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Intermittent statistics and stochastic modelling of low and high Re compressible jets

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In this work, we studied the intermittent statistics of pressure fluctuations in the near field of compressible subsonic single stream jets. The analysis has been performed considering data from numerical simulations of two isothermal jets having the same Mach number, $M=0.9$, and two different Reynolds number (3,125 and 100,000). Pressure fluctuations were measured by virtual probes located in different positions in the radial and axial directions. Intermittent events are extracted through appropriate wavelet-based indicators and two random variables are analyzed, namely the intermittent time Δt , that is the time delay between two successive events, and the energy amplitude A . Their statistics are computed with the scope of addressing the stochastic model proposed by Camussi et al. [1] that was settled up on the basis of experimental data. The present analysis extends the validity of the model to very low Reynolds number conditions and to the statistics of the 0^{th} azimuthal mode of the pressure fluctuations that exhibit a relevant degree of intermittency around the Kelvin-Helmholtz frequency.

I. Introduction

Subsonic jet noise has been a hot topic in the last 50 years because of its relevant implications in the design of modern civil aircraft tackling the problem of minimizing the noise impact. Since the publication of the Lighthill's famous paper [2], many researchers investigated jet induced pressure fluctuations both in the near-field and in the far-field in the attempt of developing models able to predict as accurately as possible the emitted noise. This large body of the literature clarified the nature of pressure events that dominate both fields. The far-field in the subsonic case is acoustic, with a Gaussian distribution of pressure fluctuations whereas hydrodynamic fluctuations induced by vortex structures generated by the interaction of the jet flow with the ambient fluid at rest dominate the near-field [3]. According to the typical statistics of vorticity in turbulence, the near-field pressure fluctuations exhibit a non Gaussian intermittent behaviour. In a statistical sense, intermittency is intended as a sequence of quiescent phases interrupted by active events inducing a non-homogeneous distribution of energy in time. This is recognized as a very efficient mechanism in generating acoustic waves and thus in contributing to jet noise. Indeed, the importance of intermittency in the noise generation mechanism of subsonic jets has been recognized since the seminal experiment undertaken by Juvé et al. [4] and confirmed in several successive studies (e.g. [5–10]). They demonstrated the existence and the dominant role played by intermittent events on the noise emitted by a jet. Recent papers (see among many, [11–13]) also highlighted the relevant contribution of turbulence-induced intermittency and the impact of non-linearities to the generation of noise.

The results reported by Kearney-Fischer et al. [14, 15] further support the idea that intermittent events are the dominant feature of jet noise. They applied a method to extract the events and developed stochastic models to reproduce their statistics in both the physical and the Fourier domain. A similar approach was adopted by Camussi et al. [1, 16] who used wavelet transform to select intermittent events from experimental data and proposed stochastic models to reproduce their relevant statistics. These analyses provided a direct measure of the degree of intermittency contained in the pressure field induced by compressible jets.

The main objective of the present work is to assess and validate the model proposed in [1, 16] by processing a pressure data-base obtained numerically. With respect to the recent paper by [17], where the same data-base was

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analyzed, the present study uses exactly the same approach proposed in [1, 16] so that the procedure and the proposed model is definitely validated and extended to flow conditions that were never tested before, including the 0^{th} mode component extracted through a Fourier azimuthal decomposition of the pressure signals.

The paper is organized as follows: section II reports the numerical setup and the post-processing procedure, results are discussed in section III, whilst final remarks are reported in section IV.

II. Numerical setup and post-processing procedure

A. Numerical setup

The data-base analyzed has been obtained numerically through DNS and LES of two single-stream circular jets, having respectively diameter-based Reynolds numbers of 3,125 and 100,000 and Mach number 0.9 . Both jets originate from a pipe nozzle of length D into a medium at ambient temperature and pressure 293 K and 10^5 Pa . The potential core length is equal to $7.3D$ for both jets (see [18] for the details). The simulation times after the transient periods are equal to $0,500D/U$ for the high Re jet and to $1,000D/U$ for the low Re jet. Pressure time series are obtained for the azimuthal angles that varies from 0 up to 315° by step of 45° for the high Re jet and from 0 up to 270° by step of 90° for the low Re jet. Signals are recorded at a sampling frequency allowing spectra to be computed up to $St_D = 6.4$. More details about the numerical simulations are reported in [18–20]. Pressure and velocity fluctuations are extracted from virtual probes positioned in the near field of the jets (of diameter D) displaced along the longitudinal (x) and radial (r) directions.

