling three Hindmarsh-Rose neurons

In the case of three neurons, we also use bidirectional connections since the number of inputs (that are two) has to be the same for all neurons to have a synchronous solution.



$$C_3 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Bidirectional synchronization for three neurons and the connectivity matrix C_3 .

System (4) with $n = 3$ reads as :

$$\left\{ \begin{array}{l} \dot{x}_1 = ax_1^2 - x_1^3 - y_1 - z_1 - (x_1 - V_s)g_s \left(\frac{1}{1 + \exp(-\lambda(x_2 - \Theta_s))} + \frac{1}{1 + \exp(-\lambda(x_3 - \Theta_s))} \right) \\ \dot{y}_1 = (a + \alpha)x_1^2 - y_1 \\ \dot{z}_1 = \mu(bx_1 + c - z_1) \\ \\ \dot{x}_2 = ax_2^2 - x_2^3 - y_2 - z_2 - (x_2 - V_s)g_s \left(\frac{1}{1 + \exp(-\lambda(x_1 - \Theta_s))} + \frac{1}{1 + \exp(-\lambda(x_3 - \Theta_s))} \right) \\ \dot{y}_2 = (a + \alpha)x_2^2 - y_2 \\ \dot{z}_2 = \mu(bx_2 + c - z_2) \\ \\ \dot{x}_3 = ax_3^2 - x_3^3 - y_3 - z_3 - (x_3 - V_s)g_s \left(\frac{1}{1 + \exp(-\lambda(x_1 - \Theta_s))} + \frac{1}{1 + \exp(-\lambda(x_2 - \Theta_s))} \right) \\ \dot{y}_3 = (a + \alpha)x_3^2 - y_3 \\ \dot{z}_3 = \mu(bx_3 + c - z_3) \end{array} \right.$$

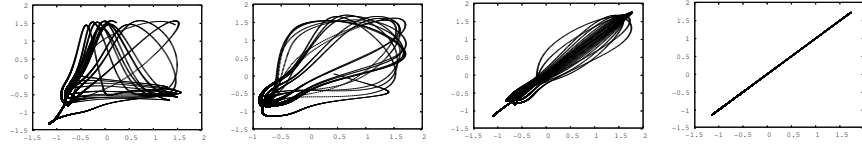


Fig. 9. x_2 according to x_1 for the following coupling strength : (a) $g_s = 0.20$, (b) $g_s = 0.40$, (c) $g_s = 0.60$, (d) $g_s = 0.63$.

The minimum coupling strength to exhibit synchronisation is about $g_s = 0.70$. Once again, we observe also this phenomenon for the variables y and z for the same numerical values of parameters.

Coupling four to eight Hindmarsh-Rose neurons

There are different possible ways for coupling four, five or more neurons with the same number of input from other neurons. Here, we decided to connect each neuron to all the others.

For system (4) with $n = 4$, $n = 5$, $n = 6$, $n = 7$, $n = 8$, we obtain :

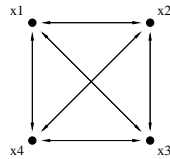


Fig. 10. Bidirectional synchronization for four neurons.

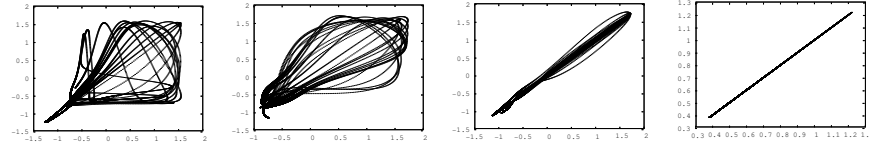


Fig. 11. x_2 according to x_1 when four neurons are connected for the coupling strength : (a) $g_s = 0.10$, (b) $g_s = 0.30$, (c) $g_s = 0.42$, (d) $g_s = 0.50$. Minimum coupling strength to observe synchronization between variables x_i and x_j , y_i and y_j , z_i and z_j ($i = 1, \dots, n$, $j = 2, \dots, n$) when coupling four neurons : $g_s = 0.50$.

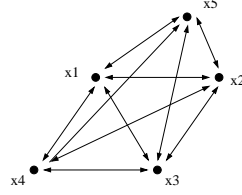


Fig. 12. Coupling five neurons.

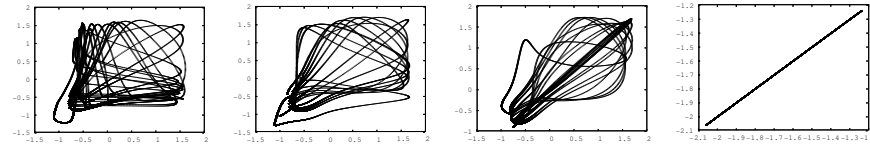


Fig. 13. x_2 according to x_1 when five neurons are connected for the coupling strength : (a) $g_s = 0.10$, (b) $g_s = 0.20$, (c) $g_s = 0.29$, (d) $g_s = 0.30$. Minimum coupling strength to observe synchronization between variables x_i and x_j , y_i and y_j , z_i and z_j ($i = 1, \dots, n$, $j = 2, \dots, n$) when coupling five neurons : $g_s = 0.30$.

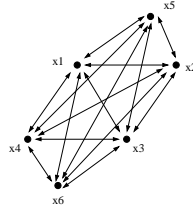


Fig. 14. Coupling six neurons.

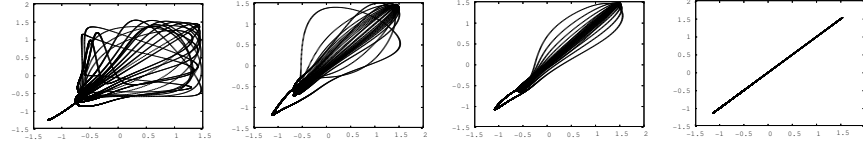


Fig. 15. x_2 according to x_1 when six neurons are connected for the coupling strength : (a) $g_s = 0.10$, (b) $g_s = 0.20$, (c) $g_s = 0.23$, (d) $g_s = 0.24$. Minimum coupling strength to observe synchronization between variables x_i and x_j , y_i and y_j , z_i and z_j ($i = 1, \dots, n$, $j = 2, \dots, n$) when coupling six neurons : $g_s = 0.24$.

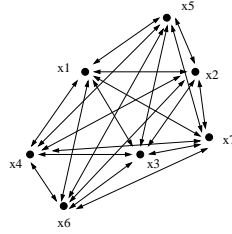


Fig. 16. Coupling seven neurons.

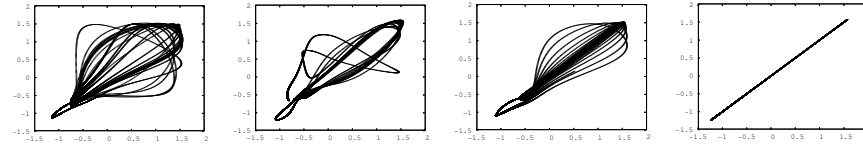


Fig. 17. x_2 according to x_1 when seven neurons are connected for the coupling strength : (a) $g_s = 0.10$, (b) $g_s = 0.17$, (c) $g_s = 0.20$, (d) $g_s = 0.21$. Minimum coupling strength to observe synchronization between variables x_i and x_j , y_i and y_j , z_i and z_j ($i = 1, \dots, n$, $j = 2, \dots, n$) when coupling seven neurons : $g_s = 0.21$.

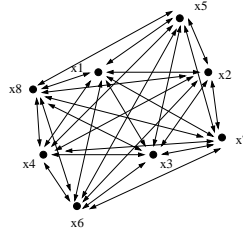


Fig. 18. Coupling eight neurons.

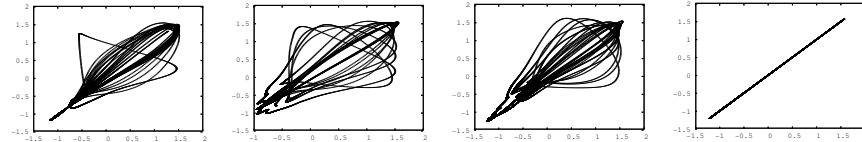


Fig. 19. x_2 according to x_1 when eight neurons are connected for the coupling strength : (a) $g_s = 0.10$, (b) $g_s = 0.15$, (c) $g_s = 0.16$, (d) $g_s = 0.17$. Minimum coupling strength to observe synchronization between variables x_i and x_j , y_i and y_j , z_i and z_j ($i = 1, \dots, n$, $j = 2, \dots, n$) when coupling eight neurons : $g_s = 0.17$.

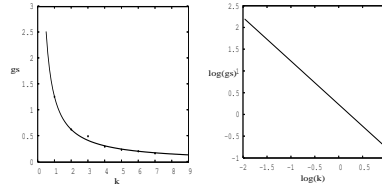


Fig. 20. (a) Comparing $g_s = g/k$ curve and our numerical results, (b) $\log(g_s)$ versus $\log(k)$.

As the previous numerical results show, the synchronization threshold follows the heuristic law : $g_s = g/k$. This result confirms the one given in [8]. In this equation, k is the number of inputs for each neuron (here $k = n - 1$ in a network of n neurons as every neuron is connected to all the others), and g is the synchronization threshold for two neurons mutually coupled.

5 Conclusion

Emergence and complexity refer to the appearance of higher-level properties and behaviours of a system that obviously comes from the collective dynamics of that system's components. These properties are not directly deductable from the lower-level motion of that system. Emergent properties are properties of the 'whole' that are not possessed by any of the individual parts making up that whole. For example, an air molecule is not a cyclone, an isolated species

does not form a food chain and an isolated neuron is not conscious : emergent behaviours are typically novel and unanticipated. Thus, if a synchronization phenomenon is exhibited in a neuron network, the transmission of information between these neurons is optimal and the network can develop consciousness. This is an emergent property which comes from the collective dynamics of n neurons. Moreover, as given in figure(20), as the number of neurons n goes larger, the synchronization threshold g gets smaller. Consciousness is more important when the number of neurons is larger. This curve (20 b) corresponds to a classical law (for n large enough) which can describe many self-organized complex systems, like earthquakes (for 1000 earthquakes of magnitude 4 on Richter scale, there are only 100 of magnitude 5 and 10 of magnitude 6), linguistic (for 1000 occurrences of 'the' in an english text, there are only 100 occurrences of 'I' and 10 of 'say'), urban systems (big cities are rare and small ones are frequent in an exponential way).

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Complexity and self-organization in a Michaelis-Menten predator-prey type

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Summary. In this paper we consider a predator-prey system modelising by a reaction-diffusion equation. It incorporates the Holling-type-II and a modified Leslie-Gower functional responses. We focus on mechanisms responsible for transitions between different kind of dynamics. We identify factors that can potentially enhance or suppress chaos and give rise to emergence of complexity and self-organisation.

1 Introduction

The dynamic relationship between species and its their complex properties are at heart of many important ecological and biological processes. Predator-prey dynamics are a classic and relatively well-studied example of interactions. This paper addresses the analysis of the emerging properties and dynamics of a system of this type. We assume that only basic qualitative features of the system are known, namely the invasion of a prey population by predators. The local dynamics has been studied in [4, 5]. Version with impulsive term is studied in [10]. Similar model with delay is studied in [7, 8], and a three dimensional similar system with the same functional responses is studied in [1, 2, 3].

This model incorporates the Holling-type-II and a modified Leslie-Gower functional responses.

Without diffusion it reads as, see [4, 5],

$$\begin{cases} \frac{dH}{dT} = \left(a_1 - b_1 H - \frac{c_1 P}{H + k_1} \right) H \\ \frac{dP}{dT} = \left(a_2 - \frac{c_2 P}{H + k_2} \right) P \end{cases} \quad (1)$$

with

$$H(0) \geq 0, P(0) \geq 0$$

H and P represent the population densities at time T . $r_1, a_1, b_1, k_1, r_2, a_2$, and k_2 are model parameters assuming only positive values.

The historical origin and applicability of this model is discussed in detail in [4, 5].

Now, we suppose that both predator and prey move in space, we represent the mathematical model by reaction-diffusion equations. It is a type of spatio-temporal model most commonly used in ecology and biology. We focus on mechanisms responsible of bifurcations and the spatio-temporal self-organization or chaos.

We first study the conditions of interior equilibrium global stability. Then, we give a bifurcation analysis when the species move on a straight line.

2 Mathematical model

The mathematical model we consider here consists of reaction-diffusion equations which expresses conservation of predator and prey densities. It has the following form,

$$\begin{cases} \frac{\partial H}{\partial T} = D_1 \Delta H + \left(a_1 - b_1 H - \frac{c_1 P}{H + k_1} \right) H \\ \frac{\partial P}{\partial T} = D_2 \Delta P + \left(a_2 - \frac{c_2 P}{H + k_2} \right) P, \end{cases} \quad (2)$$

H and P are the densities of prey and predators, respectively. ΔH (resp. ΔP) denotes $\frac{\partial^2 H}{\partial X^2}$ (resp. $\frac{\partial^2 P}{\partial X^2}$) when preys (resp. predators) move on a straight line

X or ΔH (resp. ΔP) denotes $\frac{\partial^2 H}{\partial X^2} + \frac{\partial^2 H}{\partial Y^2}$ (resp. $\frac{\partial^2 P}{\partial X^2} + \frac{\partial^2 P}{\partial Y^2}$) when the populations move on a 2 dimensional space. X (resp. (X, Y)) is the spatial position of species when they move on a straight line (resp. 2 dimensional space). D_1 and D_2 are the diffusion coefficients of prey and predator respectively. a_1 is the growth rate of preys H . a_2 describes the growth rate of predators P . b_1 measure the strength of competition among individuals of species H . c_1 is the maximum value of the per capita reduction of H due to P . c_2 has a similar meaning to c_1 . k_1 measures the extent to which environment provides protection to prey H . k_2 has a similar meaning to k_1 relatively to the predator P . To investigate problem (2), we introduce the following scaling transformations,

$$t = a_1 T, \quad u(t) = \frac{b_1}{a_1} H(T), \quad v(t) = \frac{c_2 b_1}{a_1 a_2} P(T), \quad x = X \left(\frac{a_1}{D_1} \right)^{\frac{1}{2}}, \quad y = Y \left(\frac{a_1}{D_1} \right)^{\frac{1}{2}} \quad (3)$$

$$a = \frac{a_2 c_1}{a_1 c_2}, \quad b = \frac{a_2}{a_1}, \quad e_1 = \frac{b_1 k_1}{a_1}, \quad e_2 = \frac{b_1 k_2}{r_1}, \quad \delta = \frac{D_2}{D_1} \quad (4)$$

We obtain the following equations, for local model,

$$\begin{cases} \frac{du}{dt} = u(1-u) - \frac{auv}{u+e_1} = f(u,v) \\ \frac{dv}{dt} = b\left(1 - \frac{v}{u+e_2}\right)v = g(u,v) \end{cases} \quad (5)$$

and the following system for the spatio-temporal equations:

$$\begin{cases} \frac{\partial u}{\partial t} = \Delta u + u(1-u) - \frac{auv}{u+e_1} = \Delta u + f(u,v) \\ \frac{\partial v}{\partial t} = \delta \Delta v + b\left(1 - \frac{v}{u+e_2}\right)v = \delta \Delta v + g(u,v), \end{cases} \quad (6)$$

System (5) admits a unique steady state (u^*, v^*) in the region $u > 0, v > 0$ if $ae_2 < e_1$, see [4], (u^*, v^*) for more information.

Here, we address the question of stability of (u^*, v^*) for (6). So, we assume the following conditions,

$$1 \leq e_1 \leq e_2 \quad (7)$$

$$\left(\frac{\delta}{u^* + e_1} (2u^{*2} + (e_1 - 1)u^*) + b \right)^2 \leq \frac{4\delta bu^*}{u^* + e_1} (2u^* + a + e_1 - 1) \quad (8)$$

Let us make some remarks before giving a global stability result of (u^*, v^*) for the spatio-temporal model (6).

Remark A steady state (u_e, v_e) of (5) is an equilibrium point of (6)

theorem If (7) and (8) are satisfied, then the steady state (u^*, v^*) is globally asymptotically stable for (6), (proof: see [3b]).

3 Local bifurcation in one-dimensional space

In this section, we present numerical results of our simulations in a one dimensional space. We suppose that the two species diffuse on a line. For the numerical approximation of this problem we combine the finite difference method using Crank-Nicolson scheme with Runge-Kutta scheme. The step of the numerical mesh were chosen as $dx = 0.2$, and $dt = 0.001$. At boundaries we use the zero-flux condition.

