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# A semi-empirical model for conventional crude oil energy return on energy invested

A. Lamorlette

*Aix-Marseille Univ, CNRS, IUSTI UMR 7343  
5 rue Enrico Fermi, 13 013 Marseille Cedex 13, France  
Correspondence to [aymeric.lamorlette@univ-amu.fr](mailto:aymeric.lamorlette@univ-amu.fr)  
Tel (33) 491 113 811 ; Fax (33) 491 118 502*

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## Abstract

This article aims at developing a basic physical model for crude oil energy return on energy invested (EROI). It is based on simple mechanic and energetic considerations on a typical oil extraction system and is extrapolated to world crude oil production. This extrapolation requires the fitting of two parameters, which is done on EROI data from the literature. The model gives a prediction of oil EROI which allows to estimate the decay in the net energy provided by crude oil in the near future, and the end of crude oil as an energy source for industrial societies, considering a business as usual scenario for conventional oil extraction.

*Keywords:* EROI, semi-empirical model, crude oil

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## 1 Introduction

2 Conventional crude oil can be considered as a key in industrial societies, as  
3 it accounts for nearly 30% of the world energy [1] and also because it is often  
4 considered as the cheapest and most accessible [2]. Estimating it's remaining  
5 resources is therefore a major issue. As some studies focus only on existing  
6 and potential resources [3], some authors suggest to focus on resources which  
7 will provide a significant energy surplus [2]. In order to estimate the difference  
8 between the two, the concept of EROI has been introduced [4]. Since then,  
9 numerous studies have been conducted to estimate the value of EROI for  
10 different resources, at different time [2, 4, 5, 6, 7]. A mathematical model has  
11 been suggested [8], which was fitted by [9] using a price-based methodology.  
12 Several energy-economy models, such as [10, 11], have also been developed,

where EROI is calculated through energy-economy balance considerations. However, no attempt has been made to directly calculate EROI dynamics from purely physical considerations.

It is here proposed to study EROI of conventional crude oil from a physical point of view to develop a model as simple as possible, whose parameters are fitted on EROI data from the literature. Contrary to [10, 11], it does not investigate several potential EROI dynamics through different scenarios. It only considers a business as usual for conventional oil extraction, aiming at estimating the associated dynamic trend. This trend is expected to follow the results from [9].

The article is organised as follow: A first part is dedicated to a simple physical description of conventional crude oil extraction in a typical well, showing how it can be used to estimate the energy cost and gain of extraction. It is then extrapolated to global crude oil extraction to propose a two-parameter model. These parameters are then fitted on existing data, proposing a pessimistic and an optimistic estimation. The EROI dynamic suggested by the model and its potential consequences are finally discussed.

## 1. A simple model for crude oil EROI

This section is dedicated to a mechanical modeling of conventional oil extraction, to estimate, based on physical considerations, how the energy cost and gain evolve at the wellhead. This should allow to develop a simple model to estimate EROI dynamic for a single well, and then for global crude oil production.

### 1.1. Modeling the energy cost and gain at the wellhead

In order to evaluate the energy cost and gain at a given wellhead, let us first consider a simplification of a typical crude oil extraction process, as described in classical textbook on oil as [12] or [13]. Note that all typical values (flow rate, pipe and reservoir dimensions, relative roughness, physical properties such as permeability and viscosity) are also taken from these two books:

Conventional oil extraction consists in injecting water through several pipes (injectors), down the reservoir, where a darcian flow occurs through the reservoir. After a small development zone close to the injectors, the flow could be considered as 1D. The flow eventually reaches other pipes (producers), where oil that has been pushed by water flows up the pipe. Therefore, within

the reservoir, the flow is roughly constant, within a cavity of length  $L_{res}$  (the distance between injectors and producers), of section  $S_{res} = h_{res}L_{inj}$  where  $h_{res}$  is the reservoir height and  $L_{inj}$  is the distance between injection wells. Saffman-Taylor instabilities within the reservoir cause the outflow not to be pure oil but a mix of oil and water, hence the water cut  $W_c$  defined as the volume of water on the total outflow volume is introduced [14].