The following table reports the location of the virtual probes considered for the presented analysis:

M	Re	x/D	r/D
0.9	100.00	from 1 up to 7 by step of 1	from 0.5 up to 1.5 by step of 0.5
0.9	3,125	from 1 up to 7 by step of 1	from 0.5 up to 1.5 by step of 0.5

It can be observed that the study is limited to a domain close to the jet exit, usually recognized as the noise-producing region of the jet flow and thus of relevant interest for jet–noise modelling. To this purpose, the original pressure signals are also represented in terms of their azimuthal components through the azimuthal decomposition of the Fourier modes. This approach has been extensively used in the past to extract relevant features connected with the jet noise generation [21]. The wavelet–based statistical analysis is thus applied also to the axisymmetric mode of order 0, that is known to dominate the sound field for low polar angles [22].

B. Post-processing procedure

As performed in [1], signals are analyzed using a wavelet-based approach. The wavelet decomposition allows for the simultaneous representation of a temporal signal in terms of a time shift (t) and a resolution time scale (s) whose inverse corresponds to the frequency (f). Formally, the wavelet transform of the signal $p(t)$ is given by the following expression [23]:

$$w(s, t) = C_\psi^{-\frac{1}{2}} s^{-\frac{1}{2}} \int_{-\infty}^{\infty} p(\tau) \psi^* \psi^* \left(\frac{t - \tau}{s} \right), \quad (1)$$

where s is the wavelet scale, τ is a time shift, $C_\psi^{-\frac{1}{2}}$ is a constant that takes into account the mean value of $\psi(t)$ and $\psi^* \left(\frac{t - \tau}{s} \right)$ is the complex conjugate of the dilated and translated mother wavelet $\psi(t)$. In this analysis, we applied the continuous wavelet transform (CWT) using the Morlet mother wavelet.

A relevant outcome of the wavelet transform is the wavelet scalogram, given by the square of the wavelet coefficients. An example is reported in figure 1(a) obtained by analyzing the pressure data taken at $x/D = 2$ and $r/D = 1$. It provides a decomposition of the energy onto the (f, t) plane, thus allowing us to determine the time evolution of energetic features contained into the signals (see e.g. [24]). Indeed, it should be pointed out that the Fourier power spectral density of the signal can be retrieved by integrating the scalogram into the time domain and an example of a comparison between the Fourier and the wavelet-reconstructed spectrum is provided in figure 1(b) showing a striking agreement.

In jets, in the region close to the nozzle exit, a spectral bump can be clearly identified especially, for laminar exit conditions. It is the trace of the Kelvin–Helmholtz instability mode and the corresponding frequency and spectral amplitude are denoted as f_{KH} and A_{KH} respectively.