Let us consider the two following initial conditions in connection with biological invasion. It is to say that initial domain where the prey is larger than that of the predator, in order that it diffuses to find its food.

$$\begin{aligned} u(x,0) &= u_0 \text{ for } L_{1u} < x < L_{2u}, \text{ otherwise } u(x,0) = 0 \\ v(x,0) &= v_0 \text{ for } L_{1v} < x < L_{2v}, \text{ otherwise } v(x,0) = 0 \end{aligned} \quad (9)$$

For making, during the simulation, the impact of the boundaries as small as possible, we consider

$$0 < L_{1u} \leq L_{1v} < L_{2v} \leq L_{2u} < L$$

We choose these parameters so that the populations do not disappear when the growth or regression degree of prey ((birth quantity of prey)/(death quantity of prey)) and the predator vary. Actually, if the field where the species concentrate is small enough and or the density of predator is rather high we observe an extinction of the populations. It is a property of Allee effect (see [6]). In fact this phenomenon is connected to the questions of persistence and extinction. The population is said to have an Allee effect, if the growth rate per capita is initially an increasing function, then decreases to zero at a higher density. It appears for example for the parameters and the following initial conditions.

$$e_1 = 0.08, e_2 = 0.01, b = 0.2, a = 3.0, u_0 = 0.08, v_0 = 0.01, \delta = 1$$

We fix now the parameters as follow,

$$\begin{aligned} e_1 &= 0.08, e_2 = 0.01, b = 0.2, a = 3.0, u_0 = 1.0, v_0 = 0.1, L = 100, \\ L_{1u} &= 40, L_{1v} = 48, L_{2v} = 56, L_{2u} = 60, a = 3.0, b = 0.256, \delta = 1 \end{aligned} \quad (10)$$

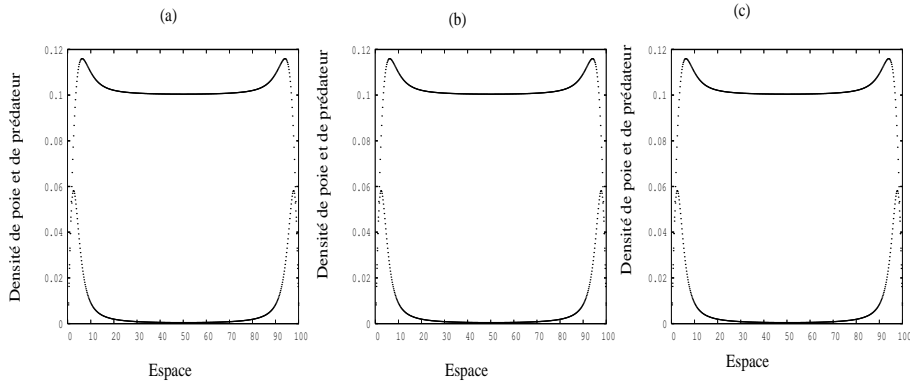


Fig.1. Spatial distribution of density of system (6), the parameter are fixed as in (10)

Figure 1 is an example of spatial distribution of the populations observed at the moment $t = 250(a)$, $t = 750(b)$, $t = 1200(c)$. With these fixed parameters, and these initial distributions, there are two patches at the beginning and the end of the field which are formed as of the first moments of simulation. Between these patches the densities remain constant with respect to the time

parameter. Similar behaviors can be observed for values of b between 0.19 and 0.23.

We fix e_1 , e_2 and a_1 as in (10), b between 0.19 and 0.263. For better examining the properties of the population dynamics as a whole we estimate the species size of prey and predator by :

$$U(t) = \int_0^L u(x, t) dx \text{ and } V(t) = \int_0^L v(x, t) dx \quad (11)$$

The aim is to study the properties of the oscillations of the dynamics of the populations when one varies a parameter of control, the choice of this parameter is then important. In the system (6), we can vary a or b . We will choose b as parameter of control since it determines the ratio of two factors which are the birth rates of the prey and the predator. Therefore, while b varies between 0.195 and 0.263 other parameter will be fixed as in (10). For each value of b the system (6) is solved with the initial conditions (10). We leave a rather large transitory time, so that the total quantities of the species U and V are in the attractor domain. We start with $b = 0.263$, value for which the system presents a focus attractor in (U, V) . The same phase plane is obtained as long as the ratio of the birth rate of the predator by the prey is higher than 0.255. We have a first bifurcation when this ratio is equals 0.255. When b is between 0.208 and 0.255 the system exhibe a periodic oscillations (see figure 2b). A second bifurcation leads to the dynamics of the species in a quasi-periodic attractor, for values of b between 0.199 and 0.208 (see figure 2c). Finally for b between 0.199 and 0.208 it becomes chaotic (see figure 2d). These results are summarized by the bifurcation diagram of *figure 3*. The focus attractor corresponds to the aligned dots on the right of the *figure 3*. The simple periodic dynamics are the two lines of points. Chaos is located in the dense part on the left, and complex behaviour between the last types of dynamics.

Remark The quantities U and V are the linear sums of the local densities u and v respectively. However, these densites do not express irregular oscillations whereas the quantity total have a chaotic dynamics for certain values of b . Thus the Whole is not any more the sum of the parts as soon as the causal interactions are nonlinear.

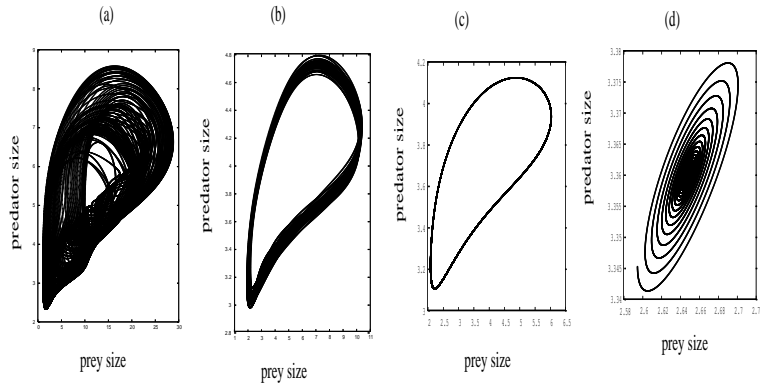


Fig.2. A cascade of bifurcations leading to the onset of chaotic oscillations crossing a complex oscillations area trajectory of the system dynamics in the phase plane (U, V) for different values of predator mortality: (a) $b = 0.197$, (b) $b = 0.203$, (c) $b = 0.23$, (d) $b = 0.26$. Other parameters are the same as in figure 1.

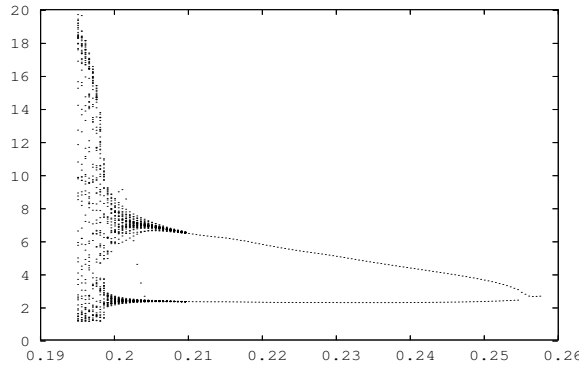


Fig.3. Bifurcation diagram when the parameter b varies.

4 Complexity and Chaos emergence in two-dimensional space

In this last part we study the problem in the limited field $D = [0, 900] \times [0, 900]$ of \mathbb{R}^2 . We are interesting in the emerging structures if the homogeneous equilibrium (u^*, v^*) of (6) is unstable. Let us denote that, if (u^*, v^*) is unstable for (5) it becomes also unstable for (6) as soon as the coefficient of diffusion is equals to one. In the case of an instability of Turing, all the eigenvalues of A are negative real parts. Thus, it is the strong diffusion of the predator who causes the manifestation of instability. In this part, we suppose that the prey and the predator diffuse in the same way. The global emerging structures are given only by the local interactions of the functional response

$(f(u, v), g(u, v))$. We start from an initial condition rather close to (u^*, v^*) , having a low disparity of the space distribution. It is given by:

$$u(x, 0) = u * -2 \cdot 10^{-7}(x - 0.1y - 231)(x - 0.1y - 632) \quad (12)$$

$$v(x, 0) = v * -3 \cdot 10^{-5}(x - 450) - 1.2 \cdot 10^{-4}(y - 150) \quad (13)$$

we now give to the parameters the following values so that the point fixes is unstable ,

$$e_1 = 0.3, e_2 = 0.1, \quad b = 0.02, \quad a = 1.1, \quad \delta = 1 \quad (14)$$

We observe the following evolution of spatial distributions :

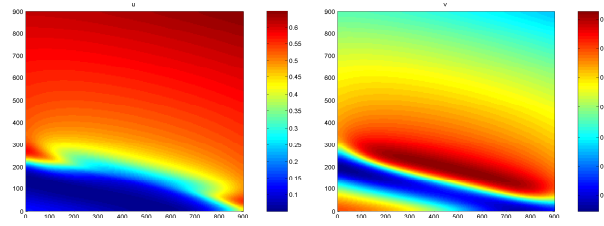


Fig.5. Spatial distribution of species at $t = 100$

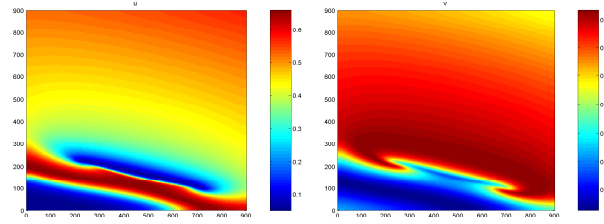


Fig.6. Spatial distribution of species at $t = 200$

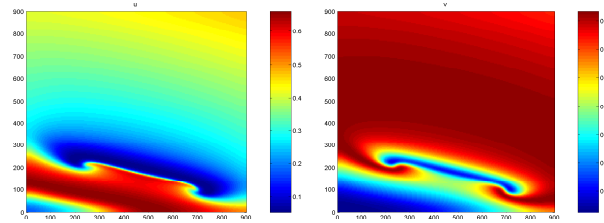


Fig.7. Spatial distribution of species at $t = 300$

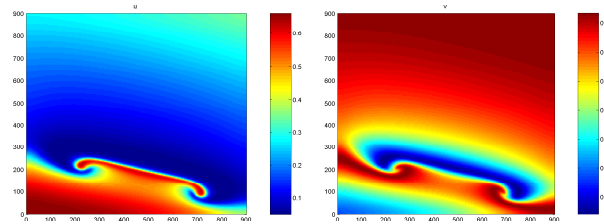


Fig.8. Spatial distribution of species at $t = 400$

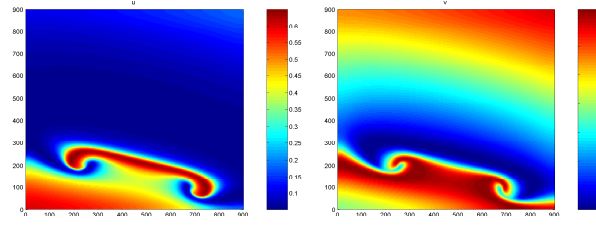


Fig.9. Spatial distribution of species at $t = 500$

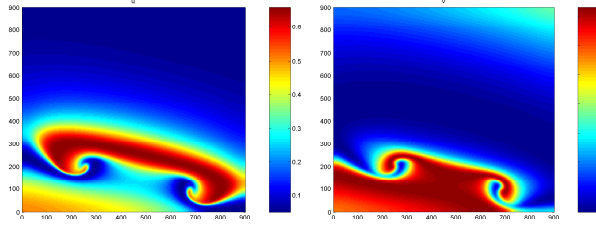


Fig.10. Spatial distribution of species at $t = 600$

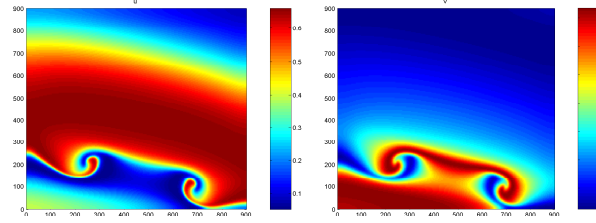


Fig.11. Spatial distribution of species at $t = 700$

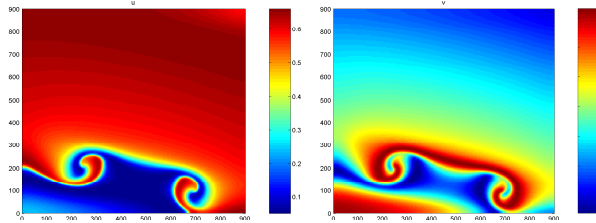


Fig.12. Spatial distribution of species at $t = 800$

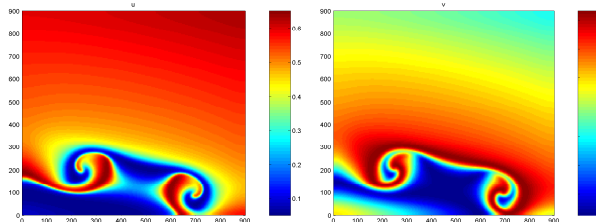


Fig.13. Spatial distribution of species at $t = 900$

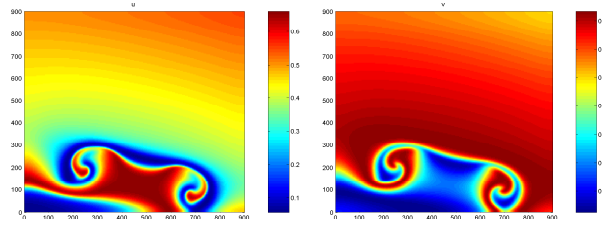


Fig.14. Spatial distribution of species at $t = 1000$

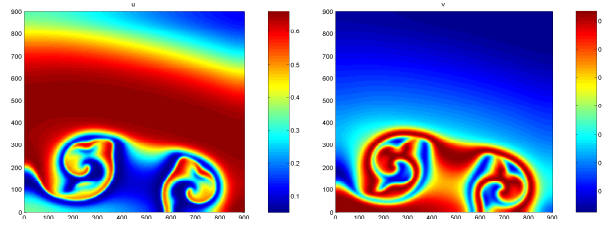


Fig.15. Spatial distribution of species at $t = 1500$

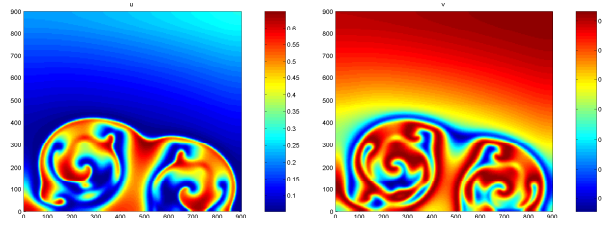


Fig.16. Spatial distribution of species at $t = 2000$

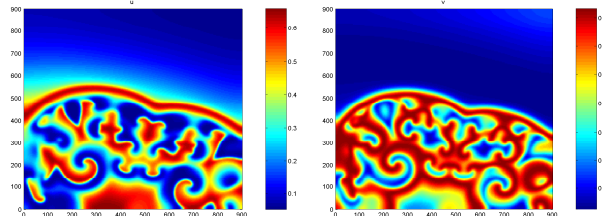


Fig.17. Spatial distribution of species at $t = 3000$

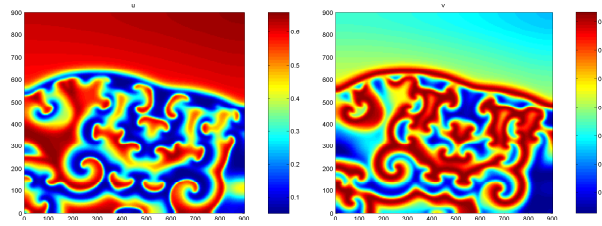


Fig.17. Spatial distribution of species at $t = 4000$

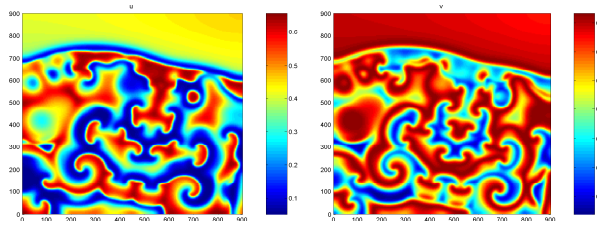


Fig.18. Spatial distribution of species at $t = 5000$

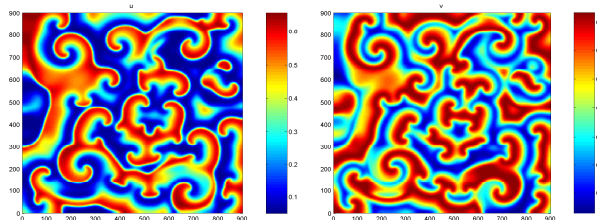


Fig.19. Spatial distribution of species at $t = 8000$

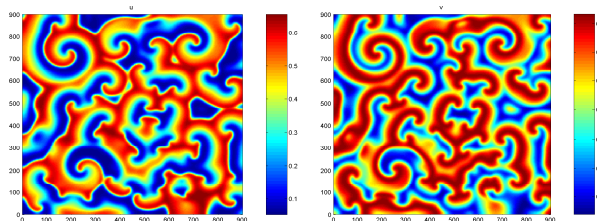


Fig.20. Spatial distribution of species at $t = 10000$

5 Conclusions and Prospect

In this paper, we have considered a spatio-temporal prey-predator system given by a reaction-diffusion equations. Bifurcations which lead to an instability of the equilibrium have been studied. We present the connection between bifurcations and the dynamics of the size species. Finally we investigate the self-organization, emerging properties and structures in two dimension space. These complex structures emerging presented here, rise from the non-linearity of the nonlinear parts of the system without diffusion. Let us denote that during all the study of the problem the parameters are selected so that the system (5) admits a limite cycle.