The energy gain with such process is proportional to the oil outflow. Hence for a given flow rate  $Q$ , the energy gain is proportional to  $Q(1 - W_c)$ .

Let us now focus on the energy cost, considering a constant flow rate: this cost is the energy given to the pumps to compensate for the pressure drop within the system. For a given pressure drop  $\Delta p$ , the energy  $E$  given to the pumps depends on their efficiency  $\eta$  (which depends on the pressure drop), with  $E \sim \Delta p/\eta$ . This pressure drop is therefore studied first, before estimating the associated energy cost. The pressure drop in this system can be considered as being the sum of two part: the regular pressure drop in the pipes  $\Delta p_p$  and the darcian pressure drop in the reservoir  $\Delta p_r$ . The darcian pressure drop can be written the following way [15]:

$$\Delta p_r = U_f \mu L_{res} / K. \quad (1)$$

Where  $U_f$  is the filtration velocity in the reservoir (in m/s),  $\mu$  is the viscosity of the liquid that flows through the reservoir (in Pa.s) and  $K$  is the reservoir permeability (in m<sup>2</sup>).  $L_{res}$  is in m.

The regular pressure drop in the pipes can be written the following way, considering mass conservation between the inflow and the outflow [16]:

$$\Delta p_p = \Lambda 2H \rho U_{inj}^2 / 2D_p. \quad (2)$$

Where  $\Lambda$  is the friction factor coefficient of the pipes (dimensionless),  $H$  is the depth of the reservoir (in m),  $\rho$  is the density of the liquid that flows through the pipes (in kg/m<sup>3</sup>),  $U_{inj}$  is the fluid velocity in the pipes (in m/s) and  $D_p$  is the pipe diameter (in m). Mass conservation is applied between the injector and the reservoir to get a relationship between the filtration velocity and the fluid velocity in the pipes.

In order to get a simple estimate of the total pressure drop, let us first study the ratio of these pressure drops  $R_p = \Delta p_r / \Delta p_p$  and get an estimation of its value based on typical parameter values taken from [12] and [13]. The ratio  $R_p$  reads:

$$R_p = \frac{\pi \mu L_{res} D_p^3}{4K \Lambda H \rho S_{res} U_{inj}}. \quad (3)$$

80 This ratio evolves during the well lifespan since both viscosity and density are  
81 changing as the water cut increases. The density evolution seems nevertheless  
82 negligible (it changes of roughly 10% from pure oil to pure water), whereas  
83 the viscosity evolution changes of about half an order of magnitude. The  
84 parameter values used to calculate this ratio are listed in table 1. The friction  
85 factor is estimated considering a 0.1% relative roughness and a flow which  
86 is turbulent enough to consider a friction factor that does not depend on  
87 Reynolds number [12].

Parameter	Unit	Value
$\mu_{oil}$	Pa.s	$2.10^{-3}$
$\mu_{water}$	Pa.s	$5.10^{-4}$
$\rho_{oil}$	kg/m <sup>3</sup>	890
$\rho_{water}$	kg/m <sup>3</sup>	990
$K$	m <sup>2</sup>	$10^{-12}$
$H$	m	1300
$L_{res}$	m	610
$D_p$	m	$56.10^{-3}$
$S_{res}$	m <sup>2</sup>	8000
$U_{inj}$	m/s	2
$\Lambda$	-	0.02

Table 1: Typical values of the different well parameters and fluid physical properties ([12] and [13])

88 This leads to  $R_p = 0.41$  at the beginning of the well life (when only oil  
89 flows through the reservoir) and  $R_p = 0.10$  in the end (when only water flows  
90 through the reservoir). As the water cut has a steep evolution in the early  
91 stage of well operation (as observed in [14] for instance),  $R_p$  becomes quickly  
92 small. Therefore, within the well lifespan, it seems acceptable to consider  
93 that the energy cost of pumping is mainly due to the regular pressure drop  
94 within the pipes, which depends mainly on the well depth, according to the  
95 previous remarks and Eq.(2).