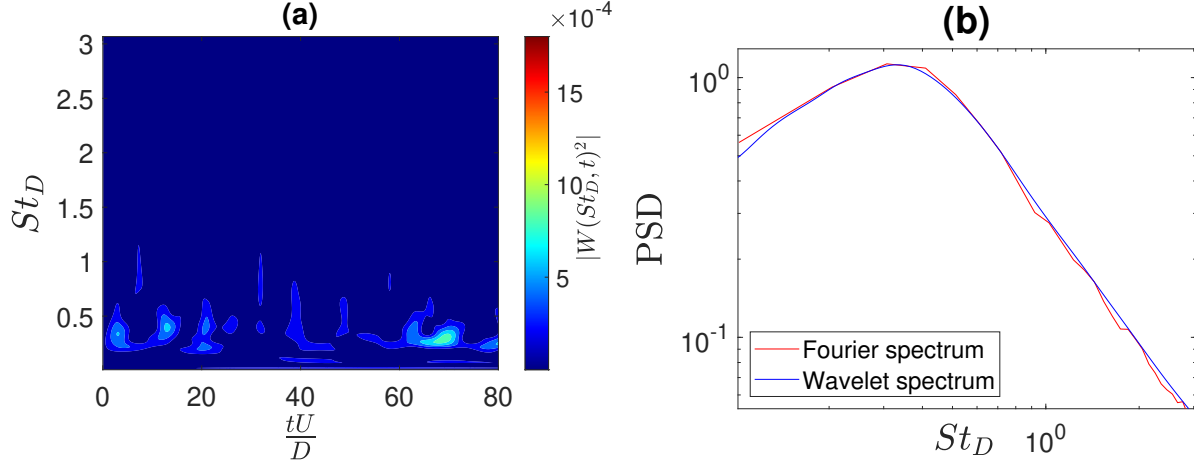


Fig. 1 (a) Wavelet scalogram at $x/D = 2$ and $r/D = 1$; (b) comparison between the Fourier spectrum and the wavelet-reconstructed spectrum obtained by integrating the case (a).

The quantity used to identify intermittent events is the so-called Local Intermittency Measure (LIM) that represents a normalized version of the wavelet scalogram. Its formal definition is the following:

$$LIM(s, t) = \frac{w^2(s, t)}{\langle w^2(s, t) \rangle_t}, \quad (2)$$

where $w^2(s, t)$ are the wavelet coefficients evaluated with eq.(1) whilst the symbol $\langle \dots \rangle_t$ indicates time average. According to [1, 16], the LIM is computed at the wavelet scale corresponding to f_{KH} and the condition $LIM > 1$ is used to identify intermittent events having a local energy greater than the average. We indicate with Δ_t the time between two consecutive intermittent events, defined as intermittent time, and with A the event energy amplitude retrieved from the square of the corresponding wavelet coefficient. These the two indicators will be characterized in details in the following.

III. Results

The number of intermittent events depends on the flow conditions considered and on the position of the selected virtual probes. As shown in figure 2, in the region close to the jet exit, the number of intermittent events decreases for increasing both the axial and the radial distances whereas it is larger for the lower Reynolds number. This is in agreement with results by Bogey et al. [17] that highlighted a large degree of intermittency at low Re .

A similar trend is observed for the statistics of the 0 mode, as reported in Figure 3. Even though the relative numbers of selected events is lower with respect to the full pressure, the 0 mode intermittency also shows a decreasing trend for increasing both x/D and r/D whereas it is larger at the lowest Re .

Following the previous work of Camussi et al. [16], averaged values of both the intermittent time and the amplitude are computed and are used as reference for a simplified model.

We report in figure 4(a) the ratio $\langle A \rangle / A_{KH}$ against x/D for the low Re case, where $\langle A \rangle$ is the mean amplitude of the detected intermittent events and A_{KH} is the amplitude of the corresponding Kelvin-Helmholtz (K-H) mode. As a first approximation, a reference constant value can be assumed by averaging the whole set of available data (the continuous line in figure 4).

The analogous results in terms of St_{int} as reported in figure 4(b). We remind that, according to [16], the quantity St_{int} is defined using as a reference time-scale the mean intermittent time Δt , and St_{KH} is the mean intermittent non-dimensional time computed at the K-H frequency.

The averaged values of the normalized intermittent time St_{int}/St_{KH} and amplitude A/A_{KH} , including the higher Re cases, are summarized in table 1 showing a weak dependence upon Re .

Similar results are obtained in the case of the 0^h mode. The normalized amplitudes and Strouhal numbers are reported in Figure 5 and the mean values are summarized in table 2 showing a very good agreement with results obtained from the full pressure signals.