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PART 4

Applications to Artificial Systems

Cooperative Medical Diagnoses Elaboration by Physicians and Artificial Agents

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Summary. Cooperative medical diagnosis systems seem to be well suited for the solving of many difficult medical diagnosis problems, like the combinations of illnesses where between the illnesses treatments are dependencies. The solving of many difficult medical problems requires knowledge from different medical domains, which cannot be detained by a single physician or a medical computational system. In this paper, a novel medical multiagent system called MASM (Medical Assistant Multiagent System) that can help physicians in their work is proposed. Are analyzed advantages of the cooperative problems solving by physicians and the proposed multiagent system. By cooperation, the human and artificial thinking advantages in the diagnostics elaboration are combined. The proposed multiagent system is a complex system. It is composed from relatively simple agents, which can discover cooperatively emergent properties that arise during the medical problems solving processes that can be ignored by the physicians. Discovered informations allows to the MASM system to increase the accuracy of the elaborated diagnostics and reduce the complexity of the diagnosis processes.

Keywords: complex system, emergent proprieties in artificial systems, software agent, expert system agent, knowledge-based system, diagnostic accuracy, medical diagnosis system, expert system

1 Introduction

Many medical diagnosis problems solving is a difficult task, which require the contribution of more medical human specialists and medical computational systems. Some difficult diagnostics elaboration requires a planning process to that must contribute more physicians with different medical knowledge. For example, we mention the planning of a surgery intervention for the elimination of a huge tumor. Such planning process can require the analysis of different situations that can occur during the treatment application (for example, the loss of a huge quantity of blood). To this problem solving must

contribute more human medical specialists. Assistant agents can be useful in such diagnostics establishment. They can verify different medical hypothesis, established autonomously by them or required by physicians. Such analyses results may allow the detection of mistakes that may appear in the physicians decisions. For example, we mention the suggestion of a medicine for treating a patient's illness in the situation when the patient is allergic to that medicine. Another possible application of the assistant agents, consist in the intelligent mediation of the physicians cooperation during different medical tasks fulfilling, which can make easier the diagnoses elaborations by the physicians (the physicians must verify less informations and data, they also may have more certitude in some elaborated medical hypotheses correctness). Many times is required for the assistant agents to cooperate with other agents in order to handle the requirements that they must fulfill.

In this paper, a novel assistant multiagent system called *MASM* (*Medical Assistant Multiagent System*) is proposed. The proposed multiagent system is a complex system, composed from relatively simple agents called *assistant expert system agents* that cooperatively can help physicians in their work. Assistant expert system agents represent a novel class of agents developed in our previous researches. Another novelty in the proposed *MASM* system consists in the novel cooperative assistance of the physicians in the medical problem solving processes offered by the system. The intelligent cooperative assistance partially hide the complexity of the medical diagnosis problems to the physicians, making easier the fulfilling of some medical tasks by them, and increasing the accuracy of the elaborated diagnostics. A physician interacts directly only with an owned assistant agent member of the system. Each assistant agent will cooperate during the problems solving, if is necessary with other assistant agents' members of the system. For a physician is necessary to knows only its owned assistant agent, the rest of the system's member agents are hidden to him. Each assistant agent know how must cooperate with different agents to offer the necessary help to its owner physician. The system's members cooperate in order to handle the complexity of the problems solving. They can discover cooperatively emergent properties that arise during the medical problem solving processes. Discovered proprieties may allow to the *MASM* system to improve the effectiveness of the medical problem solving processes. The proposed assistant multiagent system cannot substitute the physicians, but the cooperative problems solving with the physicians, may increases the accuracy of the elaborated medical diagnostics by the physicians. The final diagnostic establishment and validation in the case of an illness is the responsibility of a physician.

The paper is structured as follows: in Section 2 are presented agents and multiagent systems used for medical problems solving; Section 3 describes previous researches related with the *MASM* multiagent system; in Section 4 the novel *MASM* multiagent system is described; Section 5 presents the conclusions of the paper.

2 Agent-based medical systems

The *capability* of a cognitive system consists in the problem solving *specializations* detained by the system [1, 4]. The *capacity* of a cognitive system consists in the amount of problems that can be solved in deadline by the system using the detained resources [1, 4]. *Agents* represent artificial systems with properties, like [2]: increased autonomy in operation, capability of communication and cooperation with other systems. The systems composed from more agents are called *multiagent systems* [1, 2, 3, 4]. The solving of many difficult problems, require the cooperation of more agents with different [2] capabilities and capacities, which motivate the use of multiagent systems versus the use of agents that operate in isolation.

Agents and multiagent systems can be endowed with medical knowledge. Many medical agents that can cooperate with physicians and other agents are proposed and used [5, 6, 7, 8, 9, 10, 11, 12, 13, 36]. The paper [9] describes the state of the art medical information systems and technologies at the beginning of the 21st century. The complexity of construction of full-scaled clinical diagnoses is also analyzed. In the medical domain many type of medical problems can be solved by human medical specialists, which uses medical computational systems some times agent based medical systems [30]. As examples of problems, which appear in the medical domain that can be solved by agents, we mention: medical diagnostics elaborations, medical information collection from distributed knowledge bases, medical data collection about patients from distributed databases etc.

A *medical diagnosis problem* represents the description of a patient's illness. The solution of the problem represents the identified illness and the proposed diagnostic to cure the illness. Many medical problems that must be solved in the medical domain represent subproblems of medical diagnosis problems. As an example of subproblem, we mention a medical issue whose answer is necessary in a diagnostic establishment. For example, we mention, the issue "if exists a more effective medicine to cure an illness than a medicine known by a physician" (a physician wants to know if exists a more effective medicine to cure an illness than the medicine known by himself).

Many medical diagnosis systems must have specific proprieties, depending on the types of the medical problems that they must solve. One of the main propriety, which many times is required by a diagnosing system, consists in the adaptability realized by learning. By learning medical a medical computational system may increase the detained medical knowledge accuracy. Is not always possible to create a system, which has all the necessary knowledge and the established knowledge has the required accuracy [1, 10, 22]. In the paper [8], an intelligent medical diagnosis systems with built-in functions for knowledge discovery and data mining is described. The implementation of machine learning technology in the medical diagnosis systems seems to be well suited for medical diagnoses in specialized medical domains. By learning, a

system can autonomously construct medical diagnosis rules that can be used in diagnosis processes [8].

Difficulties of some medical diagnosis problems, motivate the use of medical multiagent systems for their solving versus medical systems that operate in isolation [52, 14]. Cooperative medical multiagent systems may combine the members capabilities and capacities in the problems solving. A medical diagnostic elaboration may have many difficulties that imply the cooperation of more human and/or artificial medical specialists in their solving [4]. In the following, are mentioned some difficulties in the establishment of a medical diagnostic. A patient may have a combination of illnesses that has different symptoms. The symptoms of more illnesses may have some similarities, which make their identification difficult. Symptoms of an illness can be different at different persons who suffer from that illness. In some situations, a patient does not exhibit the typical symptoms of a specific illness even so he suffers from it. In the case of some illnesses, the causes of the illnesses are not known. A medicine to an illness may have different effects at different persons who suffer from that illness. An illness can be in a very advanced stage that makes the diagnostic elaboration difficult. Difficult cases are those in which the patient's symptoms do not sufficiently match typical patterns known by physicians. An illness can be insufficiently known in medicine because it is either new or unusual. In such situations, the symptoms of an illness may or cannot be interpreted properly.

To model agent systems some methodologies were proposed, like: *HIM* [49], *GAIA* [50] and *PASSI* [51]. The paper [6] analyzes different aspects of the multiagent systems specialized in medical diagnosis. Understanding such systems needs a high-level visual view of how the systems operate to achieve some application related purpose. In the paper [6], a novel method of visualizing the behavior of a medical multiagent system called *Use Case Maps* is proposed. There is described a process for designing agent-based systems using a visual technique, that provide a view of the system as a whole.

In the paper [36], a medical diagnosis multiagent system that is organized accordingly to the principles of swarm intelligence is proposed. It consists of a large number of agents that interact with each other by simple indirect communication. The proposed multiagent system real power stem from the fact that a large number of simple agents collaborates with the purpose to elaborate reliable diagnostics. Agents specialized in medical diagnosis can self-organize in order to provide viable medical diagnoses.

The agents represent a very promising recent research direction in medical diagnoses elaborations [11, 12, 14, 36] and fulfilling medical tasks related with the diagnoses processes. The main motivation that confirm the necessity to use agent based applications in medical domains, consists in the possibility to combine different technologies in the same agent body. As examples of applications of the agents for fulfilling medical tasks, we mention: *patients monitoring* [15], *patients management* [16, 17], *healthcare* [18, 19, 54], *telehealth* [52],

spreads simulation of infectious disease [53], *web-enabled healthcare computing* [54], *ubiquitous healthcare* [19].

3 Related works

3.1 Medical expert system agents

Medical expert systems represent relatively classical applications in the medical diagnosis. As examples of well known medical expert systems, we mention: *MYCIN* [20], *GIDEON* [38], *CARDIAG2* [39], *PUFF* [40] and *CASNET* [41]. Expert systems had some success in specific, mainly quite narrow fields of medical expertise, but had problems to cover broader areas of expertise. Some of the problems related with the expert systems are their limited [37, 2]: flexibility, adaptability, extensibility and cooperation capability. The endowment of the expert systems with cooperation capability is an important research direction [2, 37, 10]. The paper [10] presents a system called *FELINE* composed of five autonomous medical expert systems with some proprieties of the agents. These agents cooperate to identify the causes of anemia at cats. There is also presented a development methodology for cooperating expert systems.

In the paper [52], a Web-centric extension to a previously developed expert system specialized in the glaucoma diagnosis is proposed. The proposed telehealth solution publishes services of the developed *Glaucoma Expert System* on the World Wide Web, allowing physicians and patients to interact with it from their own homes. The *Glaucoma Expert System* uses learning algorithms applied on patient data to update and improve its diagnosis rules.

The *medical expert system agents* represent a novel class of agents developed in our previous works [2, 4, 11, 12, 21, 36]. The medical expert system agents are medical expert systems endowed with agents' capabilities (1).

$$ExpertSystemAgent = ExpertSystem + Agents' Capabilities. \quad (1)$$

A medical expert system agent is endowed with a medical specializations set. As examples of specializations of a medical expert system agent, we mention specializations in subdomains of: cardiology, gastroenterology, endocrinology and rheumatology. Expert system agents can perceive and interact with the environment executing actions in the environment autonomously. Expert system agents can cooperate in the problems solving with other agents and physicians, which allows more flexible problem solving versus the expert systems. If an expert system agent cannot solve a problem (doesn't have the necessary capability and/or capacity), then he can transmit the problem for solving to another agent or physician.

In the papers [12, 48], a cooperative multiagent system specialized in medical diagnosis called *CMDS* (*Contract Net Based Medical Diagnosis System*) is proposed. *CMDS* is a complex system composed from medical expert system agents that cooperatively solve medical diagnosis problems transmitted for

solving to the system. For the problems allocation for solving, in the *CMDS* system is used a novel problem allocation protocol, which represents an adaptation of the *contract net problem allocation protocol* [1, 3].

The papers [14, 28] describe a cooperative medical diagnosis system, proposed for difficult medical diagnosis problems solving called *BMDS* (*Blackboard-based Medical Diagnosis System*). *BMDS* is a complex system, composed from: expert system agents and *assistant expert system agents*. *Assistant expert system agents* represent expert system agents endowed with capability to help intelligently physicians and other agents during the problems solving processes. The assistant agents versus the expert system agents are endowed with knowledge, about the system in which they operate and knowledge about the helped agents and/or physicians. The detained knowledge by them allows the assistance offering. The problems solving by the proposed diagnosis system is partially-based on the *blackboard-based problems solving* [29, 3].

3.2 Medical ICMA agents

The development of *large-scale medical diagnosis systems* represent an important research direction [7, 19, 11]. In the paper [7], an *Internet-based holonic medical diagnosis system* for diseases is proposed. The holonic medical diagnosis system consists of a tree-like structured alliance of agents specialized in medical diagnoses, which collaborate in order to provide viable medical diagnoses, combining the advantages of holonic systems and multiagent systems.

Another important research direction related with the development of large-scale diagnosis systems, is represented by the approaches based on software mobile agents [11, 19]. The *LMDS* system [11] that uses *medical ICMA mobile agents* developed in our previous works and the *OnkoNet* mobile agents [19] described in the literature are illustrative in this sense.

OnkoNet mobile agents have been used successfully for patient-centric medical problems solving [19]. In the paper [19], is introduced the notion *ubiquitous healthcare* (any-time/any-place access of health services via mobile computing), addressing the access of health services by individual consumers using mobile agents. This access requires medical knowledge about the individual health status (relevant recent diseases). The *OnkoNet* mobile agent architecture involve architectures on the macrolevel and microlevel as well as cooperation protocols. The work presented in the paper [19], emerged from a project covering all relevant issues, from empirical process studies in cancer diagnosis/therapy, down to system implementation and validation.

The software mobile agents are capable to change their location in the network where they operate [34, 35]. The main disadvantages of the mobile agents are related with their limited [34, 35]: communication capability, protection capability, intelligence and capability to use knowledge bases in the problems solving. These disadvantages limit the use of the mobile agents for medical problems solving in insecure networks.

In our previous researches a novel mobile agent architecture called *ICMA* (*Intelligent Cooperative Mobile Agent Architecture*) was developed [24, 25]. The proposed mobile agent architecture represents a combination of the mobile and static agent paradigms. The purpose of the research was the development of an architecture that allows the creation of mobile agents, which can solve intelligently difficult problems like medical diagnosis problems in insecure networks. Advantages of the *ICMA* mobile agents versus some of the mobile agents described in the literature consist in their increased: communication capability [25], protection possibility [26] and intelligence [25]. *ICMA* agents can solve efficiently large numbers of problems [24, 25]. These advantages of the *ICMA* agents suggest their use for different problems solving. *ICMAE* (*Intelligent Cooperative Mobile Agents with Evolutionary Problem Solving Capability*) agents represent a novel class of mobile agents developed in our previous works [31, 32, 33]. *ICMAE* agents represent agents with the *ICMA* architecture endowed with problem solving methods based on evolutionary computation. *ICMAE* agents can solve efficiently large numbers of problems, using efficiently problems solving resources (resources of the hosts) distributed in the network [31, 32, 33].

Medical ICMA agents represent a novel class of medical agents, based on the *ICMA* architecture. *Medical ICMA agents* can diagnoses illnesses. In the paper [11], a large-scale medical diagnosis system called *LMDS* (*Large-Scale Medical Diagnosis System*) is proposed. *LMDS* is a complex system composed from expert system agents and medical *ICMA* agents. The agents' members of the diagnosis system solve cooperatively overtaken diagnosis problems. A medical *ICMA* agent can cooperate with other agents in the problems solving. A medical *ICMA* agent can migrate in the network with an overtaken problem until will find an agent capable to solve the problem. The open *LMDS* system may have a large number of members.

Medical expert system agents represent a novel class of agents with medical diagnosing capability. The effectiveness of the *CMDS*, *BMDS* and *LMDS* multiagent systems demonstrate, that medical expert system agents can be used successfully as members of complex diagnosis systems. The increased intelligence of the expert system agents is proved in [21]. *Assistant medical expert system agents* that are expert system agents endowed with capability to help flexibly physicians in their work, used by the *BMDS* system, proves that expert system agents may help medical specialists during the medical diagnosis processes. Medical expert system agents require future improvements, in order to increase their intelligence in the physicians assistance during the diagnoses processes. Medical expert systems can be useful in situations when are diagnosed illnesses in more diagnosis domains.

4 The MASM multiagent system

In the following, we propose an assistant multiagent system composed from a set $AS = \{AS_1, AS_2, \dots, AS_n\}$ of *assistant expert system agents* that can help more physicians in they work. We call the proposed multiagent system *MASM* (*Medical Assistant Multiagent System*). In the following, we call the assistant expert system agents shortly *assistant agents*. Physicians own assistant agents' members of the multiagent system. Each physician is the owner of an assistant agent (in the system may exists agents that are not owned by physicians). Figure 1 presents a physician interaction with the proposed multiagent system. The arrows used in Figure 1 present the *types of cooperation* $T = \{t_a, t_c\}$. t_a is a cooperation link between a physician and the owned assistant agent. t_c is a cooperation link between assistant agents.