### 96 1.2. A simple model for a mean global EROI at the wellhead

97 For a given well at a given depth  $H$ , after the early stage of operation,  
98 the energy cost  $E$  does not changes much for a given flow rate. Its EROI  
99 should evolve in time as  $1 - W_c(t)$ , with  $W_c(t)$  the water cut of this specific

100 well. For a global modeling, based on the previous analysis on pressure drops  
 101 in a well, the mean crude oil EROI should evolve as:

$$(1 - \overline{W_c(t)}) \frac{\eta(H(t))H_0}{\eta_0 H(t)}, \quad (4)$$

102 with  $\overline{W_c(t)}$  the mean global water cut,  $\eta(H(t))$  the efficiency of the pumps,  
 103  $H(t)$  the mean depth of oil reservoir,  $H_0$  the mean depth of oil reservoir at  
 104 time  $t_0$  and  $\eta_0$  the pump efficiency at time  $t_0$ . It is noted that  $\eta$  depends on  
 105 the pressure drop in the well, and therefore on the well depth.

#### 106 1.2.1. Mean global water cut

107 As calculated numerically for a given well in [14] for instance, the function  
 108  $W_c(t)$  is not linear and changes from one well to another. A mean global  
 109 water cut seems therefore hard to model. It would be a complex production  
 110 weighted sum of type:

$$\overline{W_c(t)} = \frac{1}{\sum_N Q_{oi}(t)} \sum_N Q_{oi}(t) W_{ci}(t - \tau_i), \quad (5)$$

111 with  $Q_{oi}$  the oil production of well  $i$ ,  $W_{ci}$  the water cut of well  $i$ ,  $\tau_i$  the time  
 112 offset between operating the first well and operating well  $i$ , and  $N$  the total  
 113 amount of well in the world. Nevertheless, considering similar production and  
 114 water cut function from a well to another, Eq.(5) tends toward a rather linear  
 115 function, with a value of 0 at the beginning of water injection extraction in  
 116 1930 and a value of 0.5 at the mid-point of conventional crude oil extraction.  
 117 According to [17], this mid-point, the peak of conventional oil, has been  
 118 reached in 2008. Therefore, the simplest approximation that can be suggested  
 119 for a global water cut is:

$$\overline{W_c(t)} = \frac{0.5(t - 1930)}{78}, \quad (6)$$

120 with  $t$  the time (in year).

#### 121 1.2.2. Efficiency of pumps

122 From any textbook on pumps and turbomachines, as [18] for instance, it  
 123 is possible to estimate the evolution of a given pump efficiency as a function  
 124 of the pressure gain. However, it is rather complicated to access these data

for the pumps used in the oil industry. One can only admit this efficiency decreases when the pressure gain increases. According to the previous analysis, the pressure gain needed to extract oil is mainly (and linearly) depending on the reservoir depth  $H$ . It is here suggested to model the efficiency evolution as a negative power law of the pressure gain, therefore as a negative power law of the reservoir depth. It leads to:

$$\eta(H) \sim H^{-x}, \quad (7)$$

with  $x$  a positive constant.

### 1.2.3. EROI equation

This analysis suggests the following expression for a mean global EROI:

$$EROI(t) = EROI_0 \frac{1 - \overline{W_c(t)}}{(H(t)/H_0)^{x+1}}, \quad (8)$$

with  $H(t)$  the mean well depth. Since no data are available for this function, it is estimated based on the U.S. mean depth well, which evolve as  $H(t) = 1028.8 + 4.8(t - 1930)$ , according to the data of [17].  $EROI_0$  and  $x$  are constant which now need to be fitted based on EROI data.

In order to check the reliability of assumption  $R_p \ll 1$ , the following model is also proposed, which takes into account the darcian pressure drop in the energy cost. Considering a mean darcian pressure drop on the well lifespan (and therefore a mean global pressure drop ratio  $\overline{R_p}$ ), one gets:

$$EROI(t) = EROI_0 \frac{1 - \overline{W_c(t)}}{[(H(t)/H_0)(1 + \overline{R_p})]^{x+1}}. \quad (9)$$

For the same reason that  $\overline{W_c(t)}$  seems hazardous to calculate theoretically,  $\overline{R_p}$  can hardly be estimated. This is why it is suggested to fit this parameter and check if the fitting provides an acceptable value (that is  $\overline{R_p} \ll 1$ ) and an EROI dynamic which is close to the one obtained with Eq.(8).