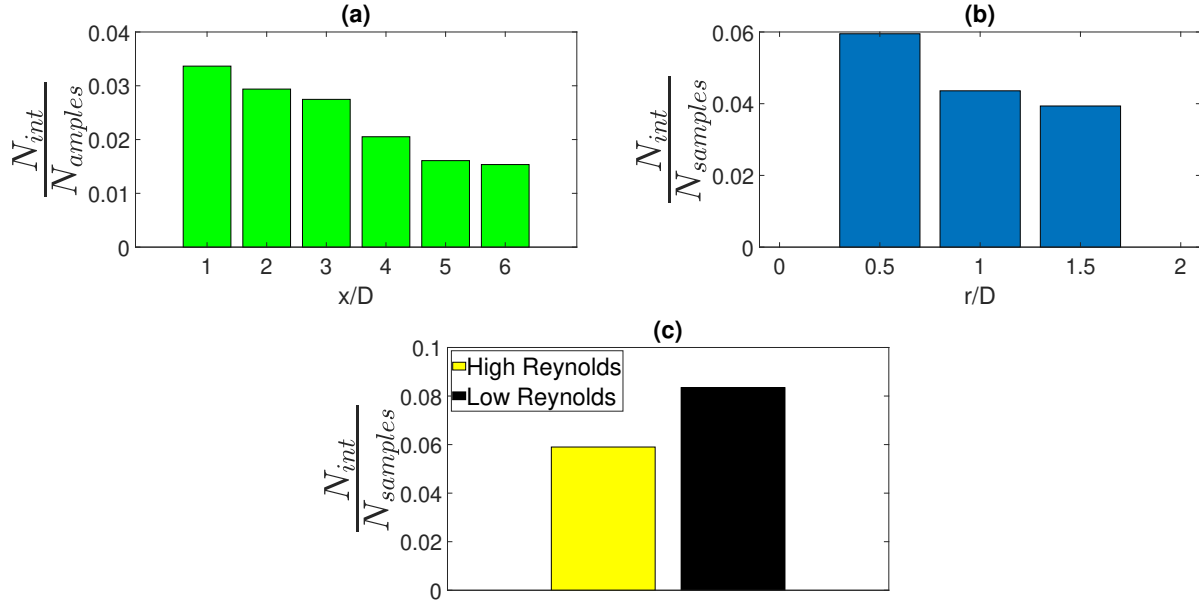


Fig. 2 Number of intermittent events normalized with respect to the number of samples: (a) as a function of x/D for all r/D and both Reynolds numbers; (b) as a function of r/D for all x/D and both Reynolds numbers; (c) as a function of Reynolds numbers for all radial and axial positions.

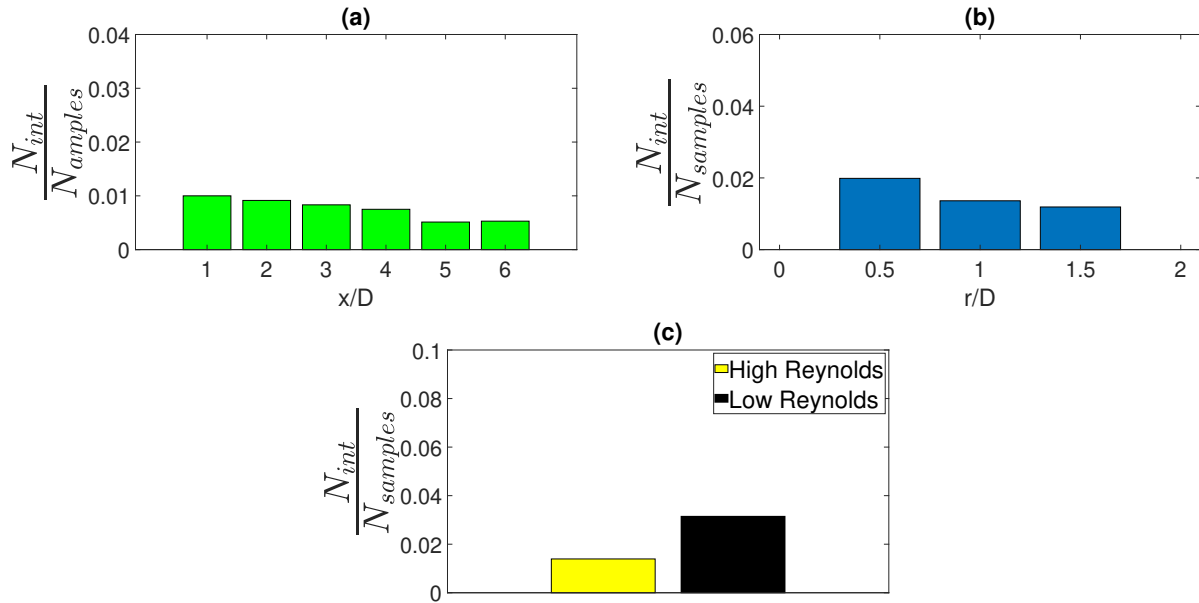


Fig. 3 Same as Figure 2 but for the 0 mode.