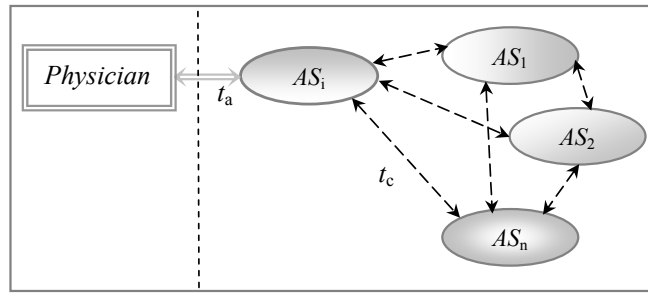


Fig. 1. A physician interaction with the MASM system

A physician denoted Ph_y interacts directly only with its owned assistant agent As_y ($As_y \in AS$, $Link(As_y, Ph_y) = t_a$). As_y can cooperate during its operation with all the other assistant agents' members of *MASM* system with which has a cooperation link by the type t_c (in the system may exists assistant agents between that doesn't exist a cooperation link). Assistant agents can cooperate in different problems solving, like: medical diagnoses elaborations, medical information search, cooperation in the solution finding to emitted medical issues by physicians etc.

4.1 MASM system operation

The algorithm *MASM Problem Solving* describes the cooperative elaboration of a medical diagnostic (a P_k medical diagnosis problem solving) by a physician denoted Ph_e and the agents members of the *MASM* system. As_e ($As_e \in AS$) represents the agent owned by Ph_e ($Link(Ph_e, As_e) = t_a$). The solution $S_k = [Il_z, Tr_v]$ of the P_k problem, represents the patient denoted Pat_c , Il_z illness and the Tr_v treatment that must be applied to cure Il_z .

Algorithm - MASM Problem Solving

{IN: P_k - the diagnosis problem}

{OUT: $S_k = [Il_z, Tr_v]$ - the solution of P_k }

Step 1. Information collection about the patient.

@ Pat_c personal informations (name, identification number etc) are collected and transmitted to As_e ($Link(Ph_e, As_e) = t_a$).

@The informations about the patient's illness (the illness symptoms, the history of the symptoms) are collected from Pat_c and transmitted to As_e and Ph_e . The rest of the informations are obtained by physical examination. During the physical examination, Ph_e looks for signs of the illness (signs are manifestations of the illness that the physician can see or feel).

@ As_e requires medical informations about Pat_c (Pat_c past illnesses descriptions, Pat_c known allergies to some medicines etc) from an assistant agent As_k ($Link(As_e, As_k) = t_c$). As_e transmits to As_k , the patient Pat_c personal informations, necessary in the Pat_c identification.

@ As_k establishes how can obtain the informations required by As_e . In the informations extraction, As_k may requires the help of other assistant agents.

@ As_k fulfill the requirement of As_e by transmitting the obtained informations.

@ As_e selects from the informations transmitted by As_k , the informations considered useful for Ph_e (informations that describe Pat_c past illnesses and different medical informations about the patient). These informations are transmitted by As_e to Ph_e .

Step 2. The problem cooperative solving.

While (P_k solution is not obtained) do

@ Ph_e continues P_k solving. Ph_e may require investigations such laboratory tests, X-rays, MRI, computed tomography etc.

If (Ph_e requires the help of As_e) then

@ As_e plans how can fulfill the Ph_e requirement cooperatively with assistant agents with that has a cooperation link by type t_c .

@ As_e tries to realize the Ph_e requirement.

If (As_e succeeded to realize the Ph_e requirement) then

@ As_e announces Ph_e about the obtained results.

else

@ As_e announces Ph_e about its incapability to fulfill the requirement.

```

    EndIf
  EndIf
  If ( $As_e$  establishes the assistance that can offer to  $Ph_e$ ) then
    @ $As_e$  plans how can help  $Ph_e$ .
    @ $As_e$  helps  $Ph_e$ .
  EndIf
EndWhile
@Let  $S_k$  be the solution obtained by  $Ph_e$ .
Step 3. The problem solution formation.
@ $Ph_e$  transmits the identified illness and the established treatment to  $As_e$ .
If ( $As_e$  finds mistakes in  $S_k$ ) then
  @ $As_e$  announces  $Ph_e$  about the mistakes contained in  $S_k$ .
  If ( $As_e$  can suggest measures that can eliminate mistakes) then
    @ $As_e$  suggests to  $Ph_e$  the measures that can offer solution to the
    elimination of the mistakes.
  EndIf
  @ $Ph_e$  eliminates the mistakes from  $S_k$ . If is necessary  $Ph_e$ 
  cooperates with  $As_e$ .
  @Let  $S_k = [Il_z, Tr_v]$  be the reviewed solution.
EndIf
Step 4. The problem solution validation.
@ $Ph_e$  validates  $S_k$ . During  $S_k$  validation,  $Ph_e$  may requires  $As_e$  help.
EndProblemSolving.

```

Figure 2 presents a cooperative diagnosis elaboration by a physician denoted Ph_i and the assistant multiagent system $MASM$, in order to identify a patient's illness and establishes a proper diagnostic to cure the illness. During the diagnostic establishment, Ph_i interacts directly only with the owned assistant agent As_i ($As_i \in AS, Link(As_i, Ph_i) = t_a$).

A problem description (2) contains a list of medical informations and data St_1, St_2, \dots, St_m that can be obtained during a diagnosis problem solving.

$$< [1 : ty_1 : nr_1 : St_1]; [2 : ty_2 : nr_2 : St_2]; \dots; [m : ty_m : nr_m : St_m] > . \quad (2)$$

St_1, St_2, \dots, St_m from (2) may have a single value or a list of values. As examples of values, we mention: illness symptoms, illness syndromes, illnesses

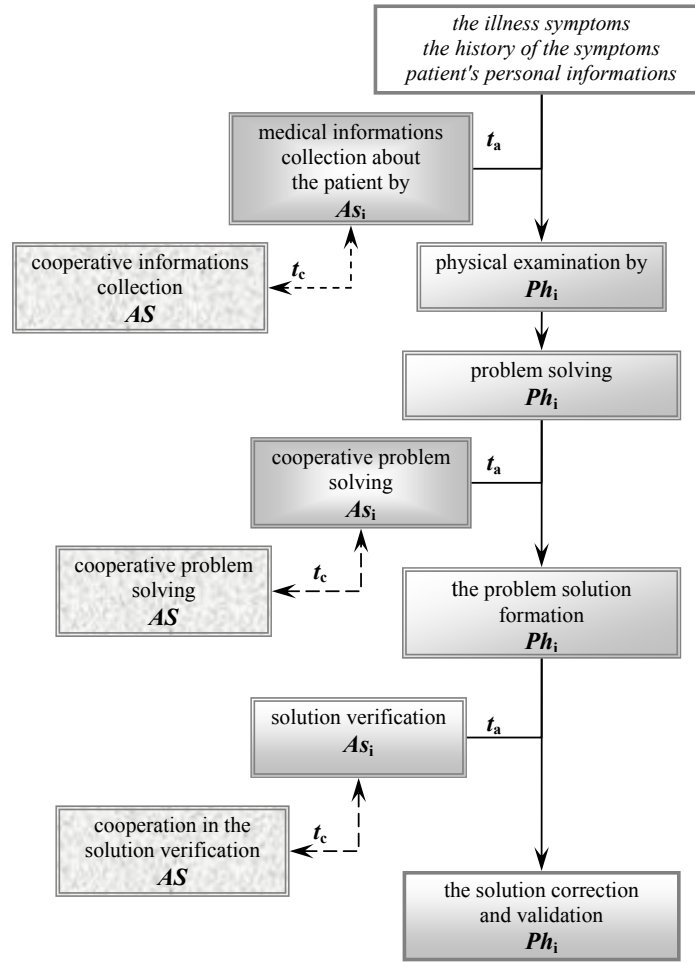


Fig. 2. Summary of a cooperative diagnosis elaboration

from the past, diagnostics etc. To each parameter St_a that can contain effective medical information is associated a numerical value a (a is completed implicitly), a type ty_a of information and/or data that can be retained in the parameter, and a numerical value nr_a that specifies how many values can be retained in St_a .

For example, we mention $[f : ty_f : nr_f : St_f]$ completed during a diagnosis problem solving with $[5 : symptoms : more : list]$; where $f = 5$ (is the parameter on the position 5), $ty_5 = symptoms$ (the parameter contain symptoms), $nr_f = more$ in the list can be retained a list of symptoms. $list$ take values during a problem solving (will be completed with a list of symptoms).

Retaining the medical informations and data about a diagnosis problem denoted P_h in the form (2), allows to the assistant agents to quickly and precisely identify the types the knowledge detained about P_h . The precise identification is necessary in the cooperation between the agents. An assistant agent based on the parameter identifier, knows with which parameter of the preconditions of its diagnosing rules must try to fit in order to establish an applicable rule.

The medical informations and data that can be retained in (2), are established based on the specifics of the illnesses that are diagnosed by the system (for example infectious disease). Initially, a problem description contains informations that describes an illness (for example the illness symptoms enumerated by the patient). During the solving of the problem, the description is changing, by adding new knowledge by physicians or artificial agents. Physicians can add, remove and modify knowledge in a problem description. The physicians can make more rational decisions versus the artificial agents, whose rationality is an automatized one. The final problem description will contains the established diagnostic. During a problem solving, some of the parameter values may not be completed (for example is not necessary to retain the history of the diagnosed illness symptoms).

An AS_b assistant agent knowledge used in medical diagnoses elaborations is composed from a set RL (3) of diagnosing rules.

$$RL = \{Rl_1, Rl_2, \dots, Rl_g\}. \quad (3)$$

An Rl_h ($Rl_h \in RL$) diagnosing rule has the form (4).

$$Rl_h : Prec_h \rightarrow Postc_h. \quad (4)$$

$Prec_h$ and $Postc_h$ have parameters that specify informations, which describe different aspects of a diagnosis process. In $Prec_h$ and $Postc_h$ appear a set Par (5) of parameters (some of the parameters specified in (2)).

$$Par = \{Par_1, Par_2, \dots, Par_y\}. \quad (5)$$

A Par_p ($Par_p \in Par$) parameter in (5) has the form (6)

$$Par_p = [f : ty_f : nr_f : St_f]. \quad (6)$$

St_f contains medical informations and/or data. $type_f$ represents the type of medical informations retained in St_f . nr_f represents the number (one or more) of values that may have St_f . f is a natural number, which specify the same type of information that specify f in (2). If Par_p appear in the precondition of a rule, then specify conditions that must be fulfilled for the applicability of the rule in whose precondition appear. If Par_p appear in the postcondition of a rule, then it specify informations that must be added into the problem description (for example a supposed illness or a suggested treatment), which

represent the informations about the problem that is diagnosed, if the precondition of the rule is verified. When an agent overtakes a medical diagnosis problem for processing, identify a diagnosing rule which postcondition specify informations that can be added in the problem description. The knowledge contained in a problem description is understandable to the physicians, they can modify it by adding, modifying or retracting knowledge.

An assistant agent As_i ($As_i \in AS$) has a knowledge base that contains different knowledge about the owner physician denoted Ph_i ($Link(Ph_i, As_i) = t_a$) and the agents' members of the *MASM* system. About Ph_i the owned assistant agent As_i may detains informations like:

- Ph_i specialization in medicine;
- Ph_i working program;
- medical informations and data about the Ph_i patients (for example the patients' medical history);
- informations (specialization in medicine, working program) about other physicians that use the system, with who Ph_i usually cooperate in the diagnoses elaborations (require and/or offer advices from/to them).

About an assistant agent As_k ($As_k \in AS$), the assistant agent As_i ($As_i \in AS$, $Link(As_i, As_k) = t_c$) may detains informations like:

- the specializations of As_k (for example information searching capability about physicians);
- the knowledge representation languages used by As_k in the detained knowledge representation. A knowledge representation language allows the representation of the used knowledge [3].

An assistant agent As_h ($As_h \in AS$) can offer autonomously assistance to its owner physician Ph_h ($Link(Ph_h, As_h) = t_a$), based on different knowledge detained in its knowledge base and the informations known about the diagnosis problem that is currently solved by Ph_h . Ph_h may requires explicitly As_h help. In the assistance offering, As_h can cooperate with other agents.

A physician Ph_g can requires its owned As_g ($Link(Ph_g, As_g) = t_a$) agent's help in solving subproblems of a diagnosis problem. As examples of subproblems of a diagnosis problem, we mention:

- the result of a medical analyzes necessary in increasing the accuracy of an illness identification;
- a supposed Il_c illness. Ph_g requires to As_g to analyze if a patient has the Il_c illness. Il_c probability of apparition is estimated to be low by Ph_g .
- the medical ontology known by As_k . A medical ontology represents a dictionary of used terms in a medical domain [42]. As examples of developed medical otologies described in the literature, we mention: *GALEN* [43], *UMLS* [44], *OntHoS* [19], *LinkBase* [45], *TAMBIS* [46] and *GENE* [47].

A physician denoted Ph_g may transmit subproblems to its As_g owned assistant agent, who if is necessary cooperates with a set As_j, As_v, \dots, As_q of

agents in order to solve them (As_g have a cooperation link by the type t_c with As_j, As_v, \dots, As_q). The received results by As_g from As_j, As_v, \dots, As_q will be transmitted to Ph_g . Ph_g establishes the diagnostic based on its different observations and the results obtained from As_g .

An assistant agent can cooperate with other agents in order to collect medical knowledge and medical informations about patients. As examples of medical informations that can be collected by an agent As_i ($As_i \in AS$) about a patient, we mention the descriptions of the patient's medical history and the patient's current known illnesses. As examples of medical knowledge that can be collected by an agent As_k ($As_k \in AS$), we mention: an illness description (the symptoms of the illness), a new medicine that can be used to cure an illness; the success of different diagnostics applied to cure an illness.

An assistant agent can verify the correctness of a problem's solution obtained by its owner physician. The agent knows the problem that is solved by the physician (the problem description is transmitted to the assistant agent). However, the agent can transmit the problem for solving to another agent that has the necessary medical specialization. For example, a physician specialized in general medicine and an agent specialized in general medicine can try to establish simultaneously a diagnostic to cure an illness. The assistant agent can compare the solution obtained by the physician with the solution obtained by the another agent. If the obtained solutions differ, the assistant agent announces the physician related about the uncertainty of the correctness of the obtained solution.

4.2 Examples of Cooperative Medical Diagnoses Elaborations

An assistant agent can manage intelligently the cooperation between the owner physician and other physicians. In the following, we present two cooperative scenarios. The scenario called *Medical Issue Solving* describes a medical issue cooperative solving by more physicians assisted by the *MASM* system. The scenario called *Medical Diagnosis Problem Solving* describes a medical diagnosis problem solving by more physicians assisted by the *MASM* system. Is considered, that the *MASM* system is used by a set $Ph = \{Ph_1, Ph_2, \dots, Ph_r\}$ of physicians. $ASR = \{As_1, As_2, \dots, As_r\}$ ($ASR \subseteq AS$) denote the assistant agents owned by Ph_1, Ph_2, \dots, Ph_r .

The scenario of a medical issue solving

We consider the situation when a physician denoted Ph_i ($Ph_i \in Ph$) wants to find the opinion of more physicians about a medical issue denoted mi . As an example of medical issue that can be emitted by a physician, we mention a hypothesis related with a diagnostic to cure an illness. Cooperation scenario - *Medical Issue Solving* describes the cooperative finding of the answer to the medical issue mi emitted by Ph_i .

*Cooperation scenario - Medical Issue Solving**Step 1*

$$Ph_i(mi) \Rightarrow As_i.$$

As_i establishes the physicians Ph_1, Ph_2, \dots, Ph_k capable to answer to mi .

As_i establishes the agents As_1, As_2, \dots, As_k owned by Ph_1, Ph_2, \dots, Ph_k .

$$As_i(mi) \Rightarrow As_1, As_2, \dots, As_k.$$

Step 2

$$As_1(mi) \Rightarrow Ph_1.$$

$$As_2(mi) \Rightarrow Ph_2.$$

...

$$As_k(mi) \Rightarrow Ph_k.$$

Step 3

$$Ph_1(rp_1) \Rightarrow As_1.$$

$$Ph_2(rp_2) \Rightarrow As_2.$$

...

$$Ph_k(rp_k) \Rightarrow As_k.$$

Step 4

$$As_1(rp_1) \Rightarrow As_i.$$

$$As_2(rp_2) \Rightarrow As_i.$$

...

$$As_k(rp_k) \Rightarrow As_i.$$

Step 5

$$learned = ASILearn(rp_1, rp_2, \dots, rp_k).$$

$$useful = Filter(rp_1, rp_2, \dots, rp_k).$$

$$As_i(useful) \Rightarrow Ph_i.$$

Ph_i establishes the answer to mi based on the *useful* knowledge.