## 2. Fitting the model parameters on published EROI data

In order to get a consistent fit for parameters  $EROI_0$  and  $x$ , it is important to consider EROI data which show consistency with the EROI definition,

with the calculation methodology and also with the set of oil well that is considered. For instance, in [2], EROI is estimated based on gross energy gain and cost, with a set of well that can be considered as representative of the whole set of well in the world. It corresponds to the approach developed in this study. On the contrary, in [4] for instance, the study focus on U.S. well; in [19], a quality correction (or “divisia”) is introduced. Therefore, in this study, the set of [2] is used. The 1930 EROI value from [4] is added to the dataset as it is the oldest data one can get.

### 2.1. A basic fitting method

The easiest way to fit the model parameter would be to consider that the value of  $EROI_0$  should be the 1930 EROI value, as it is consistent with the model development, hence  $EROI_0 = 100$ . The  $x$  value could then be estimated with only one data point. The latest available value (that is the 2006 value, EROI=18) could be a good candidate, as it would lead to a fit based on the extreme EROI values of the dataset. This rational leads to  $x + 1 = 3.39$ . This fit is presented on Fig.1 (solid line). This result is rather in line with 2009 values suggested in [20]. Values in [21] for 2014 suggest however that this fit is a bit pessimistic. It gives an EROI value of 12.5 for the year 2021, which seems acceptable, considering it is a mean value.

### 2.2. A least square fitting method

A more rigorous way to fit the model parameters would be to use a least square method to fit both parameters ( $EROI_0$  and  $x$ ) at the same time, minimizing the difference between the model and the dataset. This method leads to  $EROI_0 = 100.676$  and  $x + 1 = 2.6878$ , with a mean relative error on EROEI equal to 0.42. It is presented on Fig.1 (dashed line). This fit is rather in line with the value suggested in [21] for 2014. In opposition to the first fit this one can be considered as optimistic. It gives an EROI value of 16.2 for the year 2021.

Fitting parameters of Eq.(9) instead of Eq.(8) leads to  $EROI_0 = 103.403$ ,  $x + 1 = 2.6878$  and  $\overline{R_p} = 0.00999$ , with a mean relative error on EROEI equal to 0.42. The mean relative error between Eq.(8) and Eq.(9) is equal to  $2.10^{-10}$ , one can therefore consider the two models are equivalent. It suggests that the assumption  $R_p \ll 1$  is acceptable for the application proposed in this article and therefore only Eq.(8) is considered for the remaining of this study.