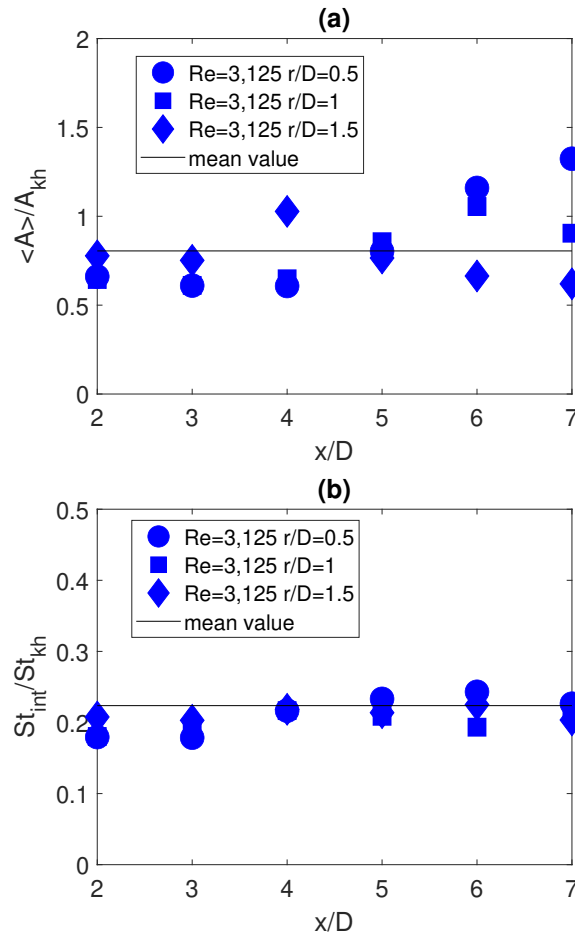


Fig. 4 Axial evolution of (a) A_{int}/A_{KH} and (b) S_{int}/S_{KH} at the lowest Re and different r/D .

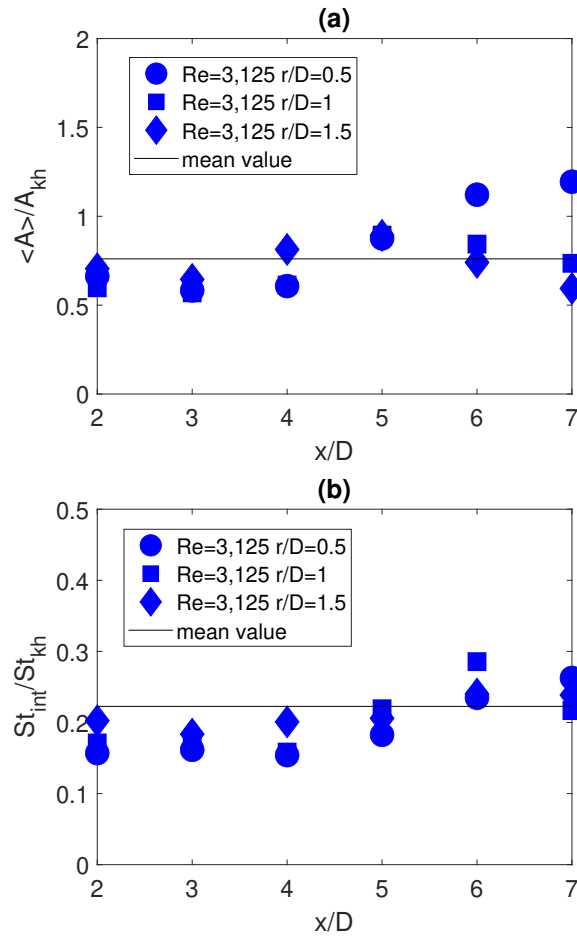


Fig. 5 Same as Figure 4 but for the mode 0.