EndMedicalIssueSolving

Ph_i transmits mi to its owned agent As_i ($Link(Ph_i, As_i) = t_a$). As_i based on its detained knowledge establishes the physicians Ph_1, Ph_2, \dots, Ph_k capable (have the necessary medical specialization and are available) to answer to the issue mi . As_i will transmit mi to the assistant agents As_1, As_2, \dots, As_k

owned by the choused physicians Ph_1, Ph_2, \dots, Ph_k (As_i have a cooperation link by the type t_c with the agents As_1, As_2, \dots, As_k). As_1, As_2, \dots, As_k will overtake the responsibility to transmit mi to their owner physicians and send back the answer to As_i . rp_1, rp_2, \dots, rp_k represent the physicians responses to mi . As examples of answers that can be received by As_i from an agent As_j ($Link(As_i, As_j) = t_c$) owned by Ph_j ($Link(Ph_j, As_j) = t_a$), we mention: Ph_j is unavailable, Ph_j answer to the issue, Ph_j cannot answer to the issue. From the received responses, As_i filters the useful responses (contains the physicians responses to the issue). As_i can learn from the received responses. *learned* represents the learned knowledge by As_i . As examples of informations that As_i can learn, we mention: what physicians are usually unavailable, what physicians can answer fast to different issues etc. The knowledge denoted *useful* obtained after the filtering process is transmitted to Ph_i . Ph_i will establishes the answer to the mi medical issue based on the *useful* knowledge received from As_i .

The scenario of a medical diagnosis problem solving

We consider the situation when a physician denoted Ph_i cannot solve a medical diagnosis problem denoted P_q . In order to solve the problem, Ph_i cooperates with other physicians. *Cooperation scenario - Medical Diagnosis Problem Solving* describes briefly the cooperative solving of the P_q problem by the physicians helped by the *MASM* system.

Cooperation scenario - Medical Diagnosis Problem Solving

Step 1

$Ph_i(P_q) \Rightarrow As_i$.

As_i establishes the physicians Ph_1, Ph_2, \dots, Ph_m considered capable to cooperate in P_q solving.

As_i establishes the As_1, As_2, \dots, As_m agents owned by Ph_1, Ph_2, \dots, Ph_m .

$As_i(P_q) \Rightarrow As_1, As_2, \dots, As_m$.

Step 2

As_1 verifies if Ph_1 is capable to cooperate in the P_q solving.

As_2 verifies if Ph_2 is capable to cooperate in the P_q solving.

...

As_m verifies if Ph_m is capable to cooperate in the P_q solving.

Step 3

$As_1(rp_1) \Rightarrow As_i$.

$As_2(rp_2) \Rightarrow As_i$.

...

$$As_m(rp_m) \Rightarrow As_i.$$

Step 4

$$PT = TeamFormation(rp_1, rp_2, \dots, rp_m).$$

P_q is solved cooperatively by the set PT ($PT \subseteq PH$) of physicians interloped by their owned agents.

S_q is formed and validated by the set PT of physicians.

EndMedicalDiagnosisProblemSolving

Ph_i requires the help of the owned assistant agent As_i ($Link(Ph_i, As_i) = t_a$) in finding physicians with who can cooperate in the P_q solving. As_i will establishes the capable physicians Ph_1, Ph_2, \dots, Ph_m based on the knowledge detained in its knowledge base. rp_1, rp_2, \dots, rp_m represent the responses (acceptance or rejection) of the physicians Ph_1, Ph_2, \dots, Ph_m to Ph_i requirement to cooperate in the P_q solving. After that, As_i will manage the cooperation of Ph_i with the set PT ($PT \subseteq PH$) of physicians. PT contains Ph_i and a set of physicians who has accepted the cooperation in the P_k solving. In some of the interactions, the physicians PT can use as interloper their owned agents. P_q solution S_q is formed and validated, cooperatively by physicians from the set of physicians PT , who are specialized in the medical domain (domains) in which the identified illness (illnesses) is included.

Figure 3 presents a diagnosis problem solving complexity distribution between the members of the *MASM* system. In Figure 3 are used the notations: *Problem* presents the problem that is solved; Ph_1, Ph_2, \dots, Ph_j represent the physicians that contribute to the problem solving; As_1, As_2, \dots, As_b represent the assistant agents (owned by Ph_1, Ph_2, \dots, Ph_j , and may contain agents that are not owned by physicians) that contribute to the problem solving.

The scenarios presented before have described cooperative medical problems solving by physicians helped by their owned agents. In the following, we mention different assistance that can offer an agent As_q to its owner physician Ph_q ($Link(Ph_q, As_q) = t_a$) in a P_t medical diagnosis problem cooperative solving by Ph_q with a set $Ph = \{Ph_1, Ph_2, \dots, Ph_m\}$ of physicians:

- As_q can transmits known information to Ph_1, Ph_2, \dots, Ph_m without consulting Ph_u . For example, may transmit medical information about the patient whose illness is diagnosed cooperatively by Ph_1, Ph_2, \dots, Ph_m and Ph_u . However, is not necessary for Ph_u to specify explicitly all the details to Ph_1, Ph_2, \dots, Ph_m necessary in the P_t solving;
- As_q can add the ontology of the used terms in a message transmitted by Ph_u to another physician Ph_v . For example, may add alternative names of the same illness;

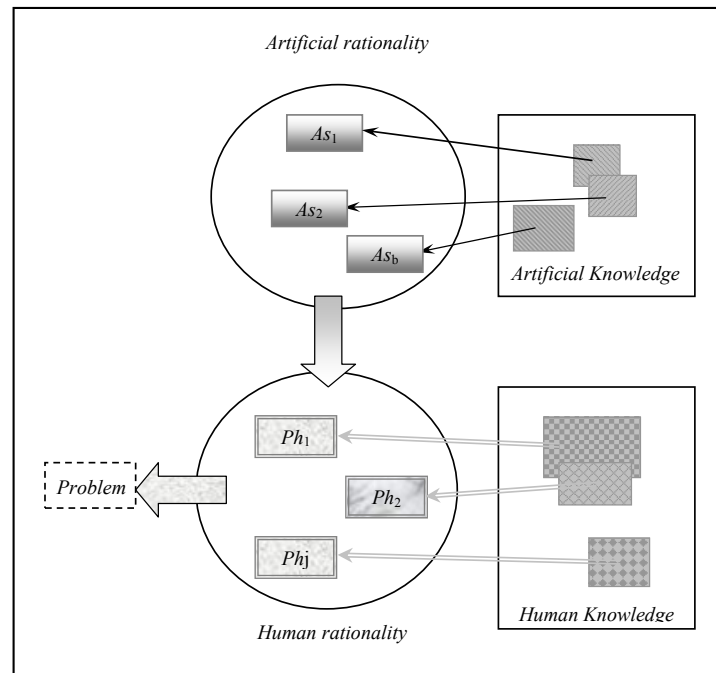


Fig. 3. Problem solving complexity distribution in the MASM system

4.3 Advantages of the MASM System

More physicians can use the proposed *MASM* system. Each physician interacts directly only with an owned assistant expert system agent. Each assistant agent will cooperate during the problems solving, if is necessary with other agents' members of the system. For a physician is necessary to knows only its owned assistant agent, the rest of the system's member agents are hidden to the physician. Each assistant agents know how must cooperate with some other agents to offer the necessary help to its owner physicians. More physicians using as interlopers their owned assistant agents can solve cooperatively medical diagnosis problems.

An external physician (is not in the set of physicians that use the *MASM system*) can use the multiagent system (for example a physician uses the multiagent system just during a problem solving). An assistant agent will manage the interactions of the new physician with the multiagent system and the physicians that already use the system. The external physician's assistant agent is endowed with an initial set of knowledge about the physician. During its life cycle, the assistant agent can learn new knowledge that improves its capability to help the physician in the future.

The *MASM* system cannot substitute the physicians, but the cooperation with the physicians may improve the elaborated diagnostics accuracy by the

physicians. A final diagnostic establishment and validation in the case of an illness is the responsibility of a physician. By cooperation, the human and artificial thinking advantages in the diagnostics elaboration are combined. Physicians can elaborate decisions using their medical knowledge and intuition formed in time. However, they can diagnoses difficult illnesses. The diagnostics accuracy and elaboration time can be improved, if in some points of decisions, the physicians use the *MASM* system that can verify medical hypotheses and execute different actions autonomously, in order to make the physicians work easier.

An assistant agent's specialization contains the description of the help that can offer the agent to physicians and other agents. An assistant agent can be endowed with new specializations; the inefficient specializations can be eliminated or improved. New assistant agents can be introduced in the multiagent system. The adaptation of a cooperative multiagent system in the efficient solving of a problem many times is easier than the adaptation of an agent that solves the same problem [1, 2, 3].

5 Conclusions

The solving of many medical diagnosis problems is a complex task that may require diagnosing knowledge from different medical domains, which cannot be detained by a single physician or a single medical computational system. Recently developed medical agents and medical multiagent systems, prove that they represents an efficient approach for many diagnosis problems solving [27, 24, 23, 28, 11, 12, 14, 36]. Agents may fulfill medical tasks, related with the diagnostics elaborations and treatments fulfilling. As examples of applications of the agents for fulfilling medical tasks, different than medical diagnoses, we mention: patients monitoring and management [15, 16, 17], healthcare related problems solving [18, 19, 54, 12] and telehealth [52].

In this paper, we have proposed a novel assistant multiagent system called *MASM* (*Medical Assistant Multiagent System*) that can help more physicians during the medical diagnosis problems solving. *MASM* system partially handle the complexity of the diagnosis problems solving, by realizing tasks that may increase the diagnostics accuracy and makes easier the medical diagnoses elaborations by physicians.

The proposed medical diagnosis system is a complex system. It is composed from assistant expert system agents, which cooperate in order to discover difficult medical diagnosis problems solutions. Difficult medical cases are those in which is difficult the illnesses identification and the corresponding efficient treatments establishment. *Medical expert system agents* represent a novel class of agents developed in our previous researches. Applications of the medical expert system agents presented in [12, 48, 11], demonstrate that they can be used as members of complex medical multiagent systems. *Assistant expert system agents* represents a novel class of agents developed in our previous

works [14, 28]. *Assistant expert system agents* represent an adaptation of the medical expert system agents, in order to help flexibly physicians and other agents in the problems solving. The main novelty in the *MASM* complex system consists in the intelligent assistance of the physicians. The intelligence of a system can be measured how "well" (flexibility, accuracy, efficiency, capability to handle uncertainties) the system can solve complex problems.

One of the purposes of the development of complex systems consist in the efficient solving of difficult problems. The developed assistant *MASM* multiagent system is illustrative in this sense. Represents a solution for helping physicians in difficult problems solving by decreasing the complexity of the medical tasks that the physicians must fulfill. The system's members cooperate in order to handle the complexity of the problems solving. They can discover cooperatively emergent properties that arise during the medical problem solving processes, which improve the system's future operation.

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AgentTime: A Distributed Multi-agent Software System for University's Timetabling

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Abstract. In the course of researching distributed timetabling problems this article applies the multi-agent paradigm of computations and presents a correspondent mathematical model for university's timetabling problem solution. The model takes into account dynamic nature of this problem and individual preferences of different remote users for time and location of classes. In the framework of that model authors propose an original problem-oriented algorithm of multi-agent communication. Developed algorithm is used as a foundation for the distributed software system AgentTime. Based on multi-agent JADE platform AgentTime provides friendly graphical interface for online design of time tables for universities.

Keywords: timetabling, multi-agent algorithms, distributed systems.

1. Introduction

In modern society time scheduling plays the outstanding role. Any schedule is the obligation, which enables to carry out authentic planning of activities for a separate person, and whole industrial systems as well. Timetabling represents an important research activity in the scheduling theory, and focuses on such problems as optimal lecture's schedules in educational institutions, week cyclic schedules of plane's flights within the framework of several airports, week or daily schedules of railway transportation, etc. For all these problems the interval of time, inside which the given set of jobs should be fulfilled, is known beforehand. Thus, the minimum of the schedule's length is not usually considered as a primary goal, - other criteria are used for estimation of quality of the schedule having been built. For example, in an educational institution the timetable design process should achieve the following goals: minimization of maximal length of a working day, minimization of the number of the "holes" in the schedules of groups and professors, maximal satisfaction of personal professor's preferences to the time and location of classes, etc. In the current situation, when many educational institutions rapidly grow in size, and distribution scale, wide application of effective software systems for distributed solution of timetabling problems becomes very important.

At present there are many various algorithms for drawing up the time tables in universities. The fundamental approaches are based on the well-known linear and integer programming paradigms [2, 3, 4, 5, 6]. However, several researches of 80s have shown that the integer programming is not equally effective from the point of view of the calculations volume. The high computational costs make integer programming poorly attractive to the large tasks of timetable design, because that method does not guarantee productivity, when the sizes and complexity of tasks grow [3, 7].

Last twenty years have shown the increased interest of the researchers to development of the approaches for the design of timetables with use of various meta-heuristics [3], like simulated annealing, Tabu Search, genetic algorithms (GAs), and their hybrids [8, 9, 10, 11, 12, 13, 14, 19]. It is affirmed, that among others, GAs have larger capacity, and allow to find the greatest number of the feasible solutions [15, 16, 17]. Nevertheless, when GAs are exploited, there are difficulties in the description of controlling parameters, in definition of exact roles of crossover and mutations, as well as in analysis of convergence [18].

Also, it should be noted, that the majority of the considered approaches follow the paradigm of centralized systems, they do not allow the remote users govern the process of timetable design. In complex distributed and evolving systems like modern virtual universities and peer-to-peer communities, that shortcoming makes impractical classical methods, and demands new timetabling principles, which take account of real-time user's preferences in complex changing environments. A multi-agent approach represents a successful paradigm for those kinds of problems, when an optimal or quasi-optimal solution is built in the result of interaction of large number of autonomous computational entities.

In general, for timetabling applications several types of multi-agent algorithms are suitable. The first type of algorithms includes economics-based models of interaction [1, 20, 21]. The second one consists of various generic algorithms for solution of Distributed Constraint Satisfaction Problems (DCSP) [22, 23]. But the most effective algorithms, comprising the third type, were specifically designed for a particular scheduling problem. Such specialized algorithms apply all domain- or problem-specific information and show unbeaten productivity. The results are known [24, 25, 26, 27], where authors propose problem-specific algorithms of agent's interaction for the meeting scheduling. Although such algorithms fit well the timetabling model, and have attractive computational efficiency, their direct application for university's timetabling is not so straightforward and requires additional efforts.

In the given article authors propose new multi-agent algorithm for university's timetabling, and describe basic principles of the distributed system AgentTime, based on that algorithm. Presentation of the results has the following structure. Section 2 describes the mathematical model of university's timetabling, which includes user's preferences. Section 3 gives overview of the corresponding multi-agent algorithm for the design of timetables. In section 4 certain topics of AgentTime's software implementation are considered. Overview of results and discussion are presented in Section 5. Section 6 contains references.

2. The Proposed Mathematical Model For the University's Timetabling Problem

The exact mathematical statement of the university's timetabling problem forms the basis of our own multi-agent algorithm for design of the educational schedule. For the sake of generality we use the term 'teacher' to denote different kinds of university employees (e.g. professors, instructors, etc), the term 'stream' to denote a stream, and the term 'subject' to denote different kinds of student's subjects. Also we give the same name of user to all of the stakeholders of the schedule (e.g. the teachers and student's groups). In our mathematical model we will use the following designations.

Student's groups and Streams. $g \in \mathbf{G}$ – the unique identifier of the group. \mathbf{G} – the set of group's identifiers. $|\mathbf{G}| = \gamma$ – the total number of groups. Each group belongs to one stream at least. Some streams can consist of a single group, but in most cases several groups form a stream with the following constraints:

1. All groups of the same stream exploit the same classrooms for lectures.
2. Lectures are delivered to all groups of the stream at the same time.
3. Each stream has as minimum one lesson.

\mathbf{R} – the set of stream's identifiers. $|\mathbf{R}| = \rho$ – the total number of streams. $r \in \mathbf{R}$ – the unique identifier of the stream. Each single group can be treated as a separate stream, thus $\rho \geq \gamma$. $\mathbf{C}_r \subset \mathbf{G}$ – the stream. $\mathbf{C} = \{\mathbf{C}_1, \mathbf{C}_2, \dots, \mathbf{C}_\rho\}$ – the set of streams.

Teachers. \mathbf{P} – the set of unique teacher's identifiers. $p \in \mathbf{P}$ – the unique teacher's identifier.

Timetable users. Union of the group's set and the teacher's set gives us the set of the timetable users: $\mathbf{M} = \mathbf{G} \cup \mathbf{P}$, $m \in \mathbf{M}$ – the unique identifier of the timetable user.