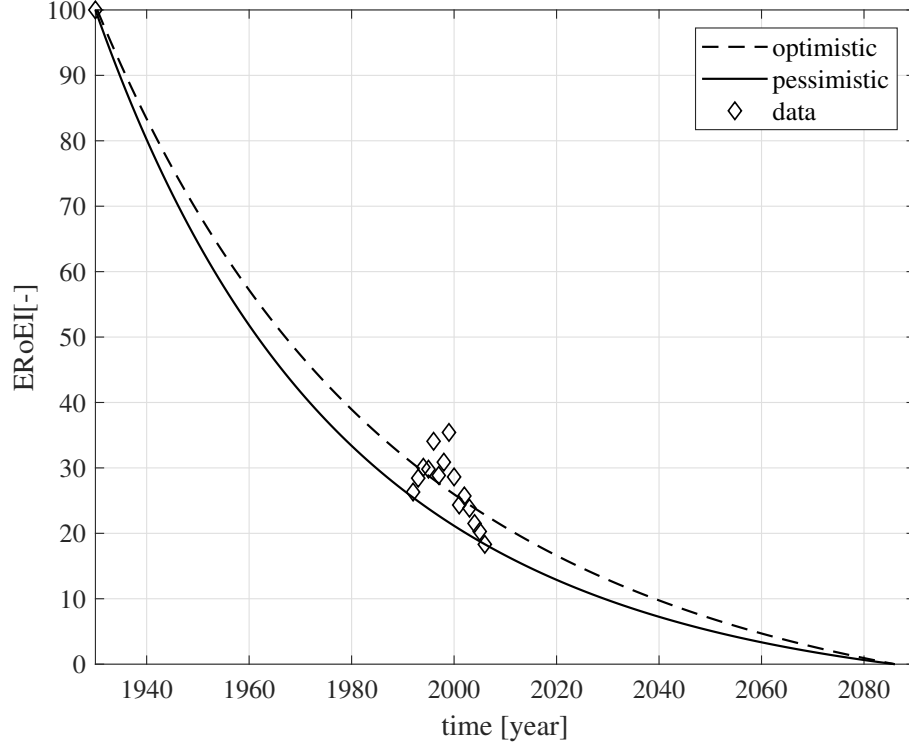


Figure 1: optimistic and pessimistic estimations of EROI evolution

### 184 3. Discussion

#### 185 3.1. Reliability

186 The main assumption in this approach is the condition  $R_p \ll 1$ , which is  
187 strictly verified for some wells, even sometimes in the early stage of operation.  
188 In many cases, this assumption seems still fairly acceptable (let us say within  
189 a 10% margin) so that the model still sounds. It is in line with the equiv-  
190 alence of Eq.(8) and Eq.(9). Nevertheless, in some cases, the assumption  
191 breaks down, hence the full rational cannot strictly be applied. For a global  
192 application, it would seem that this approximation tends to underestimate  
193 EROI values. Indeed, in cases where this approximation is not acceptable,  
194 the decrease in the energy cost when the water cut increases is not taken into  
195 account, what causes the EROI estimation to be an underestimation.

196 On the other hand, this model does not take into account the initial  
 197 investment to put the well in operation. If the initial cost is small in com-  
 198 parison to the operation cost, this should not affect the results. Since no  
 199 calculation was proposed to estimate the ratio between these costs, this as-  
 200 sumption can hardly be discussed. However, one can assess that taking into  
 201 account the initial cost would give lower EROI, hence this approximation  
 202 tends to overestimate the EROI.

203 It seems rather hazardous to claim that the first approximation discussed  
 204 in this section can compensate for the other, even if they have opposite con-  
 205 sequences on the model. Therefore the model cannot obviously be considered  
 206 as optimistic or pessimistic by construction, only the two sets of parameters  
 207 can be classified as optimistic or pessimistic. This simple approach can never-  
 208 theless provide a general trend for EROI dynamic, which is in line with data  
 209 such as [20, 21] that are contemporary to those used to fit the model. The  
 210 general trend is also in line with the results of [9] on the period 1940-2080.

211 Since all the data comes from economy analysis, it suggests an acceptable  
 212 consistency between energy-economy, mathematical and physical modeling.

### 213 3.2. *Estimating the net oil production or net energy delivered to the consumer*

214 Beside providing a trend on future EROI evolution, this model can be  
 215 applied to estimate the ratio between gross and net crude oil production or  
 216 net energy delivered to the consumer: Based on data from [17, 20, 22, 23]  
 217 to estimate the cost of refinery, transportation infrastructures, transporta-  
 218 tion to consumer and the loss in non-fuel refinery products, it is possible to  
 219 calculate explicitly the net product as a function of EROI and gross prod-  
 220 uct. Let  $c_t$  be the relative cost for all the previously listed “losses” of the  
 221 oil industry. One gets  $c_t = 0.54$ , according to [20], Table 3. Energy conser-  
 222 vation implies that the remaining part of the gross product is composed of  
 223 the extraction/production cost plus the net product (or net energy to the  
 224 consumer), hence:

$$Q_g = Q_n + Q_{ext} + Q_t, \quad (10)$$

225 with  $Q_g$  the gross product,  $Q_n$  the net product,  $Q_{ext}$  the extraction/production  
 226 cost and  $Q_t$  the previously listed costs, with  $Q_t = c_t Q_g$ . According to EROI  
 227 definition,  $Q_{ext} = Q_g / EROI$ , hence:

$$Q_n = \frac{EROI(1 - c_t) - 1}{EROI} Q_g. \quad (11)$$

228 This equation allows to estimate the future evolution of  $Q_n/Q_g$ , based on  
 229 the function EROI(t) that has been developed. This ratio is presented in  
 230 Fig.2, where the solid line corresponds to the pessimistic estimation and the  
 231 dashed line corresponds to the optimistic estimation. It is similar to the  
 232 results summarized by [21] (a non-linear, steep behavior), yet it differs in  
 233 taking into account the difference between EROI at the wellhead and EROI  
 234 “at the consumer” (i.e.  $EROI(1 - c_t)$ ) in the calculation of the net energy  
 235 delivered to the consumer.