This result provides a first indication that a stochastic model based on the full pressure signals can be extended to the 0^{th} mode intermittent statistic, a relevant feature in view of the direct correlation between the 0^{th} mode properties and the noise generation mechanisms.

Pressure	Mean
St_{int}/St_{KH}	0.223
High Reynolds $\langle A \rangle / A_{KH}$	1.00
Low Reynolds $\langle A \rangle / A_{KH}$	0.80

Table 1 Statistics of the intermittent events identified in the original pressure signals.

Pressure 0° azimuthal mode	Mean
St_{int}/St_{KH}	0.227
High Reynolds $\langle A \rangle / A_{KH}$	0.91
Low Reynolds $\langle A \rangle / A_{KH}$	0.76

Table 2 Statistics of the intermittent events identified in the 0^{th} mode pressure signals.

The detailed statistical distribution of the intermittent events is determined by calculating the Probability Density Functions (PDF) of Δt and A . The main purpose of this analysis is to evaluate the dependence of these PDFs, both for the whole pressure signals and for the 0^{th} mode, upon the considered flow parameters (Reynolds number, axial position, radial distance).

The PDFs of the numerical data are approximated using the same analytical model proposed in [16]. More specifically, the analytical forms are provided in terms of the normalized reduced variables Δt^* and A^* that are defined as follows:

$$\Delta t^* = \frac{\Delta t - \langle \Delta t \rangle}{\sigma_{\Delta t}} \quad (3)$$

$$A^* = \frac{A - \langle A \rangle}{\sigma_A} \quad (4)$$

and the functions proposed in [16] (see equations 5 and 6) consist of an hyperbolic secant to approximate the PDF of Δt and a decaying exponential function for the PDFs of A :

$$PDF(\Delta t^*) = a_1 \operatorname{sech}(b_1 \Delta t^*), \quad (5)$$

$$PDF(A^*) = a_2 e^{-b_2 A^*}, \quad (6)$$

Figure 6 (a) and (b) reports the PDFs of the variables Δt and A respectively, obtained from the whole set of virtual probes analyzed therein. The shape of the PDFs do not change significantly and the analytical approximations (5) and (6) are reported in Figure 6 (c) and (d) along with an uncertainty interval estimated from the experimental data dispersion. The amplitude of the uncertainty is not relevant thus confirming the reliability of the model. The coefficients of the approximating functions are:

$$a_1 = 0.56, b_1 = 1.337, a_2 = 0.44, b_2 = 0.92$$

that are very close to those reported in [16].

The analysis has been completed with the computation of the statistics of the intermittent events extracted from the 0^{th} mode counterpart. The PDFs are reported in figure 7(a) and (b) whereas the approximating functions and the corresponding uncertainties are given in figure 7(c) and (d).

More specifically, with reference to equations (6) and (5), the coefficients obtained from the the 0^{th} statistics are:

$$a_1 = 0.57, b_1 = 1.3, a_2 = 0.41, b_2 = 0.8$$

Even though the coefficients are slightly different, it can be concluded that the model proposed in [16] is valid for both the full pressure signals analyzed therein and their 0^{th} mode counterpart.