Time. \mathbf{W} – the set of the days of the week. $w \in \mathbf{W}_g$ – the certain day of the week.

$\mathbf{W}_g \subset \mathbf{W} = \{1, 2, \dots, 7\}$ – the set of learning days for the group g ,

$j \in \mathbf{J} = \{1, 2, \dots, 8\}$ – the lesson's number. $\mathbf{T} = \{(w, j) | w \in \mathbf{W}, j \in \mathbf{J}\}$ – the set

of timeslots, which are the elementary units in the timetabling problem. For example, the timeslot (1, 2) means the second lesson on Monday. For each timetable user m the set of free timeslots $\mathbf{T}_m^+ \subset \mathbf{T}$ is known. The set of denied timeslots $\mathbf{T}_m^- \subset \mathbf{T}$ is known also. We assume the obvious constraints are true (i.e. $\mathbf{T}_m^+ \cup \mathbf{T}_m^- = \mathbf{T}$; $\mathbf{T}_m^+ \cap \mathbf{T}_m^- = \emptyset$).

Subjects. In our model teachers conduct lectures and manage practical exercises. Lectures are delivered to the whole stream, while practical exercises are organized for a single group only. Also some practical exercises impose restrictions on allowable classrooms, like computer or chemistry labs. To describe all these peculiarities, let's introduce the following mathematical structures.

$\mathbf{S}_r = \{1, 2, \dots, \sigma_r\}$ – the set of lecture's identifiers delivered to the stream r .
 $s_r \in \mathbf{S}_r$ – the unique lecture's identifier; $\mathbf{Q}_r = \{1, 2, \dots, \theta_r\}$ – the set of practical exercise's identifiers organized to the stream r . $q_r \in \mathbf{Q}_r$ – the unique exercise's identifier;

Each lecture's assignment can be uniquely identified by a pair $(r, s_r) \in \mathbf{RS}$, where

$$\mathbf{RS} = \{(r, s_r) \mid r \in \mathbf{R}, s_r \in \mathbf{S}_r\} \quad (1)$$

The total number of lecture's assignments is computed as follows:

$$|\mathbf{RS}| = \sum_{r=1}^{\rho} \sigma_r \quad (2)$$

Each certain exercise's assignment can be uniquely identified by a triple $(r, g_r, q_r) \in \mathbf{RQG}$, where

$$\mathbf{RQG} = \{(r, g_r, q_r) \mid r \in \mathbf{R}, q_r \in \mathbf{Q}_r, g_r \in \mathbf{C}_r\} \quad (3)$$

The total number of exercise's assignments is computed as follows:

$$|\mathbf{RQG}| = \sum_{r=1}^{\rho} |\mathbf{C}_r| \cdot \theta_r, \text{ where } |\mathbf{C}_r| \text{ – is the total number of groups in the}$$

stream \mathbf{C}_r .

For further analysis differences between lectures and practical exercises can be neglected and the united set of subjects \mathbf{E} will be used:

$$\mathbf{E} = \mathbf{RS} \cup \mathbf{RQG} \quad (4)$$

Curriculum consists of subjects' assignments for each of the teacher during one semester (fall) in the following form:

$$\delta : \mathbf{E} \rightarrow \mathbf{P} \quad (5)$$

$$\delta(e) = \begin{cases} \delta_1(e), e \in \mathbf{RS} \\ \delta_2(e), e \in \mathbf{RQG} \end{cases}$$

$\delta_1 : \mathbf{RS} \rightarrow \mathbf{P}$, where \mathbf{P} – the set of teachers; \mathbf{RS} – the set of lecture's

assignments.

$\delta_2 : \mathbf{RQG} \rightarrow \mathbf{P}$, \mathbf{RQG} – the set of exercise's assignments.

For example, $\delta_1(1, 2) = 4$ means, that teacher 4 delivers lecture 2 for stream 1, and $\delta_2(1, 2, 4) = 7$ means, that teacher 7 manages practical exercise 2 for group 4, included into the stream 1.

Given the curriculum δ , we can easily compute the total number of subjects \mathbf{E}_m assigned to the teacher (or the group) with identifier m :

$$\begin{aligned} \mathbf{E}_m = & \{e \mid m \in \mathbf{P} \wedge \delta(e) = m\} \cup \{e = (r, s) \mid m \in \mathbf{C}_r \wedge s \in \mathbf{S}_r\} \cup \\ & \{e = (r, q, m) \mid m \in \mathbf{C}_r \wedge q \in \mathbf{Q}_r\} \end{aligned} \quad (6)$$

Room's stock consists of laboratories, lecture halls and classrooms available for subjects in the university. It is modeled by the set \mathbf{A} of unique room' identifiers. For each element of the set of subjects \mathbf{E} , a subset of permitted rooms \mathbf{A}_e is selected : $\mathbf{A}_e \subset \mathbf{A}$.

The primary goal of the timetabling problem in our model is formulated as looking for the feasible mapping from the set of subjects \mathbf{E} to the set of timeslots \mathbf{T} :

$$\tau : \mathbf{E} \rightarrow \mathbf{T} \quad (7)$$

For example, mapping $\tau(1, 2) = (4, 4)$ means that subject 2 for stream 1 will be given on Thursday during the fourth lesson.

Related with the mapping τ , the mapping α should assign a classroom for each subject:

$$\alpha : \mathbf{E} \rightarrow \mathbf{A}, \text{ where} \quad (8)$$

\mathbf{E} – the set of subjects; \mathbf{A} – the set of classrooms.

For example, mapping $\alpha(1, 2) = 101$ means that subject 2 for stream 1 will be conducted in the room 101.

Constraints for the university's timetabling problem are defined as follows.

1. The teacher can conduct only one subject at the single timeslot.

$$\forall p \in \mathbf{P}, \forall e_1, e_2 \in \mathbf{E} : e_1 \neq e_2 \wedge \delta(e_1) = \delta(e_2) = p \Rightarrow \tau(e_1) \neq \tau(e_2) \quad (9)$$

2. In one classroom only one subject can be given at the single timeslot.

$$\forall a \in \mathbf{A}, \forall e_1, e_2 \in \mathbf{E} : e_1 \neq e_2 \wedge \alpha(e_1) = \alpha(e_2) = a \Rightarrow \tau(e_1) \neq \tau(e_2) \quad (10)$$

3. Each group has no more than one subject at the single timeslot.

$$\begin{aligned} \forall g \in \mathbf{G} : (e_1 = (r_1, \dots), e_2 = (r_2, \dots)) \in \mathbf{E} \wedge g \in \mathbf{Cr}_1 \wedge g \in \mathbf{Cr}_2 \wedge e_1 \neq e_2) \vee \\ \vee (e_1 = (\dots, g), e_2 = (\dots, g)) \in \mathbf{E} \wedge e_1 \neq e_2) \Rightarrow \tau(e_1) \neq \tau(e_2) \end{aligned} \quad (11)$$

Subject's priority. It is obvious, not all subjects have identical importance within the framework of educational process. As such, it is necessary to set the priority order among different subjects, so subjects with higher priority will borrow the best time and location. In our model the priority is modeled as the partial order on the set of subjects \mathbf{E} :

$$e_1 \succ e_2 \Leftrightarrow U(e_1) \geq U(e_2), \text{ where} \quad (12)$$

$e_1, e_2 \in \mathbf{E}$; $U(e) = |\mathbf{M}_e| + k_1(e) + k_2(e) + k_3(p_e)$ – the “utility” of the subject e ;

$\mathbf{M}_e = \{m \mid (e = (r, s)) \in \mathbf{RS} \wedge m \in \mathbf{C}_r\} \vee \{e = (r, q, m) \in \mathbf{RQG}\}$

– the total number of groups for those the subject e is given;

$k_1(e) \in \{0, 5, 10\}$ – the measure of subject's importance for the stream (0 – optional, 5 – important in general, 10 – important for stream);

$k_2(e) \in \{0, 2\}$ – 0 – undergraduate, 2 – graduate;

$p_e = p, \delta(e) = p$ – the teacher's identifier;

$k_3(p_e) \in \{0..5\}$ – the estimation of the novelty level of the material given by the teacher p_e .

User's Preferences comprise the important part of our model. Each preference is modeled by a numeric value from the range $[0, 1]$. Value 0 corresponds to the least desired alternative, and value 1 corresponds to the most desired alternative. The model includes two kinds of the user's preferences:

- preferences of the user $m \in \mathbf{M}$ for the time of subjects:

$$f_m^1 : \mathbf{E}_m \times \mathbf{T}_m^+ \rightarrow [0, 1] \quad (13)$$

- preferences of the user $m \in \mathbf{M}$ for the location of subjects:

$$f_m^2 : \mathbf{EA}_m \rightarrow [0, 1], \text{ where} \quad (14)$$

$\mathbf{EA}_m = \{(e, a) \mid a \in \mathbf{A}_e \wedge e \in \mathbf{E}_m\}$ – the set of feasible pairs “subject-classroom”.

We use an evident representation of the user's preferences in the form of graphics tables (table 1, 2). The darker color denotes the more preferable alternative (in respect of time or location).

Table 1. Preferences of the user m for the desirable time of subjects, f_m^1 ,

		Lesson's number, j							
		1	2	3	4	5	6	7	8
Days of the week, w	Mon								
	Tue								
	Wed								
	Thu								
	Fri								
	Sat								
	Sun								

Table 2. Preferences of the user m for the desirable location of subjects, f_m^2

Classroom, a	1	2	3	4	5
f_m^2					

Criterion of timetable quality generalizes several partial criteria, and evaluates the solution found, namely the pair of mappings $\tau(e), \alpha(e)$. The first partial criterion evaluates the sum of the user's preferences for the time of the subject e :

$$F_e^1(\tau) = \sum_{m \in \mathbf{M}_e} f_m^1(e, \tau(e)) \rightarrow \max \quad (15)$$

The second partial criterion evaluates the sum of the user's preferences for the location of the subject e :

$$F_e^2(\alpha) = \sum_{m \in \mathbf{M}_e} f_m^2(e, \alpha(e)) \rightarrow \max \quad (16)$$

The generalized criterion is constructed as follows:

$$F(\tau, \alpha) = \sum_{e \in \mathbf{E}} (F_e^1 + F_e^2) \rightarrow \max \quad (17)$$

The solution of the described problem consists of the found mappings τ, α , assuming that all constraints are satisfied, and the generalized criterion has a maximum value.

3. The Multi-Agent Algorithm For Timetable Design

We took for the basis of our algorithm the well-known multi-agent algorithm MSRAC for meetings scheduling by A. Ben Hassine et al. [27]. Although some correspondences still remain, our algorithm is specifically designed for a quite different problem of university's timetable design, and together with time schedule it gives also an occupancy schedule for classrooms.

In our algorithm we recognize two roles of agents: agents-organizers and agents-participants. The agent's structure also mimics the application domain, so we classify all agents as teachers, groups and classrooms. Agents-teachers play the role of organizers; agents-groups and agents-rooms play the role of participants. The numbers of agents-teachers and agents-groups correspond to the real numbers of the teachers and the groups in the university. One agent-room corresponds to all classrooms in the context of the single time table. Collective search for the best time and location of the study involves communication between different agents. For each study the agent-teacher performs a set of actions, comprising the following state diagram (fig. 1).

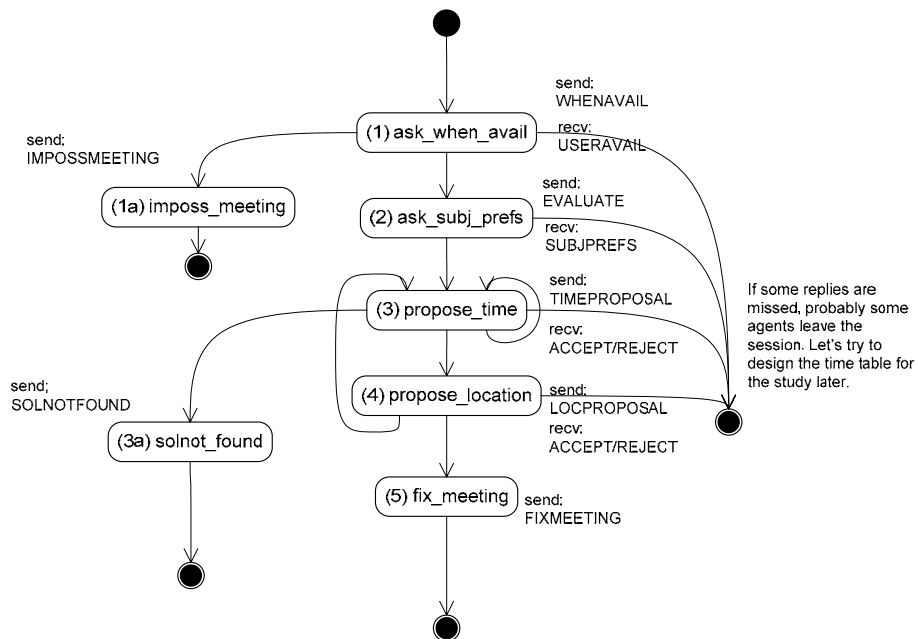


Fig. 1. The state diagram of the agent-teacher's algorithm.

The agent-teacher performs state transitions in accordance with the description given below.

1) *Ask_when_avail*. That is the first state in the algorithm. The agent-teacher sends to all agents of groups the query WHENAVAIL with the study's identifier, requesting available time for that study. The agents of groups answer by the message USERAVAIL, in which they inform when the agent is free, and has available time for the study. If all agents have informed the answer, then the agent-teacher finds intersection on time. If the intersection is empty, then the agent comes the final state *imposs_meeting*.

1a) *Imposs_meeting*. In that state the agent-teacher founds itself if intersection of available for other agents times is empty and the total solution was failed. The study is marked as "having no solution".

2) *Ask_subj_prefs*. In that state the agent-teacher requests preferences for time and location (the message EVALUATE). The agents of groups reply own preferences in the message SUBJPREFS. The agent-teacher sorts received preferences for time and for location in accordance with criteria (15) and (16).

3) *Propose_time*. The agent-teacher selects the first timeslot from the ordered list of the preferences, and sends it along with the study's identifier to the agents of groups inside the message TIMEPROPOSAL. In response the agents of groups analyze own agendas. If the proposed timeslot is free in the agent's agenda, the agent gives the positive answer, sending the message ACCEPT. Else the agent compares the priority of the study in the agenda with the priority of the study in the message. If the priority of the message's study is greater, then the agent accepts new proposal and sends the message ACCEPT. In the opposite case the agent sends the message REJECT. The agent can apply the metropolis criterion [27] for decision making when the priorities are equal. In the case of total acceptance of the proposed timeslot, the agent-teacher passes to the next state *Propose_location*; in default the agent remains in the state *propose_time*, and chooses the next timeslot to negotiate. If all timeslots were rejected, it means that the decision for the currently selected study does not exist, and the agent-teacher passes to the state (3a) *Solnot_found*.

3a) *Solnot_found*. In that state the agent-teacher founds itself if all proposed for timeslots were rejected by the agents of groups, and the total solution was failed. The study is marked as "having no solution".

4) *Propose_location*. The agent-teacher sends the sorted list of classrooms to the agent of classrooms inside the message LOCPROPOSAL. That message contains also the study's identifier and the timeslot's identifier. Using own occupancy list, the agent of classrooms searches for the first classroom in the list, which is available for the timeslot given or occupied by the study with a lower priority. If the search was successful, and the classroom is found, in reply to the agent-teacher the agent of classrooms sends the message ACCEPT with the identifier of the room found. In the failure case the agent of classrooms sends the reply REJECT. Once the positive ACCEPT reply is received, the agent-teacher moves to the next state. In the result of REJECT receiving the agent returns to the state *propose_time* for selecting the available timeslot for the study.

5) *Fix_meeting*. If the agent-teacher occurs in that state, it means that both the timeslot and the classroom for the study were successfully found. In the result the agent-teacher sends to all other agents the message FIXMEETING with the identifiers

of the study, the timeslot, and the classroom. If the agent-group does not have assignment for the received timeslot, the timeslot is fixed. By a similar way the agent of classrooms fixes the location. If the received timeslot (or the classroom) is occupied, the agent discards assignment of the study with lower priority, and sends to the agent-teacher the message CANCEL MEETING, which is forwarded further to other agents in order to modify their agendas.

In the states (1), (2), (3) and (4), if some agents did not send the answer during a predefined time period, the agent-teacher places the study, being under consideration, into the list of the cancelled subjects, to retry attempts later. Once all agents-teachers finish state processing, the common schedule is considered to be complete. One important feature of our algorithm is that of the partial timetable is always available. The complete timetable, including all the subjects, sometimes simply does not exist. In such a case, however, the considered algorithm will build the consistent time table, with some subjects of low priority ignored.