236 The following observations can also be done:

- 237 • The ratio of net on gross product had a first 10% relative decay from  
 238 1930 to 2006-2016, that is in 76 to 86 years. It will take only 22 to 23  
 239 years to get another 10% relative decay and then 11 to 12 years for the  
 240 next 10%.
- 241 • The net product will be 10% of the gross product between 2063 and  
 242 2069.
- 243 • the minimum EROI to sustain an industrial society (that is  $EROI=3$ ,  
 244 according to [20]) will be reached between 2062 and 2068.
- 245 • The net product will be nil between 2068 and 2073 at  $EROI=2.1$ .

## 246 Conclusion

247 EROI modeling has been investigated in the past through mathemati-  
 248 cal and energy-economy considerations to estimate its dynamic. This article  
 249 shows that a physical modeling of crude oil extraction provides the same  
 250 trend, considering a business as usual scenario for extraction. It highlights  
 251 the relevance of the different approaches and confirm a non-linear and steep  
 252 behavior of the ratio of net product on gross product. It also suggests that  
 253 industrial societies will not be able to fuel on oil around 2060 and that con-  
 254 ventional crude oil will switch from an energy source to an energy sink around  
 255 2070.

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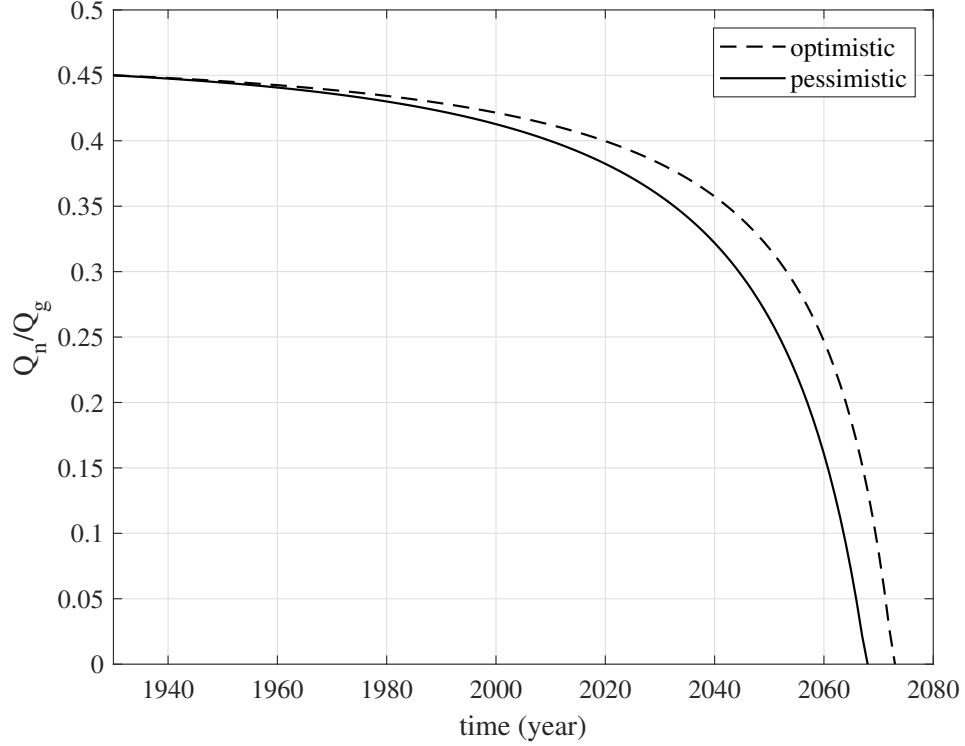


Figure 2:  $Q_n/Q_g$  ratio evolution for both the pessimistic and optimistic EROI values

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