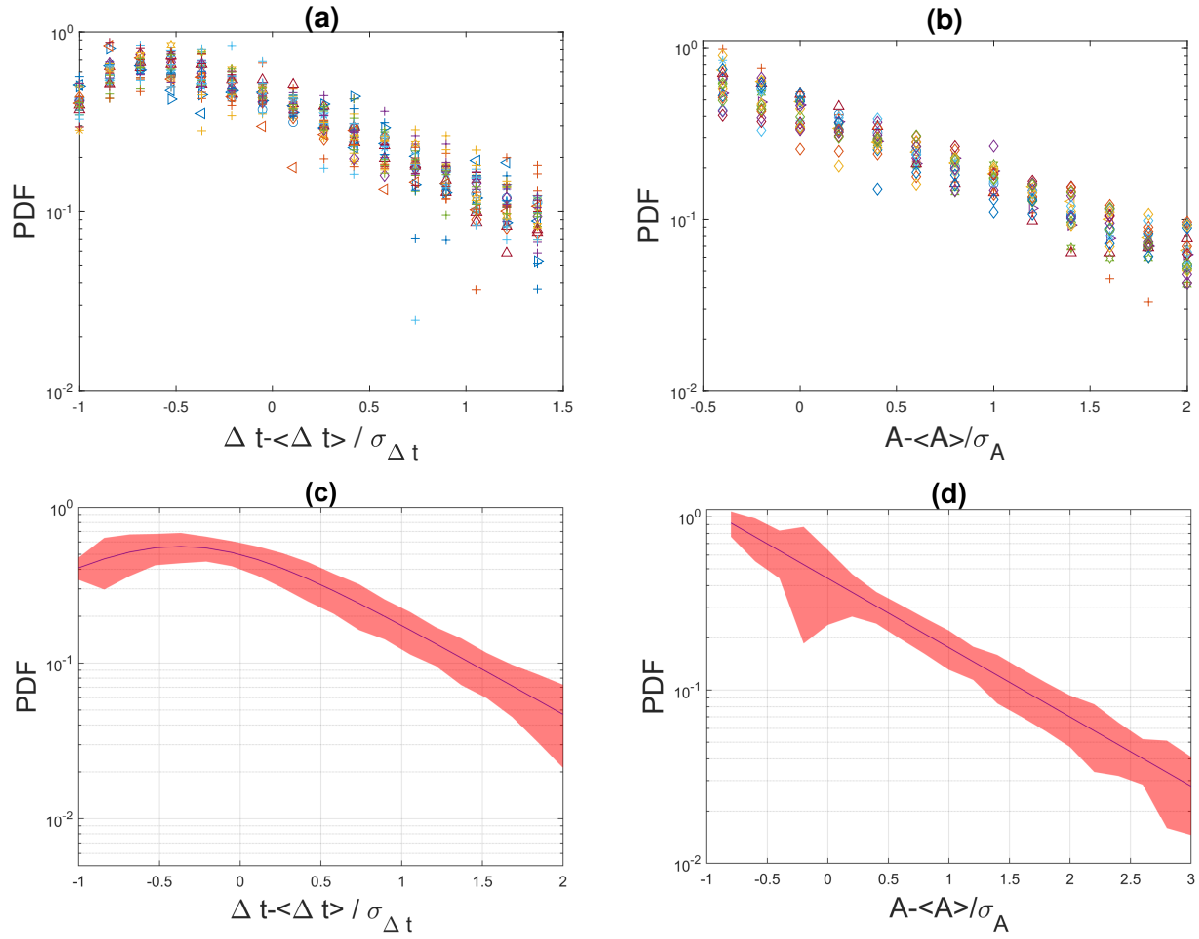


Fig. 6 Probability density functions of the considered signals: (a) Δt (b) A ; (c) and (d) models (5) and (6) with the corresponding uncertainty.