4. Implementation Details of the Software System AgentTime

The described mathematical model and the multi-agent algorithm were applied in the course of design and development the software system for time tabling called AgentTime. AgentTime uses rich communication and agent-life cycle capabilities of Java-based JADE multi-agent platform [29], and has highly distributed software architecture (fig.2). Flexible multi-tier architecture of the system supports simultaneously multiple timetable design sessions and interaction of multiple agents.

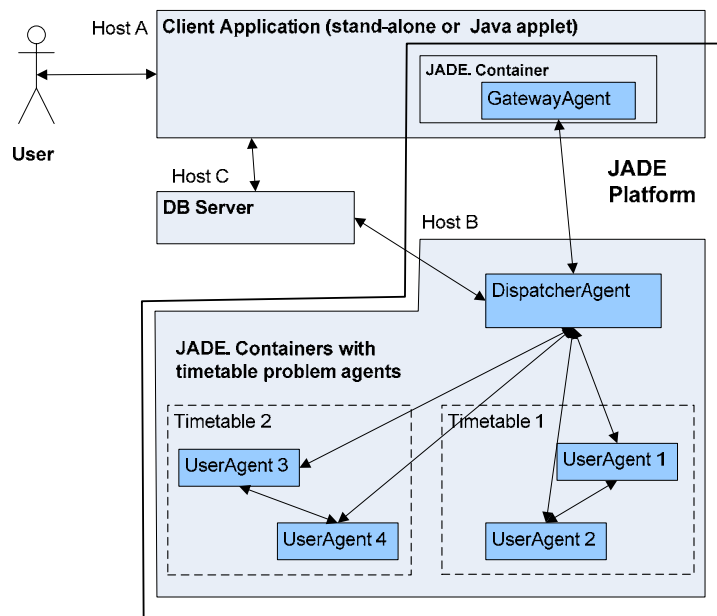


Fig. 2. The software architecture of AgentTime timetabling system.

In AgentTime agents indirectly communicate with each other by passing the messages in accordance with a problem-specific ontology (table 3).

Table 3. Multi-agent ontology for timetable design

Message	Semantics
WHENAVAIL	Inquiry to the agent-group for available timeslots.
USERAVAIL	Agent's response to the message WHENAVAIL. The message contains the vector with available timeslots. Format : $(a_{11} a_{12} \dots a_{18} a_{21} a_{22} \dots a_{28} \dots a_{71} a_{72} \dots a_{78})$, where $a_{ij} \in \{0,1\}$, 1 – working day, 0 weekend.
IMPOSSMEETING	The message to the server agent about impossibility to find a time table for the subject with id sbj_id . Format: (sbj_id) .
EVALUATE	Inquiry to the agent of group for time and location preferences related with the subject sbj_id . Format: (sbj_id) .
SUBJPREFS	Agent's response to the EVALUATE. Format: $(sbj_id (w_{11} w_{12} \dots w_{18} w_{21} w_{22} \dots w_{28} \dots w_{71} w_{72} \dots w_{78}) ((L_1 p_1) (L_2 p_2) \dots (L_n p_n)))$, where $0 \leq w_{ij} \leq 1 (i = 1..7, j = 1..8)$ – evaluation of i -th day of week and , j -th lesson; $0 \leq L_k \leq 1, k = \overline{1, n}$ – the number of the classroom; missing classrooms have the priority with value 0; $0 \leq p_k \leq 1, 0 \leq k \leq n$ – the preference of the classroom L_k .
TIMEPROPOSAL	The agent-teacher proposes time for the subject sbj_id . Format: $(sbj_id (d p))$, where d – the day of the week; p – the number of the lesson.
LOCPROPOSAL	The agent-teacher proposes location for the subject sbj_id . Format: $(sbj_id (L_1 L_2 \dots L_m))$, where $L_k, k = \overline{1, m}$ – the identifier of the classroom. The classrooms are sorted in accordance with the preferences.
ACCEPT/REJECT	Agent's response to the message TIMEPROPOSAL (ACCEPT or REJECT). If the proposal is accepted the message contains the classroom's identifier. Format: (L) , where L – is the id of classroom.
FIXMEETING	Inquiry to fix the timeslot and location for a certain subject sbj_id . Format: $(sbj_id (d p) L)$, where d – the day of week; p – the id of the lesson; L – the id of the classroom.
CANCELMEETING	Notification about cancelling a conflicting subject. Format: $(sbj_id (d p) L)$, where d – the day of the week; p – the id of the lesson; L – the id of the classroom.

Interaction of the agents during the design of timetable can be illustrated by the UML sequence diagram in fig. 3. In AgentTime apart from previously mentioned types of the agents we use the dedicated ServerAgent which is responsible for communication with external data sources, logging and other technical tasks.

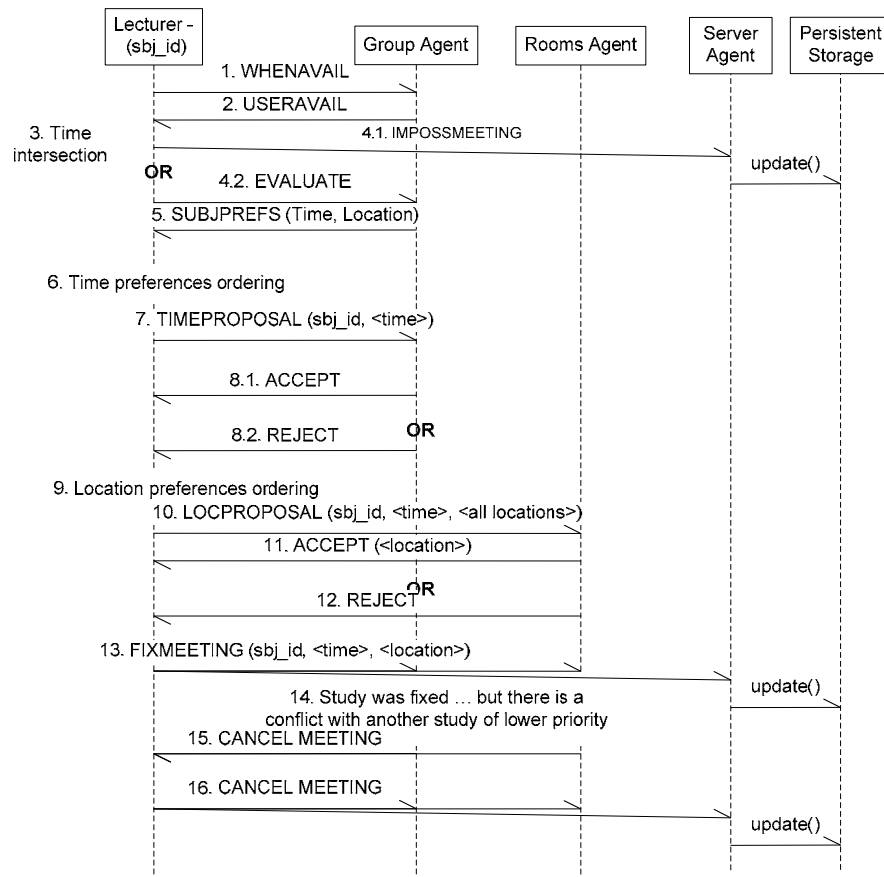
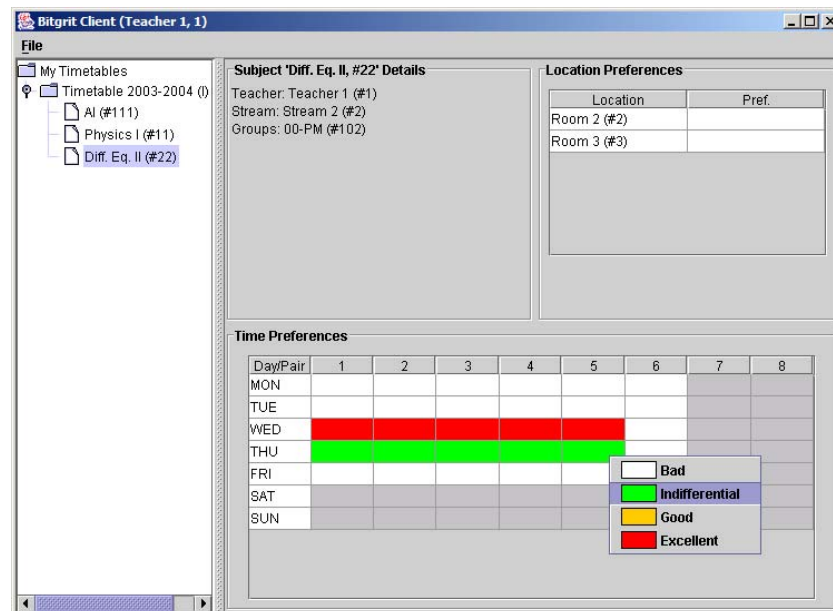
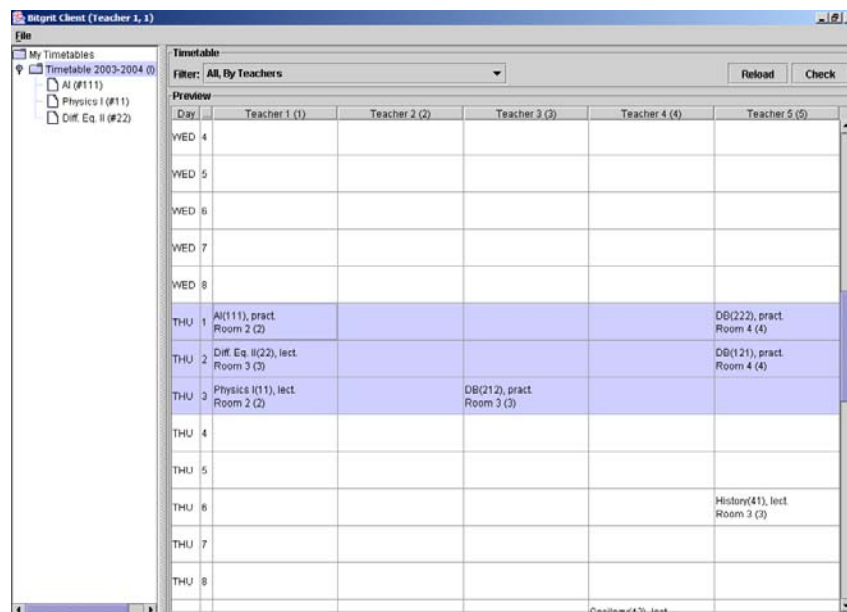


Fig. 3. The state diagram of the agent-teacher's algorithm.

Different users of AgentTime can interact with the system using different end-user tools, including web-browsers and PDAs. The mostly used way of interaction assumes application of applet-based graphical interfaces (fig.4), but also JSP-based interfaces are available.



a)



b)

Fig. 4. Examples of AgentTime's graphics interfaces: a- assignment of preferences; b – representation of the ready timetable.

5. Discussion

This article focused on the important problem of time tables' design for educational institutions. To tackle this problem in the context of modern distributed and highly dynamic universities we proposed the mathematical model and correspondent multi-agent algorithm for iterative timetabling in presence of different subjective preferences for time and location of subjects. The theoretical considerations become a foundation for development of the multi-agent software system AgentTime. That system facilitates distributed time planning and allocation of timeslots and classrooms.

The developed algorithm belongs to the class of domain-specific multi-agent algorithms and shows good performance metrics. Analysis shows that in the case of single computational node computational complexity of the algorithm C for allocation of timeslots and rooms can be estimated as follows:

$$C \leq S \cdot \log_2 S \cdot T_0(n_g, n_l), \text{ where}$$

S – is the number of subjects, $T_0(n_g, n_l)$ – a constant determined by the problem's conditions. If AgentTime is distributed among $P \leq S$ computational nodes, then estimation of processing time t_p will be $t_p \leq \frac{C}{P} = \frac{S}{P} \cdot \log_2 S \cdot T_0$. In the extreme case, when $P = S$, t_p will not be greater then $T_0 \cdot \log_2 S$.

Comparing our results with other known approaches to multi-agent timetabling like the algorithm MSRAC [27], we can note that our system is capable of solving a more general problem, allocating not only timeslots, but classrooms also. With a few modifications proposed model and algorithm will be suitable for managing other important resources as well. At the same time we need to improve theoretical background of our algorithm to rigorously prove the optimality of the solutions found in terms of the criteria (16) and (17).

In the nearest time we are going to perform wide-area field experiments with AgentTime to test its robustness and quality of timetabling in real conditions of the complex university. We are also interested in extending the proposed mathematical model and software implementation of AgentTime with other approaches to multi-agent coordination. In this context application of the paradigm "Controller-Variable Agent" [28] is seemed to be very promising.

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A Decentralised Approach for the Transportation On Demand Problem

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Summary. Public transport systems are generally organized in a static, a priori way. In such systems, the demand must be adapted to the offer. In this paper, we propose a model based on self-organization in order to dispatch a fleet of vehicles in a purely dynamic Transportation On Demand system (TOD). Our proposal consists in a decentralized approach and a multi-agent system (MAS) to model the environment. This will tackle the problem of vehicles over-concentration or the lack of service in certain areas of the city. We demonstrate that our model addresses these problems by providing vehicle agents, for a given request, to make the final decision thanks to a negotiation process and to calculate overcosts according to an original insertion heuristic.

Key words: transportation on demand, vehicle routing problem, collective intelligence, self-organization.

1 Introduction

Growing environmental concerns are linked to the difficulties of management of urban traffic. They lead to the creation of new solutions improving mobility in agglomeration. Current public transportation systems are determinist and based on frequencies and routes fixed in advance. They are built starting from opportunity studies and are not very adaptive to a request that can change in time: the demand has to adapt to the offer. It is thus advisable to complete the urban transportation services by flexible systems being more adapted to the individual needs. We propose a Transportation On Demand (TOD) system which must adapt to users need in real time. It must allow to generate an important reduction in the traffic and to offer a maximal quality of service to reduce the cost of exploitation. Lastly, it will be the basis for a decision support system, computing vehicle tours in real time, a service which is not offered by the traditional transportation systems. The stake of this article is to study the possibility of the installation of a TOD system to satisfy the

requests of the customers at any moment, by distributing the load inside the fleet of vehicles in order to achieve the goals mentioned previously. This system will adjust dynamically to the customers demand. The scenario of the execution starts with the first customer request which appears randomly in a place of the city. It sends a request indicating his departure point and his destination. The resolution consists in choosing the best located vehicle to satisfy the passengers already on board this vehicle as well as the new request by optimizing its rate of filling with respect to the maximal capacity, its time and cost of travel.

First of all, we will present some previous work of similar or neighbouring problems. Then, we will define the data of our problem. Finally we will present our approach and the preliminary results related to the initial tests to finish by a conclusion and some perspectives.

2 Previous Works

The general problem of the construction of vehicles routes is known under the name of Vehicle Routing Problem (VRP) and represents a combinatorial problem of multi-objective optimization which was the subject of many works and many alternatives in the literature. It belongs to the NP-hard category [2, 10]. In its basic version, the VRP problem (see figure 1) models a well known transportation problem which consists of pickup (and/or collect) products to serve a set of customers using a fleet of vehicles. The resolution consists in determining a set of routes which minimizes objectives as well as possible as the total traveled distance, the number of vehicles used and the sum of customers delays [8].

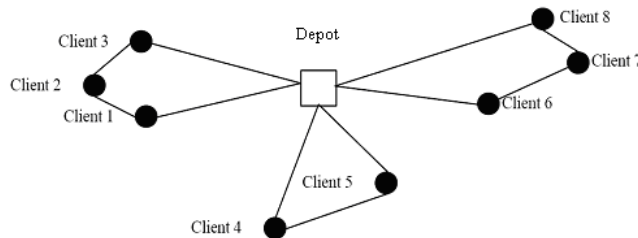


Fig. 1. Vehicle Routing Problem

A complete state of the art of the VRP problems in the static context, and in particular, the dynamic one and their applications is given by [9]. In [13], the Dynamic VRP (DVRP) problem was treated. The resolution consists in dividing time execution into slices. The DVRP problem is considered

as a succession of VRP static problems and an ant colony algorithm is used to solve these static problems. When it is about the problem of VRP with pickup and delivery of goods, one speaks about PDP (Pickup & Delivery Problem). The Dynamic PDP problem was studied by [6]. The authors adopted a multi agent approach. The conversation between agents was based on the Contract Net Protocol. The clients demand arrival was calculated with basis on Poisson distribution. A Dial-A-Ride Problem (DARP) is an extension of the PDP in which the transportation of goods is replaced by the transportation of people [4, 7]. Since we talk about people transportation, the DARP focuses better than the PDP on the satisfaction of these people. We can distinguish between a static or dynamic version of the DARP although the difference is not always strict. Indeed, in the static case, some reservations can be canceled at the last minute, which implies a degree of dynamism while in the dynamic case, most reservations are known a priori before planning [1]. The problem is then, in general, treated as a succession of static sub-problems [3]. An application in the urban transport related to the bus on demand was developed in [12]: a customer can give a time window in which he wishes to be served, instead of departure or arrival, but not both at the same time. Some requests are known in advance and other ones can arrive during the execution. The authors adopted a solution based on an insertion heuristic which gave interesting results with short execution time. This problem is known as the Dynamic Dial-A-Ride Problem (DDARP) with several places of pickup and delivery. In [11], the DARP problem was treated online by considering a homogeneous fleet of vehicles with unit capacities, i.e. a vehicle having a passenger on board, cannot serve another one except if it reached the first passenger destination. To our knowledge, the dynamic DARP is rarely studied and does not exist in its purely dynamic version. The problem we deal with in this paper is original, because the requests are dealt with in real time: we dispose of a purely dynamic case. This dynamicity is due to the fact that no reservation is known in advance. Moreover, traditional techniques suppose to have a control central of traffic which knows vehicles positions and their planned routes (the central receives customers requests, calculates new vehicles routes and orders vehicles to service customers). Moreover, these techniques suppose a perfect knowledge by the central of the states of the vehicles (including breaks, breakdowns, communication problems) in real time, which is not realistic and can involve expensive calculations in time. That's why we adopt a decentralized approach (except for the new client request reception) to face this kind of problems; since embarked system is now standardized.

3 Proposed dynamic model

3.1 Problem Description

The model tries to arbitrate between different constraints.

Each user wishes:

- To minimize the waiting time once his demand is accepted,
- To reach his destination, respecting his desired deadline.

Each vehicle tries:

- To maximize its rate of filling by changing an already planned route to service a new request,
- To deal with the evolution of the traffic load and especially unexpected events (accidents, a road becomes blocked up, new roads) and historical events: the system must be adapted to learn about repetitive events to predict similar ones in the future,
- To negotiate with the other vehicles in order to choose the best proposal to serve each new request.

The system tends, as well as possible, to pair users and vehicles by arbitrating and adjusting the previous constraints. The system is not centralized but emerges from the fleet of vehicles.

We propose an agent-oriented approach. The system is composed of the following agents: *Vehicle*, *Interface* and *Client*. The scenario of the execution is described as follows: a user connects to the system via a given support (service call, Web server.), it is then instantiated by a *Client agent* whose function consists in representing it in the system. The user indicates his departure point and his destination as indicated previously. Thus, the *Client agent* enters in interaction with the *Interface agent* (see figure 2).

The latter broadcasts the request of the user to other *Vehicle agents* located in an environment which is modelled in the following section. Our model is specific by the fact that the requests are not dealt with batch but in "real time".

3.2 Environment modeling

We represent the urban network by a directed dynamic graph $G(t) = (V(t); E(t))$ where $V(t)$ is the set of nodes and $E(t)$ the set of arcs:

- The nodes represent interesting places of the network: crossroads, stations, cinemas, commercial centers ...
- The arcs represent the roads taken by the vehicles,
- The weight on each arc represents the needed time to cross this arc, depending on the current load of the traffic,
- Dynamic aspect relates to the weights of the arcs, which can evolve in time, according to the evolution of the fluidity of circulation. It can be related to the apparition and/or disappearance of arcs.

Customers are associated to a node or vehicles, which are themselves on nodes or arcs. The size of the population of users and vehicles is variable in time, to obtain a day/night simulation mechanism for example. Once a temporal

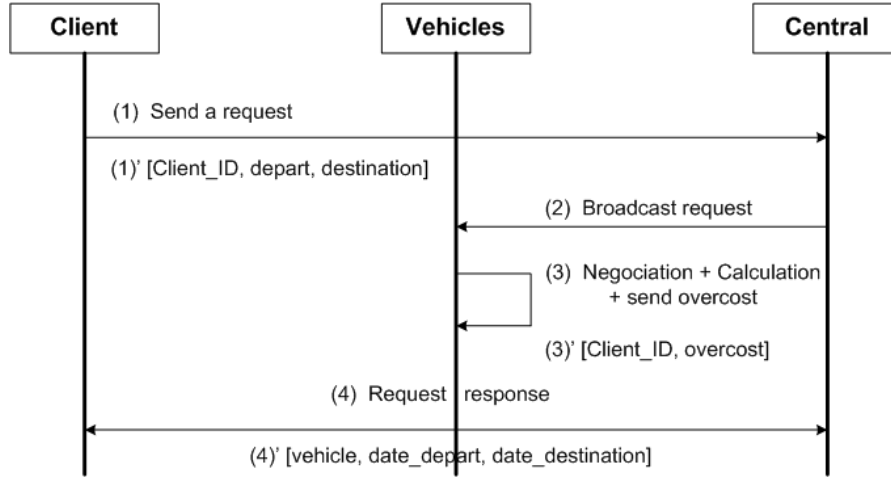


Fig. 2. UML Sequence Diagram showing interactions between the actors of the system

model of the population of users is established, the population of vehicles must be established accordingly, in order to have a satisfying average rate of occupation for vehicles.

3.3 Offers and dynamic choices

Our model is based on two simultaneous phases, an offer phase and a choice phase. We want to establish an agreement between the proposals for a transport and the needs of the customer. This is done according to a mechanism of negotiation. A key element of the system is pairing vehicles and customers. Which vehicle is the best for servicing a given request? Who determines it and how? How a vehicle knows if it has been selected to service a new demand? These questions are not independent. The best vehicle corresponding to each user will be selected; it must minimize the additional effort ΔC to service the customers. To know this additional effort, a vehicle calculates, on the one hand, the total cost (in time) of its current route, that of the route to discharge the current passengers and charge already planned ones. On the other hand, a vehicle calculates also the cost of the new route to service actual passengers by including the new one. The difference between the two costs is the additional effort or *overcost*.

Each customer request is diffused to all vehicles. When receiving a new request, each vehicle calculates its overcost to service the request and diffuses it to all the fleet. Then, it compares the received answers to its overcost. It orders the received offers and broadcasts the head of list. Finally, the determined winner is the one being the most times ranked first in the received answers. Ideally, one could exempt these last phases: if the diffusion is per-

fect, all the fleet will obtain the correct classification directly. But, because of non perfect diffusion, we proceed to this additional phase after a possible problem (a vehicle crossing a tunnel for example), and this to be sure that all the vehicles agree on the winner vehicle which will take the passenger.

3.4 The scheduling algorithm

The *vehicle agents* carry out the principal work of planning, and this thanks to the evaluation of the insertion of a travel (source and destination) in such a way to respect the deadlines of the existing passengers in the vehicles. The insertion heuristic is inspired from the ADARTW one [5]. For each vehicle, a scheduling block always starts with the first customer on its way and ends when the last customer reaches its destination. The following figure illustrates the insertion of a customer in a scheduling block of a vehicle having two customers on board ($C1$; $C2$) and going to servicing another client ($C3$) with an already planned route. Each one has a departure point (preceded by a + in figure 3) and a destination (preceded by -). In a block related to a vehicle already containing N clients, corresponds $K = 2N$ stops (2 stops per client) and $(K+1)(K+2)/2$ insertion possibilities when its pickup point must precede its delivery point.

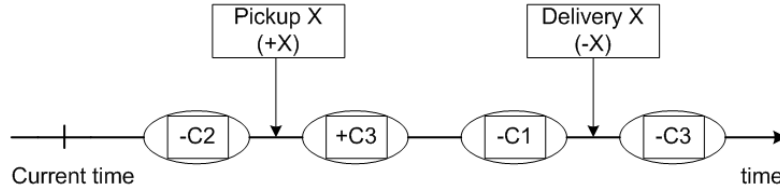


Fig. 3. Insertion heuristic

Complexity depends on the algorithm chosen to find shortest path from a vertex to another - we have considered Dijkstra's algorithm ($O(m + \log(n))$ with m edges and n vertices). It depends also on the insertion heuristic which for a vehicle V with a maximal capacity Q_v gives in the worst case a multiplying coefficient : $Q_v(2Q_v - 1)^2$.

The objective function $MinZ = \Delta C$ represents the minimal overcost due to a new client insertion. ΔC depends on the following variables: additional time to service the new demand, current capacity (number of clients on board) and proximity of the vehicle from the customer. If two or many vehicles give the same value of the overcost for the same customer, the one having the minimal capacity wins this customer. If they have also the same capacity, the winner is the nearest one (in distance).

3.5 Self-organisation

The dynamic and non deterministic aspect of the problem can lead to concentrations of demands in certain zones which are more attractive and may cause a lack of service elsewhere. Indeed, the downtown area, for example, will be a zone of strong attraction at certain hours of the day whereas certain suburban zones become badly serviced. The waiting time of clients in such a zone will then be very important. We thus have chosen to possibly degrade the performances in the attractive zones in spite of having a better service in other areas to avoid any lack of service. Several solutions are possible:

- Injecting some vehicles in the existing fleet but that can violate constraints related to environmental objectives, we could have else a maximum filling of the vehicles,
- A hierarchical centralized resolution which is opposed to the decentralized model we adopt and not very realistic,
- The use of self-organization mechanisms. We chose the last way for its distributed, local and adaptive characteristics.

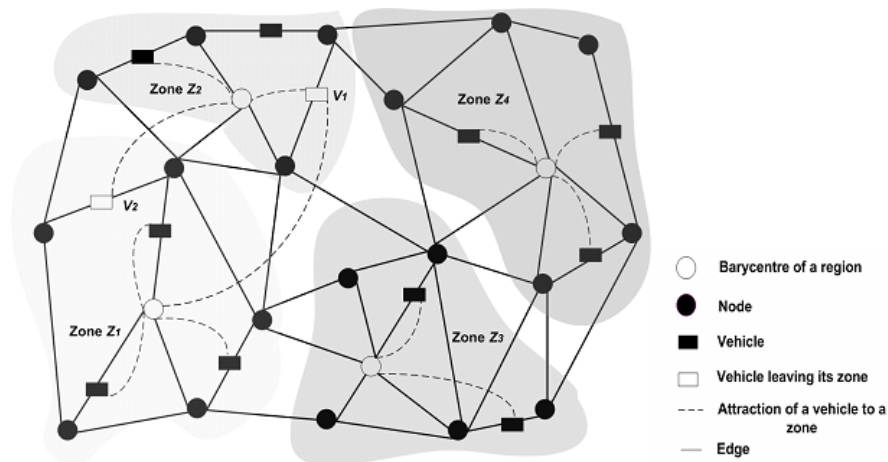


Fig. 4. Attraction of vehicles to zones

Thus, we define zones of attraction within the graph to which the vehicles are attached. These zones will evolve either geographically (the number of node they cover) or according to the number of attached vehicles. These zones have a multi-scale representation by using learning techniques since the vehicles acquire information on the road network state and are able to send the information to the graph.

When a vehicle leaves its zone, it will be penalized in its overcost function which increases while it moves away. This constitutes an exerted attraction

force so we can imagine a spring fixed on the zone center by an extremity (see figure 4) and a vehicle is attached to its other extremity. If this fights against the change of a vehicle zone, it does not prohibit the vehicle from changing its zone. Indeed, the vehicle can gain the bid for a demand coming from another zone. When a vehicle leaves its zone, it can negotiate a change of zone with others. In figure 4, the vehicles *V1* and *V2* exchange their zones. This part of the modelisation is under developpement.

4 Simulation

Multi-agent proposed architecture was developed by using the REPAST Symphony (Recursive Porus Agent Toolkit Simulation) multiagent platform written in Java which focuses on social simulation [14]. This platform developed by the Argonne laboratory of the University of Chicago, inherits main functionalities from SWARM platform (into Objective C) and offers several advanced functionalities:

- Built-in 2D, 3D, and geographical information systems (GIS) support and tools,
- Automated connections to enterprise data sources: relational databases, GIS and to external programs for statistical analysis and visualization of model results,
- Provides information about the state of each agent,
- A scheduler which supports concurrent discrete events in a sequential or parallel way.

As mentioned previously, the calculation of an overcost related to a new request, is done by each vehicle. At a time step, if a vehicle receives a request, it collects the other vehicles answers (overcost of the other vehicles) and compares their overcost to its own, broadcasts a winner message (if he is the winner), vehicles and the concerned customer before going to service him, as described in 3.3 and detailed in activity diagram (see figure 5).

5 Preliminary results

We have implemented our model on a graph with 50 nodes, 7200 time steps, with 4-passenger-seat vehicles. The customers appear at random places and hours, and give random destinations. In the table below, the optimal cost column indicates minimal total time to service all the customers from their departure until their destination nodes. The cost column represents the real time in which we serviced all customers. The client optimal itinerary column indicates the itinerary time average of the served clients. Variation per client is the percentage of difference between the two costs. Filling indicates the interval in which the capacity has oscillated.

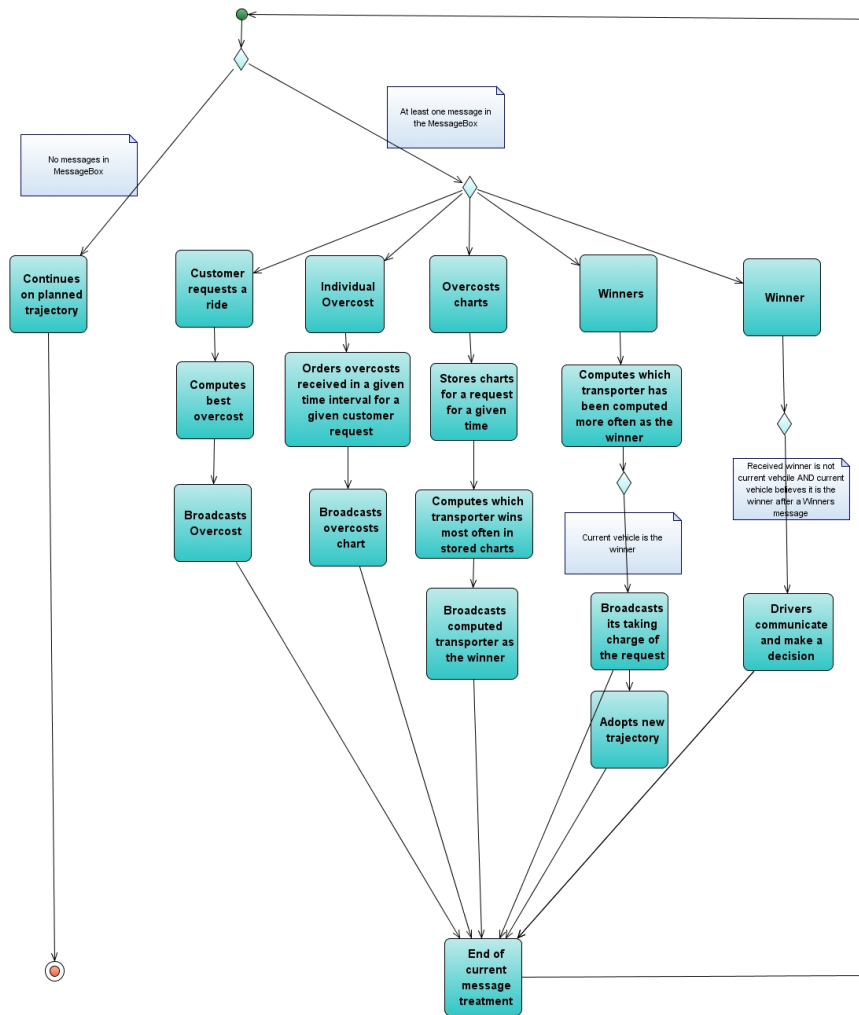


Fig. 5. Activity diagram for a vehicle

Vehicle number	Clients number	Served Clients	Optimal cost	Cost	Client optimal itinerary	Variation per client	Served Clients %	Rate of filling
4	227	227	4409	4878	19.42	10.6%	100%	0 → 4
4	47	47	7324	7922	155	8.2%	100%	0 → 4
8	357	315	26745	29381	85	9.8%	88%	0 → 4
8	200	142	16066	17243	113.14	7.31%	71%	0 → 4

Table 1. Some results of simulation

We notice that the first results are very encouraging. The variation between real cost and optimal cost is not important if we take into account the time to park an individual vehicle in the real life case and the cheaper price proposed to customers in collective transportation. We remark that, for a given number of vehicles and a given period of simulation (7200 steps in the above tableau), our model gives better results with a limited number of clients having long itineraries than with a big number of clients having short itineraries. It is because of the supplementary time due to satisfy all clients which is more important in the case of a big number of clients. The simulation must be improved: probabilistic model for the population of customers, better statistics for filling ... The self-organization mechanism is under development.

6 Conclusion and perspectives

In this paper, we presented a Transportation On Demand system which is purely dynamic, in an environment in perpetual change. We have adopted a decentralized approach based on the optimization and negotiation between vehicles. To face the lack of service in certain zones or the over-concentration of vehicles in certain other zones, we have proposed a self-organizational model which can adapt to the environmental changes. The obtained results are encouraging and the phase of self-organization is under development. We will continue our work by the complete validation of the proposed model.

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