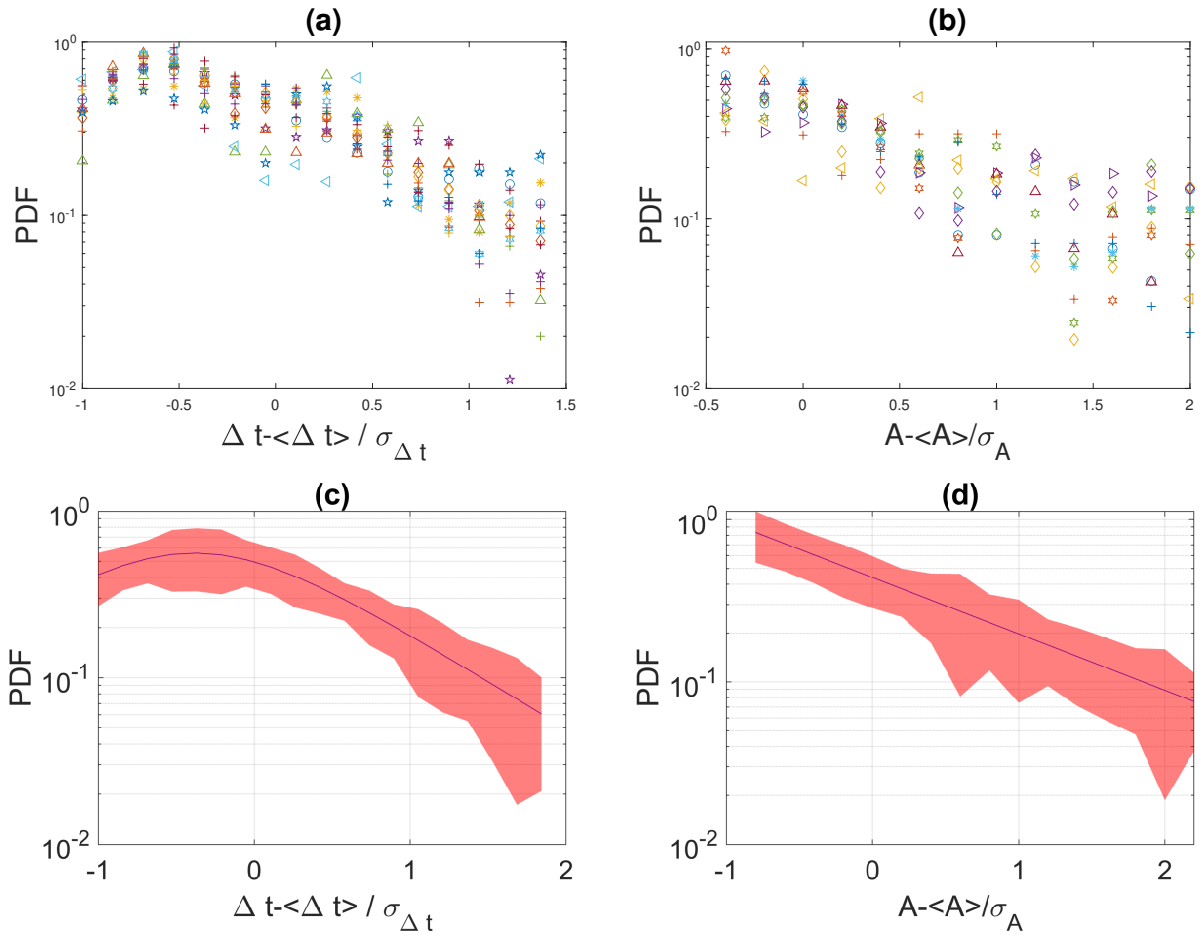


Fig. 7 Probabilty density functions of the 0^{th} mode for the considered positions: (a) Δt (b) A (c) and (d) models (5) and (6) with the respectively uncertainty.

IV. Conclusions

The pressure fluctuations in the near-field of two compressible jets at $M=0.9$ have been characterized in terms of their intermittent degree with the scope of highlighting the effects of the Reynolds number and the position in the flow. The data used are obtained numerically in a region spanning 7 diameters in the axial and 3 diameters in the radial directions. The pressure data are decomposed azimuthally, and the statistical properties of the 0-mode component are analyzed as well.

A tracking algorithm based on the computation of the wavelet quantity LIM, has been applied to extract the most energetic events at the frequency of the hydrodynamic pressure bump. By averaging the whole set of available data and following the procedure adopted in [16], a Stochastic model is determined by considering the two variables t and A denoting the intermittent time and the intermittent event amplitude, respectively. The model and the procedure adopted in the present analysis is exactly the same presented in [16] and the used functional forms are identical except for slight differences in the magnitude of the multiplicative coefficients. Specifically, the intermittent time statistics are approximated by an hyperbolic secant function whereas the amplitude statistics by a decaying pure exponential function. The results confirm the generalized validity of the model that is weakly dependent upon Re and upon the position in the near field of the jet. These predictions are shown to be valid also for the statistics extracted from the axisymmetric mode 0 that, in agreement with [17], surprisingly possesses a relevant degree of intermittency.